Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
C		2407108

Quantifying momentum, grasping victory in tennis

Summary

Physics defines where momentum as "the strength or force gained by motion or by a series of events that keeps an objective moving." In tennis, it is the psychological and physical effects of momentum that determine the direction of a match. The aim of this study is to **investigate the impact of momentum in tennis matches through a data-driven approach, and to develop a model to predict changes in momentum in matches.** Using the Wimbledon 2023 men's singles tournament as a case study, we analyzed match data to quantify momentum in matches and assess its impact on match outcomes.

Firstly, we defined a series of momentum metrics based on factors such as "score, aces, and double faults". Using these metrics, we utilized the **Random Forests Algorithm** to develop a dynamic model capable of tracking and evaluating a player's performance during a match in real time. The model takes into account the higher probability of the **serving team winning points** in a tennis match, weights the momentum score, and visualizes the flow of the match.

Secondly, with respect to the role of momentum, our model challenges the conventional wisdom that the effect of momentum on the outcome of a match is random. To this end, we designed a model that reflects the percentage of players' momentum and used a **logistic regression model** to make predictions. Some new factors were defined to quantify momentum, which referred to **"service score rate, break failure rate, net scoring rate and so on"**, proving that momentum can indeed predict the outcome of a match, and that the accuracy of our model can reach up to **90% and more.**

Thirdly, inspired by the **Sliding Window Algorithm**, we designed a new quantitative model for momentum and examined its effectiveness in predicting match outcomes. We then utilized the use of historical data to identify key factors that lead to changes in momentum and predict shifts in momentum in future games. We also proposed some model-based advice for players going into a new match accordingly. After testing, our model can predict momentum shifts with a success rate of 71% percent.

Finally, we applied the model to data from other games to test the model's ability to generalize. Although the model performed poorly in some cases, this prompted us to identify and suggest additional factors that may need to be included in future models, such as the **physical condition of the players, weather conditions**, as well as **psychological stress**.

Through this study, we have provided coaches and players with data-based insights to better understand and apply momentum shifts in matches, providing them with strategic advice going into new matches. The results of our study are not only applicable in tennis, but also informative for other sports that require an understanding of dynamic competitive states.

Keywords: Momentum Analysis; Predictive Modeling; Random Forest; Sliding Window; Logistic Regression; Data Visualization; Generalization Capability

Contents

1 Introduction

1.1 Background

“Tennis more than any other sport, is a game of momentum. The absence of a clock to do the dirty work of finishing off an opponent, and a scoring system based on units used, makes the flow of the match much more important than any lead that has been established."

——Chuck Kriese

Physics defines where momentum as “the strength or force gained by motion or by a series of events that keeps an objective moving.” **1** In tennis, it is the psychological and physical effects of momentum that determine the direction of a match. A player seemingly in the ascendancy during a match is often said to “have the momentum”. Momentum in tennis can swing wildly from point to point, game to game, set to set. Swings in momentum are referred to as turning points. These can be obvious: players switching tactics after losing a set; a brilliant winner went on the ropes in a rally or an untimely double fault causing a opponent tightening up.

However, sometimes momentum can be so small as to be imperceptible, it is difficult to measure and it is not readily apparent how various events during the match act to create or change momentum if it exists. By understanding and tapping momentum, players can employ methods and tactics in games to ensure they are in control of momentum rather than a victim of it.

1.2 Restatement of the Problem

Through in-depth analysis and research on the background of the problem, combined with topic specific constraints and requirements given, the restate of the problem can be expressed as follows:

- **Construct a model to capture the flow of play as points occur.** Identifying which player is performing better at a given time in the match, as well as how better they are performing. A visualization based on the model is required to depict the match flow. It is also noteworthy that the player to serve are supposed to be factored in to the model.
- **Use the model to assess whether “momentum” plays any role in a match**, as well as swings in play and runs of success by one player are random.
- **Identify indicators of the changing flow of play from favoring one player to the other.** Use the data provided to develop a model that predicts these swings in the match and try to probe into the most related factors. Advise a player going into a new match against a different player with the differential in past match “momentum” swings.
- **Test the developed model on other matches and identify factors that might need to be added.**
- **Produce a report of no more than 25 pages with the above findings and include a one-to two-page memo**, summarizing the results with advice for coaches on the role of “momentum”, and how to prepare players to respond to events that impact the flow of play during a tennis match.

1.3 Our Work

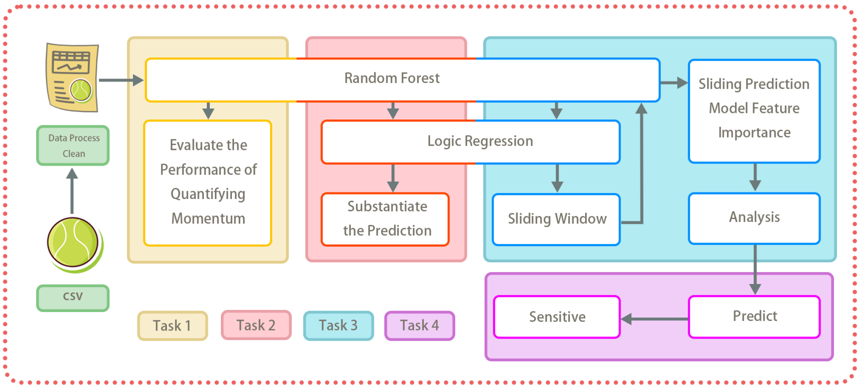


Figure 1: Modeling process

2 Assumptions and Justification

- **Athletes will not be affected by the results of previous matches while playing the current match.** Supposing that the athlete is always in a good state of mind and that his or her performance in the previous game does not affect the outcome of the current game.
- **Athletes have a fixed interval between each score.** Supposing the interval between goals can be equated to the concept of time. In this way the time cost of scoring is quantifiable and relatively stable in the tennis match and we can analyze game progression and athletes' performance from a new perspective.

3 Notations

Table 1 shows the necessary notations and signs used in this paper. Other notations and signs will be declared or defined when using.**[2]**

Table 1: **Notations**

Symbols	Descriptions
S_{P1i}	player 1's momentum integral value in one match
S_{P2i}	player 2's momentum integral value in one match
A_r	service score rate
F_r	service failure rate
B_r	break success rate
B_f	break failure rate
N_r	net scoring rate
E_r	error ratio
T_S	the total number of serve in the set
M	momentum

4 Data Preprocessing

4.1 Data Cleaning

Overview of data sets: This dataset is derived from the featured races of the Wimbledon Championship and contains detailed race statistics. It is worth noting that there are a number of missing values in the dataset, particularly in the speed_mph (752 missing), serve_width and serve_depth (54 missing each), and return_depth (1309 missing) fields.

Missing value handling: In dealing with missing values in the dataset, special attention is paid to the NA values in the speed column. In an initial check, 752 missing values were found in the speed column. These missing values can occur for a variety of reasons, including data entry errors or omissions during the data collection process. There were even instances in matches 1310 and 1311 where the whole bureau had unrecorded rally_count as well as speed values.

Further analysis showed that some of the missing speed values were associated with a rally_count of 0, which reflected a specific scenario of a no-ball exchange during the match, not missing data, as shown in **??**. In contrast, the missing data for complete matches may stem from technical problems or human negligence in the recording process.

