

Supplementary File of the paper

“A Line complex -based Evolutionary Algorithm for Many-Objective Optimization”

I. ADDITIONAL EXPERIMENTAL RESULTS

A. Validation of the Line Complex

In order to observe the impact of the line complex, we designed an experiment to compare NSGA-III with the version that only integrates line complex into NSGA-III framework, named NSGA-III/LC. Test problems are DTLZ1–DTLZ7, WFG1–WFG9, SDTLZ1–SDTLZ2 and CDTLZ2 with 5-, 10-, and 15-objectives. The statistical results of the HV indicator values are shown in Tables S-I, where the best results are highlighted.

As shown in Table S-I, NSGA-III/LC wins 37 times out of 57 instances, and 19 of them exhibit significant improvement while 23 results are statistically equivalent to those obtained by NSGA-III.

To further improve the effectiveness of our method, a new distance function is designed for the line complex-based strategy. Experimental results in Table S-II show that, compared with NSGA-III and NSGA-III/LC, NSGA-III/LCD has been enhanced by using the distance function. Its HV indicator has a significant improvement on 26 test instances compared to NSGA-III and 20 test instances significantly better than NSGA-III/LC.

B. NSGA-III/LCD vs. KnEA and BiGE

Two distance-based MaOEAs, KnEA (based on perpendicular distance) and BiGE (based on Manhattan distance) are selected to compare with NSGA-III/LCD as shown in Table S-III. Among 57 test instances, NSGA-III/LCD has obtained 40 best results. The experimental results demonstrate that the line complex-based environmental selection strategy is obviously more effective than perpendicular distance and Manhattan distance based-algorithms.

D. Experimental Comparisons of Pure Diversity Indicator

For observation of the diversity maintenance of NSGA-III/LCD, experimental comparisons of PD indicator are shown in Table S-IV. The selected competitive algorithms are MOEA/DD, RVEA, NSGA-II/SDR, MOMBI-III, θ -DEA and AR-MOEA. Test problems are DTLZ and WFG test suits. Among 57 test instances, NSGA-III/LCD has obtained 37 best results against all compared algorithms. Specifically, for DTLZ3, DTLZ6, WFG1-2 and WFG8, NSGA-III/LCD has achieved the best performance on every test instance. Experimental results demonstrate the excellent diversity maintenance of NSGA-III/LCD.

Problem	M	NSGA-III	NSGA-III/LC
DTLZ1	5	9.7982e-1 (1.19e-4) \approx	9.7988e-1 (1.51e-4)
	10	9.9223e-1 (3.31e-2) $-$	9.9966e-1 (1.89e-5)
	15	9.9954e-1 (1.39e-3) $+$	9.9151e-1 (3.03e-2)
DTLZ2	5	8.1258e-1 (3.73e-4) \approx	8.1276e-1 (3.78e-4)
	10	9.6785e-1 (7.46e-3) $-$	9.7299e-1 (8.98e-3)
	15	9.7962e-1 (8.91e-3) $-$	9.8524e-1 (8.60e-3)
DTLZ3	5	8.0988e-1 (3.38e-3) \approx	8.0966e-1 (2.79e-3)
	10	9.0893e-1 (2.16e-1) $+$	5.4181e-1 (4.64e-1)
	15	9.0183e-1 (2.14e-1) $+$	0.0000e+0 (0.00e+0)
DTLZ4	5	8.1224e-1 (3.96e-4) \approx	8.1252e-1 (5.01e-4)
	10	9.6785e-1 (8.05e-3) $-$	9.7296e-1 (9.29e-3)
	15	9.8873e-1 (4.23e-3) $-$	9.9265e-1 (2.72e-3)
DTLZ5	5	1.1288e-1 (7.52e-3) \approx	1.1466e-1 (2.59e-3)
	10	9.1168e-2 (1.93e-3) $+$	4.5946e-2 (3.81e-2)
	15	8.8362e-2 (3.09e-3) $+$	0.0000e+0 (0.00e+0)
DTLZ6	5	1.0624e-1 (8.26e-3) $+$	9.6446e-2 (7.93e-3)
	10	8.8902e-2 (8.92e-3) $+$	0.0000e+0 (0.00e+0)
	15	8.6386e-2 (2.03e-2) $+$	0.0000e+0 (0.00e+0)
DTLZ7	5	2.5777e-1 (2.94e-3) \approx	2.5956e-1 (2.44e-3)
	10	1.8591e-1 (5.30e-3) \approx	1.8633e-1 (1.40e-3)
	15	8.7963e-2 (3.83e-2) $-$	1.5416e-1 (2.62e-2)
CDTLZ2	5	9.9937e-1 (3.47e-5) $+$	9.9930e-1 (2.13e-5)
	10	9.9983e-1 (3.64e-4) $-$	1.0000e+0 (9.23e-7)
	15	9.9972e-1 (9.87e-4) $-$	1.0000e+0 (0.00e+0)
SDTLZ1	5	9.7980e-1 (3.01e-4) \approx	9.7987e-1 (1.19e-4)
	10	9.9605e-1 (1.29e-2) \approx	9.9966e-1 (1.56e-5)
	15	9.9429e-1 (2.80e-3) $-$	9.9631e-1 (1.58e-2)
SDTLZ2	5	8.1252e-1 (4.86e-4) \approx	8.1255e-1 (2.88e-4)
	10	9.6847e-1 (4.43e-3) $-$	9.7417e-1 (3.27e-3)
	15	9.8692e-1 (5.23e-3) $-$	9.9272e-1 (1.80e-3)
WFG1	5	9.9833e-1 (1.36e-4) $-$	9.9855e-1 (1.16e-4)
	10	9.9928e-1 (2.68e-4) $-$	9.9989e-1 (7.80e-5)
	15	9.9989e-1 (1.56e-4) \approx	9.9991e-1 (9.88e-5)
WFG2	5	9.9665e-1 (4.63e-4) \approx	9.9692e-1 (4.23e-4)
	10	9.9802e-1 (1.04e-3) \approx	9.9827e-1 (5.46e-4)
	15	9.9749e-1 (1.70e-3) $-$	9.9914e-1 (5.66e-4)
WFG3	5	9.7499e-1 (2.09e-3) $+$	1.7447e-1 (1.22e-2)
	10	9.0941e-1 (4.19e-3) $+$	3.2695e-2 (2.24e-2)
	15	7.8659e-1 (4.37e-2) $+$	0.0000e+0 (0.00e+0)
WFG4	5	8.0880e-1 (8.16e-4) \approx	8.0893e-1 (8.07e-4)
	10	9.5650e-1 (1.07e-2) $-$	9.6045e-1 (3.63e-3)
	15	9.8595e-1 (4.27e-3) $+$	9.7840e-1 (4.72e-3)
WFG5	5	7.6143e-1 (3.65e-4) \approx	7.6150e-1 (3.93e-4)
	10	9.0351e-1 (3.05e-4) $-$	9.0389e-1 (3.21e-4)
	15	9.1652e-1 (2.28e-3) $+$	9.1498e-1 (4.98e-4)
WFG6	5	7.4698e-1 (1.40e-2) \approx	7.4464e-1 (1.50e-2)
	10	8.8144e-1 (2.03e-2) \approx	8.8269e-1 (1.40e-2)
	15	8.9185e-1 (2.81e-2) \approx	8.8905e-1 (2.09e-2)
WFG7	5	8.0965e-1 (4.77e-4) \approx	8.0985e-1 (6.36e-4)
	10	9.6193e-1 (1.50e-2) $-$	9.6306e-1 (8.91e-3)
	15	8.9175e-1 (2.42e-2) $-$	9.8519e-1 (5.06e-3)
WFG8	5	6.9706e-1 (3.24e-3) $-$	6.9941e-1 (1.97e-3)
	10	8.7329e-1 (1.53e-2) \approx	8.6748e-1 (1.00e-2)
	15	9.2592e-1 (2.34e-2) $+$	9.0844e-1 (1.23e-2)
WFG9	5	7.6576e-1 (3.96e-3) \approx	7.6585e-1 (3.30e-3)
	10	8.8920e-1 (3.60e-2) \approx	8.7829e-1 (4.94e-2)
	15	8.9077e-1 (7.43e-2) \approx	8.9698e-1 (6.64e-2)
		+ / - / \approx	15/19/23

