Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
C		2407108

Quantifying momentum, grasping victory in tennis

Summary

Physics defines where momentum as "the strength or force gained by motion or by a series of events that keeps an objective moving." In tennis, it is the psychological and physical effects of momentum that determine the direction of a match. The aim of this study is to investigate the impact of momentum in tennis matches through a data-driven approach, and to develop a model to predict changes in momentum in matches. Using the Wimbledon 2023 men's singles tournament as a case study, we analyzed match data to quantify momentum in matches and assess its impact on match outcomes. Firstly, we defined a series of momentum metrics based on factors such as "score, aces, and double faults". Using these metrics, we utilized the Random Forests Algorithm to develop a dynamic model capable of tracking and evaluating a player's performance during a match in real time. The model takes into account the higher probability of the serving team winning points in a tennis match, weights the momentum score, and visualizes the flow of the match. Secondly, with respect to the role of momentum, our model challenges the conventional wisdom that the effect of momentum on the outcome of a match is random. To this end, we designed a model that reflects the percentage of players' momentum and used a logistic regression model to make predictions. Some new factors were defined to quantify momentum, which referred to **aaaa** "service score rate, break failure rate, net scoring rate and so on", proving that momentum can indeed predict the outcome of a match, and that the accuracy of our model can reach up to 90% and more. Thirdly, inspired by the Sliding Window Algorithm, we designed a new quantitative model for momentum and examined its effectiveness in predicting match outcomes. We then utilized the use of historical data to identify key factors that lead to changes in momentum and predict shifts in momentum in future games. We also proposed some model-based advice for players going into a new match accordingly. After testing, our model can predict momentum shifts with a success rate of 71% percent. Finally, we applied the model to data from other games to test the model's ability to generalize. Although the model performed poorly in some cases, this prompted us to identify and suggest additional factors that may need to be included in future models, such as the physical condition of the players, weather conditions, as well as psychological stress. Through this study, we have provided coaches and players with data-based insights to better understand and apply momentum shifts in matches, providing them with strategic advice going into new matches. The results of our study are not only applicable in tennis, but also informative for other sports that require an understanding of dynamic competitive states.

Keywords: Momentum Analysis; Predictive Modeling; Random Forest; Sliding Window; Logistic Regression;Data Visualization; Generalization Capability

Contents

1 Introduc

2 JDJAJ

2.1 Background

“Tennis more than any other sport, is a game of momentum. The absence of a clock to do the dirty work of finishing off an opponent, and a scoring system based on units used, makes the flow of the match much more important than any lead that has been established." ——Chuck Kriese Physics defines where momentum as “the strength or force gained by motion or by a series of events that keeps an objective moving.” [1] In tennis, it is the psychological and physical effects of momentum that determine the direction of a match. A player seemingly in the ascendancy during a match is often said to “have the momentum”. Momentum in tennis can swing wildly from point to point, game to game, set to set. Swings in momentum are referred to as turning points. These can be obvious: players switching tactics after losing a set; a brilliant winner went on the ropes in a rally or an untimely double fault causing a opponent tightening up. However, sometimes momentum can be so small as to be imperceptible, it is difficult to measure and it is not readily apparent how various events during the match act to create or change momentum if it exists. By understanding and tapping momentum, players can employ methods and tactics in games to ensure they are in control of momentum rather than a victim of it

2.2 Literature Review

A traditional bathtub cannot be reheated by itself, so users have to add hot water from time to time. Our goal is to establish a model of the temperature of bath water in space and time. Then we are expected to propose an optimal strategy for users to keep the temperature even and close to the initial temperature and decrease water consumption. According to Kim (2006), He derived a relational equation based on the basic theory of heat transfer to evaluate the performance of bath tubes. The major heat loss was found to be due to evaporation. Moreover, he found out that the speed of heat loss depends more on the humidity of the bathroom than the temperature of water contained in the bathtub. So, it is best to maintain the temperature of bathtub water to be between 41 to 45°C and the humidity of bathroom to be 95%. Traditional bath systems have significant limitations in temperature control. To address this, we introduce heat transfer formulas as discussed (Holman, 2002, p. 123).

