This Mathematica notebook contains the code to produce the supplemental figures for “Fertilization by coral-dwelling fish promotes coral growth but can exacerbate bleaching response” by A.R. Detmer, R. Cunning, F. Pfab, A.L. Brown, A. C. Stier, R.M. Nisbet, and H.V. Moeller.

Note that external irradiance is denoted with an L in the following code; this is equivalent to I in the main text

Define constant parameters (run this first)

```
In[∘]:= (* list of parameters and expressions that don't change,
run this before running any of the other code*)
(*values that were varied in the model simulations are commented out here*)
(*parameters from original Cunning et
 al. 2017 model (see Table 2 of Cunning et al. 2017)*)
nNH = .18;(*N:C molar ratio in host biomass*)
nNS = .13; (*N:C molar ratio in symbiont biomass*)
nNX = .2; (*N:C molar ratio in prey biomass*)
j0HT = .03;(*maintenance rate of host biomass*)
j0ST = .03;(*maintenance rate of symbiont biomass*)
σNH = .9;(*proportion of N turnover recycled in host*)
σCH = .1;(*proportion host metabolic CO2 recycled to photosynthesis*)
σNS = .9;(*proportion of N turnover recycled in symbiont*)
σCS = .9;(*proportion symbiont metabolic CO2 recycled to photosynthesis*)
jXm = .13;(*maximum prey assimilation rate from host feeding*)
KX = 10^-6;(*half-saturation constant for prey assimilation*)
jNm = .035; (*maximum host DIN uptake rate*)
KN = 1.5 * 10^-6; (* half-saturation constant for host DIN uptake*)
kCO2 = 10; (*efficacy of CO2 delivery to
 photosynthesis by host carbon concentrating mechanisms*)
jHGm = 1;(*maximum specific growth rate of host*)
yCL = .1; (*quantum yield of photosynthesis*)
yC = .8; (*yield of biomass formation from carbon*)
astar = 1.34; (*effective light-absorbing cross-section of symbiont*)
kNPQ = 112; (* non-photochemical quenching (NPQ) capacity of symbiont*)
kROS = 80; (* excess photon energy that
 doubles ROS production relative to baseline levels *)
jCPm = 2.8; (*maximum specific photosynthesis rate of symbiont*)
jSGm = 0.25;(*maximum specific growth rate of symbiont*)
b = 5;(*scaling parameter for bleaching response*)
(*X=0;*)(*concentration of prey in environment*)
(*N=1*10^-7;*)(* ambient concetration of nitrogen in external environment*)
L = 15; (* external irradiance, ATTENTION this is I in the main text*)
```

```
λ = 50; (* Pfab et al. (submitted)'s lambda parameter for simulating fluxes as
 ODEs (continuous time approximation method for simulating the system) *)

(* parameters for host volume*)
kv = 16.9; (* liters per C-mol H*)
vi = 0.7; (* fraction of coral volume occupied by water;
based on Zawada et al. P. damicornis data; see supplement for details *)

(* parameters for fish*)
kp = 210;(*scales fish carrying capacity with biomass of host,
calculated using kv=16.9 C-mol/L*)
rp = 0.05; (* intrinsic growth rate*)
Bp = 1; (* intraspecific competition coefficient*)
ep = 1.5 * 10^-5; (* damselfish excretion rate (total mols N per g per d);
calculated using data on D. flavicaudus*)

(* parameters for nitrogen equations*)
α = 0.23; (* proportion of coral tissue
 in direct contact with the external environment*)

d = 1660; (*flushing rate, units of 1/days*)
ξ = 1; (* fraction of waste N released
 by the symbiont that is bioavailable to the coral*)

(* derived parameters*)
```
(*jX=$\frac{\text{jXm X}}{\text{X+KX}}$;*) (* prey uptake from environment*)

(* jN=$\frac{\text{jNm N}}{\text{N+KN}}$;*) (* N uptake from environment*)

```
jHT = j0HT; (*rate of host biomass turnover*)
rNS = σNS nNS j0ST;(*nitrogen recycled from symbiont biomass turnover*)


(* initial conditions*)
H0 = 1; (*intial host biomass*)
S0 = 0.3; (*initial symbiont biomass*)
jCP0 = 1;
ρC0 = 1;
(*healthy state: jCP0 and ρC=jCPm or 1, bleached state: jCP0 and ρC=0 *)
VH0 = kv * H0; (*initial host volume*)
(*Ni0 = N;
```
jNi0 = $\frac{\text{jNm N}}{\text{N+KN}}$;*)

(*jNw0= $\frac{\text{jNm*N}}{\text{N+KN}}$*H0/S0;*)

```
(*P0=0;*) (*kp*(1-α)*H0*)
```

```
(*change P0 depending on whether or not fish are present*)


(*function to remove identifiers from simulations run in Module[]*)
str[expr_] := Module[{}, StringReplace[ToString[expr, FormatType → StandardForm],
    c : WordCharacter ~~ "$" ~~ DigitCharacter .. :→ c]];
```

Fig. S1: Bifurcation diagrams of waste N and fish excretions as a function of prey availability

Run simulations and format results

check that steady state is reached

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=
(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10^-7 (*1*10^-6*);
```

$$jN = \frac{jNm\,N}{N + KN};$$

```
Ni0 = N;
```

$$jNi0 = \frac{jNm\,N}{N + KN};$$

```
tmax = 800;

X2BifurRuns1 = Table[
   Table[
    Module[{
```

$jX = \frac{jXm\,X}{X + KX}$, (* $jN=\frac{jNm\ N}{N+KN}$,*) jHT = j0HT, rNS = σNS nNS j0ST,

VH0 = kv * H0, (* Ni0 = N,jNi0 = $\frac{jNm\ N}{N+KN}$,*) states, rNH, S, H, VH, P,

Ni, jNi, ρC, jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1,

```
 jSG, jST, jNw, F, fh, tsolve, addtimetostatevars, eqs, inis, sol, t,
 maketmax, steadyNi, steadyHGrowth, steadySH, steadyjNw, steadyep
 (*make sure to put any intermediate output values in Module*)},
```

$$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2 B + A B^2 + A^2 \rho + A B \rho + B^2 \rho};$$

(*same as $\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$ *)

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\,(\rho C\,S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH)\,nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\,yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH - nNH\,jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

$$jCO2 = jeC\,kCO2;$$

$$jL = A\,astar\,L\,;$$

$$rCH = \left(jHT + \frac{jHG\,(1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\left(j0ST + (1-yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\,cROS1)\,j0ST;$$

$$rNH = \sigma NH\,nNH\,jHT;$$

$$A = 1.256307 + 1.385969\,e^{-6.479055\,S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \Big\{$$

$$\left\{\rho C,\ jCP - jSG\,yC^{-1},\ \rho C0\right\},$$

$$\left\{jCP,\ \frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$

$$\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$

```
 };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
    Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

(*change in Ni over the last 50 days of the simulation*)
steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
    Evaluate[Ni[tmax - 50] /. sol]];

(*change in host growth*)
steadyHGrowth =
  Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
          sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]];

(* change in S/H*)
steadySH =
  Abs[(Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /.
          sol]) / Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];
```

```
        {steadyNi, steadyHGrowth, steadySH}
        (*record the changes in Ni, H growth, and S/H values as output*)],

      {P0, {0, kp * (1 - α) * H0}}], (*iterate the above
      once with no fish (P0=0) and once with fish (P0=kp*(1-α)*H0)*)
     {X, {0, 0.1 * 10^-7, 0.25 * 10^-7, 0.5 * 10^-7, 0.75 * 10^-7, 1 * 10^-7, 1.25 * 10^-7,
       1.5 * 10^-7, 1.75 * 10^-7, 2 * 10^-7}}(*different levels of prey*)
    ];
```

```
In[]:= Select[Flatten[X2BifurRuns1], # > 1 * 10^-4 &] (*nothing is greater than 10^-4*)
Out[]= {}
```

Run the simulations

```
In[]:= (*clear parameters that are changing and intermediate values*)
       ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
        jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
        tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[]:=
       (* environment: flushing rate and N level*)
       d = 1660;
       N = 1 * 10^-7 (*1*10^-6*);
```

$$jN = \frac{jNm\, N}{N + KN};$$

```
       Ni0 = N;
```

$$jNi0 = \frac{jNm\, N}{N + KN};$$

```
       tmax = 800;


       X2BifurRuns = Table[
          Table[
           Module[{
```

$$jX = \frac{jXm\, X}{X + KX}, \text{ (* } jN=\frac{jNm\ N}{N+KN}\text{,*)} jHT = j0HT,\ rNS = \sigma NS\ nNS\ j0ST,$$

$$VH0 = kv * H0,\ \text{(* } Ni0 = N, jNi0 = \frac{jNm\ N}{N+KN}\text{,*)} \text{states},\ rNH,\ S,\ H,\ VH,\ P,$$

```
             Ni, jNi, ρC, jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1,
 jSG, jST, jNw, F, fh, tsolve, addtimetostatevars, eqs, inis, sol, t,
 maketmax, steadyNi, steadyHGrowth, steadySH, steadyjNw, steadyep
 (*make sure to put any intermediate output values in Module*)},
```

$$F[\rho\_][A\_, B\_] := \frac{A\ B\ (A+B)\ \rho}{A^2\ B + A\ B^2 + A^2\ \rho + A\ B\ \rho + B^2\ \rho};$$

$$(*\text{same as } \frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}} *)$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\ (\rho C\ S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH)\ nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\ yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH - nNH\ jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\ \rho C}{H},\ 0\right];$$

$$jCO2 = jeC\ kCO2;$$

$$jL = A\ astar\ L;$$

$$rCH = \left(jHT + \frac{jHG\ (1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\ \left(j0ST + (1-yC)\ jSG\ yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\ cROS1)\ j0ST;$$

$$rNH = \sigma NH\ nNH\ jHT;$$

$$A = 1.256307 + 1.385969\ e^{-6.479055\ S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \{$$

$$\{\rho C,\ jCP - jSG\ yC^{-1},\ \rho C0\},$$

$$\left\{jCP,\ \frac{F[jCPm]\left[jL\ yCL,\ rCS + \frac{H\ (jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$

$$\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$

```
 };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧ '[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S '[t] == (S (jSG - jST) /. addtimetostatevars),
    H '[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH '[t] == fh[t, H '[t]],
    Ni '[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P '[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
 ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
    Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

(*steady state Ni*)
steadyNi = Evaluate[Ni[tmax] /. sol];

(*steady state host growth*)
steadyHGrowth = Evaluate[H '[tmax] / H[tmax] /. sol];

(*steady state S/H*)
steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

(*symbiont waste nitrogen*)
(*need to add tmax to the state variables involved in the calculation
 of jNw (S and VH need to be changed to S[t] and VH[t])*)
maketmax = ((# → #[tmax]) & /@ states);
steadyjNw =
  Evaluate@Flatten@{(ξ * (1 - α) * jNw * S) / ( vi * VH) /. maketmax /. sol};
```

```
      (*fish excretion*)
      steadyep = Evaluate[{(ep * P[tmax]) / (vi * VH[tmax])} /. sol];

      {steadyNi, steadyHGrowth, steadySH, steadyjNw, steadyep}
      (*the steady state values as output*)],

     {P0, {0, kp * (1 - α) * H0}}], (*iterate the above
     once with no fish (P0=0) and once with fish (P0=kp*(1-α)*H0)*)
    {X, {0, 0.1 * 10^-7, 0.25 * 10^-7, 0.5 * 10^-7, 0.75 * 10^-7, 1 * 10^-7, 1.25 * 10^-7,
      1.5 * 10^-7, 1.75 * 10^-7, 2 * 10^-7}}(*different levels of prey*)
   ];
```

In[ ]:=
```
(*storing these simulations so Mathematica
 saves them when this notebook is closed*)
Save["SXBifur", X2BifurRuns]
```

In[ ]:=
```
(*reload the runs: need to do this each time this notebook gets re-opened*)
Get["SXBifur"];
```

Get the results in the correct form to plot

In[ ]:=
```
(*getting the lists to plot
 (have 2 (fish/no fish) for each of the 5 outputs, so 10 lists total*)

(*transpose this to change the nested list from
 {{P1 at X1, P2 at X1}, {X2}, ...{Xn}} to {{X1, X2, ... Xn at P1}, {P2}} *)
XseparatePX = Transpose[X2BifurRuns];
(*prey no fish*)
XBifurNoPX = XseparatePX[[1]];
separateXNoPX = Transpose[XBifurNoPX];(*group each output together*)
XBifurNoPXNi = separateXNoPX[[1]];(*Ni values at each prey level for no fish*)
XBifurNoPXgrowth = separateXNoPX[[2]];
(*dH/Hdt values at each prey level for no fish*)
XBifurNoPXSH = separateXNoPX[[3]];(*S/H values at each prey level for no fish *)
(*symbiont excretion values at each prey level for no fish,
need to flatten out the list of values*)
XBifurNoPXjNw = Flatten[separateXNoPX[[4]]];
(*fish excretion values at each prey level for no fish,
need to flatten out the list of values*)
XBifurNoPXep = Flatten[separateXNoPX[[5]]];

(* prey with fish*)
```

```
XBifurPX = XseparatePX⟦2⟧;
separateXPX = Transpose[XBifurPX];(*group each output together*)
XBifurPXNi = separateXPX⟦1⟧;(*Ni values at each prey level with fish*)
XBifurPXgrowth = separateXPX⟦2⟧;(*dH/Hdt values at each prey level with fish*)
XBifurPXSH = separateXPX⟦3⟧;(*S/H values at each prey level with fish*)
(*symbiont excretion values at each prey level with fish*)
XBifurPXjNw = Flatten[separateXPX⟦4⟧];
(*fish excretion values at each prey level with fish*)
XBifurPXep = Flatten[separateXPX⟦5⟧];

(*list of X values*)
XVals = {0, 0.1 * 10 ^ -7, 0.25 * 10 ^ -7, 0.5 * 10 ^ -7, 0.75 * 10 ^ -7,
   1 * 10 ^ -7, 1.25 * 10 ^ -7, 1.5 * 10 ^ -7, 1.75 * 10 ^ -7, 2 * 10 ^ -7};
(*to add Nenv to the plots*)
N = 1 * 10 ^ -7;
XNenvVal = ConstantArray[N, Length[XVals]];
XBifurNenvLine = Transpose@{XVals, XNenvVal};

(*make list of the no prey values to plot as reference lines*)
noXnoPNiList = ConstantArray[XBifurNoPXNi⟦1⟧, Length[XVals]];
noXPNiList = ConstantArray[XBifurPXNi⟦1⟧, Length[XVals]];
noXnoPgrowthList = ConstantArray[XBifurNoPXgrowth⟦1⟧, Length[XVals]];
noXPgrowthList = ConstantArray[XBifurPXgrowth⟦1⟧, Length[XVals]];
noXnoPSHList = ConstantArray[XBifurNoPXSH⟦1⟧, Length[XVals]];
noXPSHList = ConstantArray[XBifurPXSH⟦1⟧, Length[XVals]];
noXnoPjNwList = ConstantArray[XBifurNoPXjNw⟦1⟧, Length[XVals]];
noXPjNwList = ConstantArray[XBifurPXjNw⟦1⟧, Length[XVals]];
noXnoPepList = Flatten[ConstantArray[XBifurNoPXep⟦1⟧, Length[XVals]]];
noXPepList = Flatten[ConstantArray[XBifurPXep⟦1⟧, Length[XVals]]];

(*use Table to get the lists in the form to plot*)
XBifurLists =
  Table[Module[{PlotList}, PlotList = Transpose@{XVals, list}; PlotList],
   {list, {noXnoPNiList, noXPNiList, XBifurNoPXNi, XBifurPXNi, noXnoPgrowthList,
     noXPgrowthList, XBifurNoPXgrowth, XBifurPXgrowth, (*noXnoPSHList,noXPSHList,
     XBifurNoPXSH,XBifurPXSH,*)noXnoPjNwList, noXPjNwList, XBifurNoPXjNw,
     XBifurPXjNw, noXnoPepList, noXPepList, XBifurNoPXep, XBifurPXep}}];
```

Plot the results

```
In[◦]:= (*padding for the plot margins*)
paddingS1 = {{80(*left*), 15(*right*)}, {50(*bottom*), 10(*top*)}};
```

```
(*Ni vs prey*)
plotS1a = ListLinePlot[{XBifurLists⟦1⟧,
    XBifurLists⟦2⟧, XBifurLists⟦3⟧, XBifurLists⟦4⟧, XBifurNenvLine},
   Epilog → Text[Style["a) Interstitial nitrogen", {Black, 14}],
     Offset[{5(*right from top left*), -5.5(*down from top left*)},
      Scaled[{0, 1}]], {-1, 0}], LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   ImagePadding → paddingS1, PlotRange → {Full, {0, 1.4 * 10^-7}}(*Full*),
   PlotStyle → {{ColorData[97, "ColorList"]⟦1⟧, Dashing[Large]},
     {ColorData[97, "ColorList"]⟦2⟧, Dashing[Large]}, ColorData[97, "ColorList"]⟦
      1⟧, ColorData[97, "ColorList"]⟦2⟧, Directive[GrayLevel[0.73](*, Thin*)]},
   Frame → {{True(*left*), False(*right*)},
     {True(*bottom*), False(*top*)}}, FrameTicks → (*True*)
    {{{{4 * 10^-8, "4*10^-8"}, {8 * 10^-8, "8*10^-8"}, {1.2 * 10^-7, "1.2*10^-7"}}, None},
     {(*None*){{0, "0"}, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}}, None}},
   FrameLabel → {{"mol N L^-1", None}, {None, None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 14]
   (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*dH/Hdt vs prey*)
plotS1b =
  ListLinePlot[{XBifurLists⟦5⟧, XBifurLists⟦6⟧, XBifurLists⟦7⟧, XBifurLists⟦8⟧},
   Epilog → Text[Style["b) Host specific growth rate", {Black, 14}],
     Offset[{5(*right from top left*), -6.5(*down from top left*)},
      Scaled[{0, 1}]], {-1, 0}], LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   ImagePadding → paddingS1, PlotRange → {Full, {0, 0.028}},
   PlotStyle → {{ColorData[97, "ColorList"]⟦1⟧, Dashing[Large]},
     {ColorData[97, "ColorList"]⟦2⟧, Dashing[Large]},
     ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧}, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   FrameTicks → (*True*){{{0, 0.01, 0.02}, None},
     {(*None*){{0, "0"}, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}}, None}},
   FrameLabel → {{"C-mol H C-mol H^-1 d^-1 ", None}, {"Prey (C-mol L^-1)", None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 14]
   (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*symbiont waste N vs. prey*)
plotS1c = ListLinePlot[
   {XBifurLists⟦9⟧, XBifurLists⟦10⟧, XBifurLists⟦11⟧, XBifurLists⟦12⟧},
    Epilog → Text[Style["c) Waste nitrogen per volume", {Black, 14}],
      Offset[{5(*right from top left*), -6(*down from top left*)},
       Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS1, PlotRange → {Full, {0, 3.5 * 10^-5}}(*Full*),
    PlotStyle → {{ColorData[97, "ColorList"]⟦1⟧, Dashing[Large]},
      {ColorData[97, "ColorList"]⟦2⟧, Dashing[Large]},
      ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧,
      Directive[Black, Thin]}, Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, FrameTicks → (*True*)
     {{{{1 * 10^-5, "1*10^-5"}, {2 * 10^-5, "2*10^-5"}, {3 * 10^-5, "3*10^-7"}}, None},
      {(*None*){{0, "0"}, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}}, None}},
    FrameLabel → {{"mol N L^-1 d^-1", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 14]
    (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*fish excretions vs. prey*)
plotS1d = ListLinePlot[
    {XBifurLists⟦13⟧, XBifurLists⟦14⟧, XBifurLists⟦15⟧, XBifurLists⟦16⟧},
    Epilog → Text[Style["d) Fish excretions per volume", {Black, 14}],
      Offset[{5(*right from top left*), -5.5(*down from top left*)},
       Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS1, PlotRange → {Full, {0, 0.0002}}(*Full*),
    PlotStyle → {{ColorData[97, "ColorList"]⟦1⟧, Dashing[Large]},
      {ColorData[97, "ColorList"]⟦2⟧, Dashing[Large]}, ColorData[97, "ColorList"]⟦
       1⟧, ColorData[97, "ColorList"]⟦2⟧, Directive[Black, Thin]}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    FrameTicks → (*True*){{{0.00005, 0.0001, 0.00015}, None},
      {(*None*){{0, "0"}, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}}, None}},
    FrameLabel → {{"mol N L^-1 d^-1", None}, {"Prey (C-mol L^-1)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 14]
    (*axis label size*), AspectRatio → 1 / GoldenRatio];
```

In[•]:=
```
(*make the plot legend*)
BifurLeg =
  LineLegend[{ColorData[97, "ColorList"]⟦1⟧, {ColorData[97, "ColorList"]⟦1⟧,
     Dashing[Large]}, ColorData[97, "ColorList"]⟦2⟧,
    {ColorData[97, "ColorList"]⟦2⟧, Dashing[Large]}}, {"No fish, with prey",
    "No fish, no prey", "With fish, with prey", "With fish, no prey"},
   LegendLayout → {"Row", 1}, LabelStyle → Directive[FontSize → 14]];

(*make the figure*)
SBifurFig = Show[
  Legended[GraphicsGrid[{{plotS1a, plotS1c}, {plotS1b, plotS1d}}, ImageSize → Full,
    (*Spacings→{ 0(*left to right*),-20(*Scaled[0.1]*)(*top to bottom*)}*)
    Spacings → { {0(*left?*), -20 (*labels and left col*)(*left to right*)},
      {0(*top?*), -20(*middle 2 rows*) (*labels and bottom*)
       (*top to bottom*)}}, Frame → False(*, AspectRatio→1/GoldenRatio*),
    Alignment → Bottom (*align the grid at the bottom of its frame*)],
   Placed[BifurLeg, (*Bottom*){Center(*left to right position is in the
      center*), 0(*top to bottom position*)}]], Graphics[Text["Nᵢ = Nₑₙᵥ",
    {80 (*more pos=farther right*), -54(*more neg=further down*)}]]]
```

Out[•]=

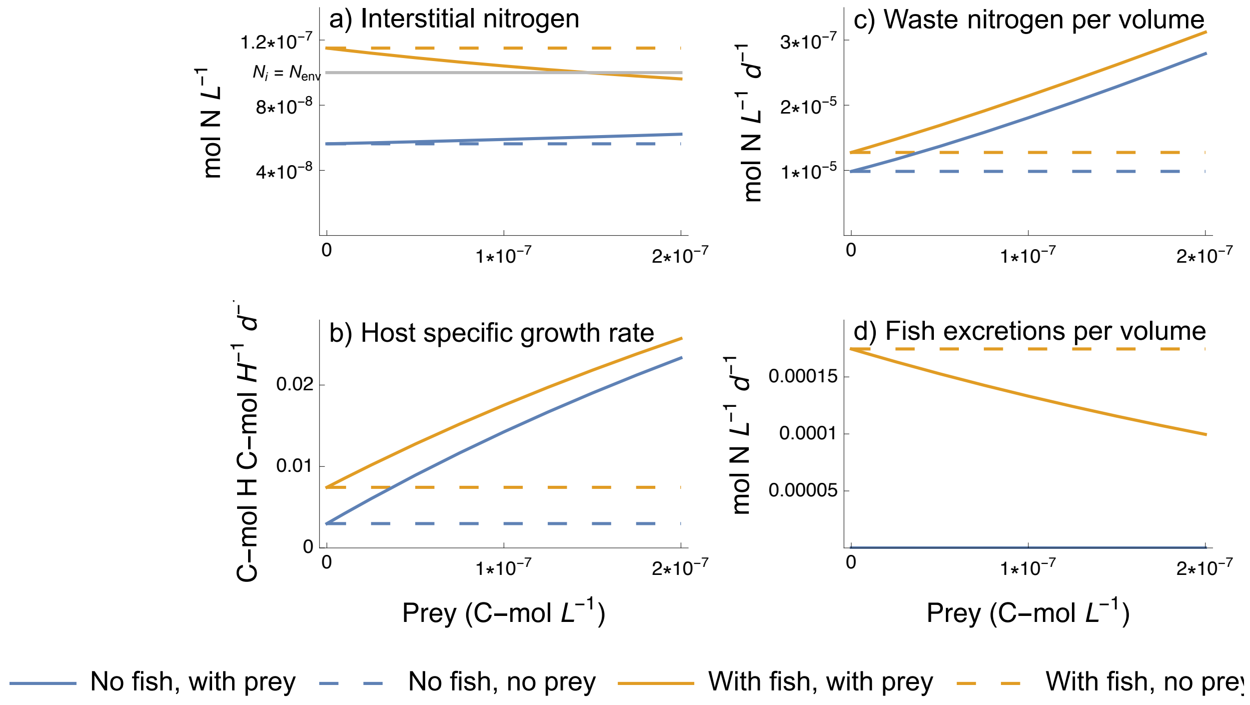


Fig. S2: Bifurcation heat maps

Run simulations and format results

Check steady state for Nenv vs prey simulations

In[∘]:=
```
(*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN, jeC, jCO2, jL, rCH, rCS, jeL, jNPQ,
 cROS1, jST, rNH, A, jNw, tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh]
```

In[∘]:=
```
(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10 ^ -7 (*1*10^-6*);

(*jN=jNm N/(N+KN);
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

tmax = 1500;

NXRunsTest = Table[
   Table[
    Table[
     Module[{

       jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
       jNi0 = jNm N/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
       jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
       addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
       (*make sure to put any intermediate output values in Module*)},


      F[ρ_][A_, B_] := A B (A + B) ρ / (A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
      (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
      (* helper function for VH*)
```

```
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar L ;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve〚All, 1〛, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#〚1〛'[t] == (λ (Max[0, #〚2〛] - #〚1〛) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
        VH'[t] == fh[t, H'[t]],
        Ni'[t] == (d * ( N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
              ( vi * VH) /. addtimetostatevars),
        P'[t] ==
          (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
       }
      ];
     (*set initial conditions*)
     inis = Join[{
        H[0] == H0,
        S[0] == S0,
        VH[0] == VH0,
        Ni[0] == Ni0,
        P[0] == P0},
       (#[[1]][0] == #[[3]]) & /@ tsolve];

     sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
        Method → {"EquationSimplification" → "Residual"}][[1]];

     (*change in Ni*)
     steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
        Evaluate[Ni[tmax - 50] /. sol]];

     (*change in host growth*)
     steadyHGrowth =
      Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] /
            H[tmax - 50] /. sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]];

     (*change in S/H*)
     steadySH =
      Abs[(Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /.
            sol]) / Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];
     {steadyNi, steadyHGrowth, steadySH}
     (*record the changes in Ni, H growth, and S/H as output*)],

    {X, {0, 0.1 * 10^-7, 0.25 * 10^-7, 0.5 * 10^-7, 0.75 * 10^-7,
      1 * 10^-7, 1.25 * 10^-7, 1.5 * 10^-7, 1.75 * 10^-7, 2 * 10^-7}}],
   (*repeat all this for different levels of prey*)
   {N, {0.1 * 10^-7, 0.25 * 10^-7, 0.5 * 10^-7, 0.75 * 10^-7,
     1 * 10^-7, 2.5 * 10^-7, 5 * 10^-7, 7.5 * 10^-7, 10 * 10^-7}}
    (*repeat all this for different Nenv values*)
  ],
  {P0, {0, kp * (1 - α) * H0}}(*run everything once
    with no fish (P0=0) and once with fish (P0=kp*(1-α)*H0)*)
  ];
```

```
In[]:= Select[Flatten[NXRunsTest], # > 1 * 10 ^ -4 &]
       (*see whether any outputs are greater than 10^-4*)
       (*no fish case: some simulations did not
          reach the steady state (changes were greater than 10^-4)*)
       Select[Flatten[NXRunsTest[[1]]], # > 1 * 10 ^ -4 &]
       (*with fish case: all simulations reached the steady state*)
       Select[Flatten[NXRunsTest[[2]]], # > 1 * 10 ^ -4 &]

