$$\hat{\eta}_\pi(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a). \tag{1}$$

we can derive Eq.1 and Eq.2 by replacing $\tilde{\pi}$ with parameterized π_θ due to $\sum_s \rho_\pi(s) \sum_a \pi_\theta(a|s) A_\pi(s,a) = 0$.

Let $p(\theta) = \sigma(\tau(r_t(\theta) - 1))$,$r_t(\theta) = \pi_\theta(a_t|s_t)/\pi_{\theta_{old}}(a_t|s_t)$, then we have

$$\begin{aligned} \nabla_\theta p(\theta) &= \sigma(\tau(r_t(\theta) - 1))(1 - \sigma(\tau(r_t(\theta) - 1)))\tau & (2) \\ &= p(\theta)(1 - p(\theta))\tau & (3) \end{aligned}$$

Then use $\nabla_\theta p(\theta)$ to abbreviate $\nabla_\theta L^{sc}$, we derive

$$\begin{aligned} \nabla_\theta L^{sc} &:= \nabla_\theta E_{a \sim \pi_{\theta_{old}}}[p(\theta)\frac{4}{\tau}\hat{A}] & (4) \\ &= \nabla_\theta \int_{\pi_{\theta_{old}}} p(\theta)\frac{4}{\tau}\hat{A}a & (5) \\ &= \int_{\pi_{\theta_{old}}} \nabla_\theta p(\theta)\frac{4}{\tau}\hat{A}a & (6) \\ &= \int_{\pi_{\theta_{old}}} 4p(\theta)(1 - p(\theta)\nabla_\theta r_t(\theta)\hat{A}a & (7) \\ &= E_{\pi_{\theta_{old}}}[4p(\theta)(1 - p(\theta)\nabla_\theta r_t(\theta)\hat{A}] & (8) \end{aligned}$$