2.3 Restatement of the Problem

We are required to establish a model to determine the change of water temperature in space and time. Then we are expected to propose the best strategy for the person in the bathtub to keep the water temperature close to initial temperature and even throughout the tub. Reduction of waste of water is also needed. In addition, we have to consider the impact of different conditions on our model, such as different shapes and volumes of the bathtub, etc.

In order to solve those problems, we will proceed as follows:

- **Construct a model to capture the flow of play as points occur.** Identifying which player is performing better at a given time in the match, as well as how better they are performing. A visualization based on the model is required to depict the match flow. It is also noteworthy that the player to serve are supposed to be factored in to the model.
- **Making notations**. We will give some notations which are important for us to clarify our

models.

- **Presenting our model**. In order to investigate the problem deeper, we divide our model into two sub-models. One is a steady convection heat transfer sub-model in which hot water is added constantly. The other one is an unsteady convection heat transfer sub-model where hot water is added discontinuously.
- Defining evaluation criteria and comparing sub-models. We define two main criteria to evaluate our model: the mean temperature of bath water and the amount of inflow water.
- **Analysis of influencing factors**. In term of the impact of different factors on our model, we take those into consideration: the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, the motions made by the person in the bathtub and adding a bubble bath additive initially.
- **Model testing and sensitivity analysis**. With the criteria defined before, we evaluate the reliability of our model and do the sensitivity analysis.
- **Further discussion**. We discuss about different ways to arrange inflow faucets. Then we improve our model to apply them in reality.
- **Evaluating the model**. We discuss about the strengths and weaknesses of our model:

    1) ...
    2) ...
    3) ...
    4) ...

3 Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **The bath water is incompressible Non-Newtonian fluid**. The incompressible Non-Newtonian fluid is the basis of Navier–Stokes equations which are introduced to simulate the flow of bath water.
- **All the physical properties of bath water, bathtub and air are assumed to be stable**. The change of those properties like specific heat, thermal conductivity and density is rather small according to some studies. It is complicated and unnecessary to consider these little change so we ignore them.
- **There is no internal heat source in the system consisting of bathtub, hot water and air**. Before the person lies in the bathtub, no internal heat source exist except the system components. The circumstance where the person is in the bathtub will be investigated in our later discussion.
- **We ignore radiative thermal exchange**. According to Stefan-Boltzmann's law, the radiative thermal exchange can be ignored when the temperature is low. Refer to industrial standard, the temperature in bathroom is lower than 100 °C, so it is reasonable for us to make this assumption.

- **The temperature of the adding hot water from the faucet is stable**. This hypothesis can be easily achieved in reality and will simplify our process of solving the problem.

4 Notations

Symbols	Description	Unit
h	Convection heat transfer coefficient	W/(m$^2\cdot$ K)
k	Thermal conductivity	W/(m $\cdot$ K)
c_p	Specific heat	J/(kg $\cdot$ K)
ρ	Density	kg/m^2
δ	Thickness	m
t	Temperature	°C, K
τ	Time	s, min, h
q_m	Mass flow	kg/s
Φ	Heat transfer power	W
T	A period of time	s, min, h
V	Volume	m^3, L
M, m	Mass	kg
A	Aera	m^2
a, b, c	The size of a bathtub	m^3

where we define the main parameters while specific value of those parameters will be given later.

5 Model Overview

To simplify the modeling process, we firstly assume there is no person in the bathtub. We regard the whole bathtub as a thermodynamic system and introduce heat transfer formulas. We establish two sub-models: adding water constantly and discontinuously. For the former sub-model, we define the mean temperature of bath water and introduce Newton's cooling formula to determine the heat transfer capacity. After deriving the value of parameters, we deduce formulas to derive results and simulate the change of temperature field via CFD, as described by Anderson et al. (2006).