Out[]= {0.000826753, 0.000233035, 0.00259319, 0.00120395, 0.00077321, 0.00492071,
        0.00405663, 0.00545449, 0.000112131, 0.00500471, 0.0158111, 0.002002, 0.0439383}

Out[]= {0.000826753, 0.000233035, 0.00259319, 0.00120395, 0.00077321, 0.00492071,
        0.00405663, 0.00545449, 0.000112131, 0.00500471, 0.0158111, 0.002002, 0.0439383}

Out[]= { }
```

```
In[]:= (*check to see which simulations did not reach the steady state*)
       (*host growth*)
       NXgrowthnoPdiff = Transpose[Flatten[NXRunsTest[[1]], 1]][[2]];
       (*Ni*)
       NXNinoPdiff = Transpose[Flatten[NXRunsTest[[1]], 1]][[1]];

       (*put the results in matrix form*)
       MatrixNXnoPgrowth = ArrayReshape[NXgrowthnoPdiff, {9, 10}] ;
       MatrixNXnoPNi = ArrayReshape[NXNinoPdiff, {9, 10}] ;

       (*check which positions of these matrices had changes greater than 10^-4*)
       Position[MatrixNXnoPgrowth, _?(# > 1 * 10 ^ -4 &)]
       Position[MatrixNXnoPNi, _?(# > 1 * 10 ^ -4 &)]

Out[]= {{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {3, 1}}

Out[]= {{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {3, 1}}
```

There are 6 cases where the steady state is not reached: at positions 1, 2, and 3 of the first column (corresponding to the 3 lowest values of Nenv with 0 prey), at positions 1 and 2 of the second column (corresponding to the 2 lowest values of Nenv with prey = $0.1*10^{-7}$, and at position 2 of the third column (corresponding to the lowest value of Nenv with prey = $0.25*10^{-7}$). All of these different situations are run individually in the following code chunks.

1. NO PREY, Nenv= $0.1*10^{-7}$

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, ℕ, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=

d = 1660;
ℕ = 1 * 10^-8 (*1*10^-6*);

X = 0;
P0 = 0;

(*jN=jNm ℕ/(ℕ+KN);
Ni0 = ℕ;
jNi0 = jNm ℕ/(ℕ+KN);*)

tmax = 2500;

NXnoXlowestN =
  Module[{

    jX = jXm X/(X + KX), jN = jNm ℕ/(ℕ + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = ℕ,
    jNi0 = jNm ℕ/(ℕ + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},
   (*light function*)
   LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
     LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

   F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ); (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B))*)
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
   jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
   jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
```

```
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + S ρC/H, 0];
jCO2 = jeC kCO2;
jL = A astar L ;
rCH = (jHT + jHG (1 - yC)/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw =  ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S]/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] ==  (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==  (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
```

```
    (*set initial conditions*)
    inis = Join[{
       H[0] == H0,
       S[0] == S0,
       VH[0] == VH0,
       Ni[0] == Ni0,
       P[0] == P0},
      (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

    (*change in Ni*)
    steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
       Evaluate[Ni[tmax - 50] /. sol]];

    (*change in host growth*)
    steadyHGrowth = If[Chop[Evaluate[H'[tmax] / H[tmax] /. sol]] == 0 &&
       Chop[Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] == 0, 0,
      Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
            sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] ];
    (*Chop[] approximates numbers that are less than 10^-10 as 0,
    so in this case, if host growth rate is asymptotically approaching 0,
    so if it is less than 10^-10 at the end of the time
     step say that the percent change is equal to zero*)

    (*change in S/H*)
    steadySH = Abs[
      (Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]) /
       Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

    {steadyNi, steadyHGrowth, steadySH}
    (*record the change in Ni, H growth, and S/H as output*)];
```

```
In[∘]:= Select[Flatten[NXnoXlowestN], # > 1 * 10 ^ -4 &] (*nothing is greater than 10^-4*)

Out[∘]= { }
```

2. NO PREY, Nenv= $0.25*10^{-7}$

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, ℕ, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=
d = 1660;
```

```
N = 2.5 * 10 ^ -8 (*1*10^-6*);

X = 0;
P0 = 0;

(*jN=jNm N/(N+KN);
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

tmax = 5000;

NXnoXsecLowN =
  Module[{

    jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
    jNi0 = jNm N/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},


   F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ); (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
   jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
   jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
   ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
   jeC = Max[jX - jHG/yC + S ρC/H, 0];
   jCO2 = jeC kCO2;
   jL = A astar L (*Lfun9*);
   rCH = (jHT + jHG (1 - yC)/yC) σCH;
   rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
```

```
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm] [jL yCL, rCS + H (jCO2+rCH)/S] / (1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) &/@ tsolve];
```

```
    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];

    (*change in Ni*)
    steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
       Evaluate[Ni[tmax - 50] /. sol]];

    (*change in host growth*)
    steadyHGrowth = If[Chop[Evaluate[H'[tmax] / H[tmax] /. sol]] == 0 &&
       Chop[Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] == 0, 0,
      Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
            sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] ];
    (*Chop[] approximates numbers that are less than 10^-10 as 0,
    so in this case, if host growth rate is less than 10^-10 at the end
     of the time step say that the percent change is equal to zero*)

    (*percent change in S/H*)
    steadySH = Abs[
      (Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]) /
       Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

    {steadyNi, steadyHGrowth, steadySH}
    (*record the change in Ni, H growth, and S/H as output*)];
```

```
In[•]:= Select[Flatten[NXnoXsecLowN], # > 1 * 10 ^ -4 &] (*nothing is greater than 10^-4*)
Out[•]= { }
```

3. NO PREY, Nenv=$0.5*10^{-7}$

```
In[•]:= (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
         jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
         tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[•]:=
        d = 1660;
        N = 5 * 10 ^ -8 (*1*10^-6*);

        X = 0;
        P0 = 0;

        (*jN=jNm N/(N+KN);
        Ni0 = N;
        jNi0 = jNm N/(N+KN);*)
```

```
tmax = 50 000;

NXnoXlowestN =
  Module[{

    jX = jXm X/(X + KX), jN = jNm ℕ/(ℕ + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = ℕ,
    jNi0 = jNm ℕ/(ℕ + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},

   F[ρ_][A_, B_] := (A B (A + B) ρ)/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ); (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
   jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
   jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
   ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
   jeC = Max[jX - jHG/yC + S ρC/H, 0];
   jCO2 = jeC kCO2;
   jL = A astar L ;
   rCH = (jHT + jHG (1 - yC)/yC) σCH;
   rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
   jeL = jL - jCP/yCL;
   jNPQ = 1/(1/jeL + 1/kNPQ);
   cROS1 = (jeL - jNPQ)/kROS;
   jST = (1 + b cROS1) j0ST;
   rNH = σNH nNH jHT;
   A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
```

```
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S]) / (1 + cROS1), jCP0},
   {jNi, (jNm * Ni) / (Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
    {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];

(*change in Ni*)
steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
    Evaluate[Ni[tmax - 50] /. sol]];

(*change in host growth*)
steadyHGrowth = If[Chop[Evaluate[H'[tmax] / H[tmax] /. sol]] == 0 &&
    Chop[Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] == 0, 0,
      Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
            sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] ];
    (*Chop[] approximates numbers that are less than 10^-10 as 0,
    so in this case, if host growth rate is less than 10^-10 at the end
     of the time step say that the percent change is equal to zero*)

    (* change in S/H*)
    steadySH = Abs[
      (Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]) /
       Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

    {steadyNi, steadyHGrowth, steadySH}
    (*record the change in Ni, H growth, and S/H as output*)];
```

```
In[•]:= Select[Flatten[NXnoXlowestN], # > 1 * 10 ^ -4 &] (*nothing is greater than 10^-4*)

Out[•]= { }
```

4. PREY=$0.1*10^{-7}$, Nenv=$0.1*10^{-7}$

```
In[•]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[•]:=
d = 1660;
N = 1 * 10 ^ -8 (*1*10^-6*);

X = 1 * 10 ^ -8;
P0 = 0;
```

(*jN=$\frac{jNm\ N}{N+KN}$;
Ni0 = N;
jNi0 = $\frac{jNm\ N}{N+KN}$;*)

```
tmax = 5000;

NXlowestXlowestN =
  Module[{
```

$jX = \frac{jXm\ X}{X + KX}$, $jN = \frac{jNm\ N}{N + KN}$, jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,

```
 jNi0 = $\frac{jNm\,\mathbb{N}}{\mathbb{N} + KN}$, states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
 jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
 addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
 (*make sure to put any intermediate output values in Module*)},


F[ρ_][A_, B_] := $\frac{A\,B\,(A + B)\,\rho}{A^2\,B + A\,B^2 + A^2\,\rho + A\,B\,\rho + B^2\,\rho}$; (*same as $\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$ *)
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, $\frac{rNS + \frac{H\,\rho N}{S}}{nNS}$];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - $\frac{jHG}{yC}$ + $\frac{S\,\rho C}{H}$, 0];
jCO2 = jeC kCO2;
jL = A astar L ;
rCH = (jHT + $\frac{jHG\,(1 - yC)}{yC}$) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - $\frac{jCP}{yCL}$;
jNPQ = $\frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}}$;
cROS1 = $\frac{jeL - jNPQ}{kROS}$;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 $e^{-6.479055\,S/H}$;
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, $\frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1}$, jCP0},
   {jNi, $\frac{jNm * Ni}{Ni + KN}$, jNi0}
 };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
  (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
  {
   S'[t] == (S (jSG - jST) /. addtimetostatevars),
   H'[t] == (H (jHG - jHT) /. addtimetostatevars),
   VH'[t] == fh[t, H'[t]],
   Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
         ( vi * VH) /. addtimetostatevars),
   P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
  }
 ];
(*set initial conditions*)
inis = Join[{
   H[0] == H0,
   S[0] == S0,
   VH[0] == VH0,
   Ni[0] == Ni0,
   P[0] == P0},
  (#[[1]][0] == #[[3]]) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
   {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];

(* change in Ni*)
steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
   Evaluate[Ni[tmax - 50] /. sol]];

(*change in host growth*)
steadyHGrowth = If[Chop[Evaluate[H'[tmax] / H[tmax] /. sol]] == 0 &&
   Chop[Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] == 0, 0,
  Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
         sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] ];
(*Chop[] approximates numbers that are less than 10^-10 as 0,
so in this case, if host growth rate is less than 10^-10 at
 the end of the time step (asymptotically approaching 0),
say that the percent change is equal to zero*)

(* change in S/H*)
steadySH = Abs[
      (Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]) /
       Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

    {steadyNi, steadyHGrowth, steadySH}
    (*record the change in Ni, H growth, and S/H as output*)];
```

```
In[]:= Select[Flatten[NXlowestXlowestN], # > 1 * 10 ^ -4 &] (*nothing is greater than 10^-4*)
Out[]= { }
```

5. PREY=$0.1*10^{-7}$, Nenv=$2.5*10^{-8}$

```
In[]:= (*clear parameters that are changing and intermediate values*)
       ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
        jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
        tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[]:=
       d = 1660(*10*);
       N = 2.5 * 10 ^ -8 (*1*10^-6*);

       X = 1 * 10 ^ -8;
       P0 = 0;
```

(*$jN=\frac{jNm\ N}{N+KN}$;
Ni0 = N;
$jNi0 = \frac{jNm\ N}{N+KN}$;*)

```
       tmax = 50 000;

       NXlowestXseclowestN =
         Module[{
```

$jX = \frac{jXm\ X}{X + KX}$, $jN = \frac{jNm\ N}{N + KN}$, jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
$jNi0 = \frac{jNm\ N}{N + KN}$, states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
(*make sure to put any intermediate output values in Module*)},

$F[ρ\_][A\_, B\_] := \frac{A\ B\ (A + B)\ ρ}{A^2\ B + A\ B^2 + A^2\ ρ + A\ B\ ρ + B^2\ ρ}$; (*same as $\frac{1}{\frac{1}{ρ}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$*)
(* helper function for VH*)

```
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S) / nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar L ;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1 / (1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ) / kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S] / (1 + cROS1), jCP0},
   {jNi, (jNm * Ni) / (Ni + KN), jNi0}
  };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
      VH'[t] == fh[t, H'[t]],
      Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
            ( vi * VH) /. addtimetostatevars),
      P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
     }
    ];
   (*set initial conditions*)
   inis = Join[{
      H[0] == H0,
      S[0] == S0,
      VH[0] == VH0,
      Ni[0] == Ni0,
      P[0] == P0},
     (#[[1]][0] == #[[3]]) & /@ tsolve];

   sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
      {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];

   (*change in Ni*)
   steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
      Evaluate[Ni[tmax - 50] /. sol]];

   (*change in host growth*)
   steadyHGrowth = If[Chop[Evaluate[H'[tmax] / H[tmax] /. sol]] == 0 &&
      Chop[Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] == 0, 0,
     Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
           sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] ];
   (*Chop[] approximates numbers that are less than 10^-10 as 0,
   so in this case, if host growth rate is less than 10^-10 at the end
    of the time step say that the percent change is equal to zero*)

   (* change in S/H*)
   steadySH = Abs[
     (Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]) /
      Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

   {steadyNi, steadyHGrowth, steadySH}
   (*record the change in Ni, H growth, and S/H as output*)];


In[]:= Select[Flatten[NXlowestXseclowestN], # > 1 * 10 ^ -4 &]
       (*nothing is greater than 10^-4*)

Out[]= { }
```

6. PREY=$2.5*10^{-8}$, Nenv=$0.1*10^{-7}$

```
In[•]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[•]:=
d = 1660;
N = 1 * 10^-8 (*1*10^-6*);

X = 2.5 * 10^-8;
P0 = 0;
```

$(*jN=\frac{jNm\ N}{N+KN};$

$Ni0\ =\ N;$

$jNi0\ =\ \frac{jNm\ N}{N+KN};*)$

```
tmax = 250 000;

NXseclowestXlowestN =
  Module[{
```

$jX=\frac{jXm\,X}{X+KX},\ jN=\frac{jNm\,N}{N+KN},$ jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,

$jNi0\ =\ \frac{jNm\,N}{N+KN},$ states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,

```
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},
```

$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2\,B+A\,B^2+A^2\,\rho+A\,B\,\rho+B^2\,\rho};\ (*\text{same as }\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}*)$

```
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
```

$jHG = F[jHGm]\left[yC\ (\rho C\ S\ /\ H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH)\ nNH^{-1}\right];$

$jSG = F[jSGm]\left[jCP\ yC,\ \frac{rNS+\frac{H\,\rho N}{S}}{nNS}\right];$

$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH - nNH\ jHG\ (*yN^{-1}*),\ 0\right];$

```
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar L ;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve〚All, 1〛, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#〚1〛'[t] == (λ (Max[0, #〚2〛] - #〚1〛) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * ( N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
      H[0] == H0,
      S[0] == S0,
      VH[0] == VH0,
      Ni[0] == Ni0,
      P[0] == P0},
     (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

   sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
      {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

   (*change in Ni*)
   steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
      Evaluate[Ni[tmax - 50] /. sol]];

   (*change in host growth*)
   steadyHGrowth = If[Chop[Evaluate[H'[tmax] / H[tmax] /. sol]] == 0 &&
      Chop[Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] == 0, 0,
     Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] / H[tmax - 50] /.
          sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]] ];
   (*Chop[] approximates numbers that are less than 10^-10 as 0,
   so in this case, if host growth rate is less than 10^-10 at the end
    of the time step say that the percent change is equal to zero*)

   (* change in S/H*)
   steadySH = Abs[
     (Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]) /
      Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

   {steadyNi, steadyHGrowth, steadySH}
   (*record the change in Ni, H growth, and S/H as output*)];
```

```
In[∘]:= Select[Flatten[NXseclowestXlowestN], # > 1 * 10 ^ -4 &]
        (*nothing is greater than 10^-4*)

Out[∘]= { }
```

Simulations for Nenv vs. prey

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN, jeC, jCO2, jL, rCH, rCS, jeL, jNPQ,
         cROS1, jST, rNH, A, jNw, tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh]

In[∘]:=
        d = 1660;
```

```
N = 1 * 10^-7 (*1*10^-6*);

(*jN=$\frac{\text{jNm N}}{\text{N+KN}}$;
Ni0 = N;
jNi0 = $\frac{\text{jNm N}}{\text{N+KN}}$;*)

tmax = 1500;

NXRuns = Table[
   Table[
    Table[
     Module[{

        jX = $\frac{\text{jXm X}}{\text{X + KX}}$, jN = $\frac{\text{jNm N}}{\text{N + KN}}$, jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
        jNi0 = $\frac{\text{jNm N}}{\text{N + KN}}$, states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
        jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
        addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
        (*make sure to put any intermediate output values in Module*)},


       F[ρ_][A_, B_] := $\frac{\text{A B (A + B) }\rho}{\text{A}^2\text{ B + A B}^2\text{ + A}^2\ \rho\text{ + A B }\rho\text{ + B}^2\ \rho}$;
       (*same as $\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$*)
       (* helper function for VH*)
       fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

       (* Calculations *)
       jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH$^{-1}$];
       jSG = F[jSGm][jCP yC, $\frac{\text{rNS + }\frac{\text{H }\rho\text{N}}{\text{S}}}{\text{nNS}}$];
       ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN$^{-1}$*), 0];
       jeC = Max[jX - $\frac{\text{jHG}}{\text{yC}}$ + $\frac{\text{S }\rho\text{C}}{\text{H}}$, 0];
       jCO2 = jeC kCO2;
       jL = A astar L ;
       rCH = (jHT + $\frac{\text{jHG (1 - yC)}}{\text{yC}}$) σCH;
```

```
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
```
$$jeL = jL - \frac{jCP}{yCL};$$
$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$
$$cROS1 = \frac{jeL - jNPQ}{kROS};$$
```
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
```
$$A = 1.256307 + 1.385969\, e^{-6.479055\, S/H};$$
```
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
  {ρC, jCP - jSG yC^-1, ρC0},
```
$$\left\{jCP,\ \frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$
$$\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$
```
 };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * ( N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
          (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

        sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
           Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

        (*steady state Ni*)
        steadyNi = Evaluate[Ni[tmax] /. sol];

        (*steady state host growth*)
        steadyHGrowth = Evaluate[H'[tmax] / H[tmax] /. sol];

        (*steady state S/H*)
        steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

        {steadyNi, steadyHGrowth, steadySH}
        (*the steady state values as output*)],

       {X, {0, 0.1 * 10^-7, 0.25 * 10^-7, 0.5 * 10^-7, 0.75 * 10^-7,
         1 * 10^-7, 1.25 * 10^-7, 1.5 * 10^-7, 1.75 * 10^-7, 2 * 10^-7}}],
      (*repeat this for different levels of prey*)
      {N, {0.1 * 10^-7, 0.25 * 10^-7, 0.5 * 10^-7, 0.75 * 10^-7,
        1 * 10^-7, 2.5 * 10^-7, 5 * 10^-7, 7.5 * 10^-7, 10 * 10^-7}}
       (*repeat this for different values of Nenv*)
     ],
     {P0, {0, kp * (1 - α) * H0}}
     (*repeat this with no fish (P0=0) and with fish (P0=kp*(1-α)*H0)*)
    ];
```

In[∘]:=
```
(*saving the output of Table[]*)
(*Save["NRunsnoPnoX1",durationRunsnoPnoX1 ]*)
Save["NXRuns", NXRuns ]
```

In[∘]:=
```
(*recover this output*)
Get["NXRuns" ];
```

Repeat this for the situations in which the system has not reached the steady state by tmax=1500:
1. NO PREY, Nenv= $0.1*10^{-7}$

In[∘]:=
```
(*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

In[∘]:=
```
d = 1660;
```

```
N = 1 * 10 ^ - 8 (*1*10^-6*);

X = 0;
P0 = 0;

(*jN=jNm N/(N+KN);
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

tmax = 2500;

NXnoXlowestN2 =
  Module[{

    jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
    jNi0 = jNm N/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},

   F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ); (*same as 1/(1/ρ+1/A+1/B-1/(A+B))*)
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
   jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
   jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
   ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
   jeC = Max[jX - jHG/yC + S ρC/H, 0];
   jCO2 = jeC kCO2;
   jL = A astar L ;
   rCH = (jHT + jHG (1 - yC)/yC) σCH;
   rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
```

```
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw =  ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S]/(1 + cROS1), jCP0},
   {jNi, jNm * Ni/(Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * ( ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) & /@ tsolve];
```

```
    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];

    (*steady state Ni*)
    steadyNi = Evaluate[Ni[tmax] /. sol];
    (*steady state host growth*)
    steadyHGrowth = Chop[Evaluate[H'[tmax] / H[tmax] /. sol]];
    (*steady state S/H*)
    steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

    {steadyNi, steadyHGrowth, steadySH}(*the steady state values as output*)];

Save["noXlowestN2", NXnoXlowestN2]
```

2. NO PREY, Nenv= $0.25*10^{-7}$

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=
d = 1660;
N = 2.5 * 10^-8 (*1*10^-6*);

X = 0;
P0 = 0;
```

$(*jN=\frac{jNm\ N}{N+KN};$

$Ni0\ =\ N;$

$jNi0\ =\ \frac{jNm\ N}{N+KN};*)$

```
tmax = 5000;

NXnoXsecLowN2 =
   Module[{
```

$jX = \frac{jXm\ X}{X + KX},\ jN = \frac{jNm\ N}{N + KN},\ jHT = j0HT,\ rNS = \sigma NS\ nNS\ j0ST,\ VH0 = kv * H0,\ Ni0\ =\ N,$

$jNi0\ =\ \frac{jNm\ N}{N + KN}$, states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,

jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,

addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH

```
 (*make sure to put any intermediate output values in Module*)},
```

$$F[\rho\_][A\_,\ B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2\,B + A\,B^2 + A^2\,\rho + A\,B\,\rho + B^2\,\rho};\ (\text{*same as } \frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}\text{*})$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\ (\rho C\ S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH)\ nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\ yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH - nNH\ jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

$$jCO2 = jeC\ kCO2;$$

$$jL = A\ astar\ L\ ;$$

$$rCH = \left(jHT + \frac{jHG\ (1 - yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\ \left(j0ST + (1 - yC)\ jSG\ yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\ cROS1)\ j0ST;$$

$$rNH = \sigma NH\ nNH\ jHT;$$

$$A = 1.256307 + 1.385969\ e^{-6.479055\ S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \Big\{$$

$$\left\{\rho C,\ jCP - jSG\ yC^{-1},\ \rho C0\right\},$$

$$\left\{jCP,\ \frac{F[jCPm]\left[jL\ yCL,\ rCS + \frac{H\ (jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$

$$\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$

$$\Big\};$$

```
states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
```

```
    addtimetostatevars = ((# → #[t]) & /@ states);
    eqs = Join[
      (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
      {
       S'[t] == (S (jSG - jST) /. addtimetostatevars),
       H'[t] == (H (jHG - jHT) /. addtimetostatevars),
       VH'[t] == fh[t, H'[t]],
       Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
             ( vi * VH) /. addtimetostatevars),
       P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
      }
     ];
    (*set initial conditions*)
    inis = Join[{
       H[0] == H0,
       S[0] == S0,
       VH[0] == VH0,
       Ni[0] == Ni0,
       P[0] == P0},
      (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

    (*steady state Ni*)
    steadyNi = Evaluate[Ni[tmax] /. sol];
    (*steady state host growth*)
    steadyHGrowth = Chop[Evaluate[H'[tmax] / H[tmax] /. sol]];
    (*steady state S/H*)
    steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

    {steadyNi, steadyHGrowth, steadySH}(*the steady state values as output*)];

Save["noXsecLowN2", NXnoXsecLowN2]
```

3. NO PREY, Nenv=$0.5*10^{-7}$

```
In[•]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, ℕ, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[•]:=
d = 1660(*10*);
```

```
N = 5 * 10 ^ - 8 (*1*10^-6*);

X = 0;
P0 = 0;

(*jN=(jNm N)/(N+KN);
Ni0 = N;
jNi0 = (jNm N)/(N+KN);*)

tmax = 50 000;

NXnoXthirdLowN2 =
  Module[{

    jX = (jXm X)/(X + KX), jN = (jNm N)/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
    jNi0 = (jNm N)/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},

   F[ρ_][A_, B_] := (A B (A + B) ρ)/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ); (*same as 1/(1/ρ+1/A+1/B-1/(A+B)) *)
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
   jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
   jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
   ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
   jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
   jCO2 = jeC kCO2;
   jL = A astar L ;
   rCH = (jHT + (jHG (1 - yC))/yC) σCH;
   rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
```

```
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw =  ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm] [jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] ==  (d * ( ℕ - Ni)  + (ξ * (1 - α) * jNw * S + ep * P  - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t]  ==  (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] ==  Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) & /@ tsolve];
```

```
    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];

    (*steady state Ni*)
    steadyNi = Evaluate[Ni[tmax] /. sol];
    (*steady state host growth*)
    steadyHGrowth = Chop[Evaluate[H'[tmax] / H[tmax] /. sol]];
    (*steady state S/H*)
    steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

    {steadyNi, steadyHGrowth, steadySH}(*the steady state values as output*)];

Save["noXthirdLowN2", NXnoXthirdLowN2]
```

4. PREY=$0.1*10^{-7}$, Nenv=$0.1*10^{-7}$

```
In[•]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[•]:=
d = 1660;
N = 1 * 10^-8 (*1*10^-6*);

X = 1 * 10^-8;
P0 = 0;

(*jN= jNm N/(N+KN) ;
Ni0 = N;
jNi0 = jNm N/(N+KN) ;*)

tmax = 5000;

NXlowestXlowestN2 =
  Module[{

    jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
    jNi0 = jNm N/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
 (*make sure to put any intermediate output values in Module*)},
```

$$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2\,B + A\,B^2 + A^2\,\rho + A\,B\,\rho + B^2\,\rho};\ (\text{*same as } \frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}\text{*})$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv *y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\,(\rho C\,S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH)\,nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\,yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH - nNH\,jHG\ (\text{*}yN^{-1}\text{*}),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

$$jCO2 = jeC\,kCO2;$$

$$jL = A\,astar\,L;$$

$$rCH = \left(jHT + \frac{jHG\,(1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\,\left(j0ST + (1-yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\,cROS1)\,j0ST;$$

$$rNH = \sigma NH\,nNH\,jHT;$$

$$A = 1.256307 + 1.385969\,e^{-6.479055\,S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \Big\{\ \{\rho C,\ jCP - jSG\,yC^{-1},\ \rho C0\},\ \left\{jCP,\ \frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},\ \left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}\ \Big\};$$

```
states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
```

```
    addtimetostatevars = ((# → #[t]) & /@ states);
    eqs = Join[
      (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
      {
       S'[t] == (S (jSG - jST) /. addtimetostatevars),
       H'[t] == (H (jHG - jHT) /. addtimetostatevars),
       VH'[t] == fh[t, H'[t]],
       Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
            ( vi * VH) /. addtimetostatevars),
       P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
      }
     ];
    (*set initial conditions*)
    inis = Join[{
       H[0] == H0,
       S[0] == S0,
       VH[0] == VH0,
       Ni[0] == Ni0,
       P[0] == P0},
      (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

    (*steady state Ni*)
    steadyNi = Evaluate[Ni[tmax] /. sol];
    (*steady state host growth*)
    steadyHGrowth = Chop[Evaluate[H'[tmax] / H[tmax] /. sol]];
    (*steady state S/H*)
    steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

    {steadyNi, steadyHGrowth, steadySH}(*the steady state values as output*)];

Save["lowestXlowestN2", NXlowestXlowestN2]
```

