Digital Media and Social Networks

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WEEK 1: NETWORKS, RANDOM GRAPHS AND METRICS

SOME SLIDES COPYRIGHT CECILIA MASCOLO (CAMBRIDGE) AND HAMED HADDADI (QMUL)

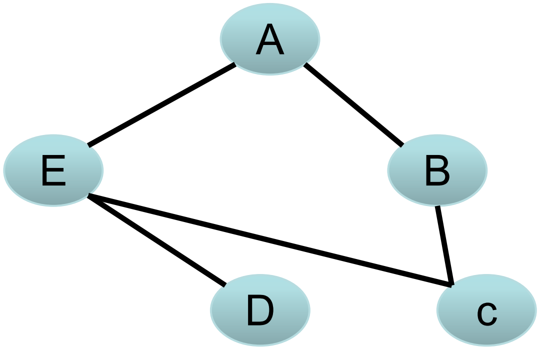
WHAT IS A GRAPH?

WHAT IS A GRAPH?

A **set of objects** where some pairs of objects are **connected by links**

A way of modelling the **connectivity** of **things**

Usually represented in either visual manner of using set theory



WHAT IS A SET?

WHAT IS A SET?

A **set** is a collection of **distinct objects**

Notation uses { }

V = {Gareth, Laurissa, Bob}

WHAT IS A SET?

Element of is an element that exists in a given set

Notation uses $a \in \mathrm{A}$

V={Gareth,Laurissa,Bob}

Gareth $\in$ V

...means that Gareth is part of set V

WHAT IS A SET?

Cardinality is the number of elements in a set

Notation uses $|V|$

V={Gareth,Laurissa,Bob}

$|V|=3$

...there are 3 elements in the set

WHAT IS A SET?

Such that means that a value is subject to certain constraints

Imagine you want to extract all people who are over the age of 65

Notation is |

$Y = \{x \mid x \in B, x < 0\}$

...Y contains all negative numbers in B

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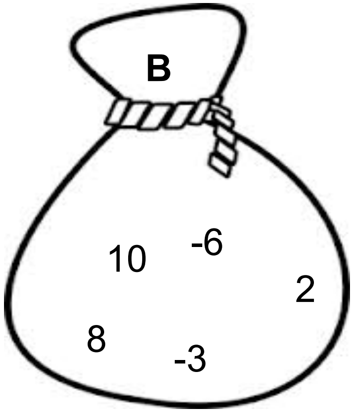

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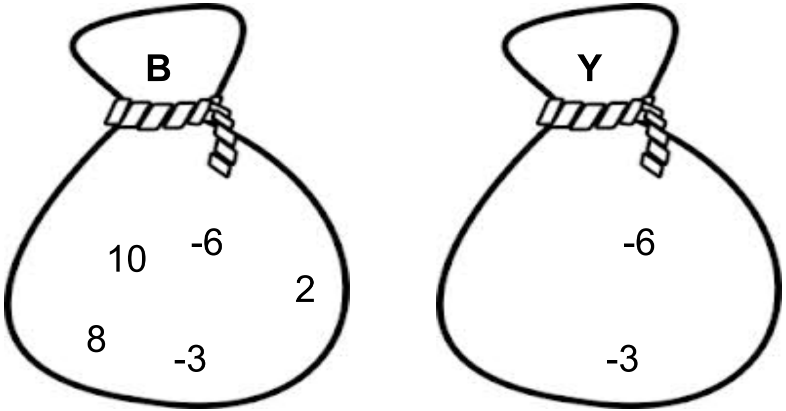

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Imagine you want to extract all people who are over the age of 65

Notation is |

We usually use uppercase letters indicate a set

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WHAT IS A SET?

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Imagine you want to extract all people who are over the age of 65

Notation is |

$\mathrm{Y} = \{x \mid x \in \mathrm{B},\ x<0\}$

We usually use lowercase letters indicate an individual element from a set

...Y contains all negative numbers in B

HOW TO DESCRIBE A GRAPH

A NETWORK IS A GRAPH

A **graph** G is a tuple (V,E) of a set of vertices V and edges E. An edge in E connects two vertices in V.

A NETWORK IS A GRAPH

V = {Gareth, Laurissa, Bob} for the vertices

E = {(Gareth,Laurissa), (Laurissa,Bob)} for the edges

G =?

A NETWORK IS A GRAPH

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G = (V, E)

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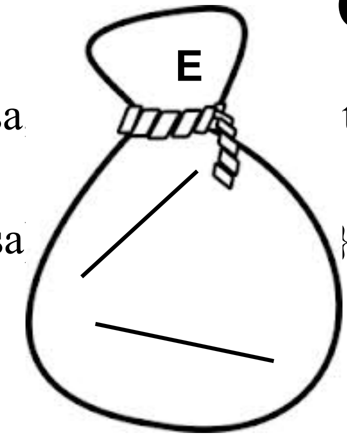
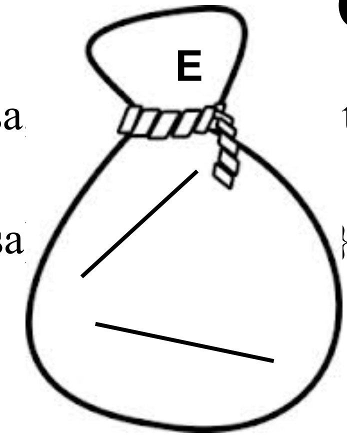


A NETWORK IS A GRAPH

G



V = {Gareth, Laurissa … tices

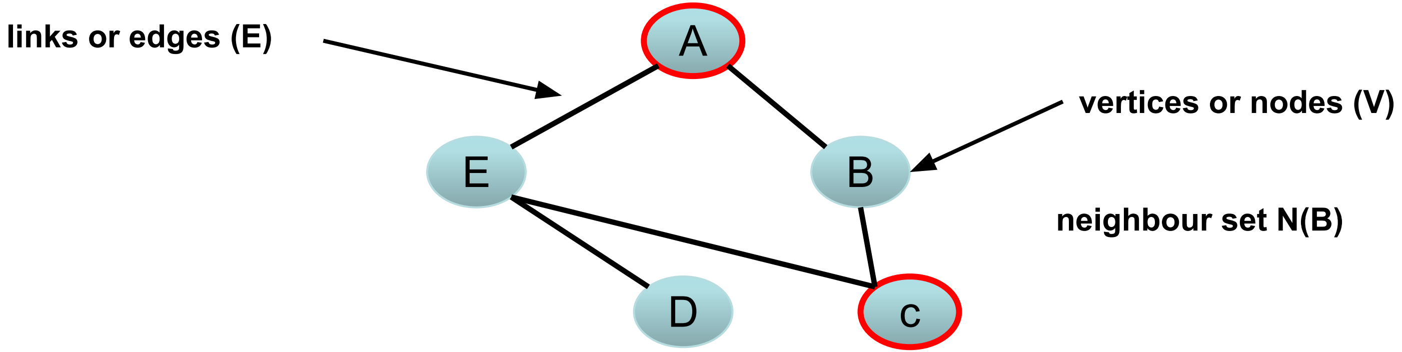
E = {(Gareth,Laurissa … } for th

G = (V, E)

|**V**| = ?

HOW TO MEASURE A GRAPH

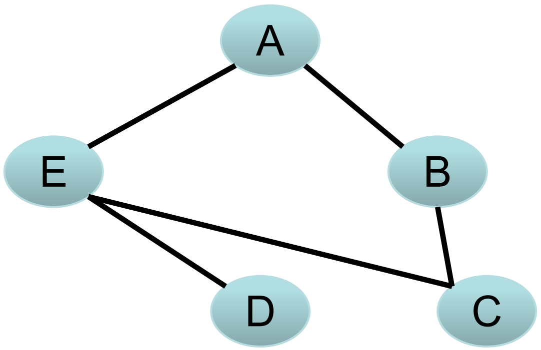
NEIGHBOUR SETS



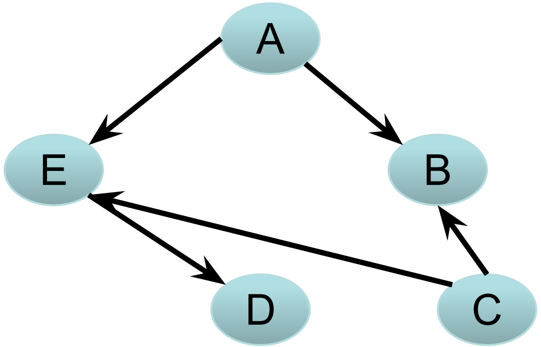
A **neighbour set** N(v) is the set of vertices adjacent to v:

$$N(v) = \{u \in V \mid u \neq v, (v,u) \in E\}$$

DIRECTED & UNDIRECTED GRAPHS



Undirected
Graph

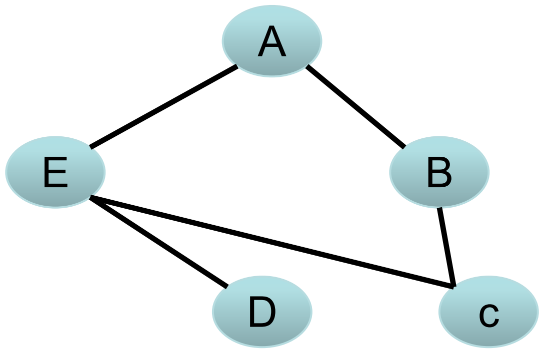


Directed
Graph

A directed graph has
arrowheads!