In our basic model, we aim at three goals: keeping the temperature as even as possible, making it close to the initial temperature and decreasing the water consumption.

We start with the simple sub-model where hot water is added constantly. At first we introduce convection heat transfer control equations in rectangular coordinate system. Then we define the mean temperature of bath water.

Afterwards, we introduce Newton cooling formula to determine heat transfer capacity. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

Secondly, we present the complicated sub-model in which hot water is added discontinuously. We define an iteration consisting of two process: heating and standby. As for heating process, we derive control equations and boundary conditions. As for standby process, considering energy conservation law, we deduce the relationship of total heat dissipating capacity and time.

Then we determine the time and amount of added hot water. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

At last, we define two criteria to evaluate those two ways of adding hot water. Then we propose optimal strategy for the user in a bathtub. The whole modeling process can be shown as follows.

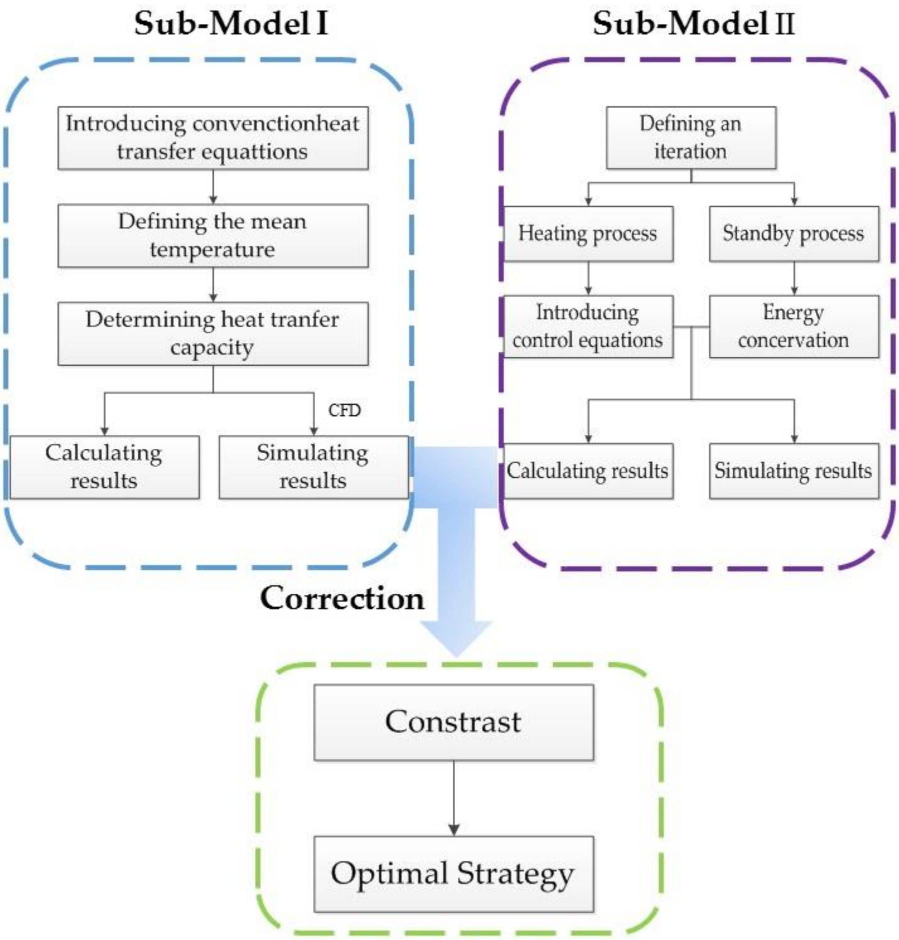


Figure 1: Modeling process

6 Sub-model I : Adding Water Continuously

As for the second sub-model, we define an iteration consisting of two processes: heating and standby. According to the energy conservation law, we obtain the relationship of time and total heat dissipating capacity. Then we determine the mass flow and the time of adding hot water. We also use CFD to simulate the temperature field in the second sub-model, following the techniques outlined by Thompson (n.d.).