5. PREY=$0.1*10^{-7}$, Nenv=$2.5*10^{-8}$

```
In[◦]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[◦]:=
d = 1660;
```

```
Ν = 2.5 * 10 ^ -8 ;

X = 1 * 10 ^ -8;
P0 = 0;

(*jN=(jNm Ν)/(Ν+KN);
Ni0 = Ν;
jNi0 = (jNm Ν)/(Ν+KN);*)

tmax = 50 000;

NXlowestXseclowestN2 =
  Module[{

    jX = (jXm X)/(X + KX), jN = (jNm Ν)/(Ν + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = Ν,
    jNi0 = (jNm Ν)/(Ν + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
    (*make sure to put any intermediate output values in Module*)},


   F[ρ_][A_, B_] := (A B (A + B) ρ)/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ); (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
   (* helper function for VH*)
   fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

   (* Calculations *)
   jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
   jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
   ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
   jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
   jCO2 = jeC kCO2;
   jL = A astar L ;
   rCH = (jHT + (jHG (1 - yC))/yC) σCH;
   rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
```

```
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S]/(1 + cROS1), jCP0},
   {jNi, jNm * Ni/(Ni + KN), jNi0}
  };

states = Join[tsolve〚All, 1〛, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#〚1〛'[t] == (λ (Max[0, #〚2〛] - #〚1〛) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * ( ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#〚1〛[0] == #〚3〛) &/@ tsolve];
```

```
    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

    (*steady state Ni*)
    steadyNi = Evaluate[Ni[tmax] /. sol];
    (*steady state host growth*)
    steadyHGrowth = Chop[Evaluate[H'[tmax] / H[tmax] /. sol]];
    (*steady state S/H*)
    steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

    {steadyNi, steadyHGrowth, steadySH}(*the steady state values as output*)];

Save["lowestXseclowestN2", NXlowestXseclowestN2]
```

6. PREY=$2.5*10^{-8}$, Nenv=$0.1*10^{-7}$

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=
d = 1660;
N = 1 * 10 ^ -8 (*1*10^-6*);

X = 2.5 * 10 ^ -8;
P0 = 0;

(*jN= jNm N / (N+KN);
Ni0 = N;
jNi0 = jNm N / (N+KN);*)

tmax = 250 000;

NXseclowestXlowestN2 =
  Module[{

    jX = jXm X / (X + KX), jN = jNm N / (N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
    jNi0 = jNm N / (N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
    jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
    addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
 (*make sure to put any intermediate output values in Module*)},
```

$$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2 B + A B^2 + A^2 \rho + A B \rho + B^2 \rho};\ (\text{*same as } \frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}\text{*})$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\,(\rho C\,S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH)\,nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\,yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = \text{Max}\left[(1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH - nNH\,jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = \text{Max}\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

$$jCO2 = jeC\ kCO2;$$

$$jL = A\ astar\ L\ ;$$

$$rCH = \left(jHT + \frac{jHG\,(1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\left(j0ST + (1-yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\ cROS1)\ j0ST;$$

$$rNH = \sigma NH\ nNH\ jHT;$$

$$A = 1.256307 + 1.385969\ e^{-6.479055\ S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \Big\{$$

$$\left\{\rho C,\ jCP - jSG\,yC^{-1},\ \rho C0\right\},$$

$$\left\{jCP,\ \frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$

$$\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$

$$\Big\};$$

```
states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
```

```
    addtimetostatevars = ((# → #[t]) & /@ states);
    eqs = Join[
      (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
      {
       S'[t] == (S (jSG - jST) /. addtimetostatevars),
       H'[t] == (H (jHG - jHT) /. addtimetostatevars),
       VH'[t] == fh[t, H'[t]],
       Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
             (vi * VH) /. addtimetostatevars),
       P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
      }
     ];
    (*set initial conditions*)
    inis = Join[{
       H[0] == H0,
       S[0] == S0,
       VH[0] == VH0,
       Ni[0] == Ni0,
       P[0] == P0},
      (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

    (*steady state Ni*)
    steadyNi = Evaluate[Ni[tmax] /. sol];
    (*steady state host growth*)
    steadyHGrowth = Chop[Evaluate[H'[tmax] / H[tmax] /. sol]];
    (*steady state S/H*)
    steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

    {steadyNi, steadyHGrowth, steadySH}(*the steady state values as output*)];

Save["seclowestXlowestN2", NXseclowestXlowestN2]
```

Get results for Nenv vs prey simulations in correct form to plot

```
In[∘]:= Get["NXRuns" ];
        Get["noXlowestN2"];
        Get["noXsecLowN2"];
        Get["noXthirdLowN2"];
        Get["lowestXlowestN2"];
        Get["lowestXseclowestN2"];
        Get["seclowestXlowestN2"];

In[∘]:= (*NXRuns is nested list, innermost is all 3 outputs for {X1, X2, ...},
        second level is this repeated for each Nenv {N1, N2, ...},
        outer levels is fish {P1, P2}, *)
        (*so first seprate the fish vs no fish cases*)
        NXnoP = NXRuns⟦1⟧;
        NXwithP = NXRuns⟦2⟧;
        separate = Transpose[Flatten[NXnoP, 1]];
        (* this separates all 3 response variables,
        but need to know that in the resulting array,
        the columns are the X values (outermost parameter tested) and the rows are
         the N levels (outermost parameter tested), so if n X values are tested,
        the first n values are the outputs for each X value at the first N value tested,
        all the way to the mth set of n outputs for the mth N value tested*)
        noPNi = separate⟦1⟧;
        (*could combine the above two lines of code*)
        noPNi = Transpose[Flatten[NXnoP, 1]]⟦1⟧;

        (*want the differences between the P and no P case*)
        (*Ni*)
        MatrixNXNinoP1 = ArrayReshape[Transpose[Flatten[NXnoP, 1]]⟦1⟧, {9, 10}] ;
        (*{n, m} dimensions are {# outermost par values, # innermost par values} *)
        MatrixNXNinoP =
          ReplacePart[MatrixNXNinoP1, {{1, 1} → NXnoXlowestN2⟦1⟧, {1, 2} → NXnoXsecLowN2⟦1⟧,
            {1, 3} → NXnoXthirdLowN2⟦1⟧, {2, 1} → NXlowestXlowestN2⟦1⟧,
            {2, 2} → NXlowestXseclowestN2⟦1⟧, {3, 1} → NXseclowestXlowestN2⟦1⟧}];
         (*elements {{1,1},{1,2},{1,3},{2,1},{2,2},{3,1}} of the array
         for the no P case had not reached the steady state,
        so need to replace these with the results of
         the separately run simulations*)
        MatrixNXNiP = ArrayReshape[Transpose[Flatten[NXwithP, 1]]⟦1⟧, {9, 10}] ;
        NXNidiff = MatrixNXNiP - MatrixNXNinoP;

        (*repeat for host growth rates*)
        MatrixNXgrowthnoP1 = ArrayReshape[Transpose[Flatten[NXnoP, 1]]⟦2⟧, {9, 10}] ;
        MatrixNXgrowthnoP = MatrixNXNinoP = ReplacePart[
            MatrixNXgrowthnoP1, {{1, 1} → NXnoXlowestN2⟦2⟧, {1, 2} → NXnoXsecLowN2⟦2⟧,
      {1, 3} → NXnoXthirdLowN2⟦2⟧, {2, 1} → NXlowestXlowestN2⟦2⟧,
      {2, 2} → NXlowestXseclowestN2⟦2⟧, {3, 1} → NXseclowestXlowestN2⟦2⟧}];
MatrixNXgrowthP = ArrayReshape[Transpose[Flatten[NXwithP, 1]]⟦2⟧, {9, 10}] ;
NXgrowthdiff = MatrixNXgrowthP - MatrixNXgrowthnoP;

(* S/H*)
MatrixNXSHnoP1 = ArrayReshape[Transpose[Flatten[NXnoP, 1]]⟦2⟧, {9, 10}] ;
MatrixNXSHnoP =
  MatrixNXNinoP = ReplacePart[MatrixNXSHnoP1, {{1, 1} → NXnoXlowestN2⟦3⟧, {1, 2} →
      NXnoXsecLowN2⟦3⟧, {1, 3} → NXnoXthirdLowN2⟦3⟧, {2, 1} → NXlowestXlowestN2⟦3⟧,
      {2, 2} → NXlowestXseclowestN2⟦3⟧, {3, 1} → NXseclowestXlowestN2⟦3⟧}];
MatrixNXSHP = ArrayReshape[Transpose[Flatten[NXwithP, 1]]⟦2⟧, {9, 10}] ;
NXSHdiff = MatrixNXSHP - MatrixNXSHnoP;
```

Check steady state for flushing rate vs carrying capacity simulations

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN, jeC, jCO2, jL, rCH, rCS, jeL, jNPQ,
 cROS1, jST, rNH, A, jNw, tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh]

In[∘]:=
d = 1660;
N = 1 * 10^-7 (*1*10^-6*);
X = 0;
```

$(*jN=\frac{jNm\ N}{N+KN};$

$Ni0\ =\ N;$

$jNi0\ =\ \frac{jNm\ N}{N+KN};*)$

```
tmax = 800;

kdRunsTest = Table[
   Table[
    Table[
     Module[{
```

$jX=\frac{jXm\ X}{X+KX},\ jN=\frac{jNm\ N}{N+KN},\ jHT = j0HT,\ rNS = \sigma NS\ nNS\ j0ST,\ VH0 = kv * H0,\ Ni0\ =\ N,$

$jNi0\ =\ \frac{jNm\ N}{N+KN},$ states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,

```
       jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
 addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
 (*make sure to put any intermediate output values in Module*)},
```

$$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2\,B + A\,B^2 + A^2\,\rho + A\,B\,\rho + B^2\,\rho};$$

$$(\text{*same as } \frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}} \text{*})$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv *y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\,(\rho C\,S\,/\,H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH)\,nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\,yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = \text{Max}\left[(1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH - nNH\,jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = \text{Max}\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

$$jCO2 = jeC\,kCO2;$$

$$jL = A\,astar\,L\ ;$$

$$rCH = \left(jHT + \frac{jHG\,(1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\,\left(j0ST + (1-yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\,cROS1)\,j0ST;$$

$$rNH = \sigma NH\,nNH\,jHT;$$

$$A = 1.256307 + 1.385969\,e^{-6.479055\,S/H};$$

$$jNw = \rho N * H\,/\,S + rNS - nNS * jSG;$$

$$tsolve = \Big\{$$

$$\quad\left\{\rho C,\ jCP - jSG\,yC^{-1},\ \rho C0\right\},$$

$$\quad\left\{jCP,\ \frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$

$$\quad\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$

```
 };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          (vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
 ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
    Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

(* change in Ni*)
steadyNi = Abs[(Evaluate[Ni[tmax] /. sol] - Evaluate[Ni[tmax - 50] /. sol]) /
    Evaluate[Ni[tmax - 50] /. sol]];

(* change in host growth*)
steadyHGrowth =
  Abs[(Evaluate[H'[tmax] / H[tmax] /. sol] - Evaluate[H'[tmax - 50] /
          H[tmax - 50] /. sol]) / Evaluate[H'[tmax - 50] / H[tmax - 50] /. sol]];

(*change in S/H*)
steadySH =
  Abs[(Evaluate[S[tmax] / H[tmax] /. sol] - Evaluate[S[tmax - 50] / H[tmax - 50] /.
        sol]) / Evaluate[S[tmax - 50] / H[tmax - 50] /. sol]];

{steadyNi, steadyHGrowth, steadySH}
```

```
        (*record the changes in Ni, H growth, and S/H as output*)],

       {kp, {150, 175, 200, 225, 250, 275, 300, 325, 350, 375, 400}}],
      (*repeat for all the kp's*)
      {d, {450, 850, 1250, 1650, 2050, 2450, 2850, 3250, 3650, 4050}}
      (*repeat for all the flushing rates*)
     ],
    {P0, {0, kp * (1 - α) * H0}}
    (*repeat the above with no fish (P0=0) and with fish (P0=kp*(1-α)*H0)*)
   ];
```

In[ ]:=
```
Select[Flatten[kdRunsTest], # > 1 * 10^-4 &]
(*check that nothing is greater than 10^-4*)
```

Out[ ]=
```
{}
```

Simulations for flushing rate vs carrying capacity

In[ ]:=
```
(*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN, jeC, jCO2, jL, rCH, rCS, jeL,
 jNPQ, cROS1, jST, rNH, A, jNw, tsolve, states, eqs, inis, sol, t, tmax, tHigh]
```

In[ ]:=
```
d = 1660;
N = 1 * 10^-7 ;
X = 0;

(*jN=jNm N/(N+KN);
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

tmax = 800;

kdRuns = Table[
   Table[
    Table[
     Module[{

        jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, Ni0 = N,
 jNi0 = (jNm N)/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
 jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, tsolve,
 addtimetostatevars, eqs, inis, sol, t, steadyNi, steadyHGrowth, steadySH
 (*make sure to put any intermediate output values in Module*)},


F[ρ_][A_, B_] := (A B (A + B) ρ)/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
(*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar L ;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S]/(1 + cROS1), jCP0},
  {jNi, jNm * Ni / (Ni + KN), jNi0}
 };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
  (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
  {
   S'[t] == (S (jSG - jST) /. addtimetostatevars),
   H'[t] == (H (jHG - jHT) /. addtimetostatevars),
   VH'[t] == fh[t, H'[t]],
   Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
         ( vi * VH) /. addtimetostatevars),
   P'[t] ==
     (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
  }
 ];
(*set initial conditions*)
inis = Join[{
   H[0] == H0,
   S[0] == S0,
   VH[0] == VH0,
   Ni[0] == Ni0,
   P[0] == P0},
  (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
   Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

(*steady state Ni*)
steadyNi = Evaluate[Ni[tmax] /. sol];

(*steady state host growth*)
steadyHGrowth = Evaluate[H'[tmax] / H[tmax] /. sol];

(*steady state S/H*)
steadySH = Evaluate[S[tmax] / H[tmax] /. sol];

{steadyNi, steadyHGrowth, steadySH}
(*record the steady state values as output*)],
      {kp, {150, 175, 200, 225, 250, 275, 300, 325, 350, 375, 400}}],
     (*repeat for the different kp's*)
     {d, {450, 850, 1250, 1650, 2050, 2450, 2850, 3250, 3650, 4050}}
     (*repeat for the different flushing rates*)
    ],
   {P0, {0, kp * (1 - α) * H0}}
   (*repeat with no fish (P0=0) and with fish (P0=kp*(1-α)*H0)*)
  ];
```

```
In[ ]:= (*saving the output of Table[]*)
        Save["kpdRuns", kdRuns ]

In[ ]:= (*recover this output*)
        Get["kpdRuns" ];
```

Get results for flushing rate vs carrying capacity correct form

```
In[ ]:= kdRuns ;(*nested list, innermost is all 3 response variables for {kp1, kp2, ...},
        second level is this repeated for each flushing rate {d1, d2, ...},
        outer levels is fish {P1, P2}, *)
        (*so first seprate the fish vs no fish cases*)
        kdnoP = kdRuns[[1]];
        kdwithP = kdRuns[[2]];
        (*want the differences between the P and no P case*)
        (*Ni*)
        kdNidiff = Transpose[Flatten[kdwithP, 1]][[1]] - Transpose[Flatten[kdnoP, 1]][[1]];
        (*host growth*)
        kdgrowthdiff =
          Transpose[Flatten[kdwithP, 1]][[2]] - Transpose[Flatten[kdnoP, 1]][[2]];
        (*S/H*)
        kdSHdiff = Transpose[Flatten[kdwithP, 1]][[3]] - Transpose[Flatten[kdnoP, 1]][[3]];

        (*turn the results into matrices for plotting*)
        MatrixkdNi = ArrayReshape[kdNidiff, {10, 11}] ;
        (*{n, m} dimensions are {# outermost par values, # innermost par values} *)
        Matrixkdgrowth = ArrayReshape[kdgrowthdiff, {10, 11}] ;
        MatrixkdSH = ArrayReshape[kdSHdiff, {10, 11}] ;
```

Plot results

```
In[ ]:= paddingShm = {{80(*left*), 15(*right*)}, {50(*bottom*), 0(*top*)}};

plotShma = MatrixPlot[Reverse[NXgrowthdiff],
   ImageSize → Full, (*PlotLegends→Automatic, *)ColorFunction →
    (ColorData[{"GrayTones", {0(*Min[MatrixNXgrowth]*), Max[NXgrowthdiff]}}]),
   ColorFunctionScaling → False(*need to specify the colorfunction to match
    the legend specified by BarLegend[], need to add colorfunctionscaling
    false to make the plot colors match these values*), Mesh → True, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   FrameLabel → {{"Prey (C-mol L^-1)", None}, {"Ambient N (mol N L^-1)", None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12],
   FrameTicks → {{{{1, "10x10^-7"}, {2, "7.5x10^-7"}, {3, "5x10^-7"},
       {4, "2.5x10^-7"}, {5, "1x10^-7"}, {6, "0.75x10^-7"},
       {7, "0.5x10^-7"}, {8, "0.25x10^-7"}, {9, "0.1x10^-7"}}, None},
     {{{1, 0}, (*{2, "0.1*10^-7"},*){3, "0.25x10^-7"}, (*{4, "0.5*10^-7"},*)
       {5, "0.75x10^-7"}, (*{6, "1*10^-7"},*){7, "1.25x10^-7"},
       (*{8, "1.5*10^-7"},*){9, "1.75x10^-7"}(*,{10, "2*10^-7"}*)}, None}},
   PlotLabel → "a) dH/Hdt with fish-dH/Hdt without fish",
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     12(*label size*)], ImagePadding → paddingShm];

plotShmb = MatrixPlot[Reverse[Matrixkdgrowth],
   ColorFunction → (ColorData[{"GrayTones", {0, Max[Matrixkdgrowth]}}]),
   ColorFunctionScaling → False, ImageSize → Full, Mesh → True, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   FrameLabel → {{"Fish:host biomass scalar (g C-mol H^-1)", None},
     {"Flushing rate (d^-1)", None}}, FrameTicksStyle →
    Directive[Black, 10(*size of ticks*)], FrameStyle → Directive[Black, 12],
   FrameTicks → {{{{1, "4050"}, {2, "3650"}, {3, "3250"}, {4, "2850"}, {5, "2450"},
       {6, "2050"}, {7, "1650"}, {8, "1250"}, {9, "850"}, {10, "450"}}, None},
     {{{1, 150}, {2, 175}, {3, 200}, {4, 225}, {5, 250}, {6, 275},
       {7, 300}, {8, 325}, {9, 350}, {10, 375}, {11, 400} }, None}},
   PlotLabel → "b) dH/Hdt with fish- dH/Hdt without fish",
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     12(*label size*)], ImagePadding → paddingShm] ;
```

```
In[•]:= plotShmaLeg =
          BarLegend[{GrayLevel, {0(*Min[MatrixNXgrowth]*), Max[NXgrowthdiff]}}];
        plotShmbLeg = BarLegend[{GrayLevel, {0, Max[Matrixkdgrowth]}}];

        Legended[GraphicsGrid[{{plotShma, plotShmb}} , ImageSize → Full,
          Spacings → { {10(*left*), 40 (*labels and left col*)(*left to right*)},
            {0(*top*), 0(*labels and bottom*)(*top to bottom*)}},
          Frame → False(*, AspectRatio→1/GoldenRatio*)],
         {Placed[plotShmaLeg, (*Bottom*){1.425(*left to right position
             is in the center*), 0.5(*top to bottom position*)}],
          Placed[plotShmbLeg, (*Bottom*){0.5(*left to right position is
             in the center*), 0.5(*top to bottom position*)}]}]
```

Out[•]=

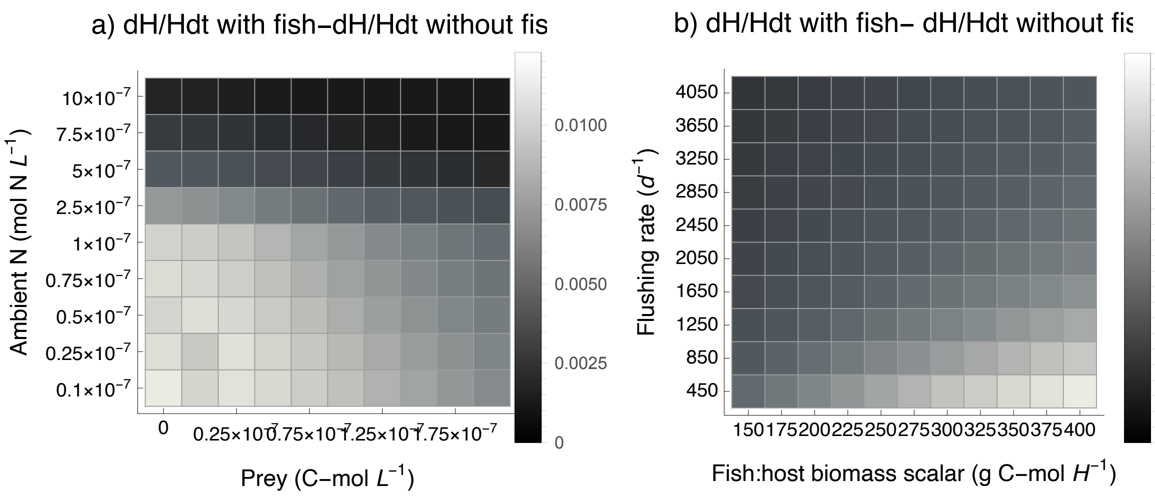


Fig. S3: Fish: host biomass at different fish growth rates

Run the simulations and format results

Run simulations

```
In[•]:= (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN, jeC, jCO2, jL, rCH, rCS, jeL, jNPQ,
         cROS1, jST, rNH, A, jNw, tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh]

In[•]:=
        (* environment: flushing rate and N level*)
        d = 1660;
        N = 1 * 10^-7 (*1*10^-6*);

        X = 0;(*no prey*)
```

```
P0 = kp * (1 - α) * H0;

(*jN=jNm N/(N+KN);
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

tmax = 2000;

(*use nested Table to simulate the model at different levels of
 nitrogen (corresponding to different steady state host growth rates)
 and different fish growth rates*)
PHrpRuns =
  Table[
   Table[
    Module[{

      jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0,
      Ni0 = N, jNi0 = jNm N/(N + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG,
      ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh,
      tsolve, addtimetostatevars, eqs, inis, sol, t, steadyHGrowth, steadyPH
      (*make sure to put any intermediate output values in Module*)},


     F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
     (*same as 1/(1/ρ+1/A+1/B-1/(A+B))*)
     (* helper function for VH*)
     fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

     (* Calculations *)
     jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
     jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
     ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
     jeC = Max[jX - jHG/yC + S ρC/H, 0];
     jCO2 = jeC kCO2;
     jL = A astar L ;
```

```
rCH = (jHT + jHG (1 - yC)/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S]/(1 + cROS1), jCP0},
   {jNi, jNm * Ni/(Ni + KN), jNi0}
  };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          (vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
      Ni[0] == Ni0,
      P[0] == P0},
     (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

   sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
      Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

   (*steady state host growth rate*)
   steadyHGrowth = Evaluate[H'[tmax] / H[tmax] /. sol];

   (*steady state fish to interstitial host biomass ratio*)
   steadyPH = Evaluate[P[tmax] / ((1 - α) * H[tmax]) /. sol];

   {steadyHGrowth, steadyPH}
   (*record the steady state host growth rate and P/H ratio as output*)],

  {rp, {0.01, 0.05, 0.1}}], (*repeat this for different fish growth rates*)
 {N, {0.1 * 10^-7, 0.5 * 10^-7, 1 * 10^-7, 2 * 10^-7, 2.5 * 10^-7, 3 * 10^-7,
   4 * 10^-7, 5 * 10^-7, 6 * 10^-7, 7 * 10^-7, 8 * 10^-7, 9 * 10^-7, 0.1 * 10^-5,
   0.125 * 10^-5, 0.15 * 10^-5, 0.175 * 10^-5, 0.2 * 10^-5, (*0.25*10^-5,*)
   0.3 * 10^-5, (*0.35*10^-5,*)0.4 * 10^-5, (*0.45*10^-5,*)0.5 * 10^-5}}
  (*repeat this for different levels of Nenv*)
];
```

Get the results in the correct form to plot and save output

```
In[ ]:=
(*transpose the output to change the nested list
 from {{rp1 at Nenv1, rp2 at N1, rp3 at N1}, {N2}, ...{Nn}}
 to {{N1, N2, ... Nn at rp1}, {rp2}} *)
PHrpseparate = Transpose[PHrpRuns];
(*fish rp=01 *)
PHrp01 = PHrpseparate⟦1⟧;
separatePHrp01 = Transpose[PHrp01];(*group each output together*)
PHrp01growth = separatePHrp01⟦1⟧;(*dH/Hdt values at each Nenv*)
PHrp01PH = separatePHrp01⟦2⟧;(*P/H values at each Nenv*)
(* fish rp=05*)
PHrp05 = PHrpseparate⟦2⟧;
separatePHrp05 = Transpose[PHrp05];(*group each output together*)
PHrp05growth = separatePHrp05⟦1⟧;(*dH/Hdt values at each Nenv*)
PHrp05PH = separatePHrp05⟦2⟧;(*P/H values at each Nenv*)

(* fish rp=1*)
PHrp1 = PHrpseparate⟦3⟧;
separatePHrp1 = Transpose[PHrp1];(*group each output together*)
PHrp1growth = separatePHrp1⟦1⟧;(*dH/Hdt values at each Nenv*)
PHrp1PH = separatePHrp1⟦2⟧;(*P/H values at each Nenv*)

(*get the lists in a form to
 plot: want to plot P/H as a function of host growth rate*)
PHgrowthrp01Plot = Transpose@{PHrp01growth, PHrp01PH};
PHgrowthrp05Plot = Transpose@{PHrp05growth, PHrp05PH};
PHgrowthrp1Plot = Transpose@{PHrp1growth, PHrp1PH};
```

```
In[ ]:= Save["rp01", PHgrowthrp01Plot]
Save["rp05", PHgrowthrp05Plot]
Save["rp1", PHgrowthrp1Plot]
```

```
In[ ]:= Get["rp01"];
Get["rp05"];
Get["rp1"];
```

Plot results

```
In[ ]:= PHGrowthLeg = LineLegend[{ColorData[97, "ColorList"][[1]],
      ColorData[97, "ColorList"][[2]], ColorData[97, "ColorList"][[3]]},
     {"0.01 d^-1", "0.05 d^-1", "0.1 d^-1"}, LabelStyle → {18},
     LegendMarkers → Automatic, LegendLabel → "Fish growth rate (rP)"];

Legended[ListLinePlot[{PHgrowthrp01Plot, PHgrowthrp05Plot, PHgrowthrp1Plot},
   PlotMarkers → {Automatic, 10},
   (*PlotLabel→"Ratio of fish to host interstitial biomass at
      varying host and fish growth rates",*)ImageSize → Full,
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     18(*label size*)], PlotRange → Full,
   Frame → {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0.00005, 0.0001, 0.00015},None},
     {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
   FrameLabel → {{"Fish: host biomass ratio (g fish C-mol H^-1)", None},
     {"Host specific growth rate (C-mol H C-mol H^-1 d^-1)", None}},
   FrameTicksStyle → Directive[Black, 14(*size of ticks*)],
   FrameStyle → Directive[Black, 18](*axis label size*),
   AspectRatio → 0.7(*PlotLegends→{"rp=0.01","rp=0.05","rp=0.1"},
   PlotMarkers→{Automatic, 6}*)],
  Placed[PHGrowthLeg, {Right(*left to right position is in the center*),
    0.8(*top to bottom position*)}]]
```

