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	3	4	6	3	5	4	
	5	2		2		3	
3	2		1	1	2		
6		3	1	1	3	2	4
	2	4	2		6	5	
		5		2	4	2	

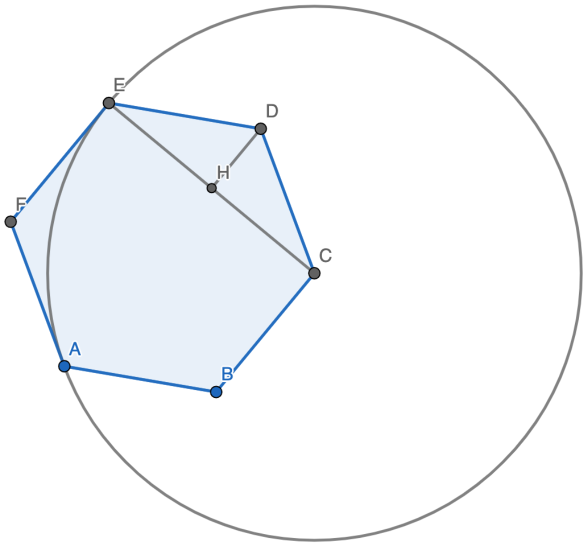

Figure 1: Diagram

Let H be the midpoint of EC. EDC is isosceles, therefore DH is an angle bisector and altitude.

$$\angle EDC = 120^\circ \implies \angle HDC = 60^\circ.$$

$$HC = \frac{1}{2} \cdot EC = \frac{1}{2}$$

Observe that DHC is a 30-60-90 triangle. Therefore,

$$DC = \frac{\sqrt{3}}{3}$$

Using the formula for area of a hexagon given side length,

$$[ABCDEF] = \frac{3\sqrt{3}}{2} \cdot DC^2 = \frac{3\sqrt{3}}{2} \cdot \frac{1}{3} = \boxed{\frac{\sqrt{3}}{2}}$$

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We first show that $M < 4$. Split the positive integers $\{1, 2, \ldots\ldots, 2024\}$ into two groups,

$$\{1, 2, 3, 7, 8, 9 \ldots\}$$

and

$$\{4, 5, 6, 10, 11, 12 \ldots\}$$

Note that the difference between two consecutive elements is either 1 or 4. Picking more than 3 consecutive elements from either set will always result in the difference between some two elements being 4. By definition, the sequence would not be fibtastic.

Therefore,

$$M < 4$$

We argue that $M = 3$ holds. Let A and B be defined as they were originally in the problem statement. Without loss of generality, assume that there exists no fibtastic sequence of length 3 in A. Then the difference between some two consecutive elements in A must not be a Fibonacci number.

Note that the least non-Fibonacci positive integer is 4. Since there exists a non-Fibonacci difference in A, there will be at least 3 consecutive positive integers in B, which are also consecutive elements, since A and B are in increasing order. Therefore, there will always exist a fibtastic sequence of 3 in either A or B no matter how they are constructed. It is unnecessary to examine $M < 3$, as $M = 3$ holds and $M < 4$.

Hence, our answer is

$$\boxed{M = 3}$$

Denote the set of all mathematicians as m. Define k to be the maximum number of mathematicians asleep at any given moment during the lecture.

Let $t(i) =$ [time m_i falls asleep, time m_i wakes up]

Put each m_i in a group, the constraint being that any m_i and m_j must not be in the same group if $t(i) \cap t(j) \neq \emptyset$.

Observe that in every case of k, there will be exactly k groups necessary to separate k mathematicians with overlapping sleep intervals.

By the Pigeonhole Principle, if the mathematicians are distributed among k groups, then at least one group is guaranteed to contain $\lceil \frac{26}{k} \rceil$ mathematicians.

Assume $k < 6$. Note that $\lceil \frac{26}{k} \rceil \geq 6$ for $k \in \{1, 2, 3, 4, 5\}$. Therefore, at least one group will contain 6 or more mathematicians. By our definition, no group can contain any two mathematicians with overlapping sleep intervals. It follows that there will exist at least 6 mathematicians whose sleep intervals never overlap in this case. Note that there are less than 6 holes, therefore there cannot exist a set of 6 mathematicians who are all asleep together at some moment during the lecture in this case.