‘+’, ‘-’ and ‘ \approx ’ indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGA-III/LC, respectively.

TABLE S-II
HV VALUES OF NSGA-III, NSGA-III/LC, AND NSGA-III/LCD.

Problem	M	NSGA-III	NSGA-III/LC	NSGA-III/LCD
DTLZ1	5	9.7982e-1 (1.19e-4) \approx	9.7988e-1 (1.51e-4) \approx	9.7987e-1 (1.61e-4)
	10	9.9223e-1 (3.31e-2) $-$	9.9966e-1 (1.89e-5) $-$	9.9969e-1 (1.71e-5)
	15	9.9954e-1 (1.39e-3) $-$	9.9151e-1 (3.03e-2) $-$	9.9994e-1 (1.42e-5)
DTLZ2	5	8.1258e-1 (3.73e-4) $-$	8.1276e-1 (3.78e-4) \approx	8.1285e-1 (2.89e-4)
	10	9.6785e-1 (7.46e-3) $+$	9.7299e-1 (8.98e-3) $+$	9.6677e-1 (1.02e-2)
	15	9.7962e-1 (8.91e-3) $-$	9.8524e-1 (8.60e-3) \approx	9.8517e-1 (4.99e-3)
DTLZ3	5	8.0988e-1 (3.38e-3) \approx	8.0966e-1 (2.79e-3) \approx	8.1081e-1 (1.29e-3)
	10	9.0893e-1 (2.16e-1) \approx	5.4181e-1 (4.64e-1) $-$	9.6196e-1 (1.99e-2)
	15	9.0183e-1 (2.14e-1) \approx	0.0000e+0 (0.00e+0) $-$	7.5910e-1 (3.76e-1)
DTLZ4	5	8.1224e-1 (3.96e-4) $-$	8.1252e-1 (5.01e-4) \approx	8.1270e-1 (3.39e-4)
	10	9.6785e-1 (8.05e-3) $-$	9.7296e-1 (9.29e-3) $-$	9.7406e-1 (6.20e-4)
	15	9.8873e-1 (4.23e-3) $-$	9.9265e-1 (2.72e-3) $+$	9.9130e-1 (1.02e-4)
DTLZ5	5	1.1288e-1 (7.52e-3) $-$	1.1466e-1 (2.59e-3) \approx	1.1618e-1 (1.18e-3)
	10	9.1168e-2 (1.93e-3) $+$	4.5946e-2 (3.81e-2) $-$	8.8170e-2 (2.50e-3)
	15	8.8362e-2 (3.09e-3) $+$	0.0000e+0 (0.00e+0) $-$	8.1804e-2 (8.86e-3)
DTLZ6	5	1.0624e-1 (8.26e-3) \approx	9.6446e-2 (7.93e-3) $-$	1.0151e-1 (8.84e-3)
	10	8.8902e-2 (8.92e-3) $+$	0.0000e+0 (0.00e+0) \approx	4.5455e-3 (2.03e-2)
	15	8.6386e-2 (2.03e-2) $+$	0.0000e+0 (0.00e+0) \approx	9.1064e-3 (2.80e-2)
DTLZ7	5	2.5777e-1 (2.94e-3) $+$	2.5956e-1 (2.44e-3) $+$	2.5472e-1 (4.36e-3)
	10	1.8591e-1 (5.30e-3) \approx	1.8633e-1 (1.40e-3) \approx	1.8816e-1 (5.12e-3)
	15	8.7963e-2 (3.83e-2) $-$	1.5416e-1 (2.62e-2) $+$	1.5290e-1 (6.11e-3)
CDTLZ2	5	9.9937e-1 (3.47e-5) $+$	9.9930e-1 (2.13e-5) $-$	9.9935e-1 (4.37e-5)
	10	9.9983e-1 (3.64e-4) $-$	1.0000e+0 (9.23e-7) \approx	1.0000e+0 (2.48e-6)
	15	9.9972e-1 (9.87e-4) \approx	1.0000e+0 (0.00e+0) $+$	9.9994e-1 (1.33e-4)
SDTLZ1	5	9.7980e-1 (3.01e-4) \approx	9.7987e-1 (1.19e-4) \approx	9.7985e-1 (1.57e-4)
	10	9.9605e-1 (1.29e-2) \approx	9.9966e-1 (1.56e-5) $-$	9.9967e-1 (3.42e-5)
	15	9.9429e-1 (2.80e-3) $-$	9.9631e-1 (1.58e-2) $-$	9.9832e-1 (2.20e-3)
SDTLZ2	5	8.1252e-1 (4.86e-4) \approx	8.1255e-1 (2.88e-4) \approx	8.1271e-1 (4.00e-4)
	10	9.6847e-1 (4.43e-3) \approx	9.7417e-1 (3.27e-3) $+$	9.6497e-1 (1.59e-2)
	15	9.8692e-1 (5.23e-3) \approx	9.9272e-1 (1.80e-3) $+$	9.8874e-1 (3.67e-3)
WFG1	5	9.9833e-1 (1.36e-4) $-$	9.9855e-1 (1.16e-4) \approx	9.9850e-1 (2.29e-4)
	10	9.9928e-1 (2.68e-4) $-$	9.9989e-1 (7.80e-5) $+$	9.9977e-1 (1.28e-4)
	15	9.9989e-1 (1.56e-4) $-$	9.9991e-1 (9.88e-5) $-$	9.9999e-1 (9.30e-6)
WFG2	5	9.9665e-1 (4.63e-4) $-$	9.9692e-1 (4.23e-4) \approx	9.9695e-1 (4.10e-4)
	10	9.9802e-1 (1.04e-3) \approx	9.9827e-1 (5.46e-4) \approx	9.9848e-1 (9.78e-4)
	15	9.9749e-1 (1.70e-3) $-$	9.9914e-1 (5.66e-4) $+$	9.9856e-1 (7.80e-4)
WFG3	5	9.7499e-1 (2.09e-3) $-$	1.7447e-1 (1.22e-2) $-$	9.7619e-1 (2.12e-3)
	10	9.0941e-1 (4.19e-3) $-$	3.2695e-2 (2.24e-2) $-$	9.1396e-1 (4.92e-3)
	15	7.8659e-1 (4.37e-2) \approx	0.0000e+0 (0.00e+0) $-$	7.9194e-1 (2.52e-2)
WFG4	5	8.0880e-1 (8.16e-4) \approx	8.0893e-1 (8.07e-4) \approx	8.0856e-1 (9.05e-4)
	10	9.5650e-1 (1.07e-2) $-$	9.6045e-1 (3.63e-3) \approx	9.6178e-1 (2.63e-3)
	15	9.8595e-1 (4.27e-3) $-$	9.7840e-1 (4.72e-3) $-$	9.8774e-1 (3.12e-3)
WFG5	5	7.6143e-1 (3.65e-4) $-$	7.6150e-1 (3.93e-4) \approx	7.6167e-1 (3.29e-4)
	10	9.0351e-1 (3.05e-4) $-$	9.0389e-1 (3.21e-4) \approx	9.0395e-1 (2.45e-4)
	15	9.1652e-1 (2.28e-3) $-$	9.1498e-1 (4.98e-4) $-$	9.1672e-1 (1.82e-3)
WFG6	5	7.4698e-1 (1.40e-2) \approx	7.4464e-1 (1.50e-2) \approx	7.5242e-1 (1.29e-2)
	10	8.8144e-1 (2.03e-2) \approx	8.8269e-1 (1.40e-2) \approx	8.8787e-1 (1.20e-2)
	15	8.9185e-1 (2.81e-2) \approx	8.8905e-1 (2.09e-2) \approx	8.9553e-1 (1.85e-2)
WFG7	5	8.0965e-1 (4.77e-4) \approx	8.0985e-1 (6.36e-4) \approx	8.0960e-1 (4.62e-4)
	10	9.6193e-1 (1.50e-2) $-$	9.6306e-1 (8.91e-3) $-$	9.6704e-1 (1.08e-3)
	15	8.9175e-1 (2.42e-2) $-$	9.8519e-1 (5.06e-3) $+$	9.1386e-1 (1.22e-2)
WFG8	5	6.9706e-1 (3.24e-3) $-$	6.9941e-1 (1.97e-3) \approx	6.9980e-1 (2.03e-3)
	10	8.7329e-1 (1.53e-2) \approx	8.6748e-1 (1.00e-2) $-$	8.7653e-1 (1.56e-2)
	15	9.2592e-1 (2.34e-2) \approx	9.0844e-1 (1.23e-2) $-$	9.3671e-1 (2.25e-2)
WFG9	5	7.6576e-1 (3.96e-3) \approx	7.6585e-1 (3.30e-3) \approx	7.6710e-1 (4.61e-3)
	10	8.8920e-1 (3.60e-2) \approx	8.7829e-1 (4.94e-2) \approx	8.6992e-1 (6.32e-2)
	15	8.9077e-1 (7.43e-2) \approx	8.9698e-1 (6.64e-2) \approx	9.1345e-1 (4.42e-2)
+ / - / \approx		7/26/24	10/20/27	