NODE DEGREE

The **node degree (k)** is the number of neighbours of a node

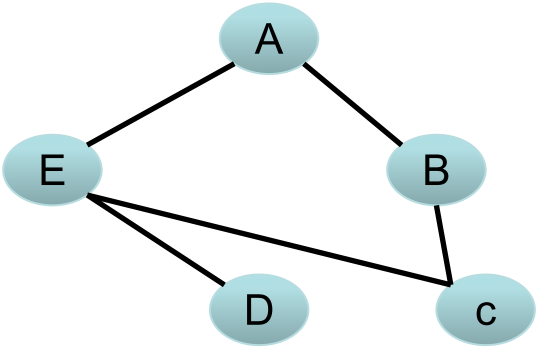


The study of the degree distribution of networks allows the classification of networks in different categories

NODE DEGREE

The **node degree (k)** is the number of neighbours of a node

If we wanted to have fun, we could say **|N(v)|**



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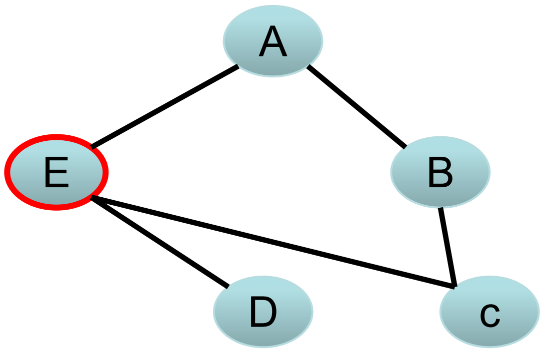
Degree of E?



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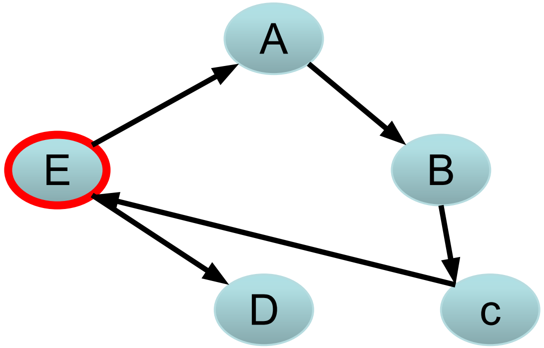
In-degree of E?



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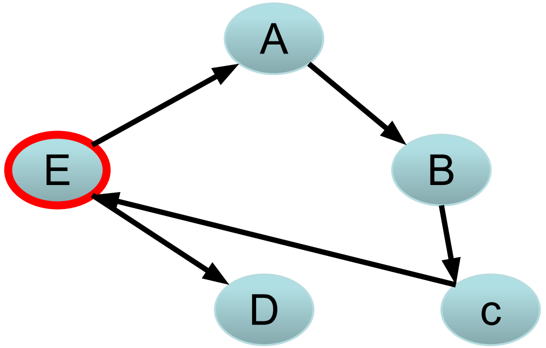
Out-degree of E?



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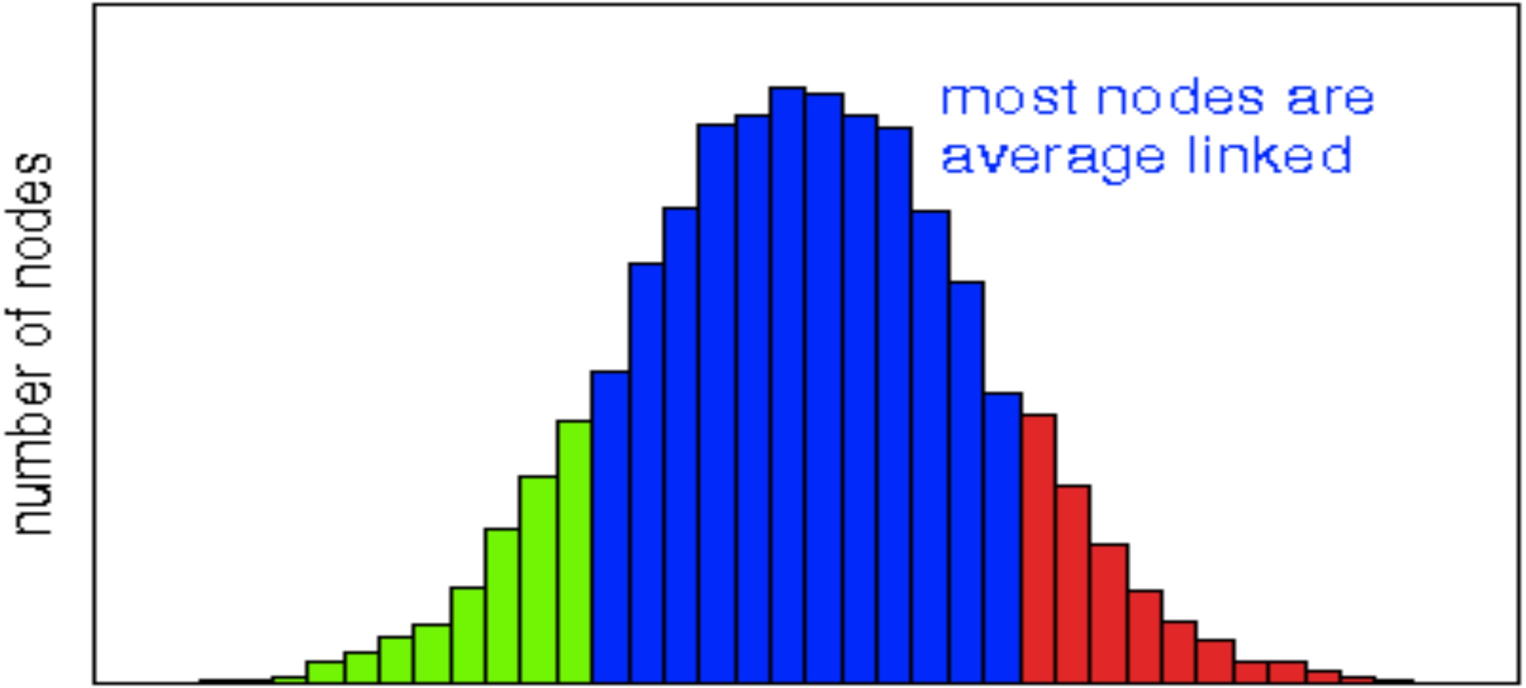