Out[∘]=

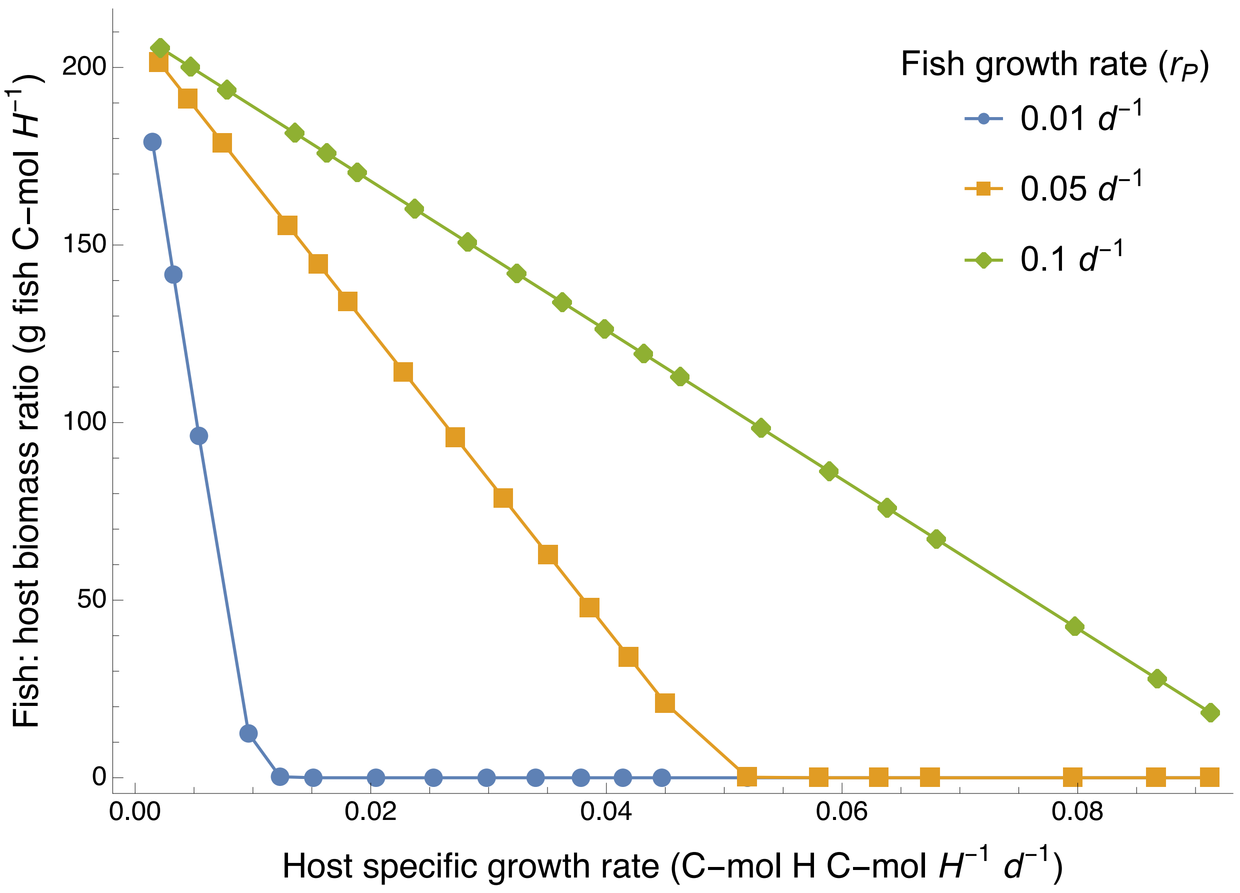


Fig. S4: Goldilocks nitrogen effects at different light magnitudes

Run simulations and format results

Run the simulations

In[∘]:=
```
(*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

In[∘]:=
```

(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10 ^ -7 (*1*10^-6*);

X = 0;
```

```
P0 = 0;

(*light function values*)
tStartStress = 600;
tHigh = 30;
(*LHigh=35;*)

(*jN=jNm N/(N+KN);
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

tmax = 800;

(*use nested Table to simulate the model with light pulses
 of different magnitudes and at different levels of Nenv*)

NRangeRuns =
  Table[
   Table[
    Module[{

      jX = jXm X/(X + KX), jN = jNm N/(N + KN), jHT = j0HT, rNS = σNS nNS j0ST,
      VH0 = kv * H0, Ni0 = N, jNi0 = jNm N/(N + KN), states, rNH, S, H, VH, P,
      Ni, jNi, ρC, jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ,
      cROS1, jSG, jST, jNw, F, fh, LfunH, tsolve, addtimetostatevars,
      eqs, inis, sol, t, tEvals, minGrowth, minSSU, minHSU(*, bleached*)
      (*make sure to put any intermediate output values in Module*)},
     (*light function*)
     LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
       LHigh * HeavisideTheta[t - (tStartStress + tHigh)];
     F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
     (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
     (* helper function for VH*)
     fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

     (* Calculations *)
     jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
```

```
jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar LfunH ;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
     }
    ];
   (*set initial conditions*)
   inis = Join[{
      H[0] == H0,
      S[0] == S0,
      VH[0] == VH0,
      Ni[0] == Ni0,
      P[0] == P0},
     (#[[1]][0] == #[[3]]) & /@ tsolve];

   sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
      Method → {"EquationSimplification" → "Residual"}][[1]];

   (*minimum host growth rate: see if growth ever went negative*)
   (*evaluate host growth rate at all
    points from the start of stress to the end of the run
    (EXCEPT when t is exactly equal to tStartStress and tStartStress +
      tHigh because at these points one of the HeavisideTheta functions
       has an argument of 0 which is not evaluated so the growth
       values are inaccurate) and record the minimum of these values*)
   tEvals = Flatten[{Table[t, {t, tStartStress + 1, tStartStress + tHigh - 1}],
      Table[t, {t, tStartStress + tHigh + 1, tmax}]}];
   minGrowth = Min[Flatten[Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tEvals}]]] ;

   (*min S SU C-lim*)
   minSSU = Min[Flatten[
      Table[Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) /
                nNS]] /. addtimetostatevars /. sol}, {t, tEvals}]]] ;

   (*min H SU C-lim*)
   minHSU =
    Min[Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[
               jHGm, ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /.
            addtimetostatevars /. sol}, {t, tEvals}]]] ;

   {minGrowth, minSSU, minHSU} (*output*)],

  {LHigh, {25, 26, 27}}], (*repeat the above for 3 different LHigh values*)
 {N, {0.1 * 10 ^ -7, 1 * 10 ^ -7, 2 * 10 ^ -7, 3 * 10 ^ -7, 3.6 * 10 ^ -7, 3.7 * 10 ^ -7, 4 * 10 ^ -7,
   5 * 10 ^ -7, 6 * 10 ^ -7, 7.2 * 10 ^ -7, 7.3 * 10 ^ -7, 8 * 10 ^ -7, 9 * 10 ^ -7, 1 * 10 ^ -6,
   2 * 10 ^ -6, 2.5 * 10 ^ -6, 2.6 * 10 ^ -6, 3 * 10 ^ -6, 4 * 10 ^ -6, 5 * 10 ^ -6,
      5.1 * 10 ^ -6, 6 * 10 ^ -6}}(*repeat all this for different Nenv*)
   ];
```

Get the results in the correct form to plot

```
In[]:= (*transpose the output to change the nested list
        from {{L1 at N1, L2 at N1, L3 at N1}, {N2}, ...{Nn}}
        to {{N1, N2, ... Nn at L1}, {L2}, {L3}} *)
       NseparateL = Transpose[NRangeRuns];
       (*lowest light*)
       LowL = NseparateL[[1]];
       (*mid light*)
       MidL = NseparateL[[2]];
       (*highest light*)
       HighL = NseparateL[[3]];

       (*separate each response variable*)
       (*low light*)
       LowLgrowth = Transpose[LowL][[1]]; (*min host growth*)
       LowLSSU = Transpose[LowL][[2]];(*min S SU lim*)
       LowLHSU = Transpose[LowL][[3]];(*min H SU lim*)

       (*mid light*)
       MidLgrowth = Transpose[MidL][[1]]; (*min host growth*)
       MidLSSU = Transpose[MidL][[2]];(*min S SU lim*)
       MidLHSU = Transpose[MidL][[3]];(*min H SU lim*)

       (*high light*)
       HighLgrowth = Transpose[HighL][[1]]; (*min host growth*)
       HighLSSU = Transpose[HighL][[2]];(*min S SU lim*)
       HighLHSU = Transpose[HighL][[3]];(*min H SU lim*)

       (*list of Nenv values*)
       NVals = {0.1 * 10 ^ -7, 1 * 10 ^ -7, 2 * 10 ^ -7, 3 * 10 ^ -7,
          3.6 * 10 ^ -7, 3.7 * 10 ^ -7, 4 * 10 ^ -7, 5 * 10 ^ -7, 6 * 10 ^ -7, 7.2 * 10 ^ -7,
          7.3 * 10 ^ -7, 8 * 10 ^ -7, 9 * 10 ^ -7, 1 * 10 ^ -6, 2 * 10 ^ -6, 2.5 * 10 ^ -6,
          2.6 * 10 ^ -6, 3 * 10 ^ -6, 4 * 10 ^ -6, 5 * 10 ^ -6, 5.1 * 10 ^ -6, 6 * 10 ^ -6};

       (*use Table to get the lists in the form to plot*)
       NLightLists =
         Table[Module[{PlotList}, PlotList = Transpose@{NVals, list}; PlotList],
          {list, {LowLgrowth, LowLSSU, LowLHSU, MidLgrowth,
            MidLSSU, MidLHSU, HighLgrowth, HighLSSU, HighLHSU}}];
```

Plot the results

```
In[]:= (*line at y=0*)
       zeros = ConstantArray[0, Length[NVals]];
```

```
zeroLine = Transpose@{NVals, zeros};

paddingS3 = {{80(*left*), 15(*right*)}, {50(*bottom*), 10(*top*)}};
(*host growth rate*)
plotS3a = ListLinePlot[{NLightLists⟦1⟧, NLightLists⟦4⟧, NLightLists⟦7⟧, zeroLine},
   PlotLabel → "a) Minimum host specific growth rate", PlotRange → Full,
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     12(*label size*)], ImagePadding → paddingS3, ImageSize → Full,
   PlotStyle → {ColorData[97, "ColorList"]⟦4⟧, ColorData[97, "ColorList"]⟦5⟧,
     {ColorData[97, "ColorList"]⟦8⟧}, Directive[Black, Thickness[0.001]]}, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
     {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
   FrameLabel → {{"C-mol H C-mol H^-1 d^-1 ", None}, {None, None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12]
   (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*S SU limitation*)
plotS3c = ListLinePlot[{NLightLists⟦2⟧, NLightLists⟦5⟧, NLightLists⟦8⟧, zeroLine},
   PlotLabel → "c) Minimum symbiont biomass C-/N-limitation", PlotRange → Full,
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     12(*label size*)], ImagePadding → paddingS3, ImageSize → Full,
   PlotStyle → {ColorData[97, "ColorList"]⟦4⟧, ColorData[97, "ColorList"]⟦5⟧,
     {ColorData[97, "ColorList"]⟦8⟧}, Directive[Black, Thickness[0.001]]}, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
     {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
   FrameLabel → {{"Relative", None}, {"Ambient nitrogen (mol N L^-1)", None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12]
   (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*H SU limitation*)
plotS3b = ListLinePlot[{NLightLists⟦3⟧, NLightLists⟦6⟧, NLightLists⟦9⟧, zeroLine},
   PlotLabel → "b) Minimum host biomass C-/N-limitation",
   PlotRange → Full, LabelStyle → Directive[FontColor → Black,
     FontFamily → "Helvetica", 12(*label size*)], ImagePadding → paddingS3,
   ImageSize → Full, PlotStyle → {ColorData[97, "ColorList"]⟦4⟧,
     ColorData[97, "ColorList"]⟦5⟧, ColorData[97, "ColorList"]⟦8⟧,
     Directive[Black, Thickness[0.001]](*,Directive[Black, Thin]*)}, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
  (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
    {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
  FrameLabel → {{"Relative", None}, {"Ambient nitrogen (mol N L^-1)", None}},
  FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
  FrameStyle → Directive[Black, 12]
  (*axis label size*), AspectRatio → 1 / GoldenRatio];
```

```
In[]:= GoldiLLeg =
   LineLegend[{ColorData[97, "ColorList"][[4]], ColorData[97, "ColorList"][[5]],
     ColorData[97, "ColorList"][[8]]}, {"40", "41", "42"}, LegendLabel →
     "Light pulse magnitude \n(mol photons m^-2 d^-1)", LabelStyle → {16}];

SGoldiLFig =
  Legended[GraphicsGrid[{{plotS3a, plotS3b}, {plotS3c}}, ImageSize → Full,
    Spacings → { {0(*left*), -140(*labels and left col*)(*left to right*)}, {0
       (*top*), 0(*middle 2 rows*) (*labels and bottom*)(*top to bottom*)}},
    Frame → False(*, AspectRatio→1/GoldenRatio*)(*, Alignment→ Bottom*)
    (*align the grid at the bottom of its frame*)],
   Placed[GoldiLLeg, (*Bottom*){Right(*left to right position
      is in the center*), 0.25(*top to bottom position*)}]]
```

Out[]=

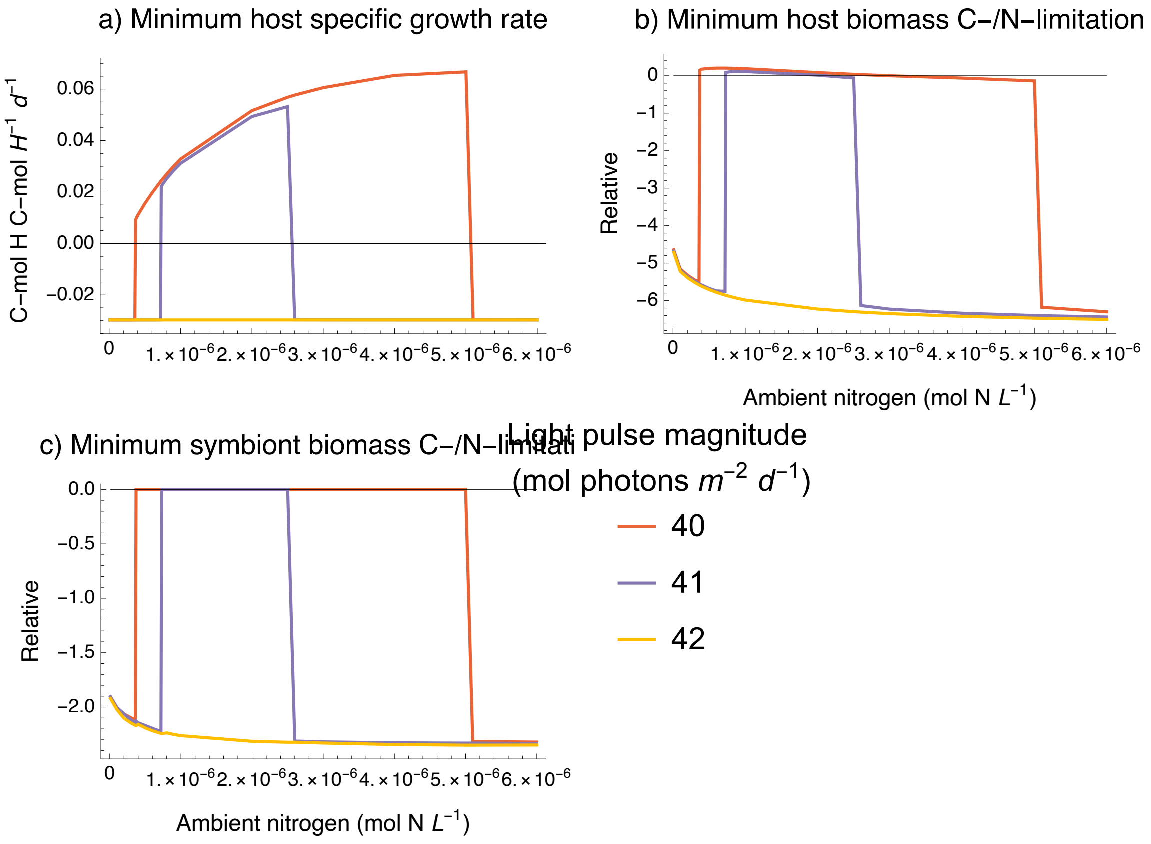


Fig. S5: Goldilocks effects of nitrogen at different levels of prey

Run simulations and format results

Run the simulations

```
In[ ]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[ ]:=
d = 1660;
N = 1 * 10 ^ -7 (*1*10^-6*);

X = 0;
P0 = 0;

(*light function values*)
tStartStress = 600;
tHigh = 30;
LHigh = 27;(*light pulse magnitude;
this is the highest value used in the above simulations where LHigh was varied*)

(*jN= jNm N / (N+KN) ;
Ni0 = N;
jNi0 = jNm N / (N+KN) ;*)

tmax = 800;

(*use nested Table to simulate the model with
 different levels of prey and at different levels of Nenv*)
NRangeRunsX =
  Table[
   Table[
    Module[{
```

```
 jX = jXm X/(X + KX), jN = jNm ℕ/(ℕ + KN), jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0,  Ni0  = ℕ,
 jNi0  = jNm ℕ/(ℕ + KN), states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN, jeC,
 jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh, LfunH, tsolve,
 addtimetostatevars, eqs, inis, sol, t, tEvals, minGrowth, minSSU, minHSU
 (*make sure to put any intermediate output values in Module*)},
(*light function*)
LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
  LHigh * HeavisideTheta[t - (tStartStress + tHigh)];
F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
(*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + S ρC/H, 0];
jCO2 = jeC kCO2;
jL = A astar LfunH ;
rCH = (jHT + jHG (1 - yC)/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
  {ρC, jCP - jSG yC^-1, ρC0},
```
$$\left\{jCP,\ \frac{F[jCPm]\left[jL\ yCL,\ rCS + \frac{H\ (jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$
$$\left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}$$
```
 };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
  (#⟦1⟧ '[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
  {
   S '[t] == (S (jSG - jST) /. addtimetostatevars),
   H '[t] == (H (jHG - jHT) /. addtimetostatevars),
   VH '[t] == fh[t, H '[t]],
   Ni '[t] == (d * ( N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
         ( vi * VH) /. addtimetostatevars),
   P '[t] ==
    (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
  }
 ];
(*set initial conditions*)
inis = Join[{
   H[0] == H0,
   S[0] == S0,
   VH[0] == VH0,
   Ni[0] == Ni0,
   P[0] == P0},
  (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
   Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

(*minimum host growth rate: see if growth ever went negative*)
(*evaluate host growth rate at all
 points from the start of stress to the end of the run
 (EXCEPT when t is exactly equal to tStartStress and tStartStress +
   tHigh because at these points one of the HeavisideTheta functions
    has an argument of 0 which is not evaluated so the growth
    values are inaccurate) and record the minimum of these values*)
tEvals = Flatten[{Table[t, {t, tStartStress + 1, tStartStress + tHigh - 1}],
   Table[t, {t, tStartStress + tHigh + 1, tmax}]}];
```

```
    minGrowth = Min[Flatten[Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tEvals}]]] ;

    (*min S SU C-lim*)
    minSSU = Min[Flatten[
       Table[Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) /
                  nNS]] /. addtimetostatevars /. sol}, {t, tEvals}]]] ;

    (*min H SU C-lim*)
    minHSU =
     Min[Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[
                jHGm, ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /.
             addtimetostatevars /. sol}, {t, tEvals}]]] ;

    {minGrowth, minSSU, minHSU}(*record output*)],

   {X, {0, 5 * 10 ^ -8, 1 * 10 ^ -7}}],
  (*iterate the above for 3 different prey levels*)
  {N, {0.1 * 10 ^ -7, 1 * 10 ^ -7, 2 * 10 ^ -7, 3 * 10 ^ -7, 4 * 10 ^ -7, 5 * 10 ^ -7, 6.1 * 10 ^ -7,
    6.2 * 10 ^ -7, 7 * 10 ^ -7, 8 * 10 ^ -7, 9 * 10 ^ -7, 1 * 10 ^ -6, 1.76 * 10 ^ -6, 1.77 * 10 ^ -6,
    2 * 10 ^ -6, 2.57 * 10 ^ -6, 2.58 * 10 ^ -6, 3 * 10 ^ -6, 4 * 10 ^ -6, 5 * 10 ^ -6, 6 * 10 ^ -6}}
   (*repeat all this for different Nenv;
  for prey=0 the host bleaches at all Nenv,
  for prey=5*10^-8 just right range is from 6.2*10^-7 to 1.76*10^-6,
  for prey=1*10^-7 just right range is from<1*10^-8 to 2.57*10^-6*)
 ];
```

Get the results in the correct form to plot

```
In[ ]:= (*transpose the output to change the nested list
         from {{X1 at N1, X2 at N1, X3 at N1}, {N2}, ...{Nn}}
         to {{N1, N2, ... Nn at X1}, {X2}, {X3}} *)
        NseparateX = Transpose[NRangeRunsX];
        (*lowest X*)
        LowX = NseparateX[[1]];
        (*mid X*)
        MidX = NseparateX[[2]];
        (*highest X*)
        HighX = NseparateX[[3]];

        (*separate each response variable*)
        (*low X*)
        LowXgrowth = Transpose[LowX][[1]]; (*min host growth*)
        LowXSSU = Transpose[LowX][[2]];(*min S SU lim*)
        LowXHSU = Transpose[LowX][[3]];(*min H SU lim*)

        (*mid X*)
        MidXgrowth = Transpose[MidX][[1]]; (*min host growth*)
        MidXSSU = Transpose[MidX][[2]];(*min S SU lim*)
        MidXHSU = Transpose[MidX][[3]];(*min H SU lim*)

        (*high X*)
        HighXgrowth = Transpose[HighX][[1]]; (*min host growth*)
        HighXSSU = Transpose[HighX][[2]];(*min S SU lim*)
        HighXHSU = Transpose[HighX][[3]];(*min H SU lim*)

        (*list of Nenv values*)
        NVals2 = {0.1 * 10^-7, 1 * 10^-7, 2 * 10^-7, 3 * 10^-7,
           4 * 10^-7, 5 * 10^-7, 6.1 * 10^-7, 6.2 * 10^-7, 7 * 10^-7, 8 * 10^-7,
           9 * 10^-7, 1 * 10^-6, 1.76 * 10^-6, 1.77 * 10^-6, 2 * 10^-6,
           2.57 * 10^-6, 2.58 * 10^-6, 3 * 10^-6, 4 * 10^-6, 5 * 10^-6, 6 * 10^-6};

        (*use Table to get the lists in the form to plot*)
        NPreyLists =
          Table[Module[{PlotList}, PlotList = Transpose@{NVals2, list}; PlotList],
           {list, {LowXgrowth, LowXSSU, LowXHSU, MidXgrowth,
             MidXSSU, MidXHSU, HighXgrowth, HighXSSU, HighXHSU}}];
```

Plot the results

```
In[ ]:=
        (*line at y=0*)
```

```
zeros2 = ConstantArray[0, Length[NVals2]];
zeroLine2 = Transpose@{NVals2, zeros2};

paddingS4 = {{80(*left*), 15(*right*)}, {50(*bottom*), 10(*top*)}};
(*host growth rate*)
plotS4a = ListLinePlot[{NPreyLists[[1]], NPreyLists[[4]], NPreyLists[[7]], zeroLine2},
   PlotLabel → "a) Minimum host specific growth rate", PlotRange → Full,
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     12(*label size*)], ImagePadding → paddingS3, ImageSize → Full,
   PlotStyle → {ColorData[97, "ColorList"][[8]], ColorData[97, "ColorList"][[5]],
     {ColorData[97, "ColorList"][[4]]}, Directive[Black, Thickness[0.001]]}, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
     {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
   FrameLabel → {{"C-mol H C-mol H^-1 d^-1 ", None}, {None, None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12]
   (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*S SU limitation*)
plotS4c = ListLinePlot[{NPreyLists[[2]], NPreyLists[[5]], NPreyLists[[8]], zeroLine2},
   PlotLabel → "c) Minimum symbiont biomass C-/N-limitation", PlotRange → Full,
   LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
     12(*label size*)], ImagePadding → paddingS3, ImageSize → Full,
   PlotStyle → {ColorData[97, "ColorList"][[8]], ColorData[97, "ColorList"][[5]],
     {ColorData[97, "ColorList"][[4]]}, Directive[Black, Thickness[0.001]]}, Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
     {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
   FrameLabel → {{"Relative", None}, {"Ambient nitrogen (mol N L^-1)", None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12]
   (*axis label size*), AspectRatio → 1 / GoldenRatio];

(*H SU limitation*)
plotS4b = ListLinePlot[{NPreyLists[[3]], NPreyLists[[6]], NPreyLists[[9]], zeroLine2},
   PlotLabel → "b) Minimum host biomass C-/N-limitation",
   PlotRange → Full, LabelStyle → Directive[FontColor → Black,
     FontFamily → "Helvetica", 12(*label size*)], ImagePadding → paddingS3,
   ImageSize → Full, PlotStyle → {ColorData[97, "ColorList"][[8]],
     ColorData[97, "ColorList"][[5]], ColorData[97, "ColorList"][[4]],
     Directive[Black, Thickness[0.001]](*,Directive[Black, Thin]*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
      {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
    FrameLabel → {{"Relative", None}, {"Ambient nitrogen (mol N L^-1)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12]
    (*axis label size*), AspectRatio → 1 / GoldenRatio];
```

```
In[•]:= GoldiXLeg =
      LineLegend[{ColorData[97, "ColorList"]⟦8⟧, ColorData[97, "ColorList"]⟦5⟧,
        ColorData[97, "ColorList"]⟦4⟧}, {"0", "0.5*10^-7", "1*10^-7"},
       LegendLabel → "Prey (C-mol L^-1)", LabelStyle → {16}];

     SGoldiXFig =
      Legended[GraphicsGrid[{{plotS4a, plotS4b}, {plotS4c}}, ImageSize → Full,
        Spacings → { {0(*left*), -140(*labels and left col*)(*left to right*)}, {0
           (*top*), 0(*middle 2 rows*) (*labels and bottom*)(*top to bottom*)}},
        Frame → False(*, AspectRatio→1/GoldenRatio*)(*, Alignment→ Bottom*)
        (*align the grid at the bottom of its frame*)],
       Placed[GoldiXLeg, (*Bottom*){Right(*left to right position
          is in the center*), 0.25(*top to bottom position*)}]]
```

Out[ ]=

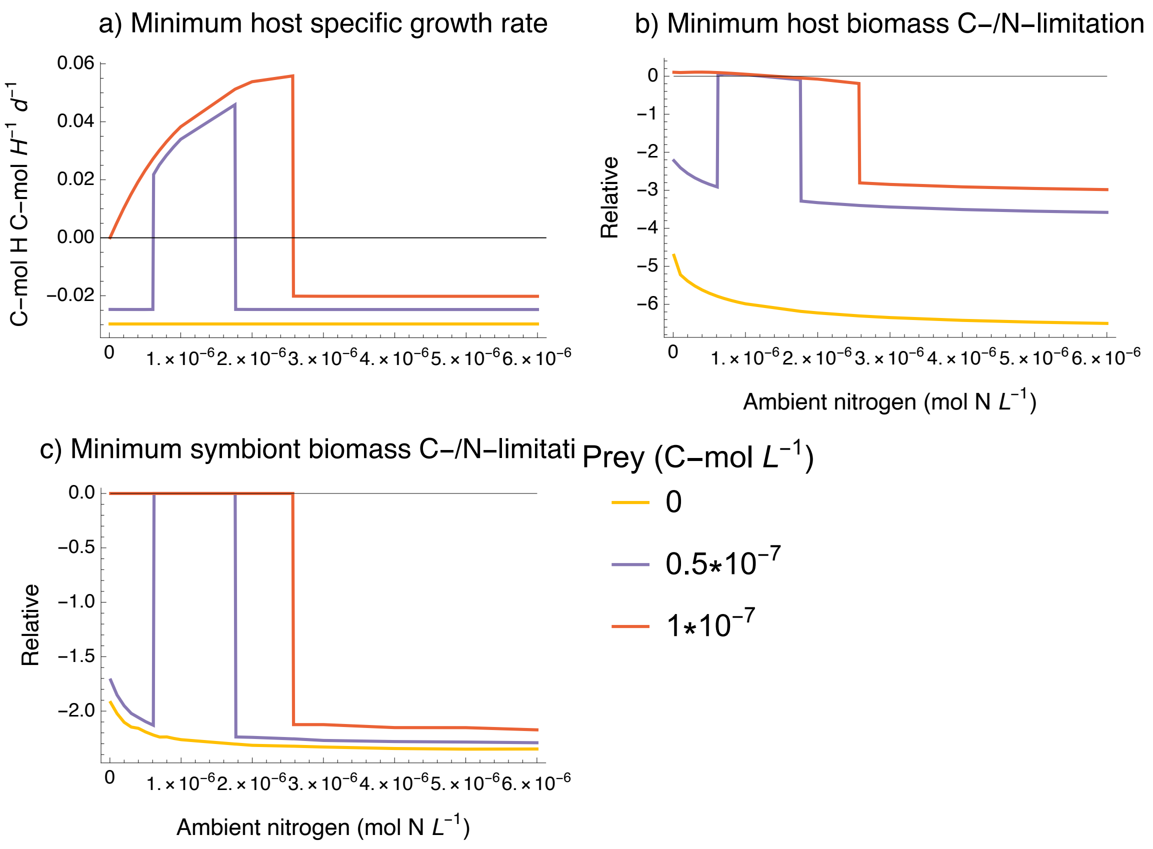


Fig. S6: Effect of nitrogen in “too low” range on bleached hosts

Run simulations and format results

In[ ]:=