'+', '-', and ' \approx ' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGA-III/LCD with distance function, respectively.

TABLE S-III
HV VALUES OF KnEA, BiGE AND NSGA-III/LCD.

Problem	M	KnEA	BiGE	NSGA-III/LCD
DTLZ1	5	6.6979e-1 (1.36e-1) –	9.1292e-1 (6.46e-2) –	9.7987e-1 (1.61e-4)
	10	0.0000e+0 (0.00e+0) –	5.9982e-1 (2.76e-1) –	9.9969e-1 (1.71e-5)
	15	0.0000e+0 (0.00e+0) –	2.9143e-1 (2.24e-1) –	9.9994e-1 (1.42e-5)
DTLZ2	5	7.9138e-1 (3.34e-3) –	7.8728e-1 (2.91e-3) –	8.1285e-1 (2.89e-4)
	10	9.4969e-1 (4.32e-2) –	5.9934e-1 (1.78e-3) –	9.6677e-1 (1.02e-2)
	15	7.3218e-1 (4.14e-1) \approx	9.8769e-1 (1.01e-3) \approx	9.8517e-1 (4.99e-3)
DTLZ3	5	5.0073e-1 (1.16e-1) –	2.9579e-1 (2.44e-1) –	8.1081e-1 (1.29e-3)
	10	0.0000e+0 (0.00e+0) –	0.0000e+0 (0.00e+0) –	9.6196e-1 (1.99e-2)
	15	0.0000e+0 (0.00e+0) –	0.0000e+0 (0.00e+0) –	7.5910e-1 (3.76e-1)
DTLZ4	5	7.9509e-1 (3.94e-3) –	7.9028e-1 (3.45e-3) –	8.1270e-1 (3.39e-4)
	10	9.5919e-1 (3.52e-3) –	8.1742e-1 (4.55e-2) –	9.7406e-1 (6.20e-4)
	15	9.9243e-1 (4.84e-4) +	8.0951e-1 (6.44e-2) –	9.9130e-1 (1.02e-4)
DTLZ5	5	9.0286e-2 (1.15e-2) –	1.1565e-1 (2.49e-3) \approx	1.1618e-1 (1.18e-3)
	10	5.1297e-2 (2.67e-2) –	9.0938e-2 (3.04e-4) +	8.8170e-2 (2.50e-3)
	15	2.1824e-2 (3.04e-2) –	9.0877e-2 (7.60e-5) +	8.1804e-2 (8.86e-3)
DTLZ6	5	9.4125e-2 (4.34e-3) –	9.2690e-2 (2.87e-3) –	1.0151e-1 (8.84e-3)
	10	1.2632e-2 (3.09e-2) \approx	9.2465e-2 (1.42e-3) +	4.5455e-3 (2.03e-2)
	15	1.2699e-3 (5.68e-3) \approx	9.1564e-2 (6.79e-4) +	9.1064e-3 (2.80e-2)
DTLZ7	5	2.6111e-1 (3.73e-3) +	2.6427e-1 (4.93e-3) +	2.5472e-1 (4.36e-3)
	10	8.1331e-2 (2.72e-2) –	1.9606e-1 (4.18e-3) +	1.8816e-1 (5.12e-3)
	15	1.0159e-3 (3.02e-3) –	1.2921e-1 (4.07e-3) –	1.5290e-1 (6.11e-3)
CDTLZ2	5	9.9885e-1 (2.33e-4) –	9.9640e-1 (1.02e-3) –	9.9935e-1 (4.37e-5)
	10	8.6420e-1 (5.09e-2) –	9.0897e-1 (5.60e-3) –	1.0000e+0 (2.48e-6)
	15	8.5908e-1 (6.75e-2) –	9.4332e-1 (1.32e-2) –	9.9994e-1 (1.33e-4)
SDTLZ1	5	6.9373e-1 (1.23e-1) –	9.0978e-1 (6.25e-2) –	9.7985e-1 (1.57e-4)
	10	0.0000e+0 (0.00e+0) –	5.2674e-1 (2.93e-1) –	9.9967e-1 (3.42e-5)
	15	0.0000e+0 (0.00e+0) –	2.3134e-1 (1.98e-1) –	9.9832e-1 (2.20e-3)
SDTLZ2	5	7.9160e-1 (4.16e-3) –	7.8792e-1 (3.60e-3) –	8.1271e-1 (4.00e-4)
	10	9.5989e-1 (1.01e-2) –	9.5979e-1 (2.33e-3) –	9.6497e-1 (1.59e-2)
	15	9.1464e-1 (2.27e-1) –	9.8762e-1 (1.19e-3) –	9.8874e-1 (3.67e-3)
WFG1	5	9.9203e-1 (1.55e-3) –	9.9218e-1 (1.54e-3) –	9.9850e-1 (2.29e-4)
	10	9.9760e-1 (9.60e-4) –	9.9719e-1 (7.39e-4) –	9.9977e-1 (1.28e-4)
	15	9.9735e-1 (1.54e-3) –	9.9776e-1 (8.71e-4) –	9.9999e-1 (9.30e-6)
WFG2	5	9.9490e-1 (5.21e-4) –	9.9593e-1 (5.67e-4) –	9.9695e-1 (4.10e-4)
	10	9.9629e-1 (6.31e-4) –	9.9743e-1 (8.82e-4) –	9.9848e-1 (9.78e-4)
	15	9.9598e-1 (7.06e-4) –	9.9685e-1 (1.20e-3) –	9.9856e-1 (7.80e-4)
WFG3	5	1.4973e-1 (2.38e-2) –	2.5236e-1 (7.26e-3) –	9.7619e-1 (2.12e-3)
	10	5.0045e-5 (2.24e-4) –	1.1397e-1 (2.60e-2) –	9.1396e-1 (4.92e-3)
	15	0.0000e+0 (0.00e+0) –	0.0000e+0 (0.00e+0) –	7.9194e-1 (2.52e-2)
WFG4	5	7.8979e-1 (2.35e-3) –	7.9397e-1 (3.60e-3) –	8.0856e-1 (9.05e-4)
	10	9.6104e-1 (9.23e-4) \approx	9.6425e-1 (1.50e-3) +	9.6178e-1 (2.63e-3)
	15	9.8895e-1 (1.07e-3) \approx	9.8986e-1 (7.46e-4) +	9.8774e-1 (3.12e-3)
WFG5	5	7.4372e-1 (2.61e-3) –	7.4157e-1 (3.73e-3) –	7.6167e-1 (3.29e-4)
	10	8.9842e-1 (6.88e-4) –	8.9698e-1 (2.40e-3) –	9.0395e-1 (2.45e-4)
	15	9.1842e-1 (3.66e-4) +	9.1608e-1 (7.63e-4) –	9.1672e-1 (1.82e-3)
WFG6	5	7.2667e-1 (1.62e-2) –	7.3054e-1 (9.92e-3) –	7.5242e-1 (1.29e-2)
	10	8.7765e-1 (2.14e-2) \approx	8.8640e-1 (1.47e-2) \approx	8.8787e-1 (1.20e-2)
	15	8.9333e-1 (2.51e-2) \approx	8.9857e-1 (1.46e-2) \approx	8.9553e-1 (1.85e-2)
WFG7	5	7.9619e-1 (2.54e-3) –	7.9391e-1 (2.74e-3) –	8.0960e-1 (4.62e-4)
	10	9.6525e-1 (3.58e-3) –	9.6523e-1 (1.02e-3) –	9.6704e-1 (1.08e-3)
	15	9.8809e-1 (2.29e-3) +	9.9103e-1 (4.14e-4) +	9.1386e-1 (1.22e-2)
WFG8	5	6.6597e-1 (3.29e-3) –	6.8015e-1 (3.43e-3) –	6.9980e-1 (2.03e-3)
	10	8.8342e-1 (2.94e-2) \approx	8.8051e-1 (4.64e-3) +	8.7653e-1 (1.56e-2)
	15	9.3952e-1 (2.61e-2) \approx	9.3722e-1 (2.25e-3) \approx	9.3671e-1 (2.25e-2)
WFG9	5	7.6966e-1 (4.11e-3) \approx	7.7760e-1 (4.47e-3) +	7.6710e-1 (4.61e-3)
	10	9.1414e-1 (1.52e-2) +	9.2561e-1 (5.01e-3) +	8.6992e-1 (6.32e-2)
	15	8.9974e-1 (7.06e-2) \approx	9.4232e-1 (6.37e-3) +	9.1345e-1 (4.42e-2)
+ / - / \approx		5/41/11	13/39/5	