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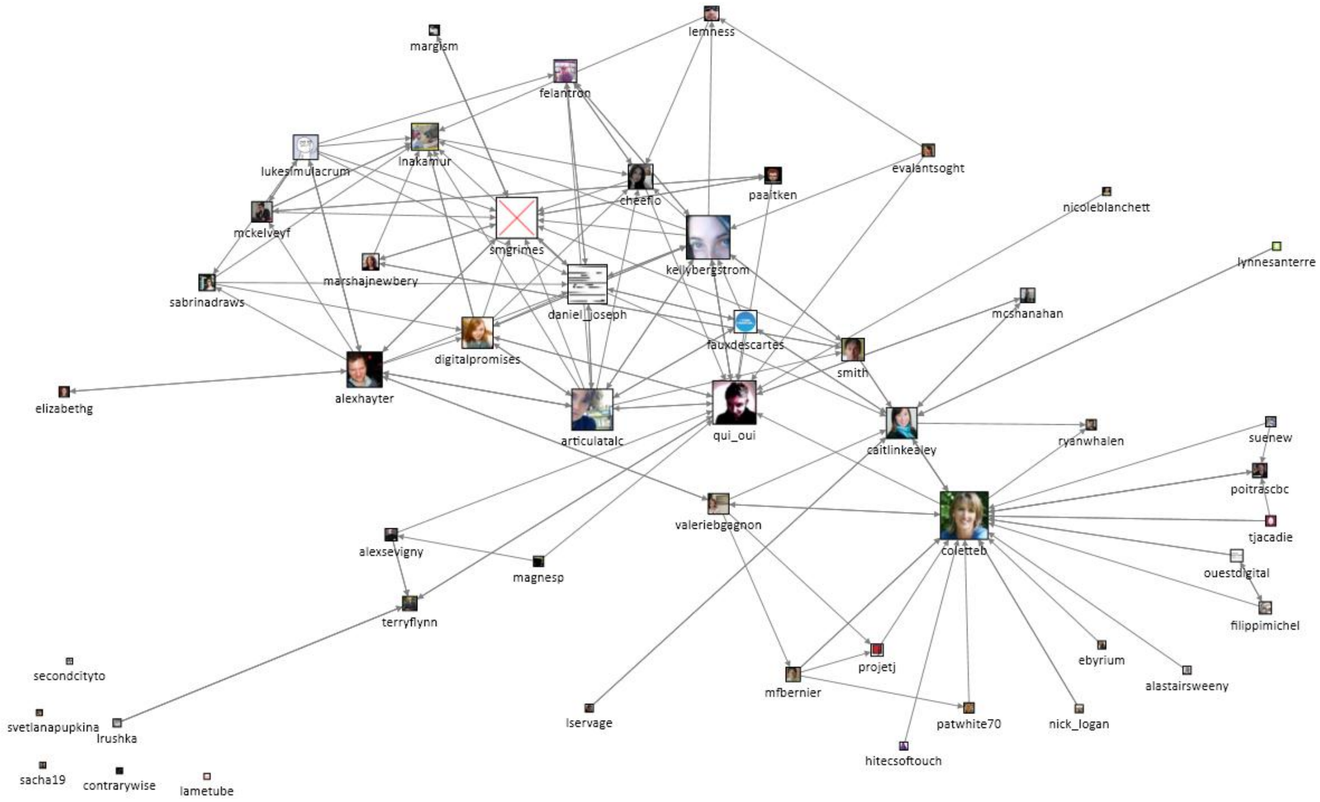
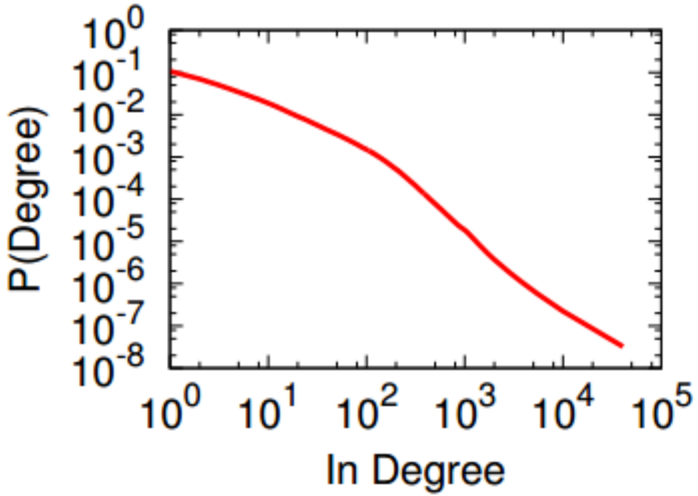
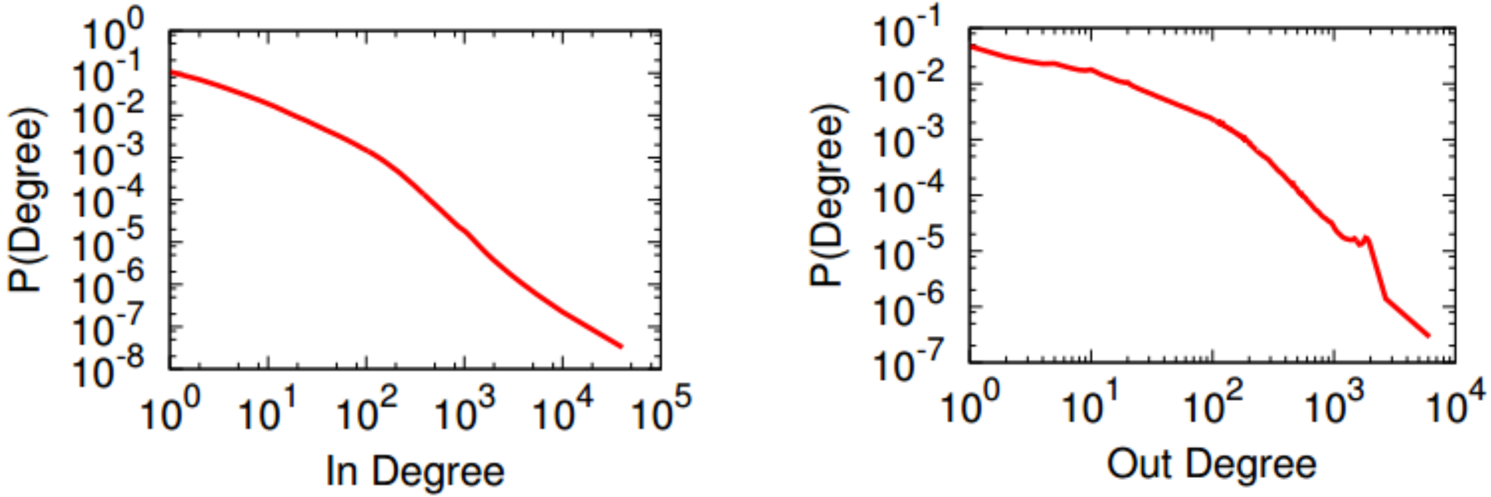

margism
lemness
felantron
lnakamur
lukesimulacrum
cheeflo
paaltken
evalantsoght
nicoleblanchett
mckelveyf
smgrimes
kellybergstrom
lynnesanterre
marshajnewbery
sabrinadraws
daniel_joseph
mcshanahan
fauxdescartes
digitalpromises
smith
elizabethg
alexhayter
articulatalc
qui_oui
caitlinkealey
ryanwhalen
suenew
poitrascbc
valeriebgagnon
coletteb
tjacadie
alexsevigny
magnesp
ouestdigital
terryflynn
filippimichel
ebyrium
secondcityto
projetj
alastairsweeny
mfbernier
lservage
patwhite70
nick_logan
svetlanapupkina
lrushka
hitecsoftouch
sacha19
contrarywise
lametube

NODE DEGREE



P(Degree)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^0 10^1 10^2 10^3 10^4 10^5
In Degree

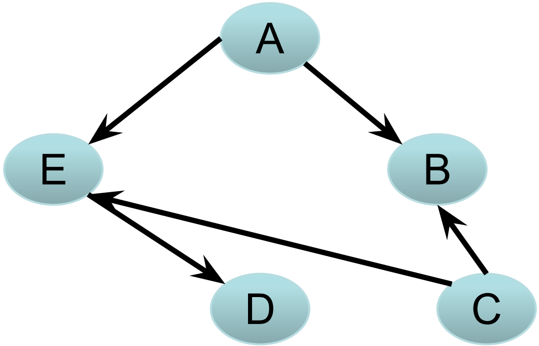
NODE DEGREE



The Structure of the Twitter Follower Graph

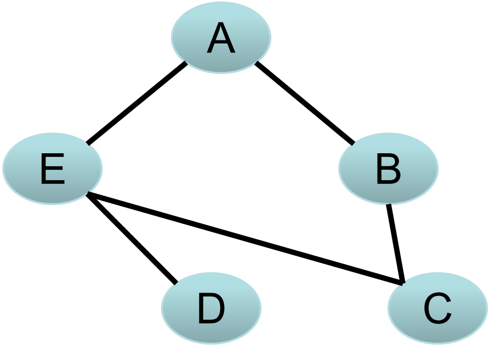
http://wwwconference.org/proceedings/www2014/companion/p493.pdf

PATHS AND CYCLES



A **path** is a sequence of nodes in which each pair of consecutive nodes is connected by an edge.