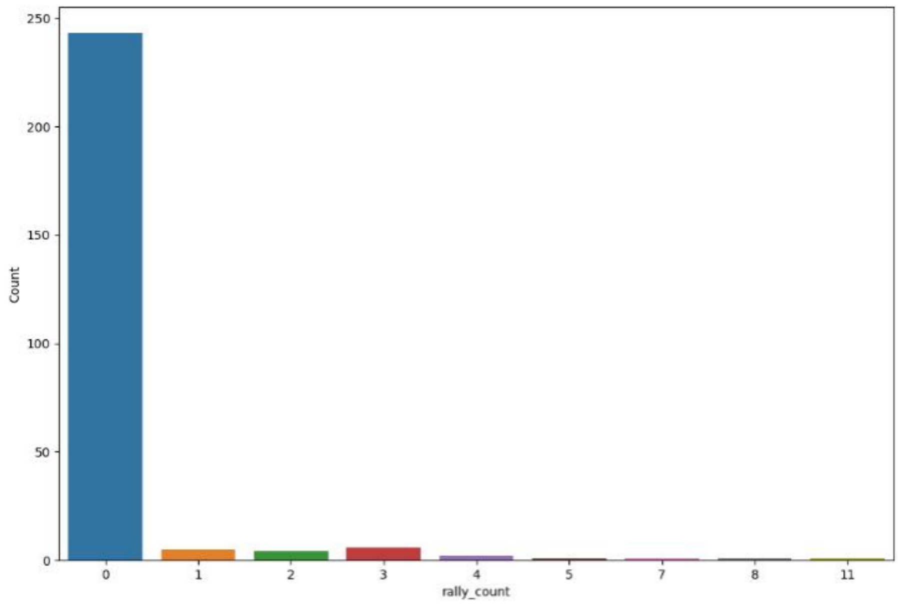


Figure 2: SpeedNA_Count_By_RallyCount

In the case of unrecorded data for an entire race, we decided to exclude these records from the dataset given the non-direct impact of this data on model training. Data where a rally_count of 0 resulted in a speed of 0 was considered a valid record and retained to accurately reflect the reality of the race.

Through this careful missing value handling strategy, we ensured the accuracy and reliability of the data analysis and laid a solid foundation for the subsequent data analysis work.

Abnormal value progress: In the dataset, there are 235 rows of records showing that when double fault is 1, the serve_width and serve_depth fields are still recorded. Theoretically, in the event of a double serve fault, the ball did not successfully cross the net and therefore the width and depth of the serve should not be recorded.

4.2 Data Analysis

We find that serve_no and speed are related. The box plot shows the distribution of serve speeds according to the number of serves (first and second). The following points can be observed from the graph:

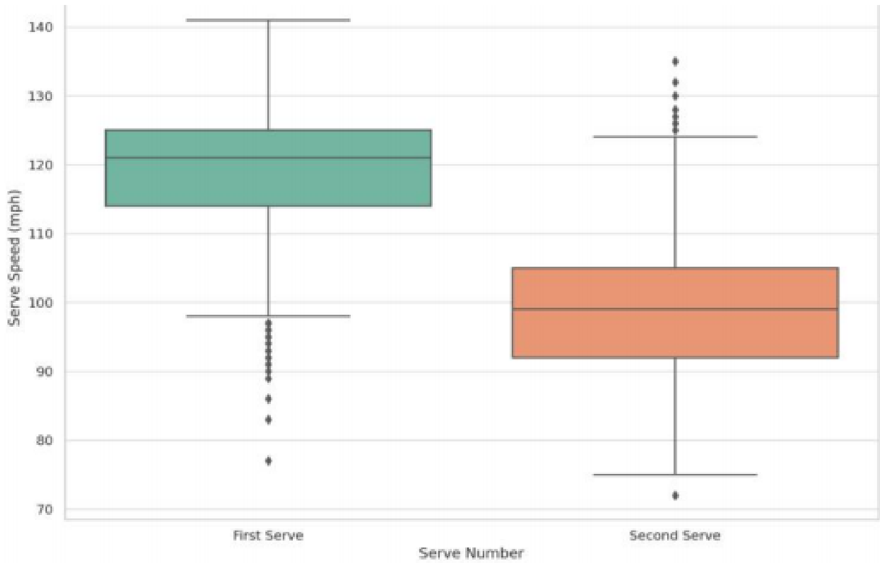


Figure 3: Boxplot of Serve Speed By Serve Number

First serve speeds were typically higher than second serve speeds, reflecting a common strategy in tennis whereby players tend to use more powerful serves on the first serve in an attempt to score points outright or to gain a favorable position, while they tend to use more secure serves on the second serve to avoid double faults.

The wider distribution of speeds on the first serve and the higher outliers at the top suggest that players' serve speeds vary more when attempting more powerful serves.

The relatively more concentrated distribution of speeds and fewer outliers on the second serve may be due to the fact that players focus more on accuracy and consistency on the second serve to minimize the risk of double faults.

These observations are consistent with the conventional strategy in tennis where the first serve is more focused on aggressiveness while the second serve is more focused on insurance. This analysis helps us to gain a deeper understanding of players' serving strategies and their potential impact on match outcomes.

5 Model Construction

5.1 Discrete Performance Evaluating Model Based on Random Forest

5.1.1 Model Preparation

According to the requirements of problem 1, this paper needs to build a model to obtain the scoring points occurring in the match. **The Random Forest model** is chosen to determine the weights of the indicators, therefore we are able to identify the better performance of the players visually, then we apply the model to as many games as possible.

- **Firstly, we acquire data information such as "ace" or "net_pt"** , which refer to a not-served shot and a player's position separately from the provided data set, result data of each set or game can also be found.
- **Next, we try to design a model to quantify "momentum".** Momentum can be perceived as strength or force during a match, since it is difficult to quantize, so we attempt to incorporate the Calculation of short-term indicators, transforming scoring concepts into momentum indicators.
- **Then, we discover that momentum is usually reflected as a change in game performance over a short period of time.** For example, consecutive scores can be seen as a direct reflection of momentum. By calculating short-term changes in scoring for each point (e.g., consecutive points, short-term serve success and break rates, etc.), these short-term performances can be quantified as indicators of momentum.

5.1.2 Random Forest Model

Although the ultimate goal is to predict the outcome of the game, the impact of momentum changes on the outcome of the game can be revealed by analyzing the relationship between short-term momentum indicators and the final outcome of the game. Short-term momentum indicators can be trained as features and match results (win/lose) as labels in the model to determine the association between momentum and match.

We can begin to quantify indicators by virtue of the above idea, but it is not yet possible to determine the specific levels of quantification. We then use a random forest model to analyze the importance of the characteristics of these indicators.

Random Forest is a supervised algorithm that uses an integrated learning method consisting of numerous decision trees. It's able to handle large and complex datasets, including multiple types of variables, which is useful for analyzing the impact of various factors in tennis matches. What is more, it can provide an effective method for assessing the importance of individual features in predicting outcomes, which allows to identify which factors have the greatest impact on player momentum and match results.**[4]**