‘+’, ‘–’ and ‘ \approx ’ indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGA-III/LCD, respectively.

TABLE S-IV
PD VALUES OF MOEA/DD, RVEA, NSGA-II/SDR, MOMBI-III, θ -DEA, AR-MOEA AND NSGA-III/LCD.

Problem	M	MOEA/DD	RVEA	NSGA-II/SDR	MOMBI-III	θ -DEA	AR-MOEA	NSGA-III/LCD
DTLZ1	5	4.4372e+5 (4.81e+4) –	7.1698e+5 (1.29e+5) –	1.4027e+7 (1.32e+6) +	3.2108e+5 (2.91e+4) –	6.0580e+5 (1.02e+5) –	5.9312e+5 (7.46e+4) –	1.0242e+7 (5.57e+5)
	10	1.1113e+9 (1.34e+8) –	2.4911e+9 (2.32e+8) –	5.7415e+9 (1.11e+9) –	2.5455e+8 (5.50e+8) –	3.0329e+9 (1.91e+9) –	3.9641e+9 (2.10e+8) –	1.7239e+10 (8.67e+9)
	15	1.1621e+11 (1.77e+10) –	2.7438e+11 (6.81e+10) –	7.0140e+10 (4.59e+10) –	3.8484e+8 (7.02e+8) –	1.7414e+11 (6.10e+10) –	2.7306e+11 (3.56e+10) –	1.2105e+12 (4.38e+11)
DTLZ2	5	2.5239e+6 (1.26e+5) –	3.1431e+6 (1.95e+5) –	2.5754e+7 (2.74e+6) ≈	4.0170e+6 (3.65e+5) –	3.1054e+6 (1.70e+5) –	3.1213e+6 (4.48e+5) –	2.5873e+7 (1.54e+6)
	10	3.6350e+9 (1.74e+8) –	6.5762e+9 (2.81e+8) –	2.0678e+9 (5.06e+8) –	6.2559e+9 (5.60e+8) –	6.7652e+9 (3.80e+8) –	1.3781e+10 (9.83e+8) –	2.5487e+10 (6.15e+9)
	15	1.9648e+11 (2.40e+10) ≈	4.3396e+11 (2.42e+10) ≈	4.8240e+9 (2.25e+9) –	7.1829e+9 (6.43e+9) –	5.0265e+11 (2.40e+10) ≈	7.0292e+11 (4.70e+10) +	2.8444e+11 (2.78e+11)
DTLZ3	5	3.9482e+6 (3.17e+5) –	4.3790e+6 (3.62e+5) –	2.3246e+7 (2.83e+6) –	5.4659e+6 (3.86e+5) –	4.6602e+6 (4.42e+5) –	5.3694e+6 (1.67e+6) –	3.6666e+7 (1.94e+7)
	10	6.0921e+9 (5.22e+8) –	1.0549e+10 (5.60e+8) –	2.0296e+9 (5.00e+8) –	1.9682e+9 (2.51e+9) –	1.2937e+10 (2.65e+9) –	5.6048e+9 (2.32e+9) –	1.0267e+11 (4.56e+10)
	15	6.8879e+11 (4.71e+11) ≈	7.4708e+11 (6.33e+10) ≈	4.7034e+9 (2.20e+9) –	9.4848e+7 (1.02e+8) –	9.1082e+11 (1.91e+11) ≈	4.1132e+11 (1.39e+11) –	9.6133e+11 (8.59e+11)
DTLZ4	5	2.7543e+6 (1.36e+5) –	4.3420e+6 (2.34e+5) –	6.7103e+3 (6.88e+3) –	4.7699e+6 (3.65e+5) –	3.7383e+6 (2.58e+5) –	3.4626e+6 (5.57e+5) –	2.4054e+7 (2.49e+6)
	10	4.3138e+9 (2.27e+8) –	9.1233e+9 (4.27e+8) +	1.6809e+4 (4.13e+3) –	7.1603e+8 (2.30e+8) –	9.5984e+9 (6.04e+8) +	3.7299e+9 (8.98e+8) –	6.8377e+9 (1.59e+9)
	15	1.8311e+11 (7.00e+9) ≈	5.9718e+11 (3.15e+10) ≈	3.8075e+5 (1.87e+6) –	1.6801e+10 (5.75e+9) ≈	6.3235e+11 (3.22e+10) ≈	1.7265e+10 (6.46e+9) ≈	4.8182e+11 (4.34e+11)
DTLZ5	5	2.4348e+7 (2.16e+6) –	3.3652e+7 (3.36e+6) –	3.1585e+7 (2.48e+6) –	1.0445e+7 (1.01e+6) –	2.1691e+7 (3.55e+6) –	6.5814e+7 (4.35e+6) +	5.0438e+7 (4.81e+6)
	10	2.0745e+10 (2.22e+9) –	2.6674e+10 (3.55e+9) –	4.8371e+10 (3.81e+9) –	6.4480e+7 (8.53e+7) –	2.5489e+10 (4.98e+9) –	8.0967e+10 (5.25e+9) +	6.1530e+10 (3.84e+9)
	15	1.5038e+12 (2.19e+11) –	3.2055e+11 (3.75e+11) –	1.4723e+12 (2.51e+11) –	1.5415e+9 (1.94e+9) –	1.3035e+12 (2.01e+11) –	1.8674e+12 (1.47e+11) –	2.4220e+12 (9.53e+11)
DTLZ6	5	3.3263e+7 (3.91e+6) –	2.2032e+7 (3.95e+6) –	1.4884e+7 (3.56e+6) –	1.1086e+7 (2.29e+6) –	4.8283e+7 (8.53e+6) –	7.2973e+7 (3.93e+6) –	7.8040e+7 (7.65e+6)
	10	3.8986e+10 (7.45e+9) –	6.0813e+10 (7.31e+9) –	4.2842e+10 (7.97e+9) –	2.8905e+8 (1.12e+9) –	5.0428e+10 (6.23e+9) –	8.4850e+10 (5.18e+9) –	1.0588e+11 (1.01e+10)
	15	2.4390e+12 (2.63e+11) –	9.6914e+11 (4.57e+11) –	1.9885e+12 (4.64e+11) –	1.6417e+9 (2.44e+9) –	2.3167e+12 (2.56e+11) –	1.5936e+12 (5.21e+11) –	6.1015e+12 (1.24e+12)