- If graph is directed the edge needs to be in the right direction.
- E.g. A-E-D is a path in both previous graphs



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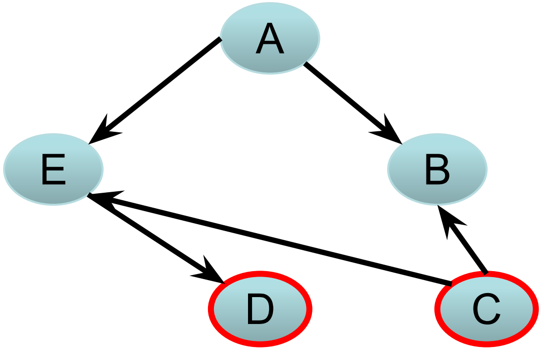

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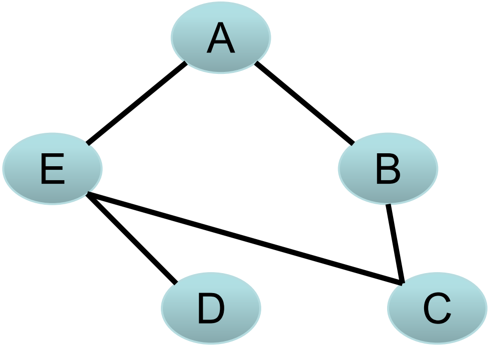

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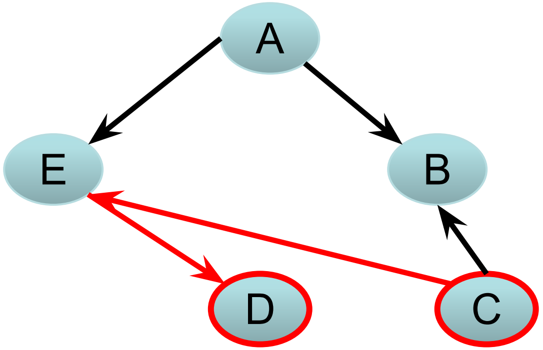

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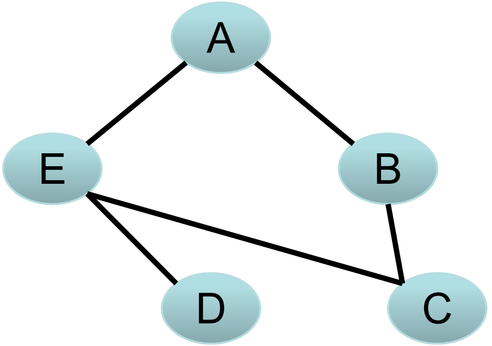

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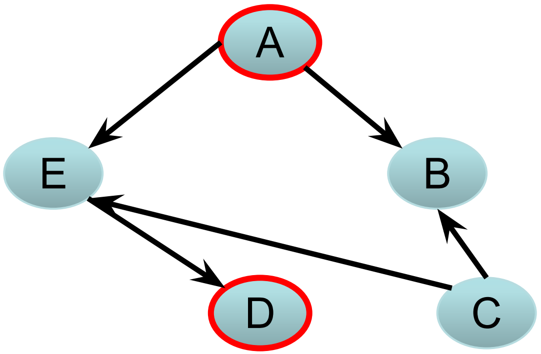

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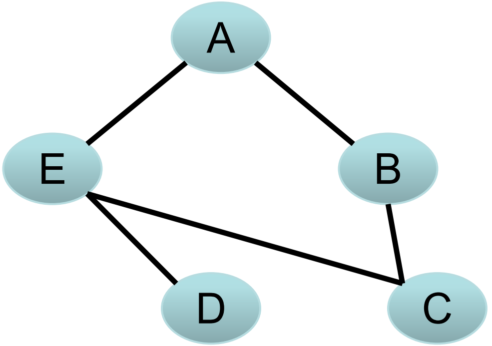

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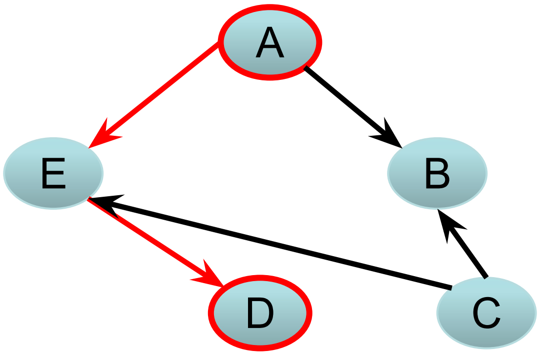

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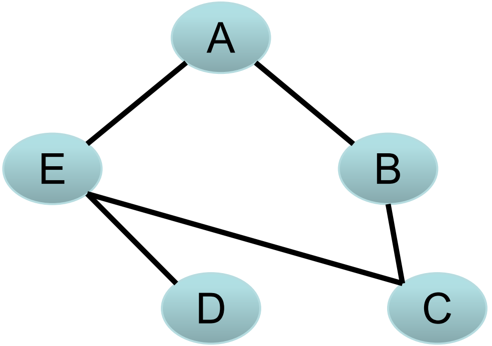

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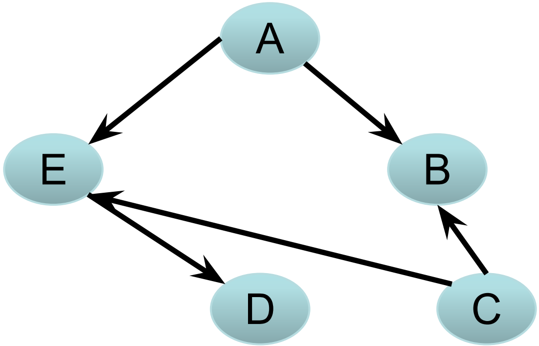

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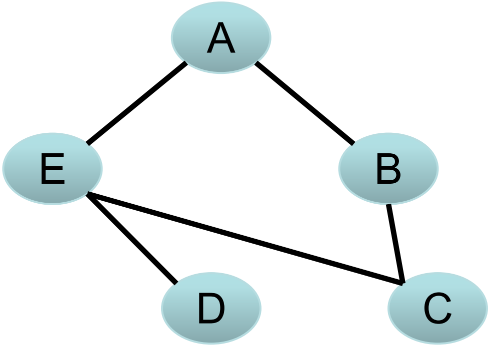
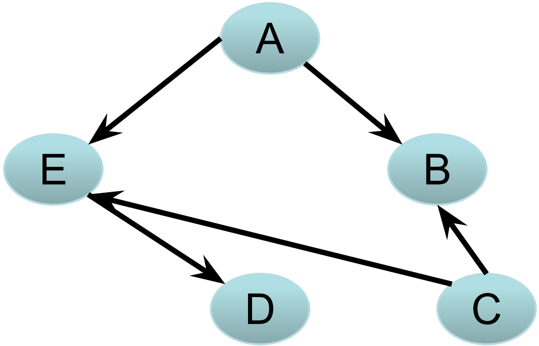


A **cycle** is a path where the start node is also the end node

E.g. E-A-B-C is a cycle in the undirected graph

PATHS AND CYCLES



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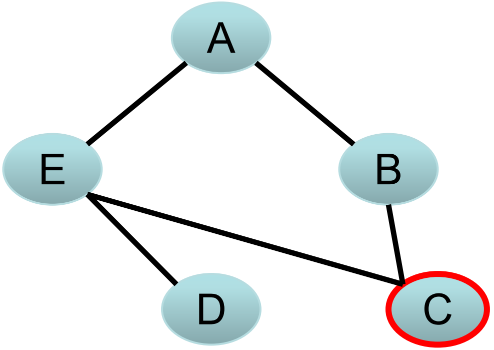

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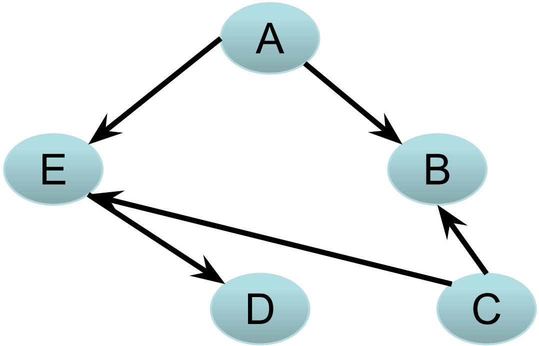

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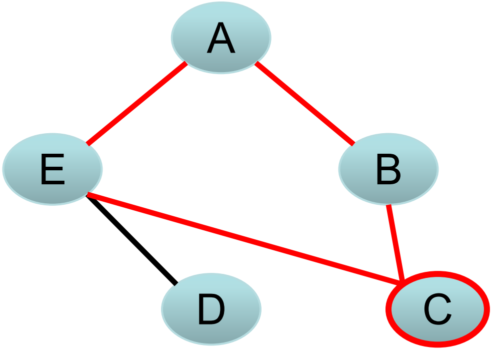

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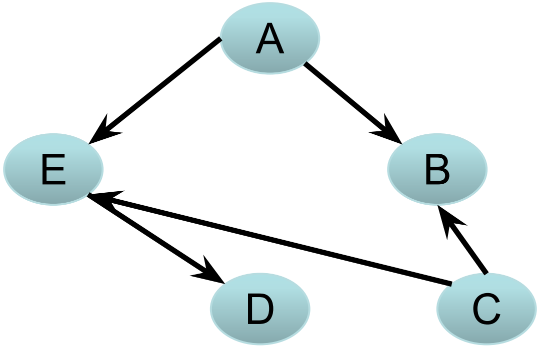