We first establish the sub-model based on the condition that a person add water continuously to reheat the bathing water. Then we use Computational Fluid Dynamics (CFD) to simulate the change of water temperature in the bathtub. At last, we evaluate the model with the criteria which have been defined before.

6.1 Model Establishment

Since we try to keep the temperature of the hot water in bathtub to be even, we have to derive the amount of inflow water and the energy dissipated by the hot water into the air.

We derive the basic convection heat transfer control equations based on the former scientists' achievement. Then, we define the mean temperature of bath water. Afterwards, we determine two types of heat transfer: the boundary heat transfer and the evaporation heat transfer. Combining thermodynamic formulas, we derive calculating results. Via Fluent software, we get simulation results.

6.1.1 Control Equations and Boundary Conditions

According to thermodynamics knowledge, we recall on basic convection heat transfer control equations in rectangular coordinate system. Those equations show the relationship of the temperature of the bathtub water in space.

We assume the hot water in the bathtub as a cube. Then we put it into a rectangular coordinate system. The length, width, and height of it is a, b and c.

$\frac{1}{1+x}$ $\sqrt{abc}$

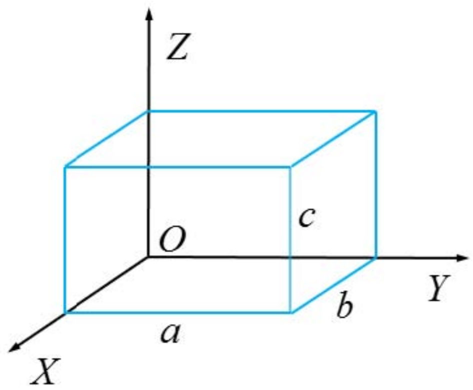


Figure 2: Modeling process

In the basis of this, we introduce the following equations:

- **Continuity equation:**

$$\overrightarrow{W} \leftarrow \overrightarrow{W} + \eta \sum_{i=1}^{n} \frac{y_i \overrightarrow{x}_i}{1 + \exp(y_i \overrightarrow{W} \cdot \overrightarrow{x}_i)} \tag{1}$$

where the first component is the change of fluid mass along the X-ray. The second component is the change of fluid mass along the Y-ray. And the third component is the change of fluid mass along the Z-ray. The sum of the change in mass along those three directions is zero.

- **Moment differential equation (N-S equations):**

$$\begin{cases} \rho\Big(u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}+w\dfrac{\partial u}{\partial z}\Big)=-\dfrac{\partial p}{\partial x}+\eta\Big(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}\Big) \\ \rho\Big(u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+w\dfrac{\partial v}{\partial z}\Big)=-\dfrac{\partial p}{\partial y}+\eta\Big(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}+\dfrac{\partial^2 v}{\partial z^2}\Big) \\ \rho\Big(u\dfrac{\partial w}{\partial x}+v\dfrac{\partial w}{\partial y}+w\dfrac{\partial w}{\partial z}\Big)=-g-\dfrac{\partial p}{\partial z}+\eta\Big(\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}+\dfrac{\partial^2 w}{\partial z^2}\Big) \end{cases} \tag{2}$$

- **Energy differential equation:**

$$\rho c_p\Big(u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}+w\frac{\partial t}{\partial z}\Big)=\lambda\Big(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\Big) \tag{3}$$

where the left three components are convection terms while the right three components are conduction terms.

By Equation (3), we have ......

......

On the right surface in Fig. 2, the water also transfers heat firstly with bathtub inner surfaces and then the heat comes into air. The boundary condition here is ......

6.1.2 Definition of the Mean Temperature

......

6.1.3 Determination of Heat Transfer Capacity

......