DTLZ7	5	6.2182e+5 (2.20e+6) –	1.9809e+7 (1.62e+6) –	3.5121e+7 (3.55e+6) –	4.5872e+6 (1.38e+6) –	2.0993e+7 (4.39e+6) –	4.2359e+7 (2.83e+6) +	3.7915e+7 (3.63e+6)
	10	1.1811e+9 (1.43e+9) –	1.4624e+10 (1.98e+9) –	1.1732e+10 (2.67e+9) –	8.5905e+9 (2.46e+9) –	2.3705e+10 (5.52e+9) –	2.4799e+10 (2.33e+9) –	4.5935e+10 (3.42e+9)
	15	3.4071e+10 (1.54e+10) –	5.9757e+11 (1.38e+11) +	9.9920e+11 (3.88e+11) +	2.6606e+11 (5.74e+10) –	8.5052e+11 (1.70e+11) +	9.7799e+11 (2.49e+11) +	3.2113e+11 (1.43e+11)
CDTLZ2	5	9.6843e+6 (3.92e+5) –	1.1447e+7 (4.59e+5) –	1.2449e+7 (9.49e+5) –	1.6632e+6 (5.29e+4) –	1.0005e+7 (3.79e+5) –	3.2614e+6 (4.79e+5) –	1.4186e+7 (5.92e+5)
	10	1.4936e+9 (5.56e+7) –	4.4708e+9 (2.06e+8) ≈	5.0180e+9 (3.55e+8) ≈	8.1070e+8 (1.76e+8) –	3.1112e+9 (5.90e+8) –	2.6276e+9 (2.28e+8) –	4.7930e+9 (1.05e+9)
	15	3.9978e+10 (4.00e+9) –	8.2993e+10 (4.90e+9) –	1.1414e+11 (7.11e+9) ≈	3.3662e+9 (5.01e+9) –	2.9083e+10 (1.14e+10) –	7.0449e+10 (1.00e+10) –	1.1528e+11 (1.62e+10)
SDTLZ1	5	3.1723e+7 (1.28e+6) –	6.4410e+6 (1.96e+6) –	5.2055e+7 (5.73e+6) –	1.1710e+6 (1.63e+5) –	2.1685e+6 (2.14e+5) –	2.8860e+6 (6.07e+5) –	7.1958e+7 (7.82e+6)
	10	1.4593e+10 (6.92e+8) –	1.9322e+10 (1.98e+9) –	2.2056e+10 (9.19e+9) –	2.4325e+9 (2.12e+9) –	7.0940e+10 (4.04e+10) ≈	8.4874e+9 (1.62e+9) –	7.4812e+10 (6.91e+10)
	15	1.0072e+12 (1.32e+11) –	6.2500e+11 (1.40e+11) –	5.8846e+10 (2.22e+10) –	1.2769e+10 (1.68e+10) –	2.8278e+13 (1.16e+13) ≈	4.0163e+11 (1.63e+11) –	3.0869e+13 (1.85e+13)
SDTLZ2	5	2.8328e+7 (3.39e+6) +	1.9222e+7 (1.60e+6) –	1.0271e+8 (1.67e+7) +	1.6007e+7 (9.09e+5) –	1.2332e+7 (4.43e+5) –	1.6090e+7 (7.50e+5) –	2.5378e+7 (3.63e+6)
	10	1.9277e+10 (1.56e+9) –	9.6279e+9 (1.35e+9) –	2.0090e+10 (9.24e+9) –	1.5553e+11 (1.56e+10) –	1.7812e+11 (9.59e+9) ≈	5.2544e+9 (3.73e+9) –	1.8042e+11 (2.81e+10)
	15	8.6359e+11 (2.20e+11) –	2.5673e+11 (4.82e+10) –	6.3275e+10 (3.98e+10) –	2.7266e+10 (3.31e+10) –	3.2021e+13 (4.44e+13) –	2.6021e+11 (3.03e+11) –	7.4095e+13 (5.55e+13)
WFG1	5	7.3687e+7 (4.71e+6) –	5.8689e+7 (3.43e+6) –	5.6233e+7 (7.47e+6) –	1.4548e+7 (1.69e+6) –	6.9796e+7 (2.96e+6) –	4.4512e+7 (3.69e+6) –	9.9782e+7 (1.40e+7)
	10	3.2040e+10 (1.86e+9) –	3.9229e+10 (1.54e+9) –	3.0209e+10 (7.21e+9) –	7.2385e+9 (1.21e+9) –	4.3053e+10 (1.84e+9) –	4.4016e+10 (1.30e+9) –	5.5660e+10 (1.16e+10)
	15	4.6499e+11 (5.73e+10) –	1.1155e+12 (8.05e+10) –	9.4390e+11 (1.31e+11) –	2.2773e+11 (6.00e+10) –	1.2023e+12 (1.46e+11) –	1.1984e+12 (4.86e+10) –	1.3732e+12 (1.23e+11)
WFG2	5	7.0849e+7 (1.94e+6) –	8.4376e+7 (3.15e+6) –	8.6362e+7 (4.20e+6) –	1.6775e+7 (1.25e+6) –	7.5200e+7 (2.23e+6) –	5.8800e+7 (3.34e+6) –	1.0158e+8 (4.56e+6)
	10	3.1256e+10 (1.30e+9) –	5.2069e+10 (2.58e+9) –	6.1987e+10 (4.78e+9) –	5.8860e+9 (4.70e+9) –	5.3552e+10 (7.77e+9) –	3.9241e+10 (1.44e+9) –	8.2959e+10 (1.20e+10)
	15	6.8723e+11 (7.18e+10) –	1.2883e+12 (8.43e+10) –	2.2351e+12 (1.76e+11) –	1.2885e+10 (1.89e+10) –	1.0974e+12 (3.29e+11) –	1.4777e+12 (1.82e+11) –	2.5316e+12 (3.29e+11)
WFG3	5	1.7466e+8 (8.06e+6) –	1.4965e+8 (8.16e+6) –	2.1943e+8 (9.29e+6) +	6.0219e+7 (7.12e+6) –	9.5874e+7 (6.26e+6) –	2.2500e+8 (9.79e+6) +	1.9143e+8 (9.32e+6)
	10	6.3530e+10 (3.75e+9) –	1.1166e+11 (1.59e+10) –	3.3928e+11 (1.74e+10) +	2.2470e+8 (1.38e+8) –	1.2076e+11 (2.65e+10) –	1.8917e+11 (1.31e+10) –	2.2081e+11 (2.44e+10)