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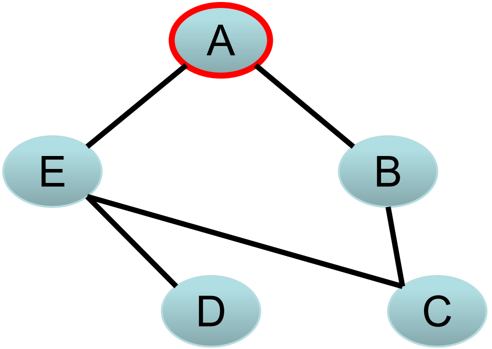

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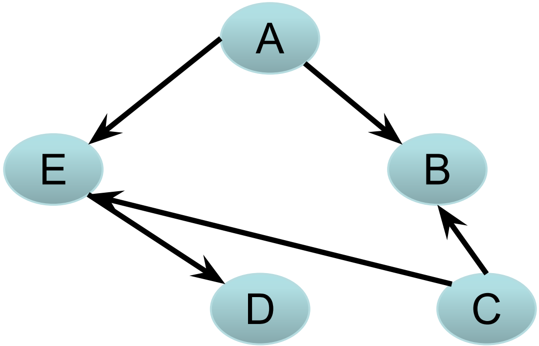

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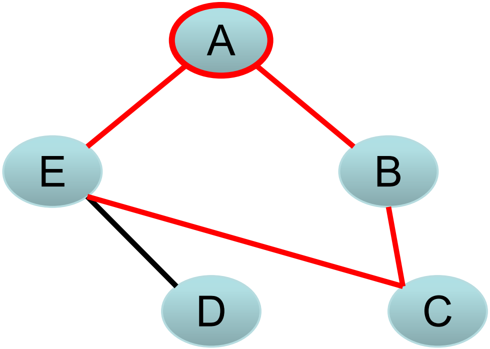

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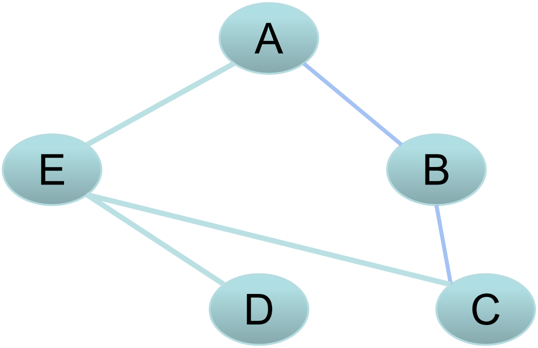
PATH LENGTH/DISTANCE

The **distance** (d) between two nodes in a graph is the length of the shortest path linking the two graphs.

The **diameter** of the graph is the maximum distance between any pair of its nodes.



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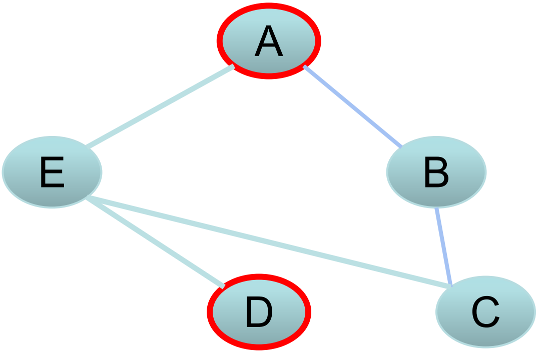

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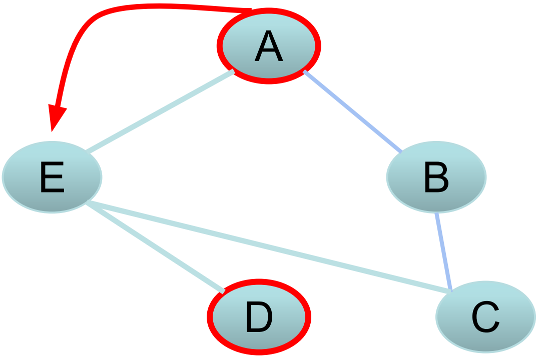

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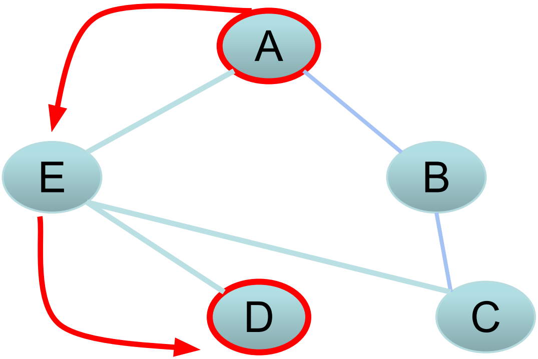

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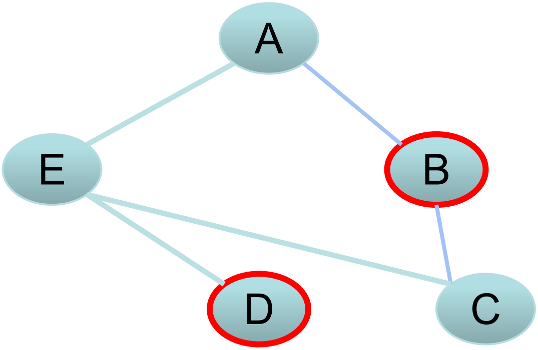

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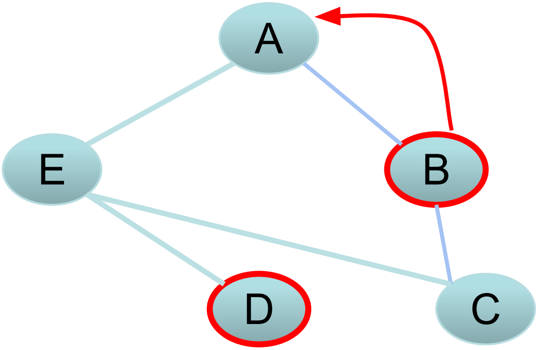

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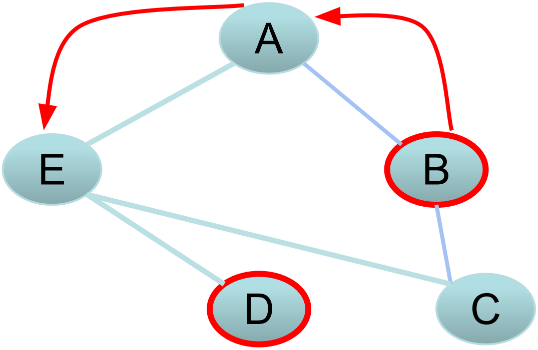

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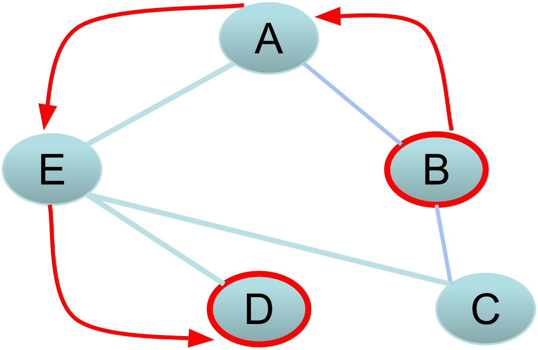

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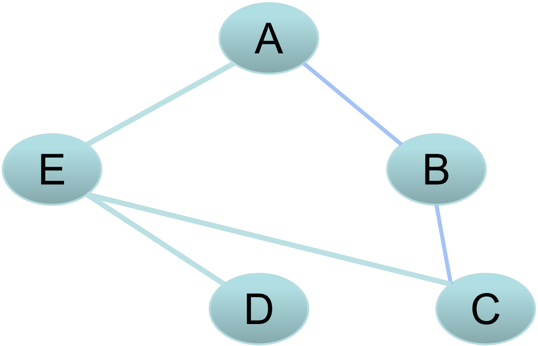

What’s the diameter?

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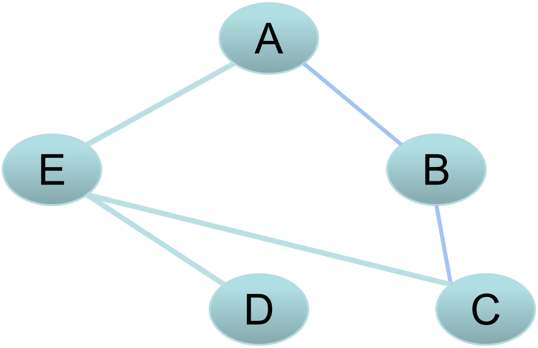


d(A,B) = 1

d(A,C) = 2

d(B,D) = 3

.