```
(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10 ^ -7 (*1*10^-6*);

(*no prey or fish*)
X = 0(*1*10^-7*);
P0 = 0(*kp*(1-α)*H0*);
jX = jXm X / (X + KX);

(*jN= jNm N / (N+KN);
Ni0 = N;
```

```
jNi0 = $\frac{jNm\ N}{N+KN}$;*)

(*light function parameters*)
tStartStress = 600;
LHigh = 25;
tHigh = 30;
tmax = 1000;

(*use Table to simulate the model at different Nenv levels*)
NRuns = Table[
   Module[{

     (*jX=$\frac{jXm\ X}{X+KX}$,*) jN = $\frac{jNm\ N}{N + KN}$, jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0,
     Ni0 = N, jNi0 = $\frac{jNm\ N}{N + KN}$, states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG,
     ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh,
     LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t, evalSSU, evalHSU
     (*make sure to put any intermediate output values in Module*)},
    (*light function*)
    LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
      LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

    F[ρ_][A_, B_] := $\frac{A\ B\ (A + B)\ \rho}{A^2\ B + A\ B^2 + A^2\ \rho + A\ B\ \rho + B^2\ \rho}$;
    (*same as $\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$*)
    (* helper function for VH*)
    fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

    (* Calculations*)
    jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH$^{-1}$];
    jSG = F[jSGm][jCP yC, $\frac{rNS + \frac{H\ \rho N}{S}}{nNS}$];
    ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN$^{-1}$*), 0];
    jeC = Max[jX - $\frac{jHG}{yC}$ + $\frac{S\ \rho C}{H}$, 0];
    jCO2 = jeC kCO2;
    jL = A astar LfunH;
    rCH = (jHT + $\frac{jHG\ (1 - yC)}{yC}$) σCH;
```

```
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
  {ρC, jCP - jSG yC^-1, ρC0},
  {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S]/(1 + cROS1), jCP0},
  {jNi, jNm * Ni/(Ni + KN), jNi0}
 };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
  (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
  {
   S'[t] == (S (jSG - jST) /. addtimetostatevars),
   H'[t] == (H (jHG - jHT) /. addtimetostatevars),
   VH'[t] == fh[t, H'[t]],
   Ni'[t] == (d * ( ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
         ( vi * VH) /. addtimetostatevars),
   P'[t] ==
    (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
  }
 ];
(*set initial conditions*)
inis = Join[{
   H[0] == H0,
   S[0] == S0,
   VH[0] == VH0,
   Ni[0] == Ni0,
   P[0] == P0},
      (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

    sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
       {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

    (*min S SU C-lim*)
    evalSSU =
     Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /.
          addtimetostatevars /. sol};

    (*min H SU C-lim*)
    evalHSU =
     Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm, ((1 - α) * jNi +
              α * jN + nNX jX + rNH) nNH ^ - 1]] /. addtimetostatevars /. sol} ;

    {sol, evalSSU, evalHSU}(*save the whole simulation
     as output and the time series of S and H SU limitations*)],

   {N, {1 * 10 ^ - 7, 3 * 10 ^ - 7, 5 * 10 ^ - 7(*4*10^-6,6*10^-6,8*10^-6*)}}];
```

```
In[ ]:=
     NSeparate = Transpose[NRuns];
```

```
In[ ]:= Sols = ToExpression[str[NSeparate⟦1⟧]];
```

```
In[ ]:= HSUev = ToExpression[str[Flatten[NSeparate⟦2⟧]]];
```

```
In[ ]:= SSUev = ToExpression[str[Flatten[NSeparate⟦3⟧]]];
```

plot results

```
In[ ]:= paddingS6 = {{80(*left*), 15(*right*)}, {50(*bottom*), 10(*top*)}};

     plotS6a =
       Show[Plot[{Evaluate[{H'[t] / H[t]} /. Sols⟦1⟧], Evaluate[{H'[t] / H[t]} /. Sols⟦2⟧],
          Evaluate[{H'[t] / H[t]} /. Sols⟦3⟧]}, {t, 590, tmax},
         PlotLabel → "a) Host specific growth rate", PlotRange → Full,
         AxesOrigin → {590, 0}, LabelStyle → Directive[FontColor → Black,
           FontFamily → "Helvetica", 12(*label size*)], ImagePadding → paddingS6,
         ImageSize → Full, (*PlotStyle→{ColorData[97,"ColorList"]⟦8⟧,
           ColorData[97,"ColorList"]⟦5⟧, {ColorData[97,"ColorList"]⟦4⟧},
           Directive[Black, Thickness[0.001]]},*)Frame →
          {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
      {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
    FrameLabel → {{"C-mol H C-mol H^-1 d^-1 ", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12]
    (*axis label size*), AspectRatio → 1 / GoldenRatio],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, tmax},
    {y, -1000, 1000}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]] ;


plotS6b = Show[Plot[{HSUev⟦1⟧, HSUev⟦2⟧, HSUev⟦3⟧},
    {t, 590, tmax}, PlotLabel → "b) Host biomass C-/N-limitation",
    PlotRange → Full, AxesOrigin → {590, 0}, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS6, ImageSize → Full,
    (*PlotStyle→{ColorData[97,"ColorList"]⟦8⟧, ColorData[97,"ColorList"]⟦5⟧,
      {ColorData[97,"ColorList"]⟦4⟧},Directive[Black, Thickness[0.001]]},*)
    Frame → {{True(*left*), False(*right*)}, {True(*bottom*), False
       (*top*)}}, (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
      {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
    FrameLabel → {{"Relative", None}, {"Time (d)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12]
    (*axis label size*), AspectRatio → 1 / GoldenRatio],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, tmax},
    {y, -1000, 1000}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]] ;

plotS6c = Show[Plot[{SSUev⟦1⟧, SSUev⟦2⟧, SSUev⟦3⟧},
    {t, 590, tmax}, PlotLabel → "c) Symbiont biomass C-/N-limitation",
    PlotRange → Full, AxesOrigin → {590, 0}, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS6, ImageSize → Full,
    (*PlotStyle→{ColorData[97,"ColorList"]⟦8⟧, ColorData[97,"ColorList"]⟦5⟧,
      {ColorData[97,"ColorList"]⟦4⟧},Directive[Black, Thickness[0.001]]},*)
    Frame → {{True(*left*), False(*right*)}, {True(*bottom*), False
       (*top*)}}, (*FrameTicks→(*True*){{{0, 0.01,0.02},None},
      {(*None*){{0, "0"},{1*10^-7, "1*10^-7"},{2*10^-7, "2*10^-7"}},None}},*)
    FrameLabel → {{"Relative", None}, {"Time (d)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12]
```

```
      (*axis label size*), AspectRatio → 1 / GoldenRatio],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, tmax},
     {y, -1000, 1000}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];
```

In[◦]:=
```
LowNLeg = LineLegend[{ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]],
     ColorData[97, "ColorList"][[3]]}, {"1*10^-7", "3*10^-7", "5*10^-7"},
    LegendLabel → "Ambient N (mol L^-1)", LabelStyle → {16}];

SLowNBleachFig =
  Legended[GraphicsGrid[{{plotS6a, plotS6b}, {plotS6c}}, ImageSize → Full,
    Spacings → { {0(*left*), -120(*labels and left col*)(*left to right*)}, {0
       (*top*), 0(*middle 2 rows*) (*labels and bottom*)(*top to bottom*)}},
    Frame → False(*, AspectRatio→1/GoldenRatio*)(*, Alignment→ Bottom*)
    (*align the grid at the bottom of its frame*)],
   Placed[LowNLeg, (*Bottom*){Right(*left to right position is in the center*),
     0.25(*top to bottom position*)}]]
```

Out[◦]=

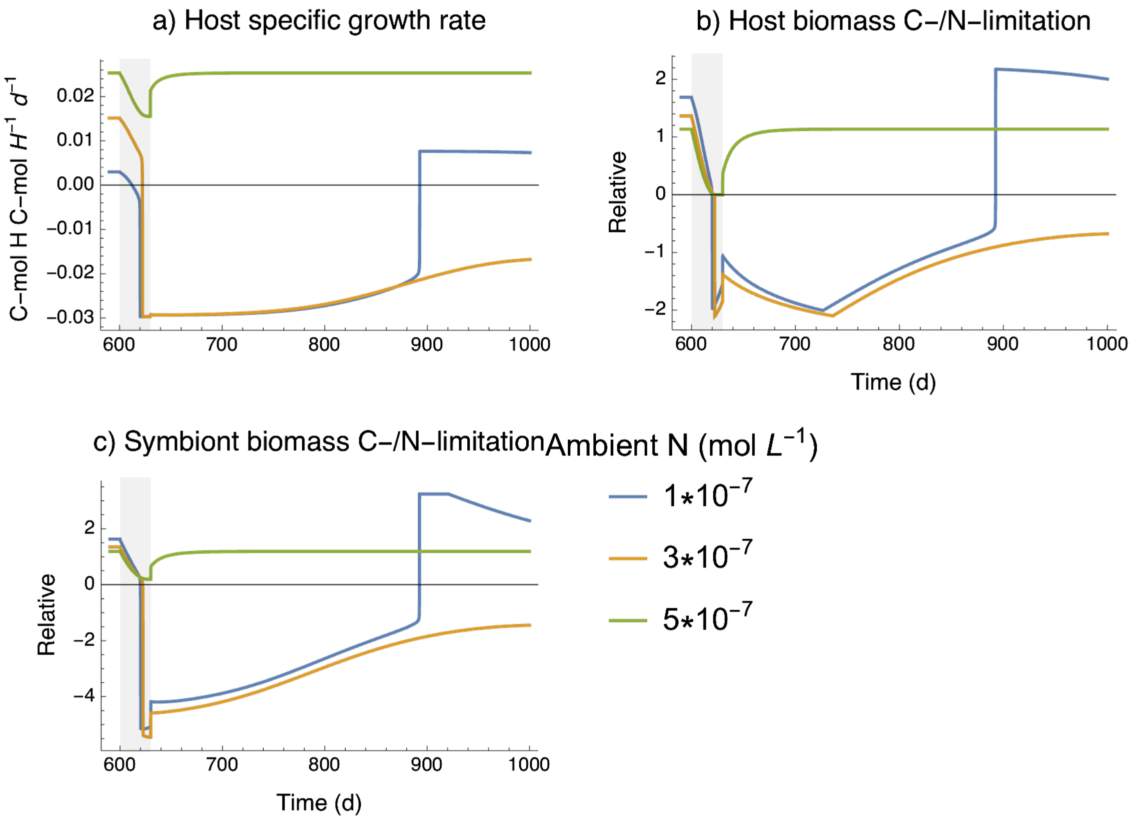


Fig. S7: Heat maps with higher ambient nitrogen and

lower flushing rate

Run simulations and format results

Low Nenv, low flushing rate, no fish

In[•]:=
```
(*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

In[•]:=
```
(* No prey no fish*)
X = 0(*1*10^-7*);
P0 = 0(*kp*(1-α)*H0*);
jX = jXm X / (X + KX);

(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10^-7;

jN = jNm N / (N + KN);
Ni0 = N;
jNi0 = jNm N / (N + KN);

(*light function parameters*)
tStartStress = 600;

(*use nested Table to run simulations at different values of LHigh and tHigh*)
stressRunsnoPnoX1 = Table[
  Table[
   Module[{

     (*jX=jXm X/(X+KX), jN=jNm N/(N+KN),*)jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, (*Ni0 = N,
     jNi0 = jNm N/(N+KN),*)states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN,
     jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh,
     LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t, tEvals, addtmax,
     minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal, tmax
```

```
 (*make sure to put any intermediate output values in Module*)},
tmax = tStartStress + tHigh + 100;
(*light function*)
LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
  LHigh * HeavisideTheta[t - (tStartStress + tHigh)];
(*light is L for t<tStartStress and L+LHigh if tStartStress ≤
 t < tStartStress+ tHigh and L if t≥ tStartStress+ tHigh *)
```

$$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2\,B + A\,B^2 + A^2\,\rho + A\,B\,\rho + B^2\,\rho};$$

(*same as $\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$*)

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\,(\rho C\,S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH)\,nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\,yC,\ \frac{rNS + \frac{H\,\rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH - nNH\,jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

```
jCO2 = jeC kCO2;
jL = A astar * LfunH;
```

$$rCH = \left(jHT + \frac{jHG\,(1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\,\left(j0ST + (1-yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\,cROS1)\,j0ST;$$

$$rNH = \sigma NH\,nNH\,jHT;$$

$$A = 1.256307 + 1.385969\,e^{-6.479055\,S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \{$$

$$\{\rho C,\ jCP - jSG\,yC^{-1},\ \rho C0\},$$

```
  {jCP, F[jCPm] [jL yCL, rCS + H (jCO2+rCH) / S] / (1 + cROS1), jCP0},
  {jNi, (jNm * Ni) / (Ni + KN), jNi0}
 };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
           (vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
    Method → {"EquationSimplification" → "Residual"}]⟦1⟧;
(*outputs to record*)
(*minimum host growth rate: see if growth ever went negative*)
(*evaluate host growth rate at all
 points from the start of stress to the end of the run
  (EXCEPT when t is exactly equal to tStartStress and tStartStress +
    tHigh because at these points one of the HeavisideTheta functions
     has an argument of 0 which is not evaluated so the growth
     values are inaccurate) and record the minimum of these values*)
tEvals = Flatten[{Table[t, {t, tStartStress + 1, tStartStress + tHigh - 1}],
    Table[t, {t, tStartStress + tHigh + 1, tmax}]}];
minGrowth = Min[Flatten[Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tEvals}]]] ;
```

```
        (*min S SU C-lim*)
        minSSU = Min[Flatten[
           Table[Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) /
                     nNS]] /. addtimetostatevars /. sol}, {t, tEvals}]]] ;

        (*min H SU C-lim*)
        minHSU =
         Min[Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[
                    jHGm, ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /.
                 addtimetostatevars /. sol}, {t, tEvals}]]] ;

        (*growth and SU limitations 100 days after end of stress (tmax)*)
        addtmax = ((# → #[tmax]) & /@ states);
        growthFinal = Evaluate[{H '[tmax] / H[tmax]} /. sol];
        SSUfinal = Evaluate@Flatten@
           {Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /. addtmax /. sol};
        HSUfinal = Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm,
                 ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /. addtmax /. sol};
        {minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal} (*output*) ],

       {LHigh, {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35}} ],
      (*pulse magnitudes tested (note total magnitude is LHigh + L) *)
      {tHigh, 7, 35, 2}
      (*pulse durations tested: from 7 to 35 in intervals of 2 days*)
     ];
```

```
In[∘]:= (*save the output*)
        Save["stressRunsnoPnoX1", stressRunsnoPnoX1 ]
```

Low Nenv, low flushing rate, with fish

```
In[∘]:= (* ----------------------------------------------------------------------------
                                                 -----------------------------------------*
         )
        (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
         jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
         tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=
        (* No prey with fish*)
        X = 0(*1*10^-7*);
        P0 = (*0*) kp * (1 - α) * H0;
```

```
jX = jXm X / (X + KX);

(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10^-7(*1*10^-6*);

jN = jNm N / (N + KN);
Ni0 = N;
jNi0 = jNm N / (N + KN);

(*light function parameters*)
tStartStress = 600;

stressRunsPnoX1 = Table[
  Table[
   Module[{

     (*jX=jXm X/(X+KX), jN=jNm N/(N+KN),*)jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, (*Ni0 = N,
     jNi0 = jNm N/(N+KN),*)states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN,
     jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh,
     LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t, tEvals, addtmax,
     minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal, tmax
     (*make sure to put any intermediate output values in Module*)},
    tmax = tStartStress + tHigh + 100;
    (*light function*)
    LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
      LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

    F[ρ_][A_, B_] := A B (A + B) ρ / (A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
    (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
    (* helper function for VH*)
    fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];
    (* Calculations *)
    jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
    jSG = F[jSGm][jCP yC, (rNS + H ρN / S) / nNS];
```

```
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar LfunH;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          (vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
```

```
     ];
    (*set initial conditions*)
    inis = Join[{
       H[0] == H0,
       S[0] == S0,
       VH[0] == VH0,
       Ni[0] == Ni0,
       P[0] == P0},
      (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

    sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
       Method → {"EquationSimplification" → "Residual"}]⟦1⟧;
    (*outputs to record*)
    (*minimum host growth rate: see if growth ever went negative*)
    tEvals = Flatten[{Table[t, {t, tStartStress + 1, tStartStress + tHigh - 1}],
       Table[t, {t, tStartStress + tHigh + 1, tmax}]}];
    minGrowth = Min[Flatten[Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tEvals}]]] ;

    (*min S SU C-lim*)
    minSSU = Min[Flatten[
       Table[Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) /
                nNS]] /. addtimetostatevars /. sol}, {t, tEvals}]]] ;

    (*min H SU C-lim*)
    minHSU =
     Min[Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[
              jHGm, ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /.
            addtimetostatevars /. sol}, {t, tEvals}]]] ;

    (*growth and SU limitations 100 days after end of stress (tmax)*)
    addtmax = ((# → #[tmax]) & /@ states);
    growthFinal = Evaluate[{H'[tmax] / H[tmax]} /. sol];
    SSUfinal = Evaluate@Flatten@
       {Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /. addtmax /. sol};
    HSUfinal = Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm,
             ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /. addtmax /. sol};
    {minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal} (*output*)],

  {LHigh, {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35}}],
 (*pulse magnitudes tested (note total magnitude is LHigh + L) *)
 {tHigh, 7, 35, 2}
 (*pulse durations tested: from 7 to 35 in intervals of 2 days*)
];
```

```
In[◦]:= (*saving the output*)
        Save["stressRunsPnoX1", stressRunsPnoX1]
```

Higher Nenv, lower flushing rate, no fish

```
In[◦]:=
        (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
         jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
         tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[◦]:=
        (* No prey no fish*)
        X = 0(*1*10^-7*);
        P0 = 0(*kp*(1-α)*H0*);
        jX = jXm X / (X + KX);

        (* environment: flushing rate and N level*)
        d = 500(*1660*);
        N = 6 * 10^-7(*1*10^-6*);

        jN = jNm N / (N + KN);
        Ni0 = N;
        jNi0 = jNm N / (N + KN);

        (*light function parameters*)
        tStartStress = 600;

        stressRunsnoPnoXN = Table[
           Table[
            Module[{

              (*jX=jXm X/(X+KX), jN=jNm N/(N+KN),*)jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, (*Ni0 = N,
              jNi0 = jNm N/(N+KN),*)states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN,
              jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh,
              LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t, tEvals, addtmax,
              minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal, tmax
```

```
 (*make sure to put any intermediate output values in Module*)},
tmax = tStartStress + tHigh + 100;
(*light function*)
LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
   LHigh * HeavisideTheta[t - (tStartStress + tHigh)];
```

$$F[\rho\_][A\_,\ B\_] := \frac{A\ B\ (A+B)\ \rho}{A^2\ B + A\ B^2 + A^2\ \rho + A\ B\ \rho + B^2\ \rho};$$

$$(*\text{same as } \frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}} *)$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\ (\rho C\ S / H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH)\ nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\ yC,\ \frac{rNS + \frac{H\ \rho N}{S}}{nNS}\right];$$

$$\rho N = \text{Max}\left[(1-\alpha) * jNi + \alpha * jN + nNX\ jX + rNH - nNH\ jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = \text{Max}\left[jX - \frac{jHG}{yC} + \frac{S\ \rho C}{H},\ 0\right];$$

$$jCO2 = jeC\ kCO2;$$

$$jL = A\ astar * LfunH;$$

$$rCH = \left(jHT + \frac{jHG\ (1 - yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\ \left(j0ST + (1 - yC)\ jSG\ yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\ cROS1)\ j0ST;$$

$$rNH = \sigma NH\ nNH\ jHT;$$

$$A = 1.256307 + 1.385969\ e^{-6.479055\ S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \Big\{$$

$$\left\{\rho C,\ jCP - jSG\ yC^{-1},\ \rho C0\right\},$$

$$\left\{jCP,\ \frac{F[jCPm]\left[jL\ yCL,\ rCS + \frac{H\ (jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},$$

```
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧ '[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S '[t] == (S (jSG - jST) /. addtimetostatevars),
    H '[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH '[t] == fh[t, H '[t]],
    Ni '[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P '[t] ==
     (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
    Method → {"EquationSimplification" → "Residual"}]⟦1⟧;
(*outputs to record*)
(*minimum host growth rate: see if growth ever went negative*)
(*evaluate host growth rate at all
 points from the start of stress to the end of the run
  (EXCEPT when t is exactly equal to tStartStress and tStartStress +
    tHigh because at these points one of the HeavisideTheta functions
     has an argument of 0 which is not evaluated so the growth
     values are inaccurate) and record the minimum of these values*)
tEvals = Flatten[{Table[t, {t, tStartStress + 1, tStartStress + tHigh - 1}],
    Table[t, {t, tStartStress + tHigh + 1, tmax}]}];
minGrowth = Min[Flatten[Table[Evaluate[{H '[t] / H[t]} /. sol], {t, tEvals}]]] ;

(*min S SU C-lim*)
minSSU = Min[Flatten[
         Table[Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) /
                   nNS]] /. addtimetostatevars /. sol}, {t, tEvals}]]] ;

      (*min H SU C-lim*)
      minHSU =
       Min[Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[
                   jHGm, ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /.
                 addtimetostatevars /. sol}, {t, tEvals}]]] ;

      (*growth and SU limitations 100 days after end of stress (tmax)*)
      addtmax = ((# → #[tmax]) & /@ states);
      growthFinal = Evaluate[{H '[tmax] / H[tmax]} /. sol];
      SSUfinal = Evaluate@Flatten@
         {Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /. addtmax /. sol};
      HSUfinal = Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm,
                 ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /. addtmax /. sol};
      {minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal} (*output*)],

     {LHigh, {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35}}],
    (*pulse magnitudes tested (note total magnitude is LHigh + L) *)
    {tHigh, 7, 35, 2}
    (*pulse durations tested: from 7 to 35 in intervals of 2 days*)
   ];
```

```
In[]:= Save["stressRunsnoPnoXN", stressRunsnoPnoXN ]
```

Higher Nenv, lower flushing rate, with fish

```
In[]:= (* ----------------------------------------------------------------------------
                                                 -----------------------------------------*
     )
     (*clear parameters that are changing and intermediate values*)
     ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
      jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
      tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[]:=
     (* No prey with fish*)
     X = 0(*1*10^-7*);
     P0 = (*0*)kp * (1 - α) * H0;
```

$$jX = \frac{jXm\ X}{X + KX};$$

```
(* environment: flushing rate and N level*)
d = 500(*1660*);
N = 6 * 10^-7(*1*10^-6*);

jN = jNm N/(N + KN);
Ni0 = N;
jNi0 = jNm N/(N + KN);

tStartStress = 600;

stressRunsPnoXN = Table[
   Table[
    Module[{

      (*jX=jXm X/(X+KX), jN=jNm N/(N+KN),*)jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0, (*Ni0 = N,
      jNi0 = jNm N/(N+KN),*)states, rNH, S, H, VH, P, Ni, jNi, ρC, jCP, jHG, ρN,
      jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST, jNw, F, fh,
      LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t, tEvals, addtmax,
      minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal, tmax
      (*make sure to put any intermediate output values in Module*)},
     tmax = tStartStress + tHigh + 100;
     (*light function*)
     LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
       LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

     F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
     (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B))*)
     (* helper function for VH*)
     fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

     (* Calculations *)
     jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
     jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
     ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
     jeC = Max[jX - jHG/yC + S ρC/H, 0];
```

```
jCO2 = jeC kCO2;
jL = A astar LfunH;
rCH = (jHT + jHG (1 - yC)/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm] [jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t,  H'[t]],
    Ni'[t] ==  (d * ( N - Ni)  + (ξ * (1 - α) * jNw * S + ep * P  - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t]  ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
        S[0] == S0,
        VH[0] == VH0,
        Ni[0] == Ni0,
        P[0] == P0},
       (#〚1〛[0] == #〚3〛) & /@ tsolve];

     sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
        Method → {"EquationSimplification" → "Residual"}]〚1〛;
     (*outputs to record*)
     (*minimum host growth rate: see if growth ever went negative*)
     tEvals = Flatten[{Table[t, {t, tStartStress + 1, tStartStress + tHigh - 1}],
        Table[t, {t, tStartStress + tHigh + 1, tmax}]}];
     minGrowth = Min[Flatten[Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tEvals}]]] ;

     (*min S SU C-lim*)
     minSSU = Min[Flatten[
        Table[Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) /
                 nNS]] /. addtimetostatevars /. sol}, {t, tEvals}]]] ;

     (*min H SU C-lim*)
     minHSU =
      Min[Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[
                jHGm, ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1]] /.
             addtimetostatevars /. sol}, {t, tEvals}]]] ;

     (*growth and SU limitations 100 days after end of stress (tmax)*)
     addtmax = ((# → #[tmax]) & /@ states);
     growthFinal = Evaluate[{H'[tmax] / H[tmax]} /. sol];
     SSUfinal = Evaluate@Flatten@
        {Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /. addtmax /. sol};
     HSUfinal = Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm,
              ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1]] /. addtmax /. sol};
     {minGrowth, minSSU, minHSU, growthFinal, SSUfinal, HSUfinal}(*output*)],

    {LHigh, {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35}}],
   (*pulse magnitudes tested (note total magnitude is LHigh + L) *)
   {tHigh, 7, 35, 2}
   (*pulse durations tested: from 7 to 35 in intervals of 2 days*)
  ];
```

In[ ]:=
```
Save["stressRunsPnoXN", stressRunsPnoXN]
```

Format results

In[ ]:=
```
(*recover the outputs*)
Get["stressRunsnoPnoX1"];
Get["stressRunsPnoX1"];
Get["stressRunsnoPnoXN"];
Get["stressRunsPnoXN"];
```

```
In[•]:=
   (*say the system only bleached if growth was neg and both SUs were C-limited,
   recovered if all returned to positive, no stress if all remained positive,
   mild stress if at least one went negative*)
   (*if the min of the three is negative but the max of the three is positive,
   then at least one is negative*)
   makeMatrixS = Table[ Module[{Separate, minGrowthVals, minSSUVals, minHSUVals,
        growthFinalVals, SSUFinalVals, HSUFinalVals, Vals, Matrix},
       Separate = Transpose[Flatten[holdm, 1]];
       (*select each set of outputs*)
       minGrowthVals = Separate⟦1⟧;
       (* mininimum growth from each simulation is the first element of Separate*)
       minSSUVals = Flatten[Separate⟦2⟧];
       minHSUVals = Flatten[Separate⟦3⟧];
       growthFinalVals = Flatten[Separate⟦4⟧];
       (*growth 100 days after stress ends*)
       SSUFinalVals = Flatten[Separate⟦5⟧];
       HSUFinalVals = Flatten[Separate⟦6⟧];
       Vals =
        Table[If[minGrowthVals⟦num⟧ > 0 && minHSUVals⟦num⟧ ≥ 0 && minSSUVals⟦num⟧ ≥ 0,
          1, If[Min[minGrowthVals⟦num⟧, minHSUVals⟦num⟧, minSSUVals⟦num⟧] < 0  &&
            Max[minGrowthVals⟦num⟧, minHSUVals⟦num⟧, minSSUVals⟦num⟧] ≥ 0, 2,
           If[ minGrowthVals⟦num⟧ < 0 && minHSUVals⟦num⟧ < 0 && minSSUVals⟦num⟧ < 0 &&
             growthFinalVals⟦num⟧ > 0 && HSUFinalVals⟦num⟧ ≥ 0 &&
             SSUFinalVals⟦num⟧ ≥ 0, 3, 4]]], {num, 1, 240, 1}];
       (*turn the values into a 15x16 matrix: rows are light levels,
       columns are light durations*)
       Matrix = ArrayReshape[Vals, {15, 16}];
       (*rows are tHigh's, columns are LHigh's *)
       {Matrix}],
      {holdm,
       {stressRunsnoPnoX1, stressRunsPnoX1, stressRunsnoPnoXN, stressRunsPnoXN}}];

   (*name the matrices to plot*)
   MatrixnoPnoX1 = Flatten[makeMatrixS⟦1⟧, 1];
   MatrixPnoX1 = Flatten[makeMatrixS⟦2⟧, 1];
   MatrixnoPXN = Flatten[makeMatrixS⟦3⟧, 1];
   MatrixPXN = Flatten[makeMatrixS⟦4⟧, 1];
```