	15	2.5568e+12 (1.81e+11) –	6.2776e+12 (4.25e+11) –	1.1852e+13 (1.55e+12) +	1.1640e+9 (1.44e+8) –	5.7841e+12 (1.97e+12) –	1.0997e+13 (7.95e+11) +	8.4606e+12 (2.79e+12)
WFG4	5	1.6603e+8 (8.95e+6) –	1.1555e+8 (4.44e+6) –	2.9937e+8 (1.19e+7) +	4.3494e+7 (2.75e+6) –	1.0757e+8 (5.80e+6) –	1.0848e+8 (5.48e+6) –	2.4869e+8 (3.67e+7)
	10	1.0395e+11 (5.27e+9) –	1.3788e+11 (8.27e+9) –	2.2666e+11 (2.34e+10) ≈	5.9077e+10 (2.73e+10) –	1.4939e+11 (4.82e+9) –	1.4662e+11 (8.37e+9) –	2.9874e+11 (1.45e+11)
	15	3.3164e+12 (4.15e+11) –	4.7660e+12 (3.33e+11) –	3.4069e+12 (5.27e+11) –	6.0133e+9 (1.24e+10) –	1.3451e+13 (1.22e+12) –	3.9863e+12 (1.58e+12) –	1.4206e+13 (2.01e+12)
WFG5	5	1.4821e+8 (3.05e+7) –	7.7792e+7 (5.80e+6) –	2.8291e+8 (1.34e+7) –	4.0190e+7 (5.28e+6) –	6.9540e+7 (3.85e+6) –	7.3927e+7 (6.12e+6) –	3.5111e+8 (1.01e+7)
	10	9.3900e+10 (1.66e+10) –	1.2038e+11 (6.70e+9) –	2.2622e+11 (1.76e+10) –	7.3965e+10 (4.65e+9) –	1.1993e+11 (4.91e+9) –	1.2273e+11 (6.42e+9) –	2.4566e+11 (3.78e+10)
	15	4.9559e+12 (2.23e+11) –	5.0327e+12 (4.63e+11) –	2.9932e+12 (5.82e+11) –	1.3203e+10 (1.28e+10) –	1.2624e+13 (8.65e+11) ≈	1.0718e+13 (5.14e+11) –	1.2260e+13 (8.88e+11)
WFG6	5	1.4543e+8 (1.33e+7) –	7.7955e+7 (7.61e+6) –	2.7359e+8 (1.60e+7) +	4.1159e+7 (4.23e+6) –	8.2911e+7 (1.04e+7) –	8.7633e+7 (9.66e+6) –	2.2769e+8 (4.90e+7)
	10	8.2528e+10 (7.37e+9) –	1.2591e+11 (1.19e+10) –	1.4507e+11 (1.71e+10) ≈	5.2971e+10 (6.43e+9) –	1.0497e+11 (6.25e+9) –	1.1045e+11 (7.40e+9) –	2.5001e+11 (1.22e+11)
	15	2.5842e+12 (5.59e+11) –	6.0866e+12 (6.93e+11) –	2.2813e+12 (5.78e+11) –	1.9685e+10 (3.49e+10) –	1.0409e+13 (5.01e+11) –	9.1049e+12 (5.14e+11) –	1.1551e+13 (1.35e+12)
WFG7	5	1.7370e+8 (1.03e+7) –	1.1908e+8 (3.12e+6) –	2.9304e+8 (1.24e+7) +	4.7772e+7 (2.56e+6) –	9.4437e+7 (3.30e+6) –	1.2842e+8 (5.81e+6) –	2.3124e+8 (3.69e+7)
	10	1.2340e+11 (9.61e+9) –	1.8209e+11 (1.07e+10) –	2.2085e+11 (2.37e+10) +	8.3806e+10 (5.00e+9) –	1.5245e+11 (7.66e+9) –	1.7870e+11 (8.31e+9) ≈	2.0651e+11 (1.05e+11)
	15	6.2235e+12 (6.55e+11) –	6.3005e+12 (1.71e+12) –	2.8421e+12 (4.88e+11) –	4.8121e+10 (5.21e+10) –	1.6781e+13 (8.75e+11) +	1.0835e+13 (2.47e+12) ≈	1.0428e+13 (4.64e+12)
WFG8	5	1.7648e+8 (2.18e+7) –	2.0570e+8 (5.70e+6) –	3.5372e+8 (1.05e+7) ≈	4.0165e+7 (3.18e+6) –	2.3825e+8 (1.59e+7) –	2.2113e+8 (7.32e+6) –	3.6006e+8 (1.70e+7)
	10	9.4743e+10 (1.03e+10) –	1.7145e+11 (1.30e+10) –	2.1327e+11 (2.02e+10) –	6.1037e+10 (3.05e+10) –	1.2399e+11 (1.98e+10) –	1.7379e+11 (2.33e+10) –	5.1023e+11 (3.90e+10)
	15	3.1276e+12 (9.15e+11) –	5.5012e+12 (1.12e+12) –	5.3237e+12 (1.91e+12) –	7.2512e+10 (7.91e+10) –	1.6585e+13 (2.77e+12) –	9.1649e+12 (2.73e+12) –	2.1082e+13 (4.09e+12)
WFG9	5	2.5340e+8 (1.47e+7) –	2.1296e+8 (5.92e+6) –	3.6938e+8 (1.65e+7) –	3.4755e+7 (7.30e+6) –	2.4843e+8 (1.19e+7) –	2.3330e+8 (7.76e+6) –	4.0366e+8 (1.10e+7)
	10	2.4305e+11 (1.81e+10) –	3.1157e+11 (1.41e+10) –	4.9159e+11 (2.09e+10) –	1.5889e+11 (1.15e+10) –	3.7872e+11 (2.94e+10) –	3.4958e+11 (1.49e+10) –	6.6939e+11 (9.87e+10)
	15	1.0465e+13 (1.42e+12) –	1.6486e+13 (1.12e+12) –	1.7306e+13 (2.09e+12) –	3.2296e+10 (2.77e+10) –	2.5913e+13 (1.77e+12) ≈	2.0631e+13 (1.40e+12) –	2.5447e+13 (1.90e+12)
+ / - / ≈		1/53/3	2/51/4	10/41/6	0/56/1	3/46/8	7/47/3	