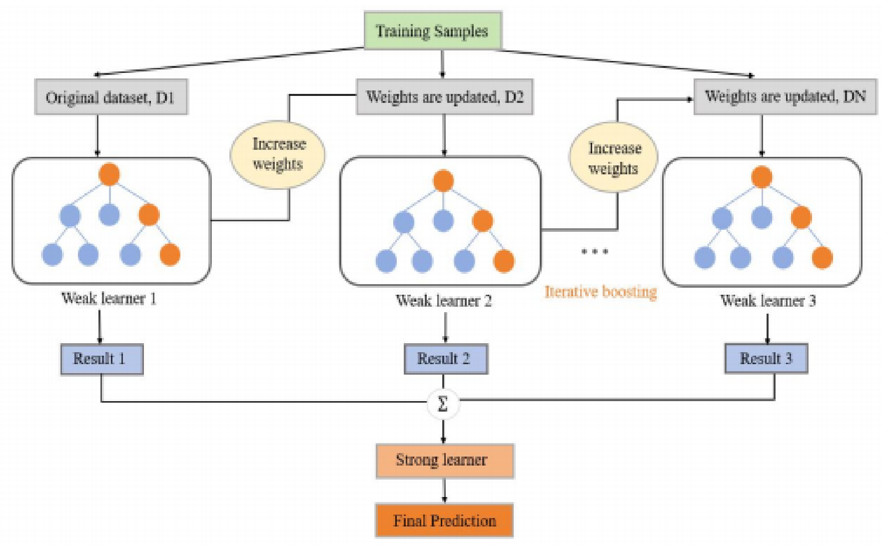


Figure 4: Concept of Random Forest

Therefore we try to describe the method by means of detail principles of the Random Forest model, including how it handles large numbers of input variables and evaluates the importance of features. We can explain the source of the dataset, the variables chosen (e.g., score, serve success rate, unforced errors, etc.), and how the data were prepared for use in the analysis accordingly.
We focus on the characteristic importance scores of each factor's impact on the outcome of the match. Specially, we weight the "serve" artificially low.
The results of the Random Forest Model analysis are in the following graphs.

Table 2: **Feature Importance Ranking in Random Forest Model**

Order	Features	Weights
1	break_pt	0.275942
2	point_victor	0.168837
3	unf_err	0.139420
4	winner	0.108685
5	ace	0.095252
6	double_fault	0.077130
7	net_pt	0.074068
8	net_pt_won	0.039238
9	break_pt_missed	0.021428

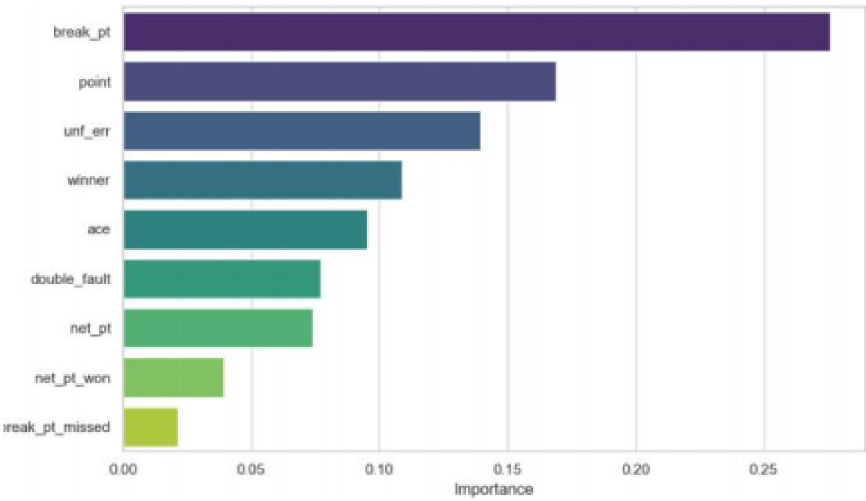
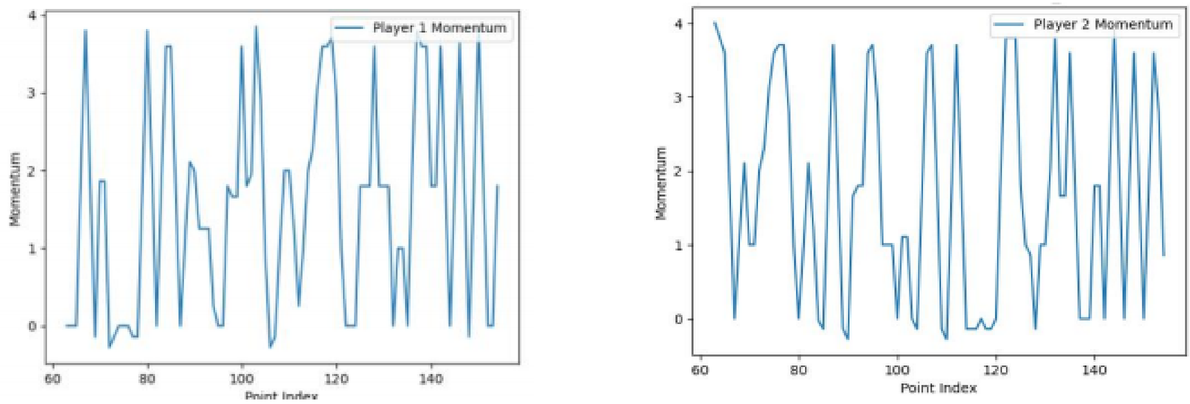


Figure 5: Feature Importance in Random Forest Model

According to the chart presented above, we define the following weights:

$$\begin{aligned} W = (&0.275942, 0.8 \times 0.168837, -0.139420, 0.108685, \\ &-0.095252, 0.077130, 0.074068, 0.074068, -0.021428) \end{aligned} \tag{1}$$

$$M = \sum_{n=1}^{9} W_i \times \alpha_i \tag{2}$$



(a) Player 1's Momentum (b) Player 2's Momentum

Figure 6: Player 1/2's Momentum in this set

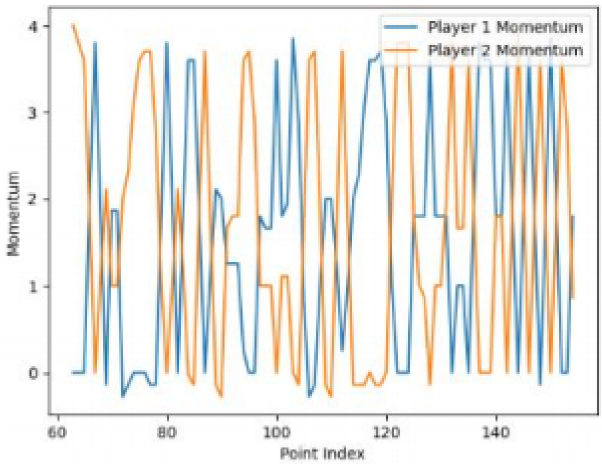


Figure 7: Player 1's vs Player 2's Momentum Comparison in the set_no

This picture above reflects the fact that the momentum values of the two players are in a state of waxing and waning, which basically corresponds with players' actual state in a real game.**3**

This picture above shows that in which time period which athlete performs better. For example, the red spot in the diagram indicates that at the 78-minute mark of the opening, Player 1's momentum minus player 2's momentum is negative, which represents clearly that at this time, player 2 performs better than player 1.

5.2 Assessment Based on Logistic Regression

5.2.1 Assessment Preparation

According to the requirements of the problem, this paper needs to assess that "momentum" actually plays an role in a match and swings in play and runs of success by one player are not random but correlative. We choose **Logistic Regression in classification algorithms** to train the model, then we can obtain the training results with higher predicted value. Finally we are able to visualize the description based on the model results to prove the crucial role "momentum" acts in the match and its high relevancy.