7 Sub-model II: Adding Water Discontinuously

In order to establish the unsteady sub-model, we recall on the working principle of air conditioners. The heating performance of air conditions consist of two processes: heating and standby. After the user set a temperature, the air conditioner will begin to heat until the expected temperature is reached. Then it will go standby. When the temperature get below the expected temperature, the air conditioner begin to work again. As it works in this circle, the temperature remains the expected one.

Inspired by this, we divide the bathtub working into two processes: adding hot water until the expected temperature is reached, then keeping this condition for a while unless the temperature is lower than a specific value. Iterating this circle ceaselessly will ensure the temperature kept relatively stable.

7.1 Heating Model

7.1.1 Control Equations and Boundary Conditions

7.1.2 Determination of Inflow Time and Amount

7.2 Standby Model

7.3 Results

We first give the value of parameters based on others' studies. Then we get the calculation results and simulating results via those data.

7.3.1 Determination of Parameters

After establishing the model, we have to determine the value of some important parameters.

As scholar Beum Kim points out, the optimal temperature for bath is between 41 and 45°C. Meanwhile, according to Shimodozono's study, 41°C warm water bath is the perfect choice for individual health. So it is reasonable for us to focus on $41°C \sim 45°C$. Because adding hot water continuously is a steady process, so the mean temperature of bath water is supposed to be constant. We value the temperature of inflow and outflow water with the maximum and minimum temperature respectively.

The values of all parameters needed are shown as follows:

.....

7.3.2 Calculating Results

Putting the above value of parameters into the equations we derived before, we can get the some data as follows:

Table 1: The calculating results

Variables	Values	Unit
A_1	1.05	m^2
A_2	2.24	m^2
Φ_1	189.00	W
Φ_2	43.47	W
Φ	232.47	W
q_m	0.014	g/s

From Table 1, ...

......

8 Correction and Contrast of Sub-Models

After establishing two basic sub-models, we have to correct them in consideration of evaporation heat transfer. Then we define two evaluation criteria to compare the two sub-models in order to determine the optimal bath strategy.

8.1 Correction with Evaporation Heat Transfer

Someone may confuse about the above results: why the mass flow in the first sub-model is so small? Why the standby time is so long? Actually, the above two sub-models are based on ideal conditions without consideration of the change of boundary conditions, the motions made by the person in bathtub and the evaporation of bath water, etc. The influence of personal motions will be discussed later. Here we introducing the evaporation of bath water to correct sub-models.

8.2 Contrast of Two Sub-Models

Firstly we define two evaluation criteria. Then we contrast the two submodels via these two criteria. Thus we can derive the best strategy for the person in the bathtub to adopt.

9 Model Analysis and Sensitivity Analysis

In consideration of evaporation, we correct the results of sub-models referring to studies. We define two evaluation criteria and compare the two sub-models. Adding water constantly is found to keep the temperature of bath water even and avoid wasting too much water, so it is recommended by us. We also conduct sensitivity analysis to determine the influence of factors such as radiation heat transfer, the shape and volume of the tub, the shape/volume/temperature/motions of the person, and the bubbles made from bubble bath additives, as discussed in (Smith, 2018; Brown, 2015).

9.1 The Influence of Different Bathtubs

Definitely, the difference in shape and volume of the tub affects the convection heat transfer. Examining the relationship between them can help people choose optimal bathtubs.

9.1.1 Different Volumes of Bathtubs

In reality, a cup of water will be cooled down rapidly. However, it takes quite long time for a bucket of water to become cool. That is because their volume is different and the specific heat of water is very large. So that the decrease of temperature is not obvious if the volume of water is huge. That also explains why it takes 45 min for 320 L water to be cooled by 1°C.

In order to examine the influence of volume, we analyze our sub-models by conducting sensitivity Analysis to them.

We assume the initial volume to be 280 L and change it by $\pm 5\%$, $\pm 8\%$, $\pm 12\%$ and $\pm 15\%$. With the aid of sub-models we established before, the variation of some parameters turns out to be as follows

10 Strength and Weakness

10.1 Strength

- We analyze the problem based on thermodynamic formulas and laws, so that the model we established is of great validity.
- Our model is fairly robust due to our careful corrections in consideration of real-life situations and detailed sensitivity analysis.