‘+’, ‘-’ and ‘≈’ indicate that the result is significantly better, significantly worse and statistically similar to that obtained by NSGA-III/LCD, respectively.

E. Experimental Comparisons of Convergence Profiles

Experimental comparisons of the convergence profiles of HV indicator values obtained with and without the line complex-based distance are performed. Specifically, the

median HV values among 30 runs obtained by the algorithms on 15-objective WFG6 and WFG8 problems are illustrated in Fig. S-1. The HV values obtained by the algorithms on 15-objective DTLZ7 and SDTLZ2 test problems are illustrated

in Fig. S-2. The red lines denote the results obtained by NSGA-III, while blue ones refer to the results by NSGA-III/LCD. The results reveal that, with the proposed line complex-based strategy, convergence is improved and better HV values are obtained.

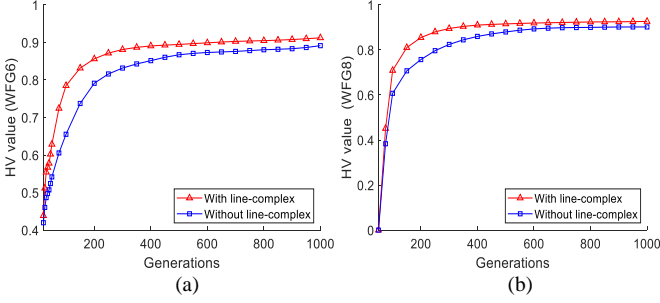


Fig. S-1. HV values of the results obtained by the algorithms on 15-objective (a) WFG6 and (b) WFG8 test problems with and without the line complex over 1000 generations.

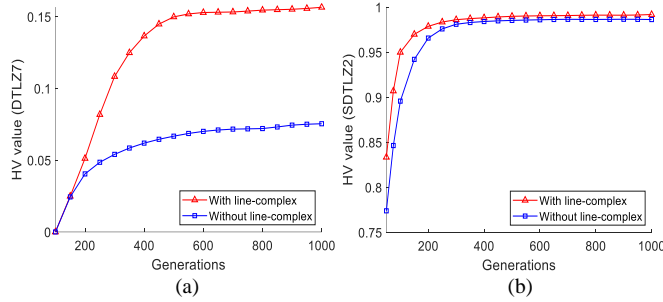


Fig. S-2. HV values of the results obtained by the algorithms on 15-objective (a) DTLZ7 and (b) SDTLZ2 test problems with and without the line complex over 1000 generations.

To further observe the convergence profiles of the proposed algorithm, Fig. S-3 present IGD values obtained by NSGA-III/LCD on 5-, 10-, 15-objective DTLZ1 and CDTLZ2 test problems. The experimental results show that NSGA-III/LCD converges to the Pareto front effectively on these problems. This means the proposed line complex strategy improves the convergence in dealing with high dimensional test problems effectively.

In summary, the experimental results demonstrate that the proposed line complex-based strategy can effectively improves both convergence and diversity of basic algorithm (NSGA-III) on DTLZ and WFG test problems.

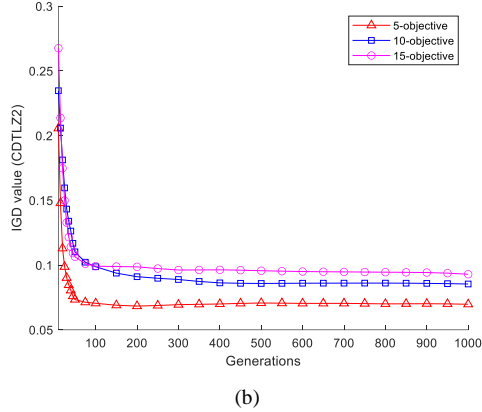
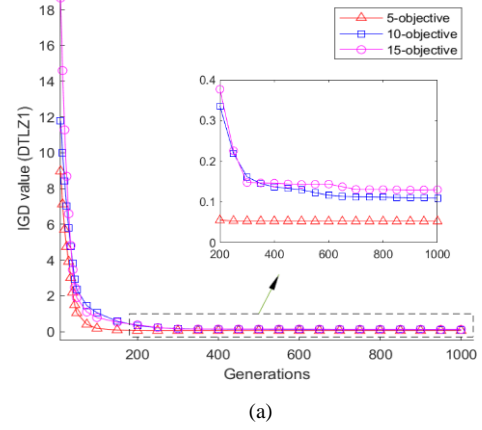
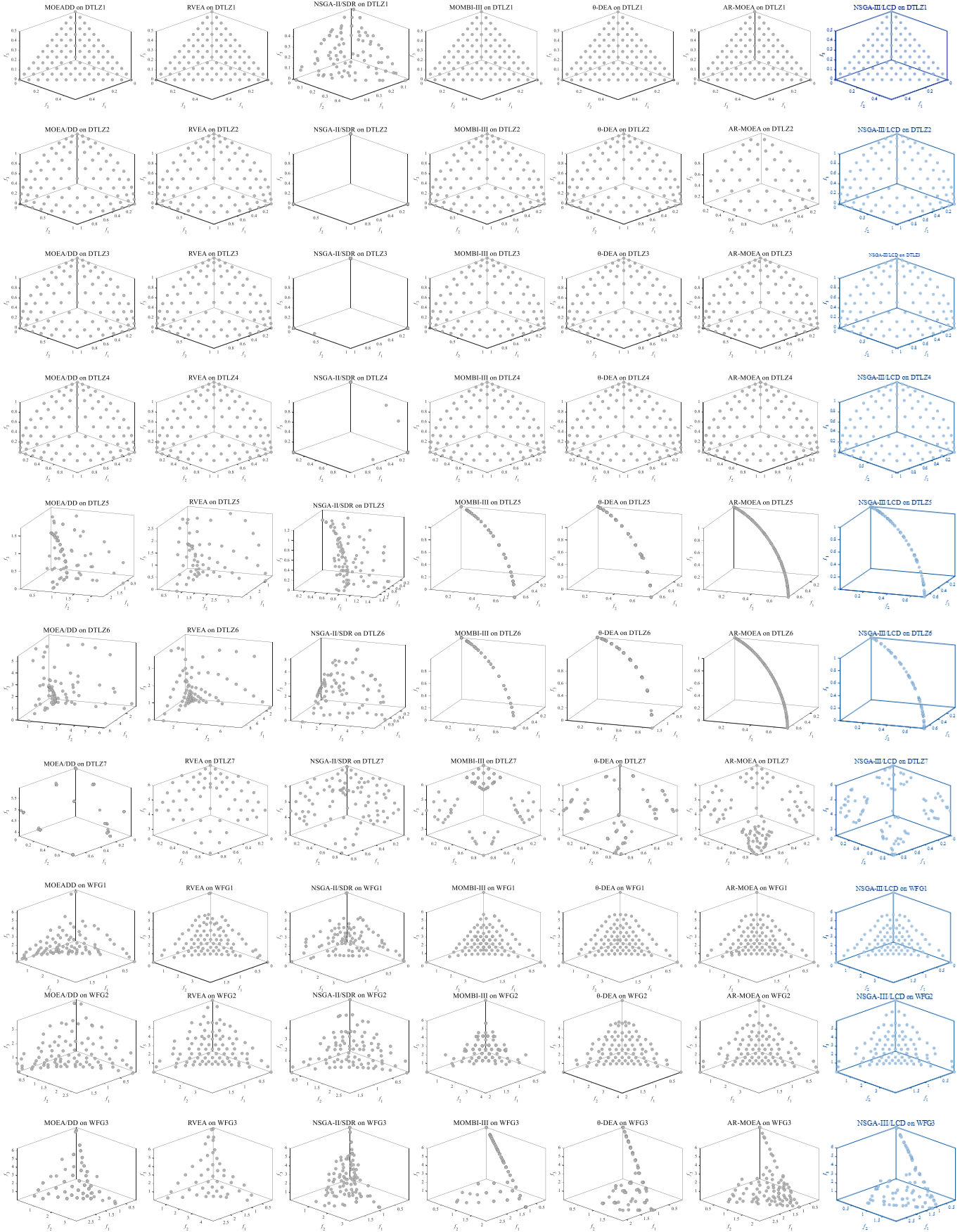