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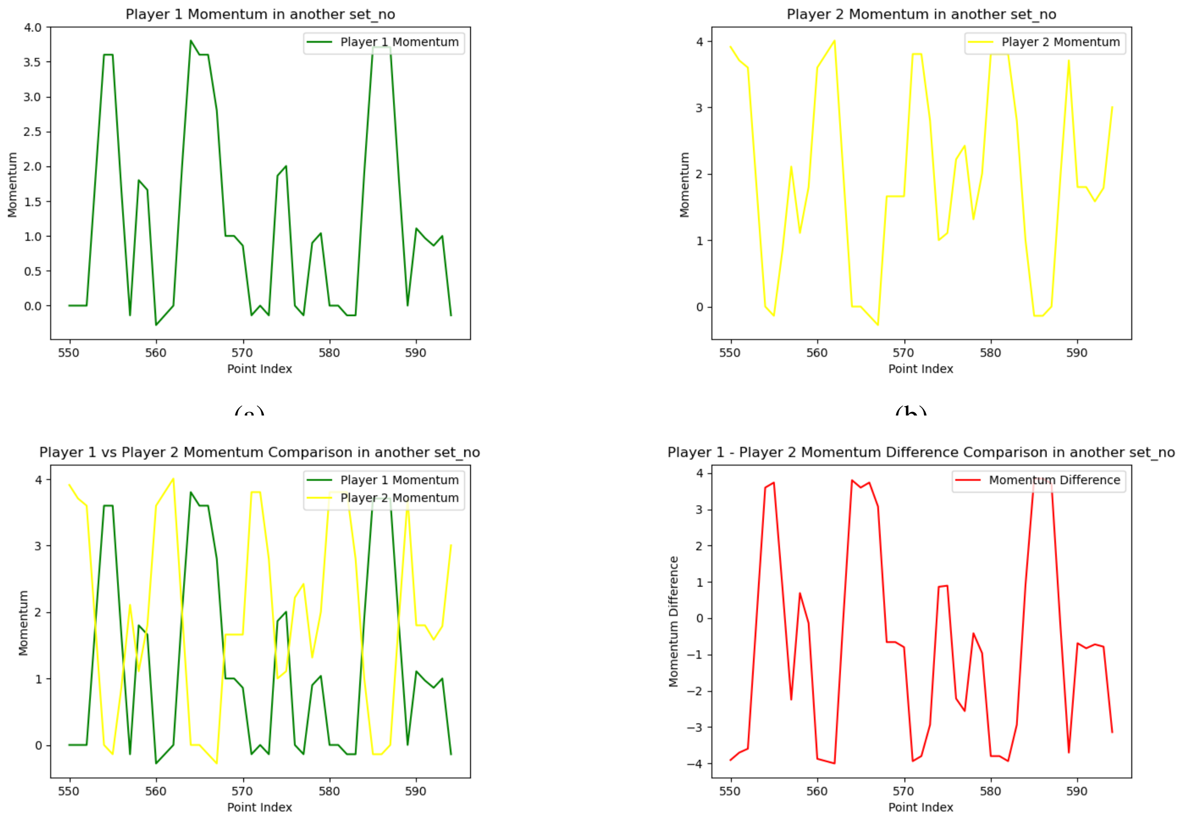
Figure 8: Player 1-Player's Momentum Difference Comparison in this set_no



(c) (d)

Figure 9: Model Application Diagram

Through the weight we determine in the previous question, we are able to derive two chart with its horizontal axis coordinates are the number of points scored per game and the vertical axis coordinates are the momentum values for each player. Then we integrate the momentum value folds in the two charts separately.

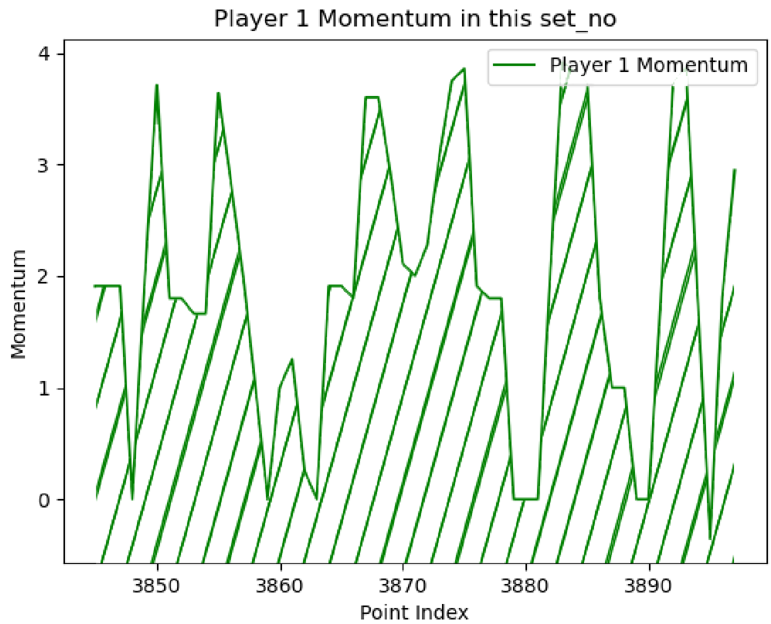


Figure 10: Player 1's Momentum in this set_no

Accordingly, we can acquire the total momentum value for each player.

By analogy with the definition "work" in physics, we figure that momentum can also accumulate over time, so we define three values:S_{P1},S_{P2},X_i

In the same way, we accordingly integral to get the cumulative value of the momentum of the two athletes in the current match, so we build this model:

$$X_i = \frac{S_{p1i}}{S_{p1i} + S_{p2i}} \tag{3}$$

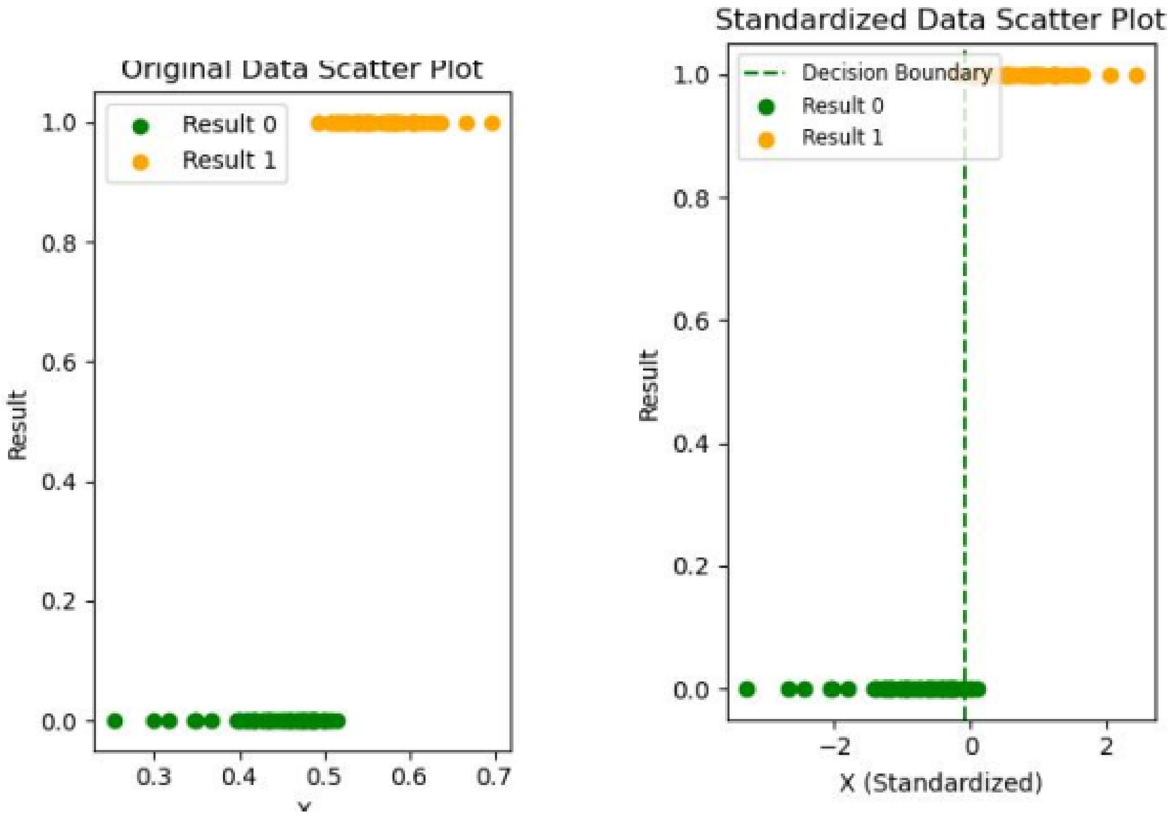
Since there are many rounds in the competition, the momentum performance of the athletes will vary accordingly, so there will be multiple X_i,$Y_i$represents the results of the set, we stipulate that :

$$Y_i = 0, P_1 \text{ loses the set} \tag{4}$$

$$Y_i = 1, P_1 \text{ wins the set} \tag{5}$$

So we can capture series of dataset(X_i,Y_i)