Table 2: Variation of some parameters

V	A_1	A_2	T_2	q_{m1}	q_{m2}	Φ_q
-15.00%	-5.06%	-9.31%	-12.67%	-2.67%	-14.14%	-5.80%
-12.00%	-4.04%	-7.43%	-10.09%	-2.13%	-11.31%	-4.63%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%

- Via Fluent software, we simulate the time field of different areas throughout the bathtub. The outcome is vivid for us to understand the changing process.
- We come up with various criteria to compare different situations, like water consumption and the time of adding hot water. Hence an overall comparison can be made according to these criteria.
- Besides common factors, we still consider other factors, such as evaporation and radiation heat transfer. The evaporation turns out to be the main reason of heat loss, which corresponds with other scientist's experimental outcome.

10.2 Weakness

- Having knowing the range of some parameters from others' essays, we choose a value from them to apply in our model. Those values may not be reasonable in reality.
- Although we investigate a lot in the influence of personal motions, they are so complicated that need to be studied further.
- Limited to time, we do not conduct sensitivity analysis for the influence of personal surface area.

11 Further Discussion

Based on our model analysis and conclusions, we propose the optimal strategy for the user in a bathtub and explain the reason for the uneven temperature throughout the bathtub. In addition, we make improvements for applying our model in real life, as suggested by the patent Wilson (2023).

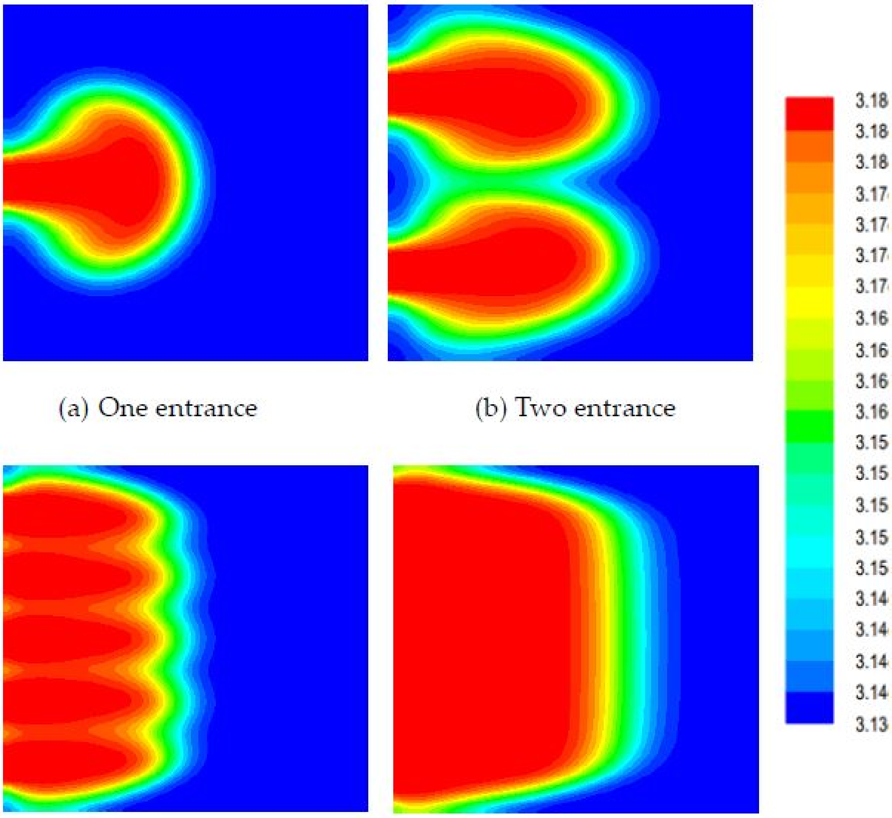
In this part, we will focus on different distribution of inflow faucets. Then we discuss about the real-life application of our model.