Now assume $k \geq 6$. By definition, it follows that there exists a set of 6 mathematicians asleep together at some moment during the lecture.

Define A as the truth value of the statement "there exists a set of 6 mathematicians all asleep together at some moment during the lecture" and

B as the truth value of "there exists a set of 6 mathematicians such that no two are asleep together at any moment during the lecture". Notice we have already proved "If B, then not A" is true. Then we must prove that "If A, then not B" is true to fully satisfy the "either" condition in the problem. Note that these two statements are contrapositives of eachother, so "If A, then not B" is true.

Hence, we have proven that there exists either a set of 6 mathematicians all asleep together at some moment during the lecture, or a set of 6 mathematicians such that no two are asleep together at any moment.

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Expanding $f(f(x)+x)$, we obtain

$$f(f(x)+x) = x^4 + 2x^3 + 3x^2 + x^2b^2 + 2x + xb^2 + 2bx^3 + 3bx^2 + 3bx + b + 2$$

which can be factored into

$$(x^2 + bx + 1)(x^2 + (b+2)x + b + 2)$$

Denote $f_1(x) = x^2 + bx + 1$ and $f_2(x) = x^2 + (b+2)x + b + 2$.

We wish to find $f_1(x) \cdot f_2(x) < 0$

Denote the roots of $f_1(x)$ as r_1, r_2 and $f_2(x)$ as r_3, r_4

We solve with the quadratic formula. Then

$$r_1 = \frac{-b + \sqrt{b^2 - 4}}{2}, r_2 = \frac{-b - \sqrt{b^2 - 4}}{2}$$

And

$$r_3 = \frac{-b - 2 + \sqrt{b^2 - 4}}{2}, r_4 = \frac{-b - 2 - \sqrt{b^2 - 4}}{2}$$

From here, we split into two cases based on the discriminant.

Case 1: $b^2 - 4 \leq 0$

Neither f_1 nor f_2 will ever be negative, therefore, there are no solutions in this case. So, one possible value is 0.

Case 2: $b^2 - 4 > 0$

Note that $r_3 = r_1 - 1, r_4 = r_2 - 1$

We also know that $f_1(x) < 0$ for $x \in (r_2, r_1)$ and $f_2(x) < 0$ for $x \in (r_4, r_3)$

We examine 5 critical intervals:

1. $(-\infty, r_4)$

   $f_1(x) > 0$ and $f_2(x) > 0$, therefore $f_1(x) \cdot f_2(x) > 0$ in this interval

2. (r_4, r_2)

   $f_1(x) > 0$ and $f_2(x) < 0$, therefore $f_1(x) \cdot f_2(x) < 0$ in this interval

3. (r_2, r_3)

   $f_1(x) < 0$ and $f_2(x) < 0$, therefore $f_1(x) \cdot f_2(x) > 0$ in this interval

4. (r_3, r_1)

   $f_1(x) < 0$ and $f_2(x) > 0$, therefore $f_1(x) \cdot f_2(x) < 0$ in this interval

5. (r_1, ∞)

   $f_1(x) > 0$ and $f_2(x) > 0$, therefore $f_1(x) \cdot f_2(x) > 0$ in this interval

Observe that $f_1(x) \cdot f_2(x) < 0$ for $x \in (r_3, r_1) \cup (r_4, r_2)$

Equivalently,

$f_1(x) \cdot f_2(x) < 0$ for $x \in (r_1 - 1, r_1) \cup (r_2 - 1, r_2)$

If r_1 and r_2 are both integers, then exactly 0 solutions lie in each of $(r_1 - 1, r_1)$ and $(r_2 - 1, r_2)$

If exactly one of (r_1, r_2) is an integer (e.g. b = 2.5), exactly one solution will lie in either $(r_1 - 1, r_1)$ or $(r_2 - 1, r_2)$, leading to a total of 1 solution.

If both r_1 and r_2 are not integers, exactly one solution will lie in each of $(r_1 - 1, r_1)$ and $(r_2 - 1, r_2)$, leading to a total of 2 solutions.

Obviously, no more than two integers can exist in

$$(r_1 - 1, r_1) \cup (r_2 - 1, r_2)$$

Therefore, our answer is

$$\boxed{0, 1, 2}$$