Through the original data scatter plot, we notice that the distribution of $X_i$is quite compact, so we choose to standardize $X_i$to make it more discrete, as it shown in the following diagram:



(a) Player 1's Momentum (b) Player 2's Momentum

Figure 11: Two Types of Data Scatter

We then use **Logistic Regression**:

$$\vec{w} \leftarrow \vec{w} + \eta \sum_{i=1}^{n} \frac{y_i \vec{x}_i}{1 + exp(y_i \vec{w} \cdot \vec{x}_i)} \tag{6}$$

We apply the above function to approximate the optimal value. Then we take 80% of the dataset as training set, the rest 20% of the dataset as test set.**[5]** After the training, we can derive the outcome. The result of the training can be shown in the following figure:

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Figure 12: Logistic Regression Decision Boundary

We are able to observe that the predictions of the remaining test set using this method have an **rather high accuracy which even reach 90% or more**.
The visualization of the accuracy rate can be seen as follows:

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Figure 13: ROC Curve

The ROC Curve demonstrate that the momentum of the players in the match do show strong correlation. The blue diagonal line represents the random situation, and the orange laddered straight line represents the normal situation of the swings in play and runs of success by one player. Differences are distinct and convincing. **Therefore, swings in play and runs of success by one player are not random but correlative.**

5.3 Continuous Performance Evaluating Model Based on Sliding Window

5.3.1 Model Preparation

According to the requirements of the problem, this paper needs to build another model to analyze the swings and performance of the player in one match. Given that the hypothesis of our assumption is no longer applicable, we have to reconsider the features to make our new model much more practical. Inspired by the **Sliding Window Algorithm**, combining with the new features or factors and new weights, we successfully construct a model and illustrate it visually. Therefore we are able to predict the swings and the flow of the match, and make preparations for providing an useful competition suggestion.

The previous model will not be able to solve the current problem because we did not consider the impact of the previous game's results on the current game. Therefore, we remove the previous assumption, and incorporate the performance and results of the previous game into the impact of the momentum of the current game, and redefine the new indicators with corresponding new weights to measure momentum. The redefined indicators are as follows:

$$A_r = \frac{Sum(ace)}{Ts} \tag{7}$$

$$F_r = \frac{Sum(double_fault)}{Ts} \tag{8}$$

$$B_r = \frac{Sum(break_pt_won)}{Sum(break_pt)} \tag{9}$$

$$B_{fr} = \frac{Sum(break_pt_missed)}{Sum(break_pt)} \tag{10}$$

$$N_r = \frac{Sum(net_pt_won)}{Sum(net_pt)} \tag{11}$$

$$E_r = \frac{Sum(p2_winner)}{Sum(unf_err)} \tag{12}$$

In the previous question, we only considered the single-inning case, which only involves the accumulation of the number of points scored by a simple player as a quantitative indicator of momentum; whereas now we need to **consider the quantification of momentum within a scoring interval, i.e., reflected by an intuitive ratio.** For example,A_r refers to Service Score Rate, which consists of the ratio between the number of points served to the total number of serves made in the set. Therefore, we can define other ratios such as F_r,B_r, B_{fr},N_r,E_r that presented in the notation table.

In addition, it's worth noting that the formation of E_r is different. E_r is composed of the ratio of forced errors to unforced errors. We make a conversion to better illustrate the formula. Here, forced errors refer to the total sum of errors caused by a player failed to receive an opponent's untouchable shot, i.e., as a numerator, forced errors are represented by the total sum of the other player's winners in the match.

After we define the new indicators, we apply the Random Forest algorithm to find the new feature weights of the above feature indicators. The results are presented as follows:

Table 3: **Feature Importance Ranking in Random Forest Model**

Order	Features	Weights
1	break_fail_ratio	0.247522
2	point_victor	0.168837
3	unf_err	0.139420
4	winner	0.108685
5	ace	0.095252
6	double_fault	0.077130
7	net_pt	0.074068
8	net_pt_won	0.039238
9	break_pt_missed	0.021428

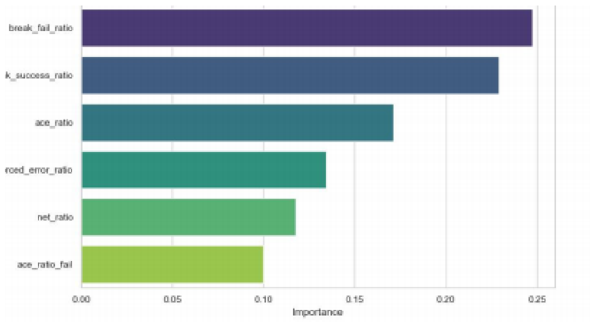


Figure 14: Feature Importance in Random Forest Model

According to the chart presented above, we define the following weights:

$$\begin{aligned} W = (&-0.247422, 0.229059, 0.139077, 0.171377, \\ &-0.134501, 0.117746, -0.099895) \end{aligned} \tag{13}$$

$$M = \sum_{i=1} 6W_i \times \beta_i \tag{14}$$

From this, we can further analyze the conclusions we visualized: we can see from the ranking of the weights of the feature indicators that the breakout failure rate, who has the greatest impact on quantifying the momentum indicators, turns out to be the most related factor among all; While the relative breakout success rate also should not be underestimated in terms of its impact on momentum quantification. It's in line with the reality that **an athlete's success in blocking a powerful attack against an opponent has a huge impact on his morale in subsequent matches**. What is more, the weaker impact of the net scoring rate indicator on momentum quantification is due to the fact that players feel it risky when hitting the net, and it is more difficult to be able to receive a baseline ball from an opponent, **so players mostly do not incline to stand in a more forward position to hit the ball**.

Such analysis is more consistent with reality, and measurement over time can further reflect trends in momentum over time in preparation for subsequent modeling.

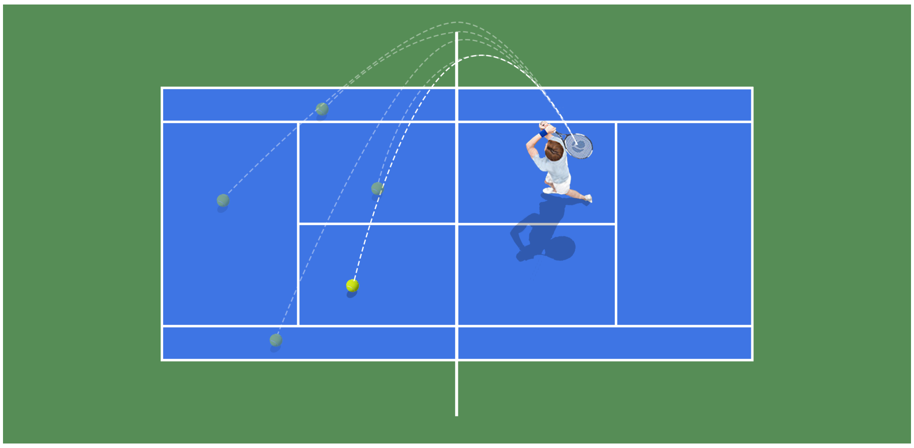


Figure 15: sketch of net scoring

5.3.2 Sliding Window Modeling

Based on our derived new momentum impact indicator weights, we use the **Sliding Window Algorithm** to build a new quantitative model of momentum. The Sliding Window Algorithm is a common technique used in data processing and statistical analysis to analyze or modify a series of data points in a sequential manner. The algorithm processes data incrementally by sliding a fixed- size "window" over the data set, performing a specific operation or computation at each window position. The size of the moving window and the step size of the slide can be adjusted according to the specific application scenario.