Fig. S-3. Convergence profiles of IGD values obtained by NSGA-III/LCD on 5-, 10-, 15-objective DTLZ1 with regular PFs and 5-, 10-, 15-objective CDTLZ2 with irregular PFs, all results are averaged over 30 runs.

F. Plot Results on Three-objective Problems

Comparison of all methods (with the same experimental settings, the population size of all the algorithms is set to 92) on 3-objective DTLZ1-7, WFG1-9, SDTLZ1-2 and CDTLZ are shown in Fig. S-4. It can be seen that, NSGA-III/LCD can obtain better distributed solutions in most cases, although it does not perform very well on DTLZ7 and WFG3. Furthermore, we found that NSGA-II/SDR doesn't perform well on most 3-objective test problems, although it did achieve better HV and PD performance. This indicates that SDR considering convergence in terms of the sum of objectives and diversity based on the tailed niching technique does not generalize well to fewer objectives.



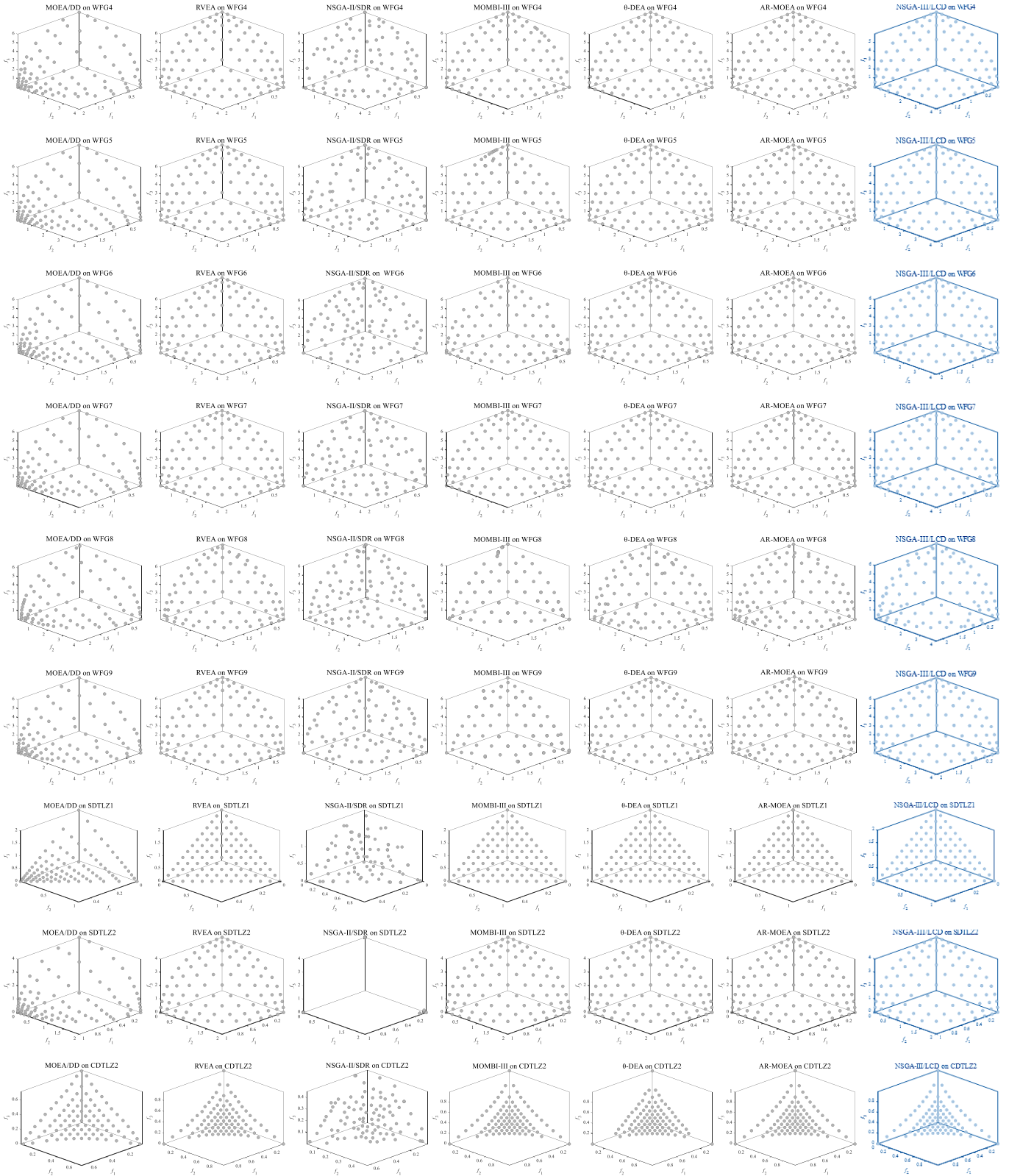


Fig. S-4. Plot results for each algorithm on 3-objective DTLZ1-7, WFG1-9, SDTLZ1-2 and CDTLZ