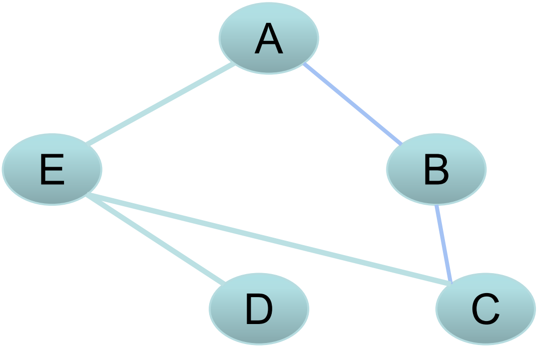
.

.

PATH LENGTH/DISTANCE

The **distance** (d) between two nodes in a graph is the length of the shortest path linking the two graphs.

The **diameter** of the graph is the maximum distance between any pair of its nodes.



d(A,B) = 1

d(A,C) = 2

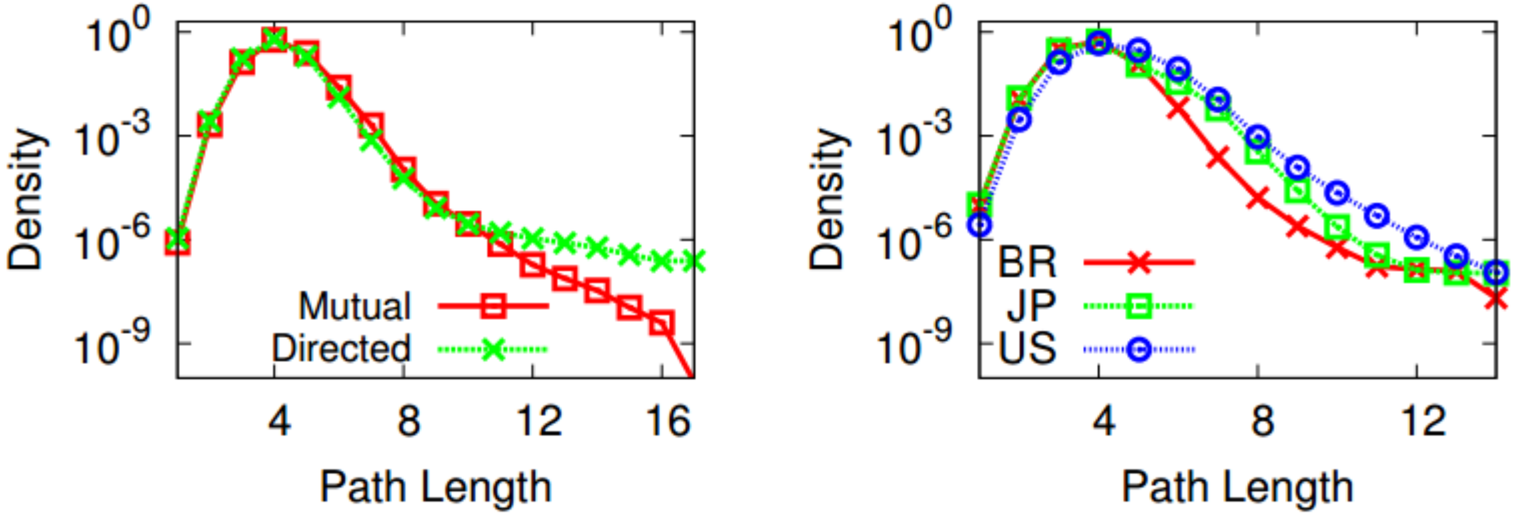
d(B,D) = **3**

.

.

.

PATH LENGTH/DISTANCE



CONNECTIVITY

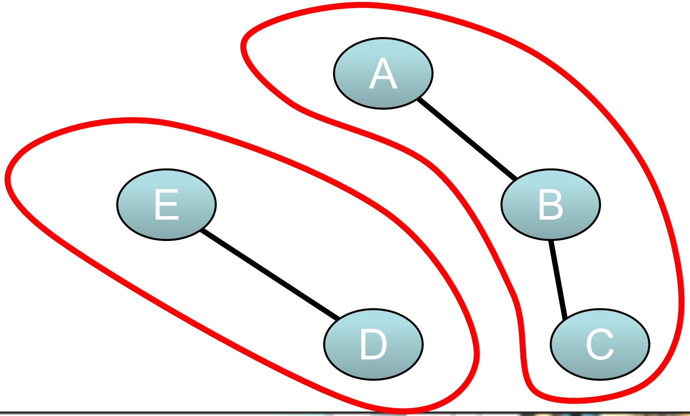
A graph is **connected** if there is a path between *each pair* of nodes.

Example of **disconnected** graph:

COMPONENTS

A **connected component** of a graph is the subset of nodes for which each of them has a path to all others (and the subset is not part of a larger subset with this property).

Connected components: A-B-C and E-D



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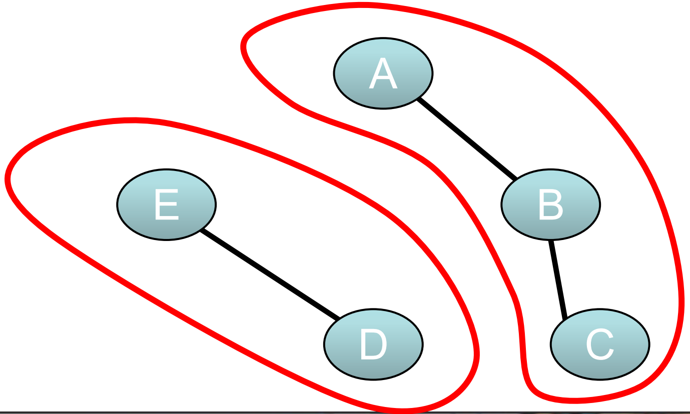
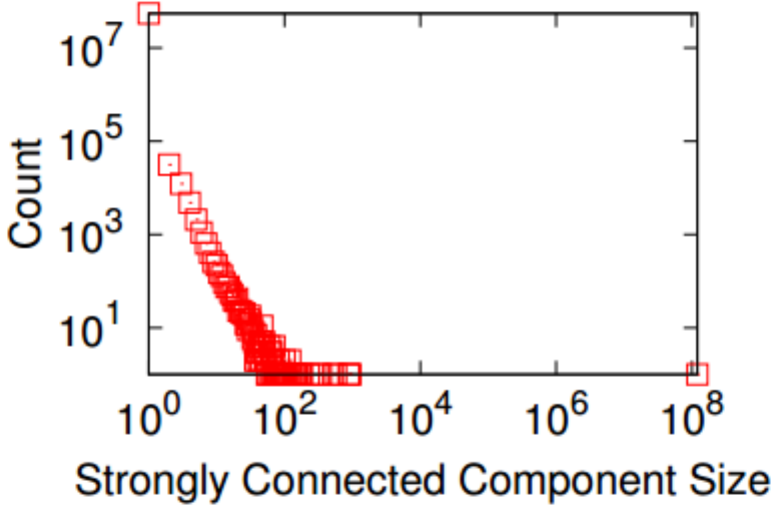
Connected components: A-B-C and E-D



What's the largest connected component?

COMPONENTS



Count
10^7
10^5
10^3
10^1
10^0
10^2
10^4
10^6
10^8
Strongly Connected Component Size

FORMALLY: CLUSTERING COEFFICIENT

Local Clustering Coefficient

$$C_i = \frac{2\,|\{e_{jk}\}|}{k_i(k_i - 1)} : v_j, v_k \in N_i, e_{j,k} \in E$$

Network Clustering Coefficient

$$CG = \frac{1}{N}\sum_i C_i$$

FORMALLY: CLUSTERING COEFFICIENT

Local Clustering Coefficient

$$C_i = \frac{2|\{e_{jk}\}|}{k_i(k_i - 1)} : v_j, v_k \in N_i, e_{j,k} \in E$$

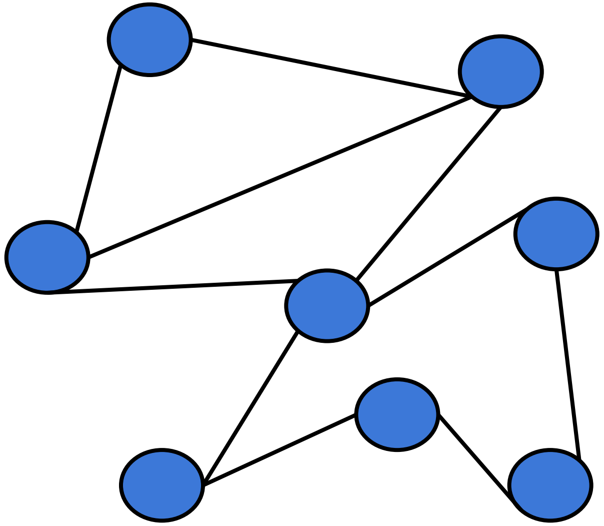
Proportion of my friends who are also friends with my other friends...