Plot results

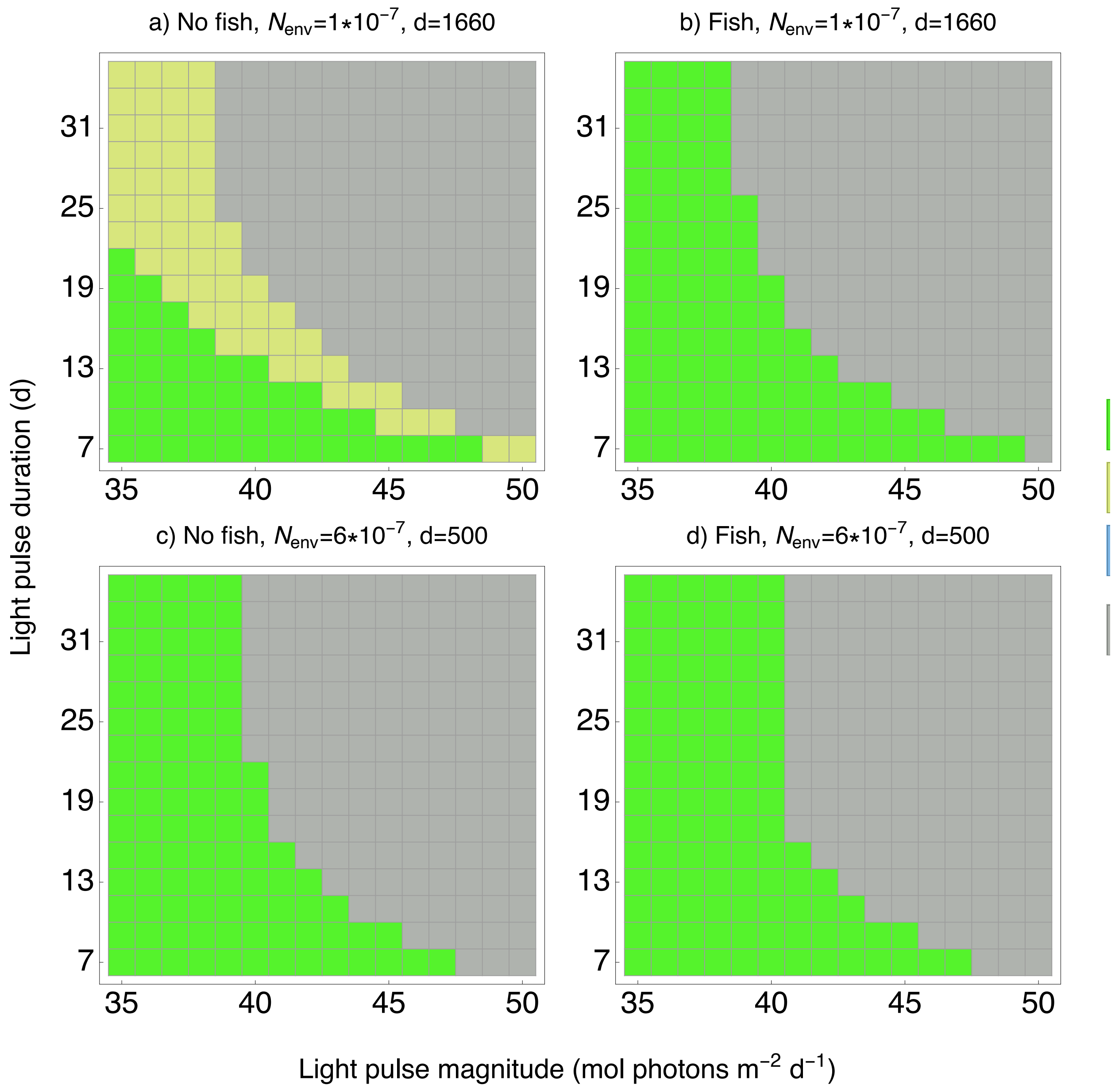
```
In[•]:= padding5 = {{25(*left*), 10(*right*)}, {25(*bottom*), 10 (*top*)}};
   (*low Nenv*)
```

```
plotS7a = MatrixPlot[Reverse[MatrixnoPnoX1],
   PlotLabel → "a) No fish, Nenv=1*10^-7, d=1660", LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   ImagePadding → padding5, ImageSize → Full, Mesh → True,
   ColorRules → {1 → RGBColor["#55F32C"(*hexadecimal*)],
     2 → RGBColor["#D8E67D"(*#57D138*)],
     3 → RGBColor["#7DB7E6"(* #98C28D*)], 4 → RGBColor["#AFB3AE"]},
   FrameTicks → {{{{15, "7"}, {12, "13"}, {9, "19"}, {6, "25"}, {3, "31"}}, None},
     {{{1, 35(*20*)}, {6, 40(*25*)}, {11, 45(* 30*)}, {16, 50(*35*)}},
      None}}, FrameTicksStyle → Directive[18](*change font size*)] ;

plotS7b = MatrixPlot[Reverse[MatrixPnoX1],
   PlotLabel → "b) Fish, Nenv=1*10^-7, d=1660", LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   ImagePadding → padding5, ImageSize → Full, Mesh → True(*,Mesh→{{1, 2}, {1}}*)
   (*True*), ColorRules → {1 → RGBColor["#55F32C"(*hexadecimal*)],
     2 → RGBColor["#D8E67D"(*#57D138*)],
     3 → RGBColor["#7DB7E6"(* #98C28D*)], 4 → RGBColor["#AFB3AE"]},
   FrameTicks → {{{{15, "7"}, {12, "13"}, {9, "19"}, {6, "25"}, {3, "31"}}, None},
     {{{1, 35(*20*)}, {6, 40(*25*)}, {11, 45(* 30*)}, {16, 50(*35*)}},
      None}}, FrameTicksStyle → Directive[18]] ;

plotS7c = MatrixPlot[Reverse[MatrixnoPXN],
   PlotLabel → "c) No fish, Nenv=6*10^-7, d=500", LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   ImagePadding → padding5, ImageSize → Full, Mesh → True,
   ColorRules → {1 → RGBColor["#55F32C"(*hexadecimal*)],
     2 → RGBColor["#D8E67D"(*#57D138*)],
     3 → RGBColor["#7DB7E6"(* #98C28D*)], 4 → RGBColor["#AFB3AE"]},
   FrameTicks → {{{{15, "7"}, {12, "13"}, {9, "19"}, {6, "25"}, {3, "31"}}, None},
     {{{1, 35(*20*)}, {6, 40(*25*)}, {11, 45(* 30*)}, {16, 50(*35*)}},
      None}}, FrameTicksStyle → Directive[18](*change font size*)] ;

plotS7d = MatrixPlot[Reverse[MatrixPXN],
   PlotLabel → "d) Fish, Nenv=6*10^-7, d=500", LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   ImagePadding → padding5, ImageSize → Full, Mesh → True,
   ColorRules → {1 → RGBColor["#55F32C"(*hexadecimal*)],
     2 → RGBColor["#D8E67D"(*#57D138*)],
     3 → RGBColor["#7DB7E6"(* #98C28D*)], 4 → RGBColor["#AFB3AE"]},
   FrameTicks → {{{{15, "7"}, {12, "13"}, {9, "19"}, {6, "25"}, {3, "31"}}, None},
     {{{1, 35(*20*)}, {6, 40(*25*)}, {11, 45(* 30*)}, {16, 50(*35*)}},
      None}}, FrameTicksStyle → Directive[18]] ;
```

```
In[◦]:= (*make legend*)
heatmapLeg = SwatchLegend[
   {RGBColor["#55F32C"(*hexadecimal*)], RGBColor["#D8E67D"(*#57D138*)],
    RGBColor["#7DB7E6"(*#98C28D*)], RGBColor["#AFB3AE"]},
   {"Not stressed", "Mildly stressed", "Bleached and \n recovered",
    "Bleached with \n mortality"}, LegendMarkerSize → 30];
```

```
In[∘]:= SheatmapFig = Legended[
          Labeled[GraphicsGrid[{{plotS7a, plotS7b}, {plotS7c, plotS7d}}, ImageSize → Full,
            Spacings → { {0, -20(*left to right*)}, {0, 10, 10, 10(*top to bottom*)}},
            Frame → False(*, AspectRatio→1/GoldenRatio*), Alignment → Bottom],
           {"Light pulse duration (d)", "Light pulse magnitude (mol photons m^-2 d^-1)"},
           {Left, Bottom}, RotateLabel → True,
           LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica", 16]],
          Placed[heatmapLeg, (*Bottom*){1.5, 0.8(*x, y position,
            0,0 is the lower left corner of the plot*)}]]
```

Out[∘]=



```
In[ ]:= Export["S_bleach_heatmaps.pdf", SheatmapFig, "PDF"]
Out[ ]= S_bleach_heatmaps.pdf
```

Fig. S8 and S11: Time series of post-bleaching dynamics with and without fish

Run Simulations

No Fish

```
In[ ]:= (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
         jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
         tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[ ]:=
        (*no fish*)
        P0 = 0;

        d = 1660;
        N = 1 * 10 ^ - 7 (*1*10^-6*);
```

$$jN = \frac{jNm\ N}{N + KN};$$

`Ni0 = N;`

$$jNi0 = \frac{jNm\ N}{N + KN};$$

`X = 1.1 * 10 ^ - 7;`

$$jX = \frac{jXm\ X}{X + KX};$$

```
        (*light function parameters*)
        tStartStress = 600;
        LHigh = 35;
        tHigh = 30;
        tmax = 1000;

        (*light function*)
        LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
```

```
   LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

F[ρ_][A_, B_] := (A B (A + B) ρ)/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);(*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar LfunH;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, (F[jCPm][jL yCL, rCS + (H (jCO2+rCH))/S])/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve〚All, 1〛, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
```

```
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) / ( vi * VH) /.
        addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) & /@ tsolve];

solRecovNoP = NDSolve[{Join[eqs, inis]}, # & /@ states,
    {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];
```

With Fish

```
In[ ]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[ ]:=
(*with fish*)
P0 = kp * (1 - α) H0;

d = 1660;
N = 1 * 10^-7 (*1*10^-6*);
```

$$jN = \frac{jNm\,N}{N + KN};$$

Ni0 = N;

$$jNi0 = \frac{jNm\,N}{N + KN};$$

```
X = 1.1 * 10^-7;
```

$$jX = \frac{jXm\ X}{X + KX};$$

```
(*light function parameters*)
tStartStress = 600;
LHigh = 35;
tHigh = 30;
tmax = 1000;



(*light function*)
LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
   LHigh * HeavisideTheta[t - (tStartStress + tHigh)];
```

$$F[\rho\_][A\_, B\_] := \frac{A\ B\ (A + B)\ \rho}{A^2\ B + A\ B^2 + A^2\ \rho + A\ B\ \rho + B^2\ \rho}; (*\text{same as } \frac{1}{\frac{1}{\rho} + \frac{1}{A} + \frac{1}{B} - \frac{1}{A+B}} *)$$

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\ (\rho C\ S / H + jX),\ ((1 - \alpha) * jNi + \alpha * jN + nNX\ jX + rNH)\ nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\ yC,\ \frac{rNS + \frac{H\ \rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1 - \alpha) * jNi + \alpha * jN + nNX\ jX + rNH - nNH\ jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\ \rho C}{H},\ 0\right];$$

$$jCO2 = jeC\ kCO2;$$

$$jL = A\ astar\ LfunH;$$

$$rCH = \left(jHT + \frac{jHG\ (1 - yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\ \left(j0ST + (1 - yC)\ jSG\ yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\ cROS1)\ j0ST;$$

$$rNH = \sigma NH\ nNH\ jHT;$$

```
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S] / (1 + cROS1), jCP0},
   {jNi, jNm * Ni / (Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * ( N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) / ( vi * VH) /.
        addtimetostatevars),
    P'[t] == (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
     H[0] == H0,
     S[0] == S0,
     VH[0] == VH0,
     Ni[0] == Ni0,
     P[0] == P0},
    (#[[1]][0] == #[[3]]) &/@ tsolve];

solRecovP = NDSolve[{Join[eqs, inis]}, # &/@ states,
     {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];
```

With fish, carrying capacity depends on interstitial volume

```
In[ ]:= (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, ℕ, jN, Ni0, jNi0, jHG, jSG, ρN,
         jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
         tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[ ]:=
        (*constant scaling fish biomass to interstitial volume
         (from qunatile regression, see Supplementary information on parameterization)*)
        kpv = 13.66;

        (*with fish*)
        P0 = kpv * (vi) VH0;

        d = 1660;
        ℕ = 1 * 10^-7 (*1*10^-6*);

              jNm ℕ
        jN = ────────;
              ℕ + KN
        Ni0 = ℕ;
                jNm ℕ
        jNi0 = ────────;
                ℕ + KN

        X = 1.1 * 10^-7;

              jXm X
        jX = ────────;
              X + KX

        (*light function parameters*)
        tStartStress = 600;
        LHigh = 35;
        tHigh = 30;
        tmax = 1000;

        (*light function*)
        LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
           LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

                                   A B (A + B) ρ                                       1
        F[ρ_][A_, B_] := ─────────────────────────────────────────;(*same as ─────────────────── *)
                          A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ           1/ρ + 1/A + 1/B - 1/(A+B)
        (* helper function for VH*)
        fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

        (* Calculations *)
```

```
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + H ρN / S) / nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG / yC + S ρC / H, 0];
jCO2 = jeC kCO2;
jL = A astar LfunH;
rCH = (jHT + jHG (1 - yC) / yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP / yCL;
jNPQ = 1 / (1 / jeL + 1 / kNPQ);
cROS1 = (jeL - jNPQ) / kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH) / S] / (1 + cROS1), jCP0},
   {jNi, jNm * Ni / (Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) / (vi * VH) /.
        addtimetostatevars),
      P'[t] == (P * (rp * (kpv * vi * VH - Bp * P) / (kpv * vi * VH)) /. addtimetostatevars)
     }
    ];
  (*set initial conditions*)
  inis = Join[{
      H[0] == H0,
      S[0] == S0,
      VH[0] == VH0,
      Ni[0] == Ni0,
      P[0] == P0},
     (#[[1]][0] == #[[3]]) & /@ tsolve];

  solRecovPv = NDSolve[{Join[eqs, inis]}, # & /@ states,
      {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}][[1]];
```

Plot results (Fig. S8: fish carrying capacity scaled to interstitial biomass)

```
In[ ]:= paddingS8 = {{60(*left*), 15(*right*)}, {30(*bottom*), 10 (*top*)}};
  plotS8a = Show[
     Plot[{Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm, ((1 - α) * jNi +
                   α * jN + nNX jX + rNH) nNH ^ - 1]] /. addtimetostatevars /. solRecovNoP },
        Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm, ((1 - α) * jNi +
                   α * jN + nNX jX + rNH) nNH ^ - 1]] /. addtimetostatevars /. solRecovP }},
      {t, 500, 850(*tmax*)}, PlotLabel → "a) Host biomass C-/N-limitation",
      ImagePadding → paddingS8,
      LabelStyle → Directive[FontColor → Black,
        FontFamily → "Helvetica", 12(*label size*)],
      PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
      PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
        (*, {ColorData[97,"ColorList"][[1]],Dashed},
        {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
       {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
      (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
        {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
          {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
      FrameLabel → {{"Relative", None}, {None, None}},
      FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
      FrameStyle → Directive[Black, 12](*axis label size*)],
     RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
      {y, -10, 10}, Frame → False, Mesh → None,
      BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];
```

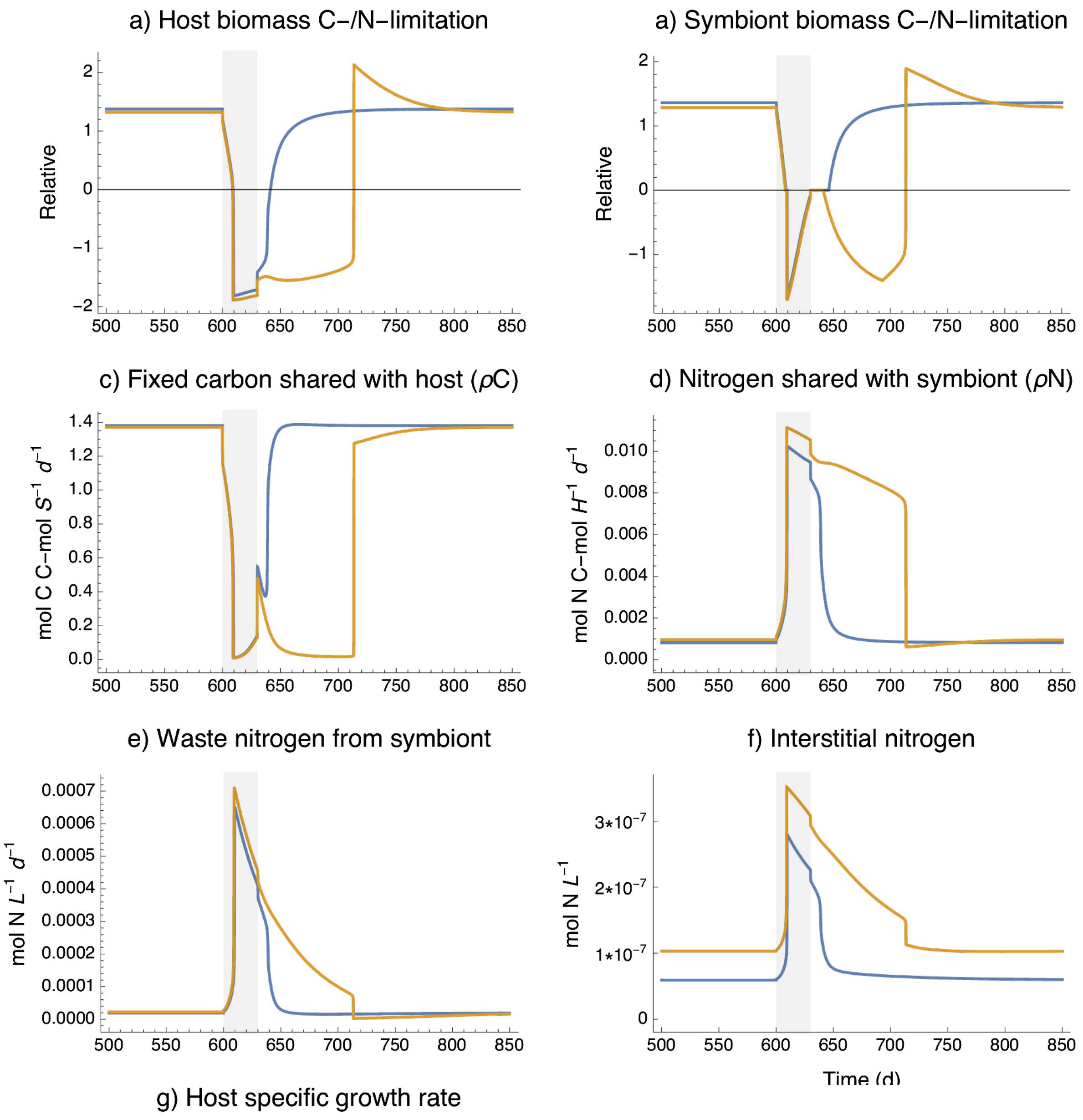
```
plotS8b = Show[Plot[
    {Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /.
          addtimetostatevars /. solRecovNoP },
     Evaluate@Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /.
          addtimetostatevars /. solRecovP }},
    {t, 500, 850(*tmax*)}, PlotLabel → "a) Symbiont biomass C-/N-limitation",
    ImagePadding → paddingS8, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
      (*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"Relative", None}(*\n makes line break*), {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS8c = Show[Plot[{Evaluate@Flatten@{ρC /. addtimetostatevars /. solRecovNoP },
     Evaluate@Flatten@{ρC /. addtimetostatevars /. solRecovP }},
    {t, 500, 850(*tmax*)}, PlotLabel → "c) Fixed carbon shared with host (ρC)",
    ImagePadding → paddingS8, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
      (*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},{6*10^-7, "0.6"},
        {8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)FrameLabel →
     {{"mol C C-mol S^-1 d^-1", None}(*\n makes line break*), {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];
```

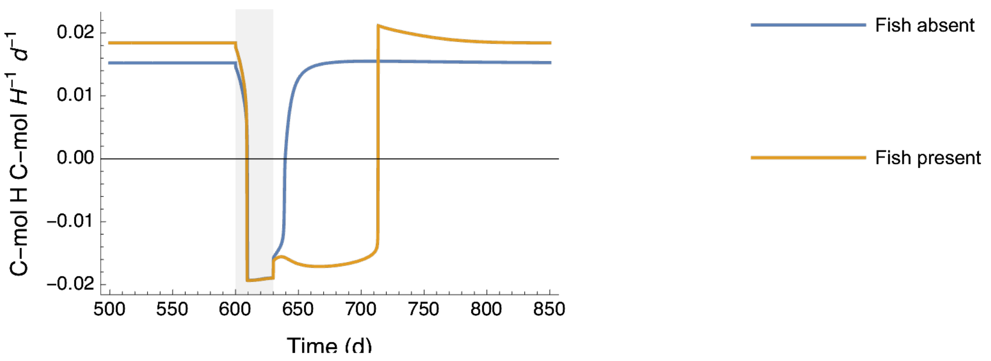
```
plotS8d = Show[Plot[{Evaluate@Flatten@{ρN /. addtimetostatevars /. solRecovNoP },
      Evaluate@Flatten@{ρN /. addtimetostatevars /. solRecovP }},
    {t, 500, 850(*tmax*)}, PlotLabel → "d) Nitrogen shared with symbiont (ρN)",
    ImagePadding → paddingS8, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
      (*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"mol N C-mol H^-1 d^-1", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS8e = Show[Plot[{Evaluate@Flatten@
        {ξ * (1 - α) jNw * S / ( vi * VH) /. addtimetostatevars /. solRecovNoP }, Evaluate@
       Flatten@{ξ * (1 - α) jNw * S / ( vi * VH) /. addtimetostatevars /. solRecovP }},
    {t, 500, 850(*tmax*)}, PlotLabel → "e) Waste nitrogen from symbiont",
    ImagePadding → paddingS8, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
      (*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"mol N L^-1 d^-1", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None, BoundaryStyle → None,
    PlotStyle → GrayLevel[0.09, 0.06](*Opacity[0.05, Gray]*)]];
```

```
plotS8f =
  Show[Plot[{Evaluate@Flatten@{Ni /. addtimetostatevars /. solRecovNoP }, Evaluate@
      Flatten@{Ni /. addtimetostatevars /. solRecovP }}, {t, 500, 850(*tmax*)},
    PlotLabel → "f) Interstitial nitrogen", ImagePadding → paddingS8, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]](*,
      {ColorData[97,"ColorList"][[1]],Dashed}, {ColorData[97,"ColorList"][[2]],
       Dashed}*)}, Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, FrameTicks → (*True*)
     {{{0, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}, {3 * 10^-7, "3*10^-7"}}, None},
      {(*None*){500, 550, 600, 650, 700, 750, 800, 850}, None}}, FrameLabel →
     {{"mol N L^-1", None}(*\n makes line break*), {"Time (d)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None, BoundaryStyle → None,
    PlotStyle → GrayLevel[0.09, 0.06](*Opacity[0.05, Gray]*)]];

plotS8g = Show[Plot[
    {Evaluate[{H'[t] / H[t]} /. solRecovNoP ], Evaluate[{H'[t] / H[t]} /. solRecovP]},
    {t, 500, 850(*tmax*)}, PlotLabel → "g) Host specific growth rate",
    ImagePadding → paddingS8, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    PlotRange → Full, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
      (*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"C-mol H C-mol H^-1 d^-1", None}(*\n makes line break*),
      {"Time (d)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

recovTSLeg =
  LineLegend[{ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]},
{"Fish absent", "Fish present"}, LegendMarkerSize → 60
(*, LegendLayout→{"Row",1}*)];
```

```
In[∘]:= recovTSFig = Legended[GraphicsGrid[{{plotS8a, plotS8b},
          {plotS8c, plotS8d}, {plotS8e, plotS8f}, {plotS8g}}, ImageSize → Full,
        Spacings → { {0(*left*), -30 (*first (leftmost) and second col*)
           (*left to right*)}, {0(*top*), 0(*first and second rows*),
           0 (*second and third*) , 0(*top to bottom*)}},
        Frame → False(*, AspectRatio→1/GoldenRatio*), Alignment → Bottom],
      Placed[recovTSLeg, (*Bottom*){0.95, 0.22(*x, y position of legend*)}]]
```