With the help of the algorithm of sliding window, we can further explain the suitability of this concept to this model. For the previous problem, we just picked a window to quantitatively analyze the magnitude of the momentum in a discrete way, whereas now we are able to observe the trend change of the momentum with the sliding window change.

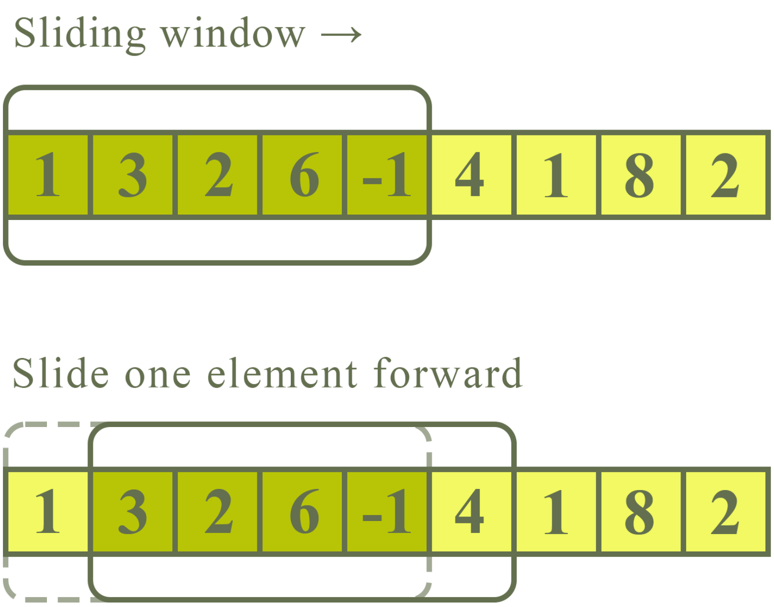


Figure 16: Sketch of Sliding Window Algorithm

We can briefly describe the process of the sliding window algorithm.

- **We start by initializing the parameters:**
- Window size:W:10 (consider the last 10 scoring points)

  We notice that determining the sliding window size in units of 10 helps to fit the actual situation. It is possible to accurately measure the impact of the athlete's past performance on the present, but it is also possible to effectively control the size of the window to avoid the transitional impact of the past situation on the present, and to imply that the athlete will be positively adjusting his mindset for the following competitions.

  Starting value of attenuation factor D_{start}:10

  attenuation step D_{step}:0.1

- **For each score point, we perform the following steps:**

a. **Determine the sliding window**: Select a sliding window of W score points counting forward from the current score point i. If the current score point i is less than the previous score point, then take as many score points as possible. Take as many score points as possible if the previous score points are less than W.

b. **Calculating the attenuation weights:**For each score point j in the window, we calculate its decay weight D_j.D_j refers to the magnitude of the momentum influencing factors decrease as the interval to the target time increases, also, the decreasing trend demonstrates a linear pattern, which in accordance with D_{step}. This notation is also in consistent with the actuality of what we feel in real life. The further back in time an event occurs, the less impact it will cause on the present. So the weight closest to the current score point is D_{step}, and for each score point moved forward, the weight decreases by D_{step}. At the same time, we guarantee that the minimum value of D_j is D_{step}.

$$D_j = D_{start} - (D_{step} \times (W - J)) \tag{15}$$

c. **Weighted feature calculation:**For each score within the window, we calculate the weighted value of each feature F_k and multiplied by the corresponding decay weight D_j,

$$F^{'}_{k,j} = F_{k,j} \times D_j \tag{16}$$

where$F^{'}_{k,j}$, is the weighted eigenvalue and $F_{k,j}$ is the original eigenvalue.

d. **Calculation of momentum indicators:** For each feature, the weighted eigenvalues of all the scores in the window are summed to obtain a momentum indicator for that feature M_k:

$$M_k = \sum_{j=1}^{W} F^{'}_{k,j} \tag{17}$$

Then, based on the feature importance weightsM_k, we calculate the weighted Momentum Indicator:

$$M = \sum_{k=1}^{n} M_k \times M_k \tag{18}$$

- **Momentum Indicator Updates and Model Training:**We train the model using the momentum indicator M calculated in the above steps as one of the input features of the model. Additionally, we also adjust the sliding window size W, the decay factors D_{start}

and D_{step} to optimize the model performance according to the model training results and performance evaluation needs.

In conclusion, this algorithmic process utilizes the concept of time decay to give greater weight to the most recent scoring points, allowing for more sensitive capture of immediate changes in game momentum. With this approach, a dynamic predictive model can be constructed that reflects changes in game momentum. Results are presented as follows:

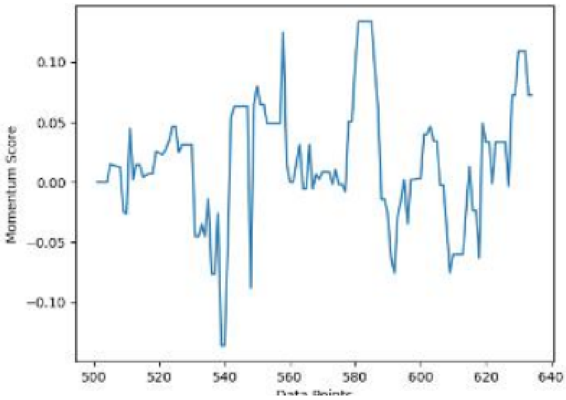


Figure 17: Momentum Score Over Time for 2023-wimbledon-1303

According to the requirements of Problem 3, we need to prove the validity of the prediction results of the new model we built to quantify momentum. We continue to use Logistic Regression for predictive testing of the model's accuracy.

We note that unlike problem one, when we quantify momentum by directly using the characteristic elements, we can observe the result that most of the compared momentum value integrals are positive, which implies that the previously built model may amplify the good performance of a particular athlete while relatively ignoring the worse performance.

When we quantify the momentum values in Problem 3 by using the newly defined values of the various ratios, we find that most of the compared momentum value integrals are negative, and this will have a negative effect on our measurement of momentum. To avoid this problem, we input S_{p1i},S_{p2i} separately instead of input X_i as a ratio.

Next, we input (S_{p1i},S_{p2i},Y_i) as a sample set and train it, the results are presented in the following graph:

Based on the above process, we can illustrate the validity of the model with full affirmation of the correctness of the model's predictions.

5.3.3 Identifying Turning Points in Swings

Next, we are going to predict the swings in the match. Our research object shifts from predictions on the outcome of the match to factors affecting momentum swings, in other words, our research perspective transferred from a holistic grasp to a partly and refined analysis of the subject.

We continue to use the ratios from problem 1 and apply Random Forests to recreate a model. Firstly, we redefine Y_i. Different from the previous one, $Y_i$presented here refers to the turning point in the trend fluctuation curve of the athlete's momentum value, where a value of 0 means that the athlete's momentum has no turning point at this scoring point, and 1 means that the athlete's momentum has turned at this scoring point. And X_i is defined as before.