Network Clustering Coefficient

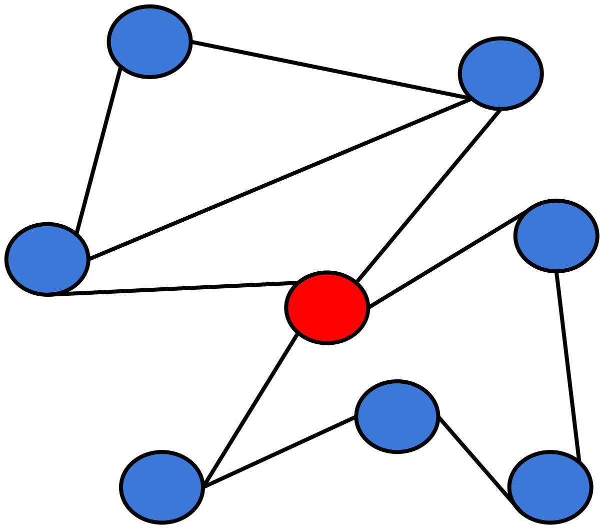
$$CG = \frac{1}{N}\sum_i C_i$$

The average all all the node’s local clustering coefficients

CLUSTERING COEFFICIENT: EXAMPLE

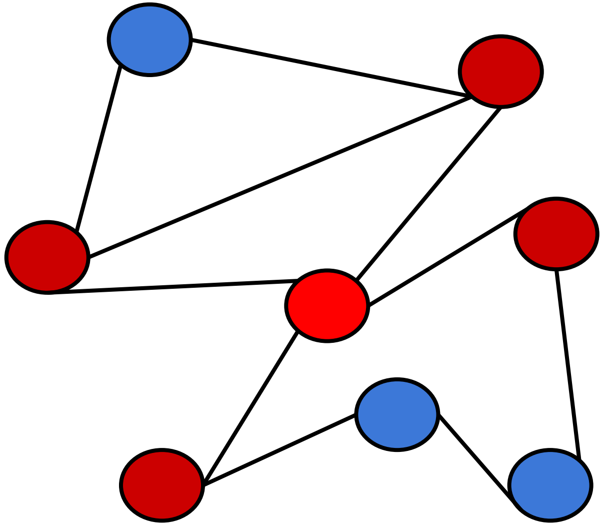


CLUSTERING COEFFICIENT: EXAMPLE



CLUSTERING COEFFICIENT: EXAMPLE

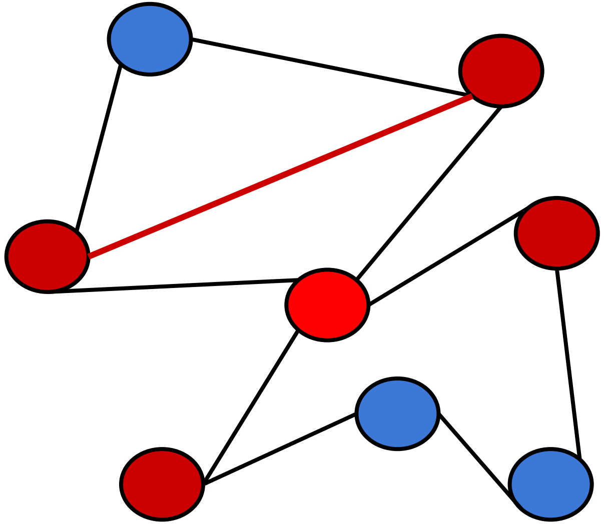
Degree = 4



CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

Links between neighbours = 1

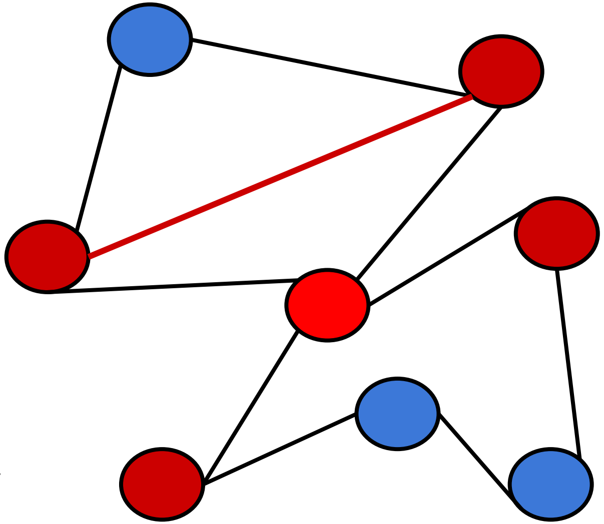


CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

Links between neighbours = 1

$$C_i = \frac{2\,|\,\{e_{jk}\}\,|}{k_i(k_i - 1)} : v_j, v_k \in N_i, e_{j,k} \in E$$



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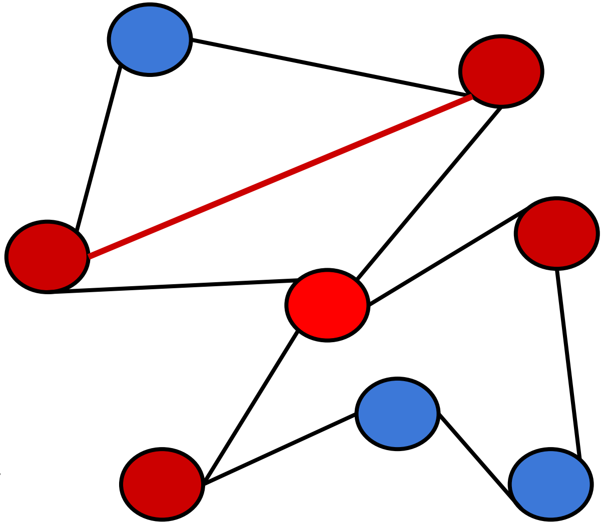

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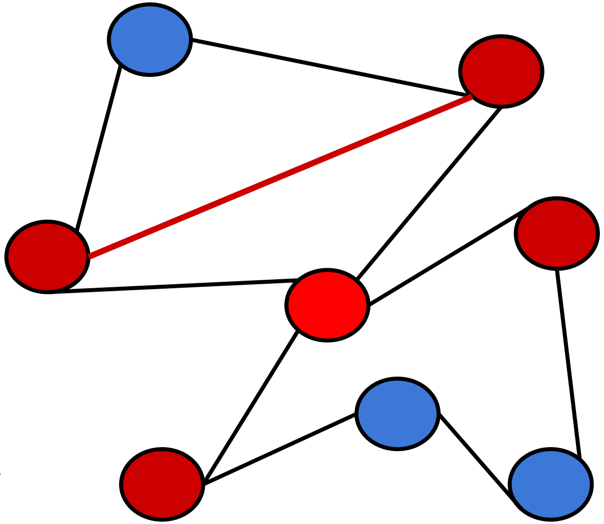

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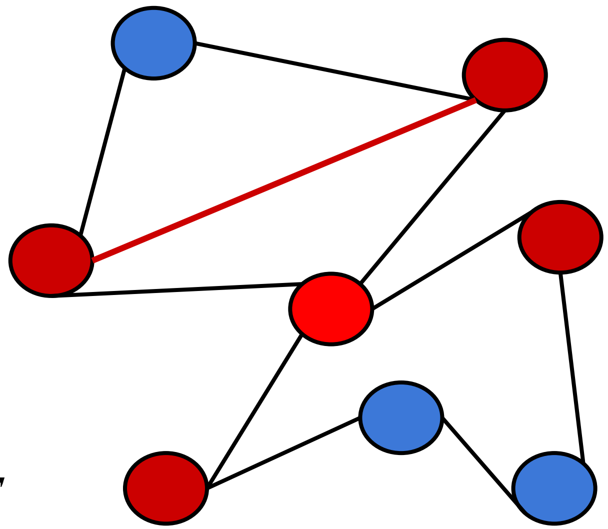
$$C_i = \frac{2}{\quad}$$