Out[∘]=

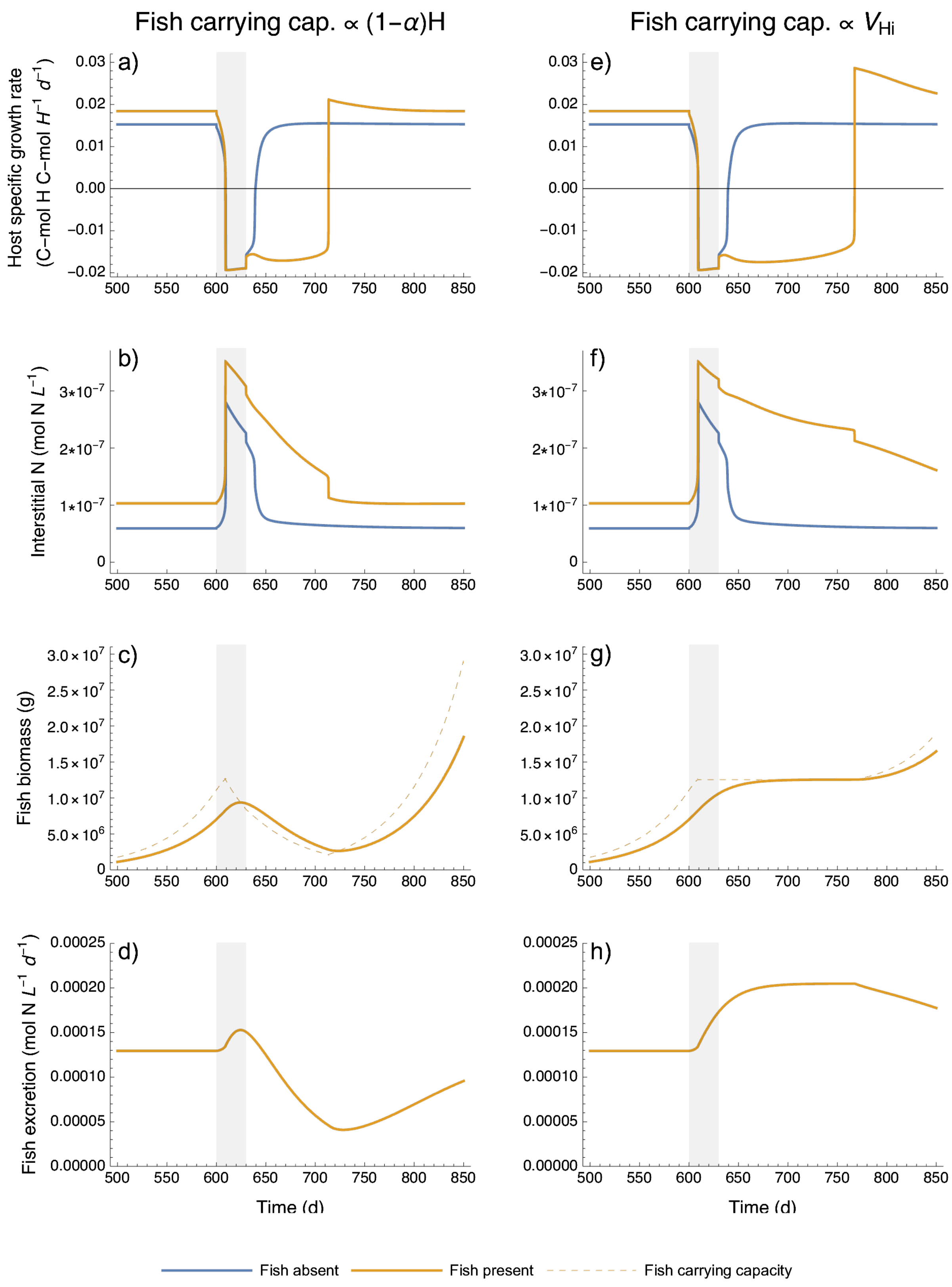




Plot results (Fig. S11: comparison of fish carrying capacity scaling to interstitial biomass vs. volume)

```
In[∘]:= paddingS9 = {{80(*left*), 15(*right*)}, {30(*bottom*), 10 (*top*)}};
plotS9a = Show[Plot[{Evaluate[{H'[t] / H[t]} /. solRecovNoP ],
     Evaluate[{H'[t] / H[t]} /. solRecovP]}, {t, 500, 850(*tmax*)},
    PlotLabel → "Fish carrying cap. ∝ (1-α)H", PlotRange → {Full, {-0.021, 0.032}},
    Epilog → Text[Style["a)", {Black, 16}], Offset[{5(*right from top left*),
        -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 14(*label size*)],
    ImagePadding → paddingS9, ImageSize → Full, AxesOrigin → {500, 0},
    PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]
      (*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"Host specific growth rate \n (C-mol H C-mol H^-1 d^-1)", None}
      (*\n makes line break*), {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9b = Show[Plot[{Evaluate@Flatten@{Ni /. addtimetostatevars /. solRecovNoP },
     Evaluate@Flatten@{Ni /. addtimetostatevars /. solRecovP }},
    {t, 500, 850(*tmax*)}, (*PlotLabel→"f) Interstitial nitrogen"*)
    Epilog → Text[Style["b)", {Black, 16}], Offset[{5(*right from top left*),
        -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS9, PlotRange → Full, ImageSize → Full,
    AxesOrigin → {500, 0}, PlotStyle → {ColorData[97, "ColorList"][[1]],
      ColorData[97, "ColorList"][[2]](*, {ColorData[97,"ColorList"][[1]],Dashed},
      {ColorData[97,"ColorList"][[2]],Dashed}*)},
    Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, FrameTicks → (*True*)
     {{{0, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}, {3 * 10^-7, "3*10^-7"}}, None},
      {(*None*){500, 550, 600, 650, 700, 750, 800, 850}, None}},
    FrameLabel → {{"Interstitial N (mol N L^-1)", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9c =
  Show[Plot[{Evaluate[P[t] /. solRecovP], Evaluate[kp * (1 - α) * H[t] /. solRecovP]},
    {t, 500, 850}, (*PlotLabel→"c) Fish biomass"*)
    PlotRange → {Full, {0, 3.1 * 10^7}},
    Epilog → Text[Style["c)", {Black, 16}], Offset[{5(*right from top left*),
        -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS9, ImageSize → Full, AxesOrigin → {500, 0}, PlotStyle →
     {{ColorData[97, "ColorList"][[2]]}, {ColorData[97, "ColorList"][[2]], Dashed}},
    Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, (*FrameTicks→(*True*)
     {{{0, 0.01,0.02,0.03,0.04,0.05},None},{(*None*){{0, "0"},{2*10^-7, "0.2"},
        {4*10^-7, "0.4"},{6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"Fish biomass (g)", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)(*,
    PlotLegends→{"fish biomass", "fish carrying capacity"}*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
    {y, 0, 1 * 10^10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9d = Show[Plot[{Evaluate[ep * P[t] / ( vi * VH[t]) /. solRecovP]}, {t, 500, 850},
    (*PlotLabel→"d) Fish excretion per unit interstitial volume"*)
    Epilog → Text[Style["d)", {Black, 16}], Offset[{5(*right from top left*),
         -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImagePadding → paddingS9, PlotRange → {Full, {0, 0.00025}}, ImageSize → Full,
    AxesOrigin → {500, 0}, PlotStyle → {{ColorData[97, "ColorList"]⟦2⟧}}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"Fish excretion (mol N L^-1 d^-1)", None}, {"Time (d)", None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)(*,
    PlotLegends→{"fish biomass", "fish carrying capacity"}*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
    {y, -1000, 1000}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9e = Show[Plot[{Evaluate[{H'[t] / H[t]} /. solRecovNoP ],
     Evaluate[{H'[t] / H[t]} /. solRecovPv]}, {t, 500, 850(*tmax*)},
    PlotLabel → "Fish carrying cap. ∝ V_Hi", PlotRange → {Full, {-0.021, 0.032}},
    Epilog → Text[Style["e)", {Black, 16}], Offset[{5(*right from top left*),
         -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 14(*label size*)],
    ImagePadding → paddingS9, ImageSize → Full, AxesOrigin → {500, 0},
    (*GridLines→{{{tStartStress,Directive[Black]},
       {tStartStress+tHigh,Directive[Black]}},{}},*)
    PlotStyle → {ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧
      (*, {ColorData[97,"ColorList"]⟦1⟧,Dashed},
      {ColorData[97,"ColorList"]⟦2⟧,Dashed}*)}, Frame →
     {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
    (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
      {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
        {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{None, None}(*\n makes line break*), {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
    {y, -10, 10}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9f = Show[Plot[{Evaluate@Flatten@{Ni /. addtimetostatevars /. solRecovNoP },
      Evaluate@Flatten@{Ni /. addtimetostatevars /. solRecovPv}},
     {t, 500, 850(*tmax*)}, (*PlotLabel→"f) Interstitial nitrogen"*)
     Epilog → Text[Style["f)", {Black, 16}], Offset[{5(*right from top left*),
         -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
      Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
     ImagePadding → paddingS9, PlotRange → Full, ImageSize → Full,
     AxesOrigin → {500, 0}, PlotStyle → {ColorData[97, "ColorList"][[1]],
       ColorData[97, "ColorList"][[2]](*, {ColorData[97,"ColorList"][[1]],Dashed},
       {ColorData[97,"ColorList"][[2]],Dashed}*)},
     Frame → {{True(*left*), False(*right*)},
       {True(*bottom*), False(*top*)}}, FrameTicks → (*True*)
      {{{0, {1 * 10^-7, "1*10^-7"}, {2 * 10^-7, "2*10^-7"}, {3 * 10^-7, "3*10^-7"}}, None},
       {(*None*){500, 550, 600, 650, 700, 750, 800, 850}, None}},
     FrameLabel → {{None, None}, {None, None}},
     FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
     FrameStyle → Directive[Black, 12](*axis label size*)],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 500, 850},
     {y, -10, 10}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9g =
  Show[Plot[{Evaluate[P[t] /. solRecovPv], Evaluate[kpv * vi * VH[t] /. solRecovPv]},
     {t, 500, 850}, (*PlotLabel→"c) Fish biomass"*)
     Epilog → Text[Style["g)", {Black, 16}], Offset[{5(*right from top left*),
         -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
      Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
     ImagePadding → paddingS9, PlotRange → {Full, {0, 3.1 * 10^7}},
     (*AxesLabel→{"t"},*)ImageSize → Full, AxesOrigin → {500, 0}, PlotStyle →
      {{ColorData[97, "ColorList"][[2]]}, {ColorData[97, "ColorList"][[2]], Dashed}},
     Frame → {{True(*left*), False(*right*)},
       {True(*bottom*), False(*top*)}}, (*FrameTicks→(*True*)
      {{{0, 0.01,0.02,0.03,0.04,0.05},None},{(*None*){{0, "0"},{2*10^-7, "0.2"},
         {4*10^-7, "0.4"},{6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
     FrameLabel → {{None, None}(*\n makes line break*), {None, None}},
     FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
     FrameStyle → Directive[Black, 12](*axis label size*)(*,
     PlotLegends→{"fish biomass", "fish carrying capacity"}*)],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
     {y, 0, 1 * 10^10}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS9h = Show[Plot[{Evaluate[ep * P[t] / ( vi * VH[t]) /. solRecovPv]}, {t, 500, 850},
      (*PlotLabel→"d) Fish excretion per unit interstitial volume"*)
      Epilog → Text[Style["h)", {Black, 16}], Offset[{5(*right from top left*),
          -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}], LabelStyle →
       Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
      ImagePadding → paddingS9, PlotRange → {Full, {0, 0.00025}},
      (*AxesLabel→{"t"},*)ImageSize → Full, AxesOrigin → {500, 0},
      PlotStyle → {{ColorData[97, "ColorList"][[2]]}}, Frame →
       {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
      (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
        {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
          {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
      FrameLabel → {{None, None}, {"Time (d)", None}},
      FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
      FrameStyle → Directive[Black, 12](*axis label size*)(*,
      PlotLegends→{"fish biomass", "fish carrying capacity"}*)],
     RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
      {y, -1000, 1000}, Frame → False, Mesh → None,
      BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

In[•]:= recovPVHLeg = LineLegend[{ColorData[97, "ColorList"][[1]],
      ColorData[97, "ColorList"][[2]], {ColorData[97, "ColorList"][[2]], Dashed}},
     {"Fish absent", "Fish present", "Fish carrying capacity"},
     LegendMarkerSize → 60, LegendLayout → {"Row", 1}];

   recovPVHFig = Legended[GraphicsGrid[{{plotS9a, plotS9e}, {plotS9b, plotS9f},
      {plotS9c, plotS9g}, {plotS9d, plotS9h}}, ImageSize → Full,
     Spacings → { {0(*left*), -60 (*first (leftmost) and second col*)
        (*left to right*)}, {0(*top*), -30(*first and second rows*),
        -30, -30(*second and third*) (*top to bottom*)}},
     Frame → False(*, AspectRatio→1/GoldenRatio*), Alignment → Bottom],
    Placed[recovPVHLeg, (*Bottom*){0.5, 0(*x, y position of legend*)}]]
```

Out[●]=



Fish carrying cap. ∝ (1−α)H
Fish carrying cap. ∝ V_{Hi}
a)
b)
c)
d)
e)
f)
g)
h)
Host specific growth rate (C−mol H C−mol H^{-1} d^{-1})
Interstitial N (mol N L^{-1})
Fish biomass (g)
Fish excretion (mol N L^{-1} d^{-1})
Time (d)
Fish absent
Fish present
Fish carrying capacity

Fig. S9: Post-bleaching dynamics when prey is low

Run simulations

```
In[•]:= (*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

```
In[•]:=
(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10^-7 (*1*10^-6*);
```

$jN = \frac{jNm\,N}{N + KN};$

Ni0 = N;

$jNi0 = \frac{jNm\,N}{N + KN};$

```
X = 0.5 * 10^-7;(*low prey*)
```

$jX = \frac{jXm\,X}{X + KX};$

```
tmax = 1000;
tStartStress = 600;
tHigh = 30;
LHigh = 35;

(*use Table to simulate the model with and without fish*)
LowXRuns =
  Table[
   Module[{
```

(*jX=$\frac{jXm\ X}{X+KX}$,*) (* jN=$\frac{jNm\ N}{N+KN}$,*) jHT = j0HT, rNS = σNS nNS j0ST,
VH0 = kv * H0, (* Ni0 = N,jNi0 = $\frac{jNm\ N}{N+KN}$,*) states, rNH, S, H, VH, P, Ni,
jNi, ρC, jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG,
jST, jNw, F, fh, LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t
(*make sure to put any intermediate output values in Module*)},
(*light function*)

```
LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
   LHigh * HeavisideTheta[t - (tStartStress + tHigh)];
```

$$F[\rho\_][A\_, B\_] := \frac{A\,B\,(A+B)\,\rho}{A^2 B + A B^2 + A^2 \rho + A B \rho + B^2 \rho};$$

(*same as $\frac{1}{\frac{1}{\rho}+\frac{1}{A}+\frac{1}{B}-\frac{1}{A+B}}$ *)

```
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv *y, y ≥ 0}}];

(* Calculations *)
```

$$jHG = F[jHGm]\left[yC\,(\rho C\,S/H + jX),\ ((1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH)\,nNH^{-1}\right];$$

$$jSG = F[jSGm]\left[jCP\,yC,\ \frac{rNS + \frac{H\rho N}{S}}{nNS}\right];$$

$$\rho N = Max\left[(1-\alpha) * jNi + \alpha * jN + nNX\,jX + rNH - nNH\,jHG\ (*yN^{-1}*),\ 0\right];$$

$$jeC = Max\left[jX - \frac{jHG}{yC} + \frac{S\,\rho C}{H},\ 0\right];$$

$$jCO2 = jeC\,kCO2;$$

$$jL = A\,astar\,LfunH;$$

$$rCH = \left(jHT + \frac{jHG\,(1-yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\left(j0ST + (1-yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\,cROS1)\,j0ST;$$

$$rNH = \sigma NH\,nNH\,jHT;$$

$$A = 1.256307 + 1.385969\,e^{-6.479055\,S/H};$$

$$jNw = \rho N * H/S + rNS - nNS * jSG;$$

$$tsolve = \left\{\left\{\rho C,\ jCP - jSG\,yC^{-1},\ \rho C0\right\},\ \left\{jCP,\ \frac{F[jCPm]\left[jL\,yCL,\ rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\ jCP0\right\},\ \left\{jNi,\ \frac{jNm * Ni}{Ni + KN},\ jNi0\right\}\right\};$$

```
        states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
        addtimetostatevars = ((# → #[t]) & /@ states);
        eqs = Join[
          (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
          {
           S'[t] == (S (jSG - jST) /. addtimetostatevars),
           H'[t] == (H (jHG - jHT) /. addtimetostatevars),
           VH'[t] == fh[t, H'[t]],
           Ni'[t] ==  (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
                 (vi * VH) /. addtimetostatevars),
           P'[t] ==
            (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
          }
         ];
        (*set initial conditions*)
        inis = Join[{
           H[0] == H0,
           S[0] == S0,
           VH[0] == VH0,
           Ni[0] == Ni0,
           P[0] == P0},
          (#⟦1⟧[0] == #⟦3⟧) & /@ tsolve];

        sol = NDSolve[{Join[eqs, inis]}, # & /@ states,
           {t, 0, tmax}, Method → {"EquationSimplification" → "Residual"}]⟦1⟧;

        {sol}(*save the whole simulation as output*)],

      {P0, {0, kp * (1 - α) * H0}}];
    (*repeat once with no fish (P0=0) and once with fish (P0=kp*(1-α)*H0)*)


In[ ]:= (*get the state variables*)
    LowXnoP = ToExpression[str[Flatten[LowXRuns, 1]⟦1⟧]];
    LowXP = ToExpression[str[Flatten[LowXRuns, 1]⟦2⟧]];
```

Plot results

```
In[ ]:= plotS10a =
      Show[Plot[{Evaluate[{H'[t] / H[t]} /. LowXnoP], Evaluate[{H'[t] / H[t]} /. LowXP]},
        {t, 400, tmax}, PlotLabel → "a) Host specific growth rate", PlotRange → Full,
        LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
          12(*label size*)], ImagePadding → paddingS9, LabelStyle →
```

```
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImageSize → Full, AxesOrigin → {400, 0},
    PlotStyle → {ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧},
    Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, (*FrameTicks→(*True*)
     {{{0, 0.01,0.02,0.03,0.04,0.05},None},{(*None*){{0, "0"},{2*10^-7, "0.2"},
        {4*10^-7, "0.4"},{6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"C-mol H C-mol H^-1 d^-1", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
    {y, -1000, 1000}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS10b =
  Show[Plot[{Evaluate[{H[t] / VH[t]} /. LowXnoP], Evaluate[{H[t] / VH[t]} /. LowXP]},
    {t, 400, tmax}, PlotLabel → "b) Host biomass per unit host volume",
    PlotRange → Full, LabelStyle → Directive[FontColor → Black,
      FontFamily → "Helvetica", 12(*label size*)], ImagePadding → paddingS9,
    LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
      12(*label size*)], ImageSize → Full, AxesOrigin → {400, 0},
    PlotStyle → {ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧},
    Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, (*FrameTicks→(*True*)
     {{{0, 0.01,0.02,0.03,0.04,0.05},None},{(*None*){{0, "0"},{2*10^-7, "0.2"},
        {4*10^-7, "0.4"},{6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
    FrameLabel → {{"C-mol H L^-1", None}, {None, None}},
    FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
    FrameStyle → Directive[Black, 12](*axis label size*)],
   RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
    {y, -1000, 1000}, Frame → False, Mesh → None,
    BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS10c = Show[Plot[{Evaluate[{P[t]} /. LowXnoP], Evaluate[{P[t]} /. LowXP]},
    {t, 400, tmax}, PlotLabel → "c) Fish biomass", PlotRange → Full,
    LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
      12(*label size*)], ImagePadding → paddingS9, LabelStyle →
     Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
    ImageSize → Full, AxesOrigin → {400, 0},
    PlotStyle → {ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧},
    Frame → {{True(*left*), False(*right*)},
      {True(*bottom*), False(*top*)}}, (*FrameTicks→(*True*)
      {{{0, 0.01,0.02,0.03,0.04,0.05},None},{(*None*){{0, "0"},{2*10^-7, "0.2"},
         {4*10^-7, "0.4"},{6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
     FrameLabel → {{"g Fish", None}, {"Time (d)", None}},
     FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
     FrameStyle → Directive[Black, 12](*axis label size*)],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
     {y, -1 * 10^10, 1 * 10^10}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

plotS10d = Show[Plot[{Evaluate[{Ni[t]} /. LowXnoP], Evaluate[{Ni[t]} /. LowXP]},
     {t, 400, tmax}, PlotLabel → "d) Interstitial N", PlotRange → Full,
     LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
       12(*label size*)], ImagePadding → paddingS9, LabelStyle →
      Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
     ImageSize → Full, AxesOrigin → {400, 0},
     PlotStyle → {ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]},
     GridLines → {{}, {N}}, GridLinesStyle → Directive[Black], Frame →
      {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
     (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
       {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
         {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
     FrameLabel → {{"mol L^-1", None}, {"Time (d)", None}},
     FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
     FrameStyle → Directive[Black, 12](*axis label size*)],
    RegionPlot[tStartStress < t < tStartStress + tHigh,
     {t, 590, 640}, {y, -1, 1}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];
```

```
In[∘]:= lowXrecovLeg =
      LineLegend[{ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]},
       {"Fish absent", "Fish present"}, LegendMarkerSize → 60
       (*, LegendLayout→{"Row",1}*)];
    lowXrecovFig = Legended[
      GraphicsGrid[{{plotS10a, plotS10b}, {plotS10c, plotS10d}}, ImageSize → Full,
       Spacings → { {0(*left*), -30 (*first (leftmost) and second col*)
          (*left to right*)}, {0(*top*), 0(*first and second rows*),
          0 (*second and third*) (*top to bottom*)}},
       Frame → False(*, AspectRatio→1/GoldenRatio*), Alignment → Bottom],
      Placed[lowXrecovLeg, (*Bottom*){1.2, 1.3(*x, y position of legend*)}]]
```

Out[∘]=

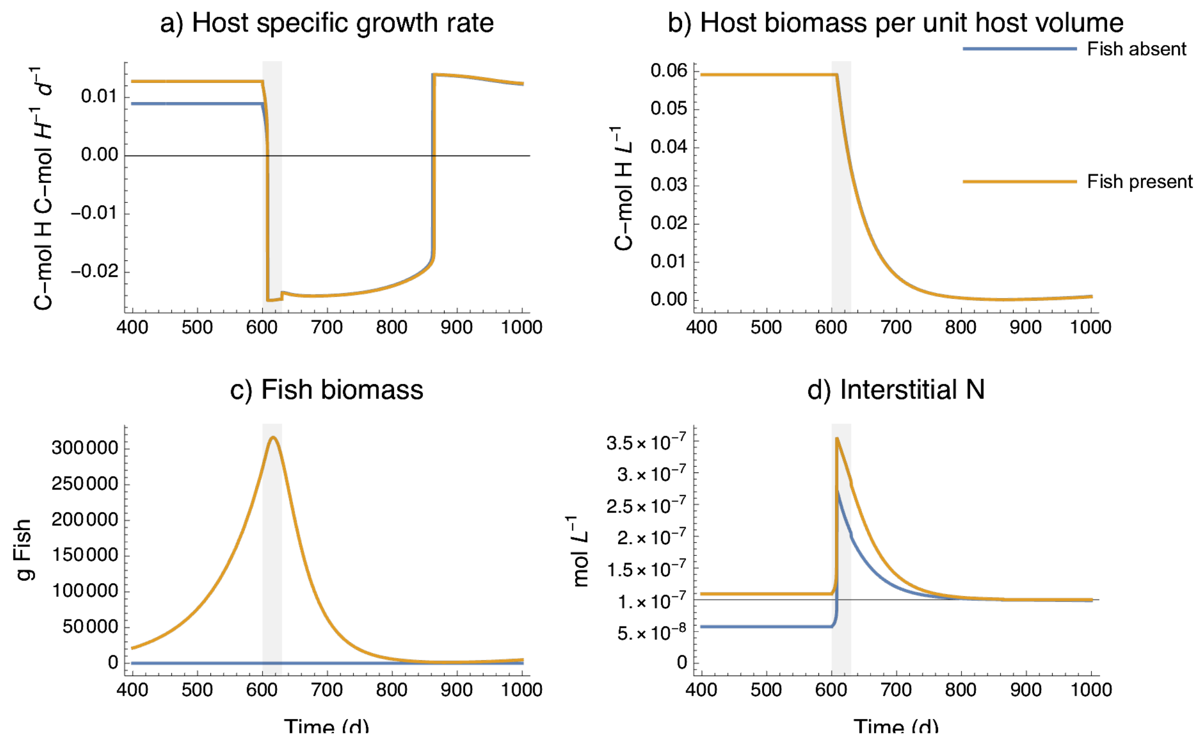


Fig. S10: Effects of prey and N on recovery time

Vary prey at lower Nenv

Run the simulations

In[∘]:=

```
(*clear parameters that are changing and intermediate values*)
ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

In[∘]:=

```
(* environment: flushing rate and N level*)
d = 1660;
N = 1 * 10^-7 (*1*10^-6*);
```

$$jN = \frac{jNm\,N}{N + KN};$$

```
Ni0 = N;
```

$$jNi0 = \frac{jNm\,N}{N + KN};$$

```
tmax = 1000;

(*light function parameters*)
tStartStress = 600;
tHigh = 30;
LHigh = 35;

XRecovRuns = Table[
   Table[
    Module[{

      jX = jXm X / (X + KX), (* jN= jNm N / (N+KN) ,*)jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0,
      (* Ni0 = N,jNi0 = jNm N / (N+KN) ,*)states, rNH, S, H, VH, P, Ni, jNi, ρC,
      jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST,
      jNw, F, fh, LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t,
      HGrowthVals, PosGrowth, HSULim, HNLim, SSULim, SNLim, RecoveryTime
      (*make sure to put any intermediate output values in Module*)},
     (*light function*)
     LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
       LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

     F[ρ_][A_, B_] := A B (A + B) ρ / (A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
     (*same as 1 / (1/ρ + 1/A + 1/B - 1/(A+B)) *)
     (* helper function for VH*)
     fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

     (* Calculations *)
     jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
     jSG = F[jSGm][jCP yC, (rNS + H ρN / S) / nNS];
     ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
     jeC = Max[jX - jHG / yC + S ρC / H, 0];
     jCO2 = jeC kCO2;
     jL = A astar LfunH ;
     rCH = (jHT + jHG (1 - yC) / yC) σCH;
     rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
```

```
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 e^(-6.479055 S/H);
jNw =  ρN * H / S + rNS - nNS * jSG;

tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm][jL yCL, rCS + H (jCO2+rCH)/S]/(1 + cROS1), jCP0},
   {jNi, (jNm * Ni)/(Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t,  H'[t]],
    Ni'[t] ==  (d * ( N - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) &/@ tsolve];
```

```
    sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
       Method → {"EquationSimplification" → "Residual"}]〚1〛;

    (*host growth values*)
    (*all host growth values from the end of stress*)
    HGrowthVals = Flatten[
      Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tStartStress + tHigh + 1, tmax}]];
    PosGrowth = Flatten[Position[HGrowthVals, _? (# > 0 &)]]〚1〛;
    (* position of these values at which growth is positive, which
     corresponds to the number of days after stress: if 1st position is pos.,
    then growth was pos. 1 day after stress ended, etc. *)

    (*host biomass SU limitation*)
    HSULim =
     Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm,
              ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH ^ -1]] /.
           addtimetostatevars /. sol}, {t, tStartStress + tHigh + 1, tmax}]];
    HNLim = Flatten[Position[HSULim, _? (# > 0 &)]]〚1〛;

    (*symbiont biomass SU limitation*)
    SSULim = Flatten[Table[Evaluate@
         Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /.
           addtimetostatevars /. sol}, {t, tStartStress + tHigh + 1, tmax}]];
    SNLim = Flatten[Position[HSULim, _? (# > 0 &)]]〚1〛;

    (* time to recovery=
     when H growth is pos and both H and S SU are back at N-lim,
    so take the maximum of the timepoints at which each of these occur *)
    RecoveryTime = Max[PosGrowth, HNLim, SNLim];


    {RecoveryTime} (*record time to recovery*)],

   {P0, {0, kp * (1 - α) * H0}}],
  (*repeat with no fish (P0=0) and with fish (P0=kp*(1-α)*H0)*)
  {X, {0, 0.1 * 10 ^ -7, 0.2 * 10 ^ -7, 0.3 * 10 ^ -7, 0.4 * 10 ^ -7, 0.5 * 10 ^ -7, 0.6 * 10 ^ -7,
    0.7 * 10 ^ -7, 0.8 * 10 ^ -7, 0.9 * 10 ^ -7, 1 * 10 ^ -7, 1.1 * 10 ^ -7, 1.2 * 10 ^ -7,
    1.3 * 10 ^ -7, 1.4 * 10 ^ -7, 1.5 * 10 ^ -7, 1.6 * 10 ^ -7, 1.7 * 10 ^ -7, 1.8 * 10 ^ -7,
    1.9 * 10 ^ -7, 2 * 10 ^ -7}} (*repeat for different levels of prey*)
 ];
```

Get the results in the correct form to plot

```
In[∘]:= (*getting the lists to plot (should have 2 lists total,
          one for the outputs without fish and one for the outputs with fish*)

        (*transpose this to change the nested list from
         {{P1 at X1, P2 at X1}, {X2}, ...{Xn}} to {{X1, X2, ... Xn at P1}, {P2}} *)
        RecovSeparatePX = Transpose[XRecovRuns];
        (*prey no fish*)
        XRecovNoPX = Flatten[RecovSeparatePX⟦1⟧];
        (*with prey with fish*)
        XRecovPX = Flatten[RecovSeparatePX⟦2⟧];

        (*list of X values*)
        XVals3 = {0, 0.1 * 10^-7, 0.2 * 10^-7, 0.3 * 10^-7, 0.4 * 10^-7,
            0.5 * 10^-7, 0.6 * 10^-7, 0.7 * 10^-7, 0.8 * 10^-7, 0.9 * 10^-7,
            1 * 10^-7, 1.1 * 10^-7, 1.2 * 10^-7, 1.3 * 10^-7, 1.4 * 10^-7, 1.5 * 10^-7,
            1.6 * 10^-7, 1.7 * 10^-7, 1.8 * 10^-7, 1.9 * 10^-7, 2 * 10^-7};


        (*use Table to get the lists in the form to plot*)
        XRecovLists =
          Table[Module[{PlotList}, PlotList = Transpose@{XVals3, list}; PlotList],
           {list, {XRecovNoPX, XRecovPX}}];
```