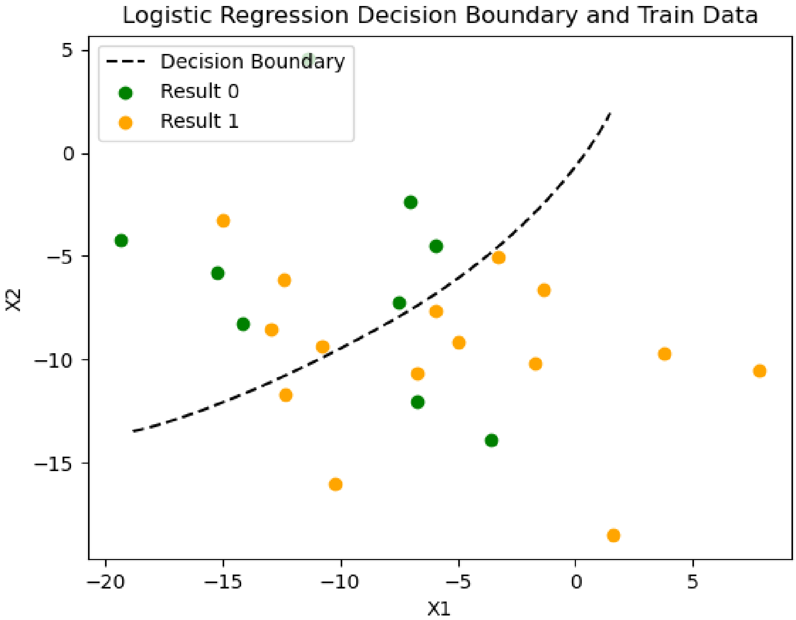


Figure 18: Logistic Regression Decision Boundary

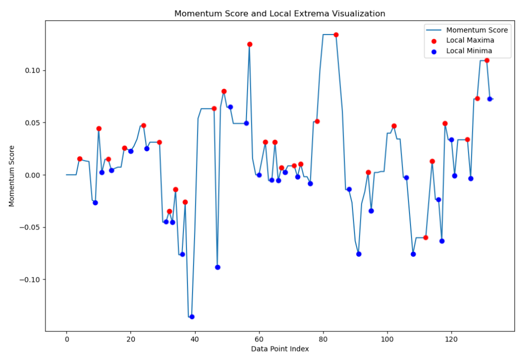


Figure 19: Momentum Score and Local Extreme Visualization

Then we determine the tendency of momentum and find the sequence of Y_i. We first locate all the extreme points in the graph of the momentum value. We design a program to process the curve, from which we found all the turning points in the curve and obtain the sequence of Y_i, which is Y values: [0.0, 0.0, 1.0,1.0. . .]. Next we utilize sliding window algorithm to acquire the sequence of X_i.In this way, we are able to produce the dataset $(X_i,Y_i)$and put it into the random forest to generate a new set of weights for swings in momentum.

Results are listed in the diagram as follows:

Table 4: **Feature Importance Ranking in Random Forest Model**

Order	Features	Weights
1	break_fail_ratio	0.247522
2	point_victor	0.168837
3	unf_err	0.139420
4	winner	0.108685
5	ace	0.095252
6	double_fault	0.077130
7	net_pt	0.074068
8	net_pt_won	0.039238
9	break_pt_missed	0.021428

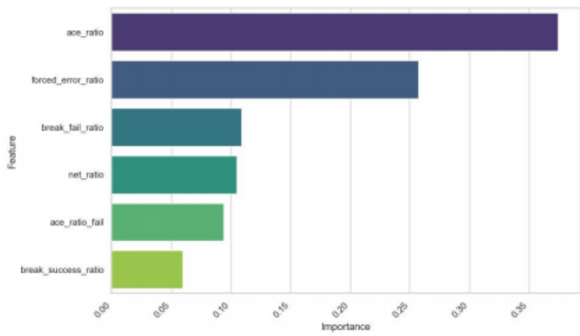


Figure 20: Factors Influencing Momentum Swings

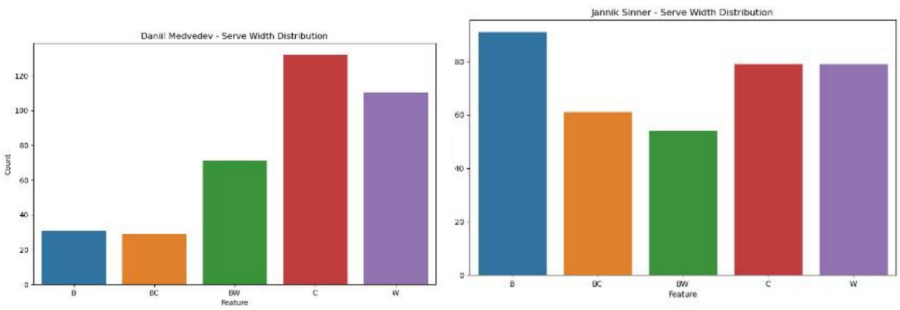
From the data in the graph, we can conclude that an unstoppable serve as the factor A_r , which refers to service score rate, with the largest weighting, i.e., as the element most associated with fluctuations in momentum changes, will have the potential to influence or even determine key changes or processes in a match. In the contrast, the factor B_r , which refers to break success rate, has a lower weight value, meaning that the athlete's momentum does not fluctuate greatly after winning a game. The reason to explain that result is that while B_r is important for overall match results, switching points to see that breaks don't matter in the middle. Another interesting finding is that the factor Fr, referring to the service failure rate, turns out to be insignificant in momentum fluctuation. This may be due to the fact that the athletes are prone to maintain a better state of mind and believe in their own strengths in order to prepare themselves for the score later on.

5.3.4 Advices Towards a Different Player

Based on the weights of the factors we derived above, we can more accurately determine the changes in momentum fluctuations of the athletes in the course of the game, try to analyze the reasons for the changes in momentum fluctuations of the athletes, and provide suggestions for the game performance of the athletes during the match.

- **A high level of self-confidence is a prerequisite for a victory:**As we can see from the results of the weighting, the percentage of serving points is in the first place of the weighting, which means that the confidence brought by successful serving points is the primary factor for athletes to keep their morale at a rather high level during the match. True self-confidence enables athletes to have a reasonable expectation of success, to actively and orderly deal with their game behavior, to think more quickly and rationally, and to have a clearer and more coordinated sense of technique, which lays the foundation for creating a good performance.

- **Competitive strength and thorough preparation are the foundation for a victory:**We can tell that the change of the error ratio to the momentum fluctuation is still large. A good athlete should train actively to minimize the errors during the game, improve the technical level and tactical adjustment ability, so that he or she has a strong competitive strength and full preparation. Collecting opponents' performance information during the season is also included in the preparations.



(a) (b)

Figure 21: Width of player's serve

- **Be cautious consistently throughout the whole match is indispensable for a victory:** We know from the weight of the breaks that an exceptional athlete will remain cautious even when he or she is scoring points, not letting down the guard against the opponent, but continuing to play steadily and looking for opportunities to break through the opponent's defenses.

- **Stress management also plays an important role for a victory:**The calculated weighting values indicate that the service failure rate also present a rather high value towards the swings of momentum. A brilliant athlete is supposed to perform even if they are faced with great pressure from opponents with fierce strength. It is necessary to face this pressure as a common feeling correctly and do well under pressure.

5.4 Model Testing and Expansion

According to the requirements of Problem 4, we want to further test the predictive power of the model built previously. Unlike the previous one, here we artificially divide the time 80% for the training set and 20% for the testing set, and the training results are shown as follows:

Ultimately, we arrive at a model with 71% predictive accuracy. It is rather high for predictive modeling. Therefore its reliability can also be confirmed.