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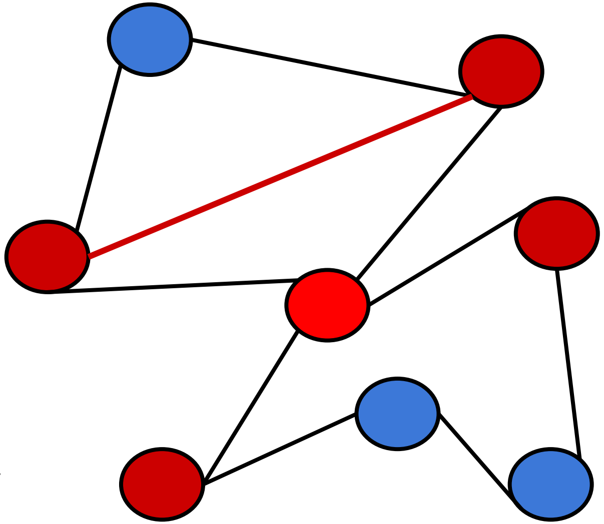
$$C_i = \frac{2 * 1}{}$$



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Degree = 4

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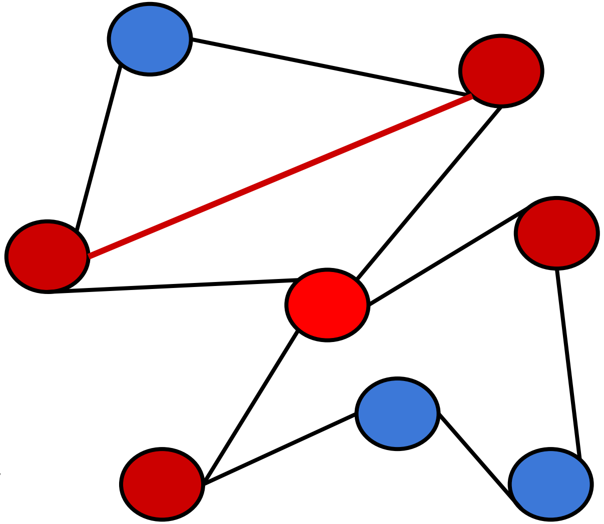
$$C_i = \frac{2\,|\{e_{jk}\}|}{\boxed{k_i}(k_i - 1)} : v_j, v_k \in N_i, e_{j,k} \in E$$

$$C_i = \frac{2 * 1}{4}$$



CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

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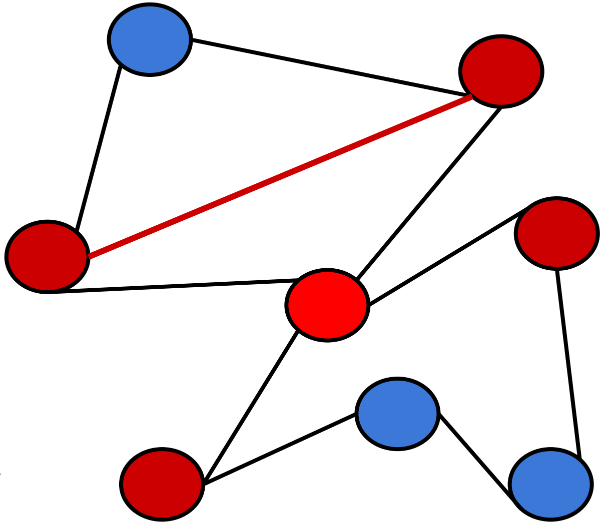
$$C_i = \frac{2|\{e_{jk}\}|}{k_i\boxed{(k_i - 1)}} : v_j, v_k \in N_i, e_{j,k} \in E$$

$$C_i = \frac{2 * 1}{4 * 3}$$



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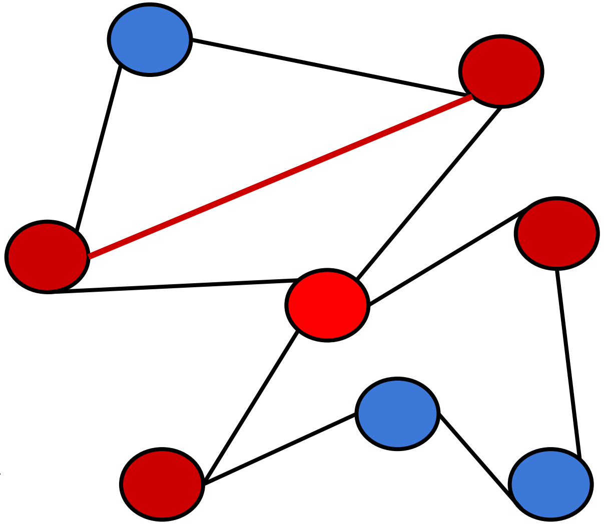


$$C_i = \frac{2}{12}$$

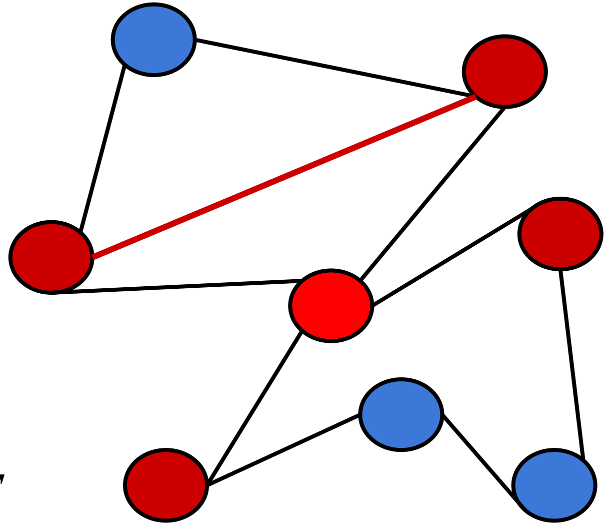
CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

Links between neighbours = 1

$$C_i = \frac{2\,|\{e_{jk}\}|}{k_i(k_i-1)} : v_j, v_k \in N_i, e_{j,k} \in E$$

$$C_i = \frac{1}{6}$$



CLUSTERING COEFFICIENT: EXAMPLE

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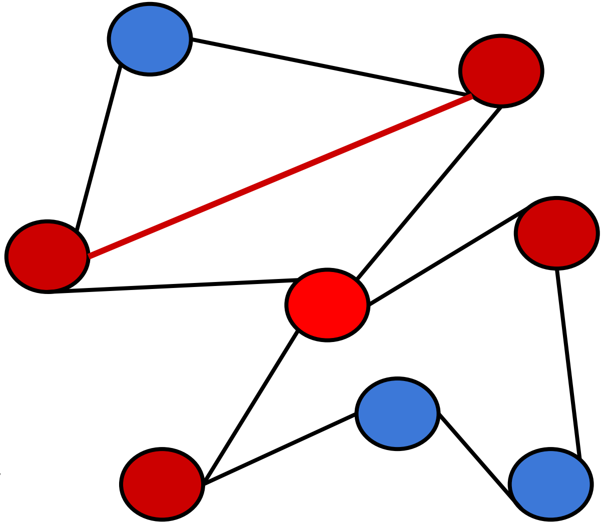


$$C_i = \frac{1}{6}$$

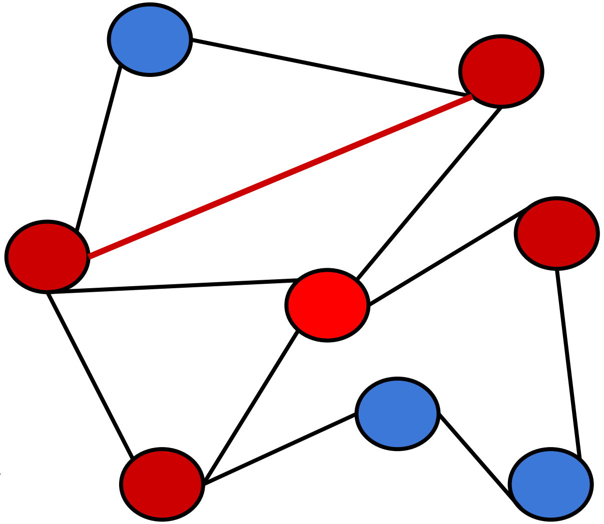
Fraction of possible interconnections between my neighbour!

CLUSTERING COEFFICIENT: EXAMPLE

Degree =

Links between neighbours =

$$C_i = \frac{2|\{e_{jk}\}|}{k_i(k_i-1)} : v_j, v_k \in N_i, e_{j,k} \in E$$



CLUSTERING COEFFICIENT: EXAMPLE

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Links between neighbours =

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