Vary prey at higher Nenv

Run the simulations

```
In[∘]:= (*clear parameters that are changing and intermediate values*)
        ClearAll[X, P0, jX, N, jN, Ni0, jNi0, jHG, jSG, ρN,
         jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
         tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]

In[∘]:=
        (* environment: flushing rate and N level*)
        d = 1660;
        N = 2.5 * 10^-7 (*1*10^-7 *);
```

$$jN = \frac{jNm\,N}{N + KN};$$

```
Ni0 = N;
jNi0 = jNm N/(N + KN);

tmax = 1100;

(*light function values*)
tStartStress = 600;
tHigh = 30;
LHigh = 35;

XRecovRuns2 = Table[
   Table[
    Module[{

       jX = jXm X/(X + KX), (* jN=jNm N/(N+KN),*)jHT = j0HT, rNS = σNS nNS j0ST, VH0 = kv * H0,
       (* Ni0 = N,jNi0 = jNm N/(N+KN),*)states, rNH, S, H, VH, P, Ni, jNi, ρC,
       jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1, jSG, jST,
       jNw, F, fh, LfunH, tsolve, addtimetostatevars, eqs, inis, sol, t,
       HGrowthVals, PosGrowth, HSULim, HNLim, SSULim, SNLim, RecoveryTime
       (*make sure to put any intermediate output values in Module*)},
     (*light function*)
     LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
       LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

     F[ρ_][A_, B_] := A B (A + B) ρ/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
     (*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B)) *)
     (* helper function for VH*)
     fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

     (* Calculations *)
     jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
     jSG = F[jSGm][jCP yC, (rNS + H ρN/S)/nNS];
     ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
     jeC = Max[jX - jHG/yC + S ρC/H, 0];
     jCO2 = jeC kCO2;
```

$$jL = A\,astar\,LfunH\,;$$

$$rCH = \left(jHT + \frac{jHG\,(1 - yC)}{yC}\right)\sigma CH;$$

$$rCS = \sigma CS\,\left(j0ST + (1 - yC)\,jSG\,yC^{-1}\right);$$

$$jeL = jL - \frac{jCP}{yCL};$$

$$jNPQ = \frac{1}{\frac{1}{jeL} + \frac{1}{kNPQ}};$$

$$cROS1 = \frac{jeL - jNPQ}{kROS};$$

$$jST = (1 + b\,cROS1)\,j0ST;$$

$$rNH = \sigma NH\,nNH\,jHT;$$

$$A = 1.256307 + 1.385969\,e^{-6.479055\,S/H};$$

$$jNw = \rho N * H / S + rNS - nNS * jSG;$$

$$tsolve = \Big\{ \{\rho C,\, jCP - jSG\,yC^{-1},\, \rho C0\},\; \left\{jCP,\, \frac{F[jCPm]\left[jL\,yCL,\, rCS + \frac{H\,(jCO2+rCH)}{S}\right]}{1 + cROS1},\, jCP0\right\},\; \left\{jNi,\, \frac{jNm * Ni}{Ni + KN},\, jNi0\right\} \Big\};$$

```
states = Join[tsolve⟦All, 1⟧, {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) & /@ states);
eqs = Join[
   (#⟦1⟧'[t] == (λ (Max[0, #⟦2⟧] - #⟦1⟧) /. addtimetostatevars)) & /@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * ( ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
(*set initial conditions*)
inis = Join[{
    H[0] == H0,
    S[0] == S0,
      VH[0] == VH0,
      Ni[0] == Ni0,
      P[0] == P0},
     (#[[1]][0] == #[[3]]) & /@ tsolve];

  sol = NDSolve[{Join[eqs, inis]}, # & /@ states, {t, 0, tmax},
      Method → {"EquationSimplification" → "Residual"}][[1]];

  (*host growth values*)
  (*all host growth values from the end of stress*)
  HGrowthVals = Flatten[
    Table[Evaluate[{H'[t] / H[t]} /. sol], {t, tStartStress + tHigh + 1, tmax}]];
  PosGrowth = Flatten[Position[HGrowthVals, _?(# > 0 &)]][[1]];
  (* position of these values at which growth is positive, which
   corresponds to the number of days after stress: if 1st position is pos.,
  then growth was pos. 1 day after stress ended, etc. *)

  (*host biomass SU limitation*)
  HSULim =
   Flatten[Table[Evaluate@Flatten@{Log[Min[jHGm, yC (ρC S / H + jX)] / Min[jHGm,
              ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1]] /.
          addtimetostatevars /. sol}, {t, tStartStress + tHigh + 1, tmax}]];
  HNLim = Flatten[Position[HSULim, _?(# > 0 &)]][[1]];

  (*symbiont biomass SU limitation*)
  SSULim = Flatten[Table[Evaluate@
       Flatten@{Log[Min[jSGm, jCP yC] / Min[jSGm, (rNS + (H ρN) / S) / nNS]] /.
          addtimetostatevars /. sol}, {t, tStartStress + tHigh + 1, tmax}]];
  SNLim = Flatten[Position[HSULim, _?(# > 0 &)]][[1]];

  (* time to recovery= when H growth is pos and both H and S SU are back at N-
    lim, so take the maximum of the timepoints at which each of these occur*)
  RecoveryTime = Max[PosGrowth, HNLim, SNLim];


  {RecoveryTime}(*record time to recovery*)],

 {P0, {0, kp * (1 - α) * H0}}],
(*repeat the above with no fish (P0=0) and with fish (P0=kp*(1-α)*H0)*)
{X, {0, 0.1 * 10^-7, 0.2 * 10^-7, 0.3 * 10^-7, 0.4 * 10^-7, 0.5 * 10^-7, 0.6 * 10^-7,
  0.7 * 10^-7, 0.8 * 10^-7, 0.9 * 10^-7, 1 * 10^-7, 1.1 * 10^-7, 1.2 * 10^-7,
  1.3 * 10^-7, 1.4 * 10^-7, 1.5 * 10^-7, 1.6 * 10^-7, 1.7 * 10^-7, 1.8 * 10^-7,
  1.9 * 10^-7, 2 * 10^-7}}(*repeat for different levels of prey*)
```

```
];
```

Get the results in the correct form to plot

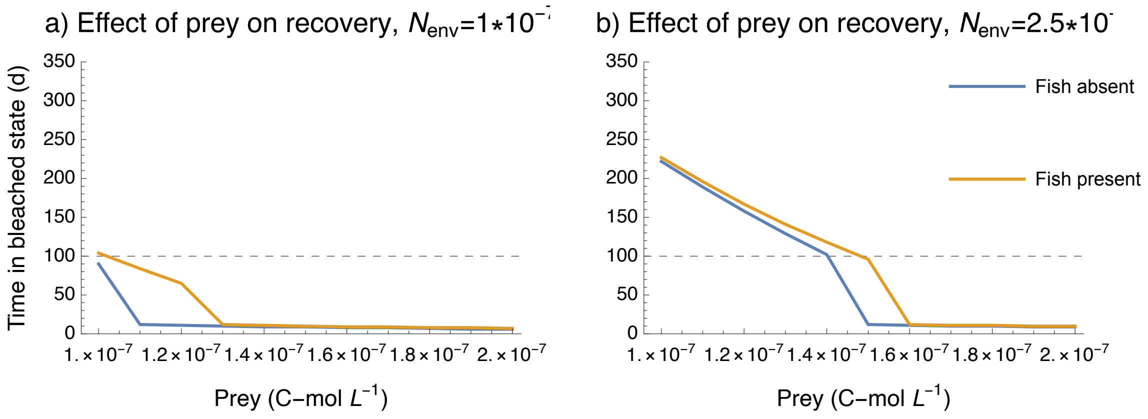
```
In[ ]:= (*getting the lists to plot (should have 2 lists total,
          one for the outputs without fish and one for the outputs with fish*)

        (*transpose this to change the nested list from
         {{P1 at X1, P2 at X1}, {X2}, ...{Xn}} to {{X1, X2, ... Xn at P1}, {P2}} *)
        RecovSeparatePX2 = Transpose[XRecovRuns2];
        (*prey no fish*)
        XRecovNoPX2 = Flatten[RecovSeparatePX2[[1]]];
        (*with prey with fish*)
        XRecovPX2 = Flatten[RecovSeparatePX2[[2]]];

        (*list of X values*)
        XVals3 = {0, 0.1 * 10^-7, 0.2 * 10^-7, 0.3 * 10^-7, 0.4 * 10^-7,
            0.5 * 10^-7, 0.6 * 10^-7, 0.7 * 10^-7, 0.8 * 10^-7, 0.9 * 10^-7,
            1 * 10^-7, 1.1 * 10^-7, 1.2 * 10^-7, 1.3 * 10^-7, 1.4 * 10^-7, 1.5 * 10^-7,
            1.6 * 10^-7, 1.7 * 10^-7, 1.8 * 10^-7, 1.9 * 10^-7, 2 * 10^-7};

        (*use Table to get the lists in the form to plot*)
        XRecovLists2 =
          Table[Module[{PlotList}, PlotList = Transpose@{XVals3, list}; PlotList],
           {list, {XRecovNoPX2, XRecovPX2}}];
```

Plot the results

```
In[◦]:= paddingS11 = {{60(*left*), 15(*right*)}, {60(*bottom*), 10 (*top*)}};

plotS11a = ListLinePlot[{XRecovLists⟦1⟧⟦11 ;; 21⟧, XRecovLists⟦2⟧⟦11 ;; 21⟧},
   PlotLabel → "a) Effect of prey on recovery, N_env=1*10^-7", PlotRange →
    {Full, {0, 350}}, ImageSize → Full, ImagePadding → paddingS11, LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   PlotStyle → {ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧},
   GridLines → {{}, {100}}, GridLinesStyle → Directive[Black, Dashed], Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
     {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
       {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
   FrameLabel → {{"Time in bleached state (d)", None}, {"Prey (C-mol L^-1)", None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12](*axis label size*)];

plotS11b = ListLinePlot[{XRecovLists2⟦1⟧⟦11 ;; 21⟧, XRecovLists2⟦2⟧⟦11 ;; 21⟧},
   PlotLabel → "b) Effect of prey on recovery, N_env=2.5*10^-7", PlotRange →
    {Full, {0, 350}}, ImageSize → Full, ImagePadding → paddingS11, LabelStyle →
    Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
   PlotStyle → {ColorData[97, "ColorList"]⟦1⟧, ColorData[97, "ColorList"]⟦2⟧},
   GridLines → {{}, {100}}, GridLinesStyle → Directive[Black, Dashed], Frame →
    {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
   (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
     {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
       {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
   FrameLabel → {{None, None}, {"Prey (C-mol L^-1)", None}},
   FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
   FrameStyle → Directive[Black, 12](*axis label size*)];
```

In[ ]:=

```
(*make the legend*)
XNRecovLeg =
  LineLegend[{ColorData[97, "ColorList"][[1]], ColorData[97, "ColorList"][[2]]},
   {"Fish absent", "Fish present"}, LegendMarkerSize → 40
   (*, LegendLayout→{"Row",1}*)];

XNRecovFig =
 Legended[GraphicsGrid[{{plotS11a, plotS11b}}, ImageSize → Full, Spacings → { {0
      (*left?*), -150 (*first (leftmost) and second col*)(*left to right*)},
     {0(*top*), 0(*first and second rows*)(*top to bottom*)}},
    Frame → False(*, AspectRatio→1/GoldenRatio*), Alignment → Bottom],
  Placed[XNRecovLeg, (*Bottom*){1.2, 0.85(*x, y positionof legend*)}]]
```



Out[ ]=

Fig. S12: Effects of ζ on bleaching

Run Simulations and format results

Run simulations

In[ ]:=
```
(*clear parameters that are changing and intermediate values from simulations*)
ClearAll[N, X, P0, jX, jN, Ni0, jNi0, jHG, jSG, ρN,
 jeC, jCO2, jL, rCH, rCS, jeL, jNPQ, cROS1, jST, rNH, A, jNw,
 tsolve, states, eqs, inis, sol, t, tmax, tHigh, LHigh, LfunH]
```

In[ ]:=
```
(* with prey*)
X = (*0*)1 * 10^-7;
jX = jXm X / (X + KX);

(* environment: flow and N level*)
d = 1660;
N = 1 * 10^-7;

(*jN=jNm N/(N+KN); (* these all need to be within Table[] because N is changing*)
Ni0 = N;
jNi0 = jNm N/(N+KN);*)

(*light function parameters: same light pulse as in section 2*)
tStartStress = 600;
LHigh = 30;
tHigh = 30;

tmax = 800;

(*use Table[] to run simulations at
 different values of zeta and with and without fish*)
zetaRuns =
  Table[
   Table[
    (*everything that is evaluated
     each iteration needs to be included in Module*)
    Module[{
```

```
 (*jX=(jXm X)/(X+KX),*) jN = (jNm ℕ)/(ℕ + KN), jHT = j0HT, rNS = σNS nNS j0ST,
 VH0 = kv * H0, Ni0 = ℕ, jNi0 = (jNm ℕ)/(ℕ + KN), states, rNH, S, H, VH, P, Ni,
 jNi, ρC, jCP, jHG, ρN, jeC, jCO2, A, jL, rCH, rCS, jeL, jNPQ, cROS1,
 jSG, jST, jNw, F, fh, LfunH, tsolve, addtimetostatevars, eqs,
 inis, sol(*put any intermediate output values in Module*)},
(*light function*)
LfunH = L + LHigh * HeavisideTheta[t - tStartStress] -
  LHigh * HeavisideTheta[t - (tStartStress + tHigh)];

F[ρ_][A_, B_] := (A B (A + B) ρ)/(A^2 B + A B^2 + A^2 ρ + A B ρ + B^2 ρ);
(*same as 1/(1/ρ + 1/A + 1/B - 1/(A+B))*)
(* helper function for VH*)
fh[t_?NumericQ, y_] := Piecewise[{{0, y < 0}, {kv * y, y ≥ 0}}];

(* Calculations *)
jHG = F[jHGm][yC (ρC S / H + jX), ((1 - α) * jNi + α * jN + nNX jX + rNH) nNH^-1];
jSG = F[jSGm][jCP yC, (rNS + (H ρN)/S)/nNS];
ρN = Max[(1 - α) * jNi + α * jN + nNX jX + rNH - nNH jHG (*yN^-1*), 0];
jeC = Max[jX - jHG/yC + (S ρC)/H, 0];
jCO2 = jeC kCO2;
jL = A astar LfunH;
rCH = (jHT + (jHG (1 - yC))/yC) σCH;
rCS = σCS (j0ST + (1 - yC) jSG yC^-1);
jeL = jL - jCP/yCL;
jNPQ = 1/(1/jeL + 1/kNPQ);
cROS1 = (jeL - jNPQ)/kROS;
jST = (1 + b cROS1) j0ST;
rNH = σNH nNH jHT;
A = 1.256307 + 1.385969 ⅇ^(-6.479055 S/H);
jNw = ρN * H / S + rNS - nNS * jSG;
```

```
tsolve = {
   {ρC, jCP - jSG yC^-1, ρC0},
   {jCP, F[jCPm] [jL yCL, rCS + H (jCO2+rCH)/S] / (1 + cROS1), jCP0},
   {jNi, jNm * Ni / (Ni + KN), jNi0}
  };

states = Join[tsolve[[All, 1]], {H, S, VH, Ni, P}];
addtimetostatevars = ((# → #[t]) &/@ states);
eqs = Join[
   (#[[1]]'[t] == (λ (Max[0, #[[2]]] - #[[1]]) /. addtimetostatevars)) &/@ tsolve,
   {
    S'[t] == (S (jSG - jST) /. addtimetostatevars),
    H'[t] == (H (jHG - jHT) /. addtimetostatevars),
    VH'[t] == fh[t, H'[t]],
    Ni'[t] == (d * (ℕ - Ni) + (ξ * (1 - α) * jNw * S + ep * P - jNi * (1 - α) * H) /
          ( vi * VH) /. addtimetostatevars),
    P'[t] ==
      (P * (rp * (kp * (1 - α) H - Bp * P) / (kp * (1 - α) H)) /. addtimetostatevars)
   }
  ];
 (*set initial conditions*)
 inis = Join[{
    H[0] == H0,
    S[0] == S0,
    VH[0] == VH0,
    Ni[0] == Ni0,
    P[0] == P0},
   (#[[1]][0] == #[[3]]) &/@ tsolve];

 (*simulate the model and call the output sol*)
 sol = NDSolve[{Join[eqs, inis]}, # &/@ states, {t, 0, tmax},
    Method → {"EquationSimplification" → "Residual"}][[1]];

 (* outputs to save each iteration of Table *)
 {sol}],

{P0, {0, kp * (1 - α) * H0}}],
{ξ, {1, 0.5, 0.1}}];(* values of zeta tested*)
```

Format results

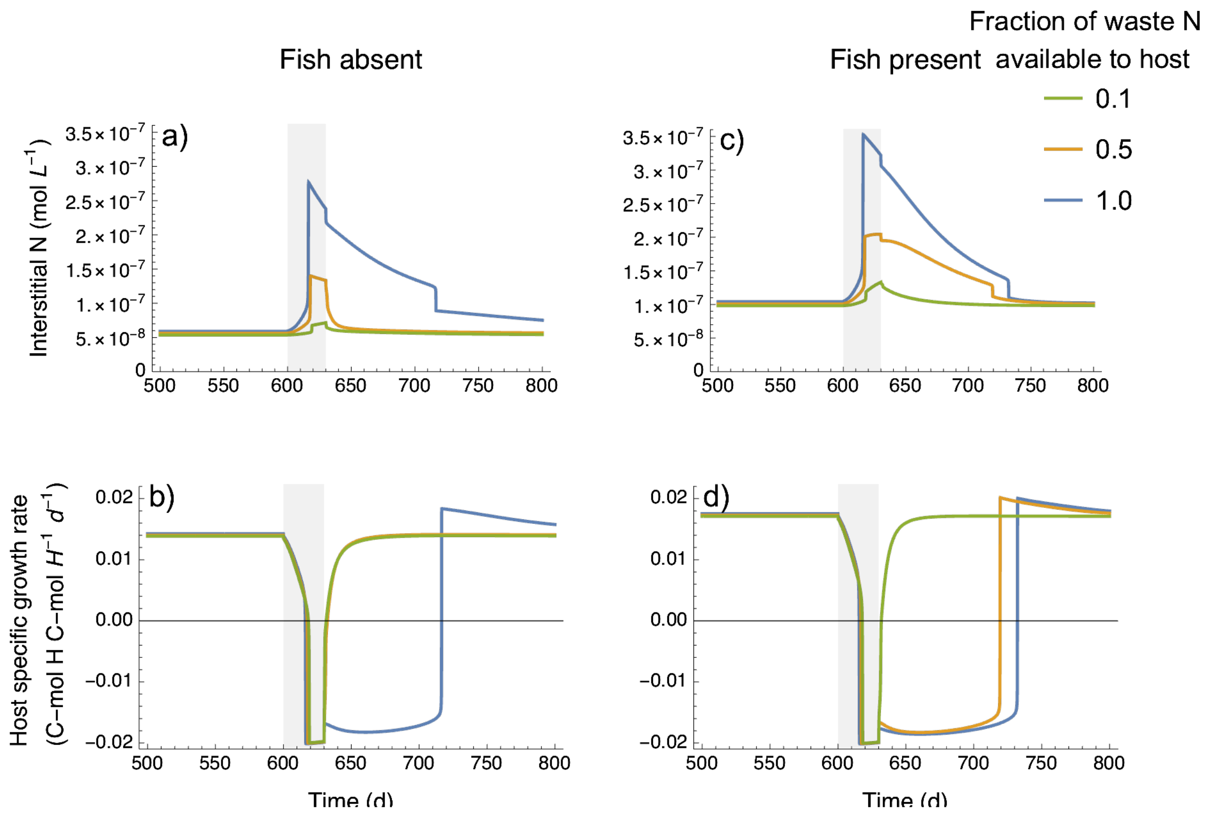
```
In[•]:= (*transpose this to change the nested list from
         {{P1 at z1, P2 at z1}, {z2}, {z3}} to {{z1, z2, z3 at P1}, {P2}} *)
        SeparateP = Transpose[zetaRuns];
        (*no fish*)
        zetaRunsNoP = Flatten[SeparateP⟦1⟧, 1];
        (*with fish*)
        zetaRunsP = Flatten[SeparateP⟦2⟧, 1];

        (*no fish, different zetas*)
        zeta100 = ToExpression[str[zetaRunsNoP⟦1⟧]];(*zeta=1*)
        zeta50 = ToExpression[str[zetaRunsNoP⟦2⟧]];(*zeta=0.5*)
        zeta10 = ToExpression[str[zetaRunsNoP⟦3⟧]];(*zeta=0.1*)

        (*fish, different zetas*)
        zeta100P = ToExpression[str[zetaRunsP⟦1⟧]];
        zeta50P = ToExpression[str[zetaRunsP⟦2⟧]];
        zeta10P = ToExpression[str[zetaRunsP⟦3⟧]];
```

Plot results

```
In[•]:= paddingS12 = {{80(*left*), 15(*right*)}, {30(*bottom*), 10 (*top*)}};

        (*no fish case*)
        (* plot interstitial nitrogen for the three values of zeta *)
        plotS12a = Show[Plot[{Evaluate[Ni[t] /. zeta100],
              Evaluate[Ni[t] /. zeta50], Evaluate[Ni[t] /. zeta10]}, {t, 500, tmax},
            PlotLabel → "Fish absent", PlotRange → {Full, {0, 3.6 * 10^-7}},
            Epilog → Text[Style["a)", {Black, 16}], Offset[{5(*right from top left*),
                -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}],
            LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
              12(*label size*)], ImagePadding → paddingS12, LabelStyle →
             Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
            ImageSize → Full, AxesOrigin → {500, 0}, Frame →
             {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
            (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
              {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},{6*10^-7, "0.6"},
        {8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)FrameLabel →
      {{"Interstitial N (mol L^-1)", None}(*\n makes line break*), {None, None}},
     FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
     FrameStyle → Directive[Black, 12](*axis label size*),
     PlotStyle → {{ColorData[97, "ColorList"]⟦1⟧}, {ColorData[97, "ColorList"]⟦2⟧
        (*,Dashed*)}, {ColorData[97, "ColorList"]⟦3⟧}}],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
     {y, -10, 10}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

(* plot host growth rate for the three values of zeta *)
plotS12b = Show[Plot[{Evaluate[H'[t] / H[t] /. zeta100],
      Evaluate[H'[t] / H[t] /. zeta50], Evaluate[H'[t] / H[t] /. zeta10]},
     {t, 500, tmax}, PlotRange → {Full, {-0.021, 0.022}},
     Epilog → Text[Style["b)", {Black, 16}], Offset[{5(*right from top left*),
         -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}],
     LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
       12(*label size*)], ImagePadding → paddingS12, LabelStyle →
      Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
     ImageSize → Full, AxesOrigin → {500, 0}, Frame →
      {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
     (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
       {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
         {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
     FrameLabel → {{"Host specific growth rate \n (C-mol H C-mol H^-1 d^-1)", None}
       (*\n makes line break*), {"Time (d)", None}},
     FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
     FrameStyle → Directive[Black, 12](*axis label size*),
     PlotStyle → {{ColorData[97, "ColorList"]⟦1⟧}, {ColorData[97, "ColorList"]⟦2⟧
        (*,Dashed*)}, {ColorData[97, "ColorList"]⟦3⟧}}],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
     {y, -10, 10}, Frame → False, Mesh → None,
     BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

(*with fish case*)
(* plot interstitial nitrogen for the three values of zeta  *)
plotS12c = Show[Plot[{Evaluate[Ni[t] /. zeta100P],
      Evaluate[Ni[t] /. zeta50P], Evaluate[Ni[t] /. zeta10P]}, {t, 500, tmax},
     PlotLabel → "Fish present", PlotRange → {Full, {0, 3.6 * 10^-7}},
     Epilog → Text[Style["c)", {Black, 16}], Offset[{5(*right from top left*),
         -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}],
     LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
        12(*label size*)], ImagePadding → paddingS12, LabelStyle →
       Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
      ImageSize → Full, AxesOrigin → {500, 0}, Frame →
       {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
      (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
        {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
          {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
      FrameLabel → {{None, None}, {None, None}}, FrameTicksStyle →
       Directive[Black, 10(*size of ticks*)],
      FrameStyle → Directive[Black, 12](*axis label size*),
      PlotStyle → {{ColorData[97, "ColorList"][[1]]}, {ColorData[97, "ColorList"][[2]]
         (*,Dashed*)}, {ColorData[97, "ColorList"][[3]]}}],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
      {y, -10, 10}, Frame → False, Mesh → None,
      BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];

(* plot host growth rate for the three values of zeta *)
plotS12d = Show[Plot[{Evaluate[H'[t] / H[t] /. zeta100P],
       Evaluate[H'[t] / H[t] /. zeta50P], Evaluate[H'[t] / H[t] /. zeta10P]},
      {t, 500, tmax}, PlotRange → {Full, {-0.021, 0.022}},
      Epilog → Text[Style["d)", {Black, 16}], Offset[{5(*right from top left*),
          -5(*down from top left*)}, Scaled[{0, 1}]], {-1, 0}],
      LabelStyle → Directive[FontColor → Black, FontFamily → "Helvetica",
        12(*label size*)], ImagePadding → paddingS12, LabelStyle →
       Directive[FontColor → Black, FontFamily → "Helvetica", 12(*label size*)],
      ImageSize → Full, AxesOrigin → {500, 0}, Frame →
       {{True(*left*), False(*right*)}, {True(*bottom*), False(*top*)}},
      (*FrameTicks→(*True*){{{0, 0.01,0.02,0.03,0.04,0.05},None},
        {(*None*){{0, "0"},{2*10^-7, "0.2"},{4*10^-7, "0.4"},
          {6*10^-7, "0.6"},{8*10^-7, "0.8"},{1*10^-6, "1.0"}},None}},*)
      FrameLabel → {{None, None}, {"Time (d)", None}},
      FrameTicksStyle → Directive[Black, 10(*size of ticks*)],
      FrameStyle → Directive[Black, 12](*axis label size*),
      PlotStyle → {{ColorData[97, "ColorList"][[1]]}, {ColorData[97, "ColorList"][[2]]
         (*,Dashed*)}, {ColorData[97, "ColorList"][[3]]}}],
    RegionPlot[tStartStress < t < tStartStress + tHigh, {t, 590, 640},
      {y, -10, 10}, Frame → False, Mesh → None,
      BoundaryStyle → None, PlotStyle → GrayLevel[0.09, 0.06]]];
```

In[◦]:=
```
zetaLeg = LineLegend[{ColorData[97, "ColorList"]⟦3⟧, ColorData[97, "ColorList"]⟦2⟧,
    ColorData[97, "ColorList"]⟦1⟧}, {"0.1", "0.5", "1.0"},
   LegendLabel → "Fraction of waste N \n available to host", LabelStyle → {14}];

zetaFig = Legended[
  GraphicsGrid[{{plotS12a, plotS12c}, {plotS12b, plotS12d}}, ImageSize → Full,
   Spacings → { {0(*left?*), -80(*labels and left col*)(*left to right*)}, {0
      (*top*), 0(*middle 2 rows*) (*labels and bottom*)(*top to bottom*)}},
   Frame → False(*, AspectRatio→1/GoldenRatio*)(*, Alignment→ Bottom*)
   (*align the grid at the bottom of its frame*)],
  Placed[zetaLeg, (*Bottom*){1.2(*left to right position is in the center*),
    1(*top to bottom position*)}]]
```

Out[◦]=


Fish absent
Fish present
Fraction of waste N available to host
a)
b)
c)
d)
Interstitial N (mol L^{-1})
Host specific growth rate (C–mol H C–mol H^{-1} d^{-1})
3.5×10^{-7}
$3.\times10^{-7}$
2.5×10^{-7}
$2.\times10^{-7}$
1.5×10^{-7}
$1.\times10^{-7}$
$5.\times10^{-8}$
Time (d)