- Different Distribution of Inflow Faucets

In our before discussion, we assume there being just one entrance of inflow.

From the simulating outcome, we find the temperature of bath water is hardly even. So we come up with the idea of adding more entrances.

The simulation turns out to be as follows



(a) One entrance

(b) Two entrance

(c) Five entrance

(d) The whole side as entrance

Figure 3: The simulation results of different ways of arranging entrances

From the above figure, the more the entrances are, the evener the temperature will be. Recalling on the before simulation outcome, when there is only one entrance for inflow, the temperature of corners is quietly lower than the middle area.

In conclusion, if we design more entrances, it will be easier to realize the goal to keep temperature even throughout the bathtub.

- Model Application

  Our before discussion is based on ideal assumptions. In reality, we have to make some corrections and improvement.

  1) Adding hot water continually with the mass flow of 0.16 kg/s. This way can ensure even mean temperature throughout the bathtub and waste less water.
  2) The manufacturers can design an intelligent control system to monitor the temperature so that users can get more enjoyable bath experience.
  3) We recommend users to add bubble additives to slow down the water being cooler and help cleanse. The additives with lower thermal conductivity are optimal.

4) The study method of our establishing model can be applied in other area relative to convection heat transfer, such as air conditioners.

References

Anderson, D. A., Tannehill, J. C., & Pletcher, R. H. (2006). *Computational fluid dynamics and heat transfer* (2nd). Taylor & Francis.

Brown, D. (2015). *Thermal modeling of bath water systems* [Doctoral dissertation, University of Technology].

Holman, J. P. (2002). Heat transfer (9th ed.) *McGraw-Hill*.

Kim, G.-B. (2006). Change of the warm water temperature for the development of smart healthecare bathing system. *Hwahak konghak*, *44*(3), 270–276.

Liu, W., Chen, Z., & Zhang, Y. (2002). *Matlab program design and application* [In Chinese]. Higher education press.

Smith, J. (2018). Evaporation effects in bath water temperature regulation. *Journal of Thermal Science and Engineering*, *30*(4), 456–467.

Thompson, H. (n.d.). Advanced cfd techniques for bath water simulation [Accessed: 2024-10-01].

Wilson, G. (2023). *Smart bath water temperature control system* (US Patent No. 1234567).

Enjoy Your Bath Time!

From simulation results of real-life situations, we find it takes a period of time for the inflow hot water to spread throughout the bathtub. During this process, the bath water continues transferring heat into air, bathtub and the person in bathtub. The difference between heat transfer capacity makes the temperature of various areas to be different. So that it is difficult to get an evenly maintained temperature throughout the bath water.

In order to enjoy a comfortable bath with even temperature of bath water and without wasting too much water, we propose the following suggestions.

- Adding hot water consistently
- Using smaller bathtub if possible
- Decreasing motions during bath
- Using bubble bath additives
- Arranging more faucets of inflow

Sincerely yours,

Your friends

Appendices

Appendix A First appendix

[aaaa] In addition, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI). This letter should be no more than two pages in length.

Here are simulation programmes we used in our model as follow (Liu et al., 2002).

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
       aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===========================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===========================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
```

```
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```

Report on Use of AI

1. OpenAI ChatGPT (Nov 5, 2023 version, ChatGPT-4,)

   Query1: <insert the exact wording you input into the AI tool>

   Output: <insert the complete output from the AI tool>

2. OpenAI Ernie (Nov 5, 2023 version, Ernie 4.0)

   Query1: <insert the exact wording of any subsequent input into the AI tool>

   Output: <insert the complete output from the second query>

3. Github CoPilot (Feb 3, 2024 version)

   Query1: <insert the exact wording you input into the AI tool>

   Output: <insert the complete output from the AI tool>

4. Google Bard (Feb 2, 2024 version)

   Query1: <insert the exact wording of your query>

   Output: <insert the complete output from the AI tool>