Next, we analyze further to figure out the expansion of the model in other conditions. If the model performs poorly in certain situations, this may indicate that the model is missing some key predictors or that the model is unable to capture all of the complex dynamics that affect the

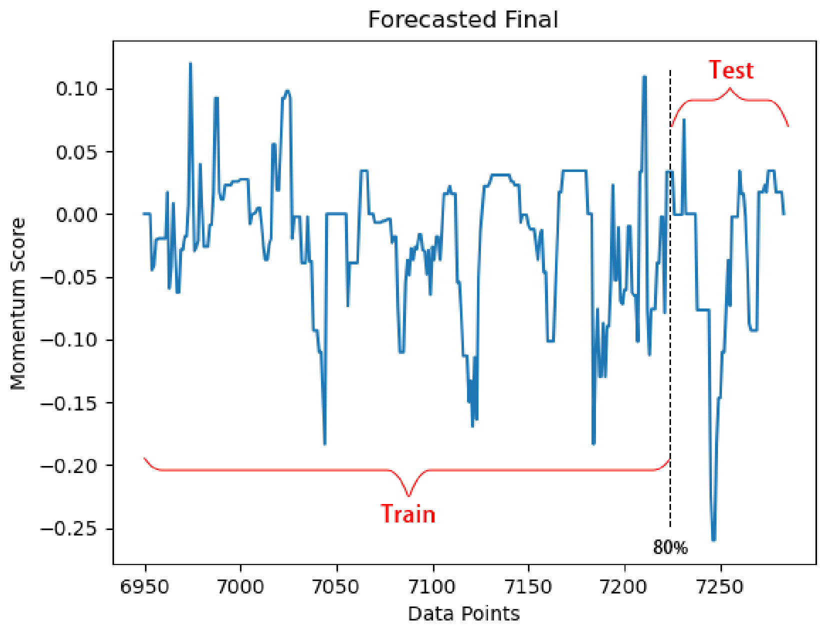


Figure 22: Sketch of Segment

momentum of the game. In response, we need to identify and supplement the possible missing factors, and perform a series of more in-depth data analysis and adjustments to improve the model's predictive power and generalizability.

- **Player Status:**Consider the player's physical condition, injury history, and mental state.
- **Environmental factors:**e.g. weather conditions, location of the match, spectator support, etc.
- **Historical performance:**a player's past match record, especially against specific opponents.
- **Real-time data:**e.g. heart rate monitoring data during a match, which can provide information about a player's stress and fitness status.
- **Style of Play:**Players' playing styles and strategies and how they affect the momentum of the game.
- **Technical Statistics:**: more detailed technical statistics such as first serve success rate, net scoring rate, unforced errors, etc.
- **Stress coping:**a player's ability to manage stress at critical moments.
- **Dynamic mental states:**players' confidence and motivation levels during the game.

For women's tournaments, the model may need to analyze the data to identify the different characteristics of momentum changes in men's and women's tournaments. For tournaments, their level and type may affect player performance and the model needs to be able to accommodate these differences. For different court surfaces (hardcourt, grass, red clay) affecting ball bounce and speed of play, this needs to be captured in the model. For other sports, such as table tennis, the model needs to be adapted to match different scoring systems and pace of play.

Through the above process, we can identify key factors that may be missing from the current model and propose targeted improvements. At the same time, we can also assess the model's ability to generalize under different conditions and adjust the model accordingly to fit different playing environments and rules. This iterative process will help build stronger and more accurate predictive models, and ultimately provide more valuable strategic advice to coaches and players!

6 Sensitivity Analysis

In the sensitivity analysis of the momentum prediction model for the 2023 Wimbledon tennis tournament, we explored the impact of different sliding window sizes (8, 10, 15) on the model's accuracy and the importance of its features. The charts illustrate the variation in momentum scores over time for each of the three different window sizes, as well as the corresponding importance of feature scores.

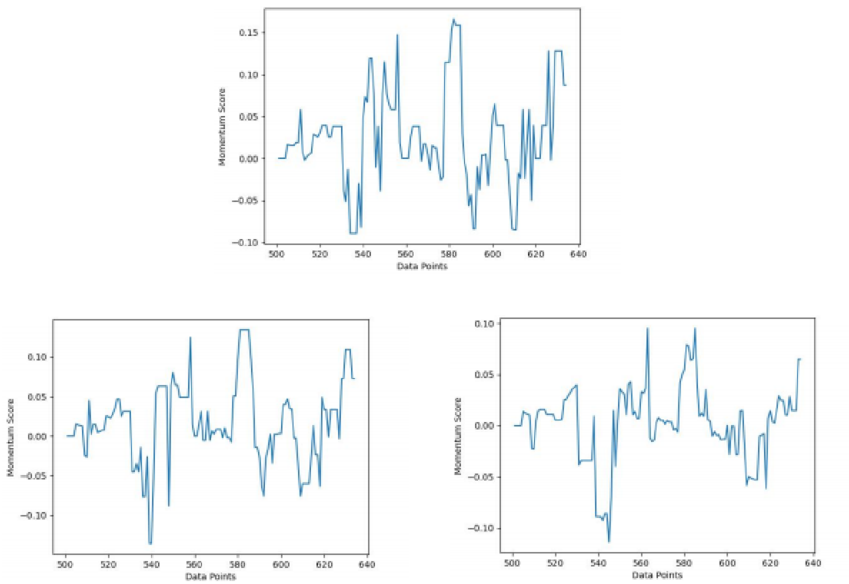


Figure 23: Momentum scores at different sizes

Table 5: **Feature Importance Ranking in Random Forest Model**

Order	Features	Weights
1	break_fail_ratio	0.247522
2	point_victor	0.168837
3	unf_err	0.139420
4	winner	0.108685
5	ace	0.095252
6	double_fault	0.077130
7	net_pt	0.074068
8	net_pt_won	0.039238
9	break_pt_missed	0.021428

Model Accuracy: 0.5925925925925926

Table 6: **Feature Importance Ranking in Random Forest Model**

Order	Features	Weights
1	break_fail_ratio	0.247522
2	point_victor	0.168837
3	unf_err	0.139420
4	winner	0.108685
5	ace	0.095252
6	double_fault	0.077130
7	net_pt	0.074068
8	net_pt_won	0.039238
9	break_pt_missed	0.021428

Model Accuracy: 0.7777777777777778

With a window size of 8, the model accuracy was 59.26%, suggesting that the model may not capture enough information within shorter time windows to accurately predict momentum shifts. The ranking of feature importance shows that the ace_ratio (ace scoring rate) has the highest weight of 0.353956, indicating that it is a key factor in influencing model predictions even within a shorter timeframe.

As the window size increases to 10, we observe a significant improvement in accuracy, and the distribution of feature importance shifts as well. At a window size of 15, the accuracy of the model slightly decreases to 70.37%, possibly because the longer time window includes too much information, diluting the impact of some key short-term changes.

Across all window sizes, the ace_ratio remains the most important feature, especially at a window size of 15, where its weight increases to 48.73%, further emphasizing the importance of the ace scoring rate in predicting momentum changes. The forced_error_ratio (forced error rate) maintains high importance across all window sizes, particularly at window sizes of 8 and 15, suggesting that forced errors are a consistent factor affecting the momentum shifts in matches.

When selecting a window size, it is necessary to balance changes in accuracy and feature importance to find the optimal window size that enhances the predictive performance of the model. As the charts suggest, a moderate window size (such as 10) appears to provide the best balance between accuracy and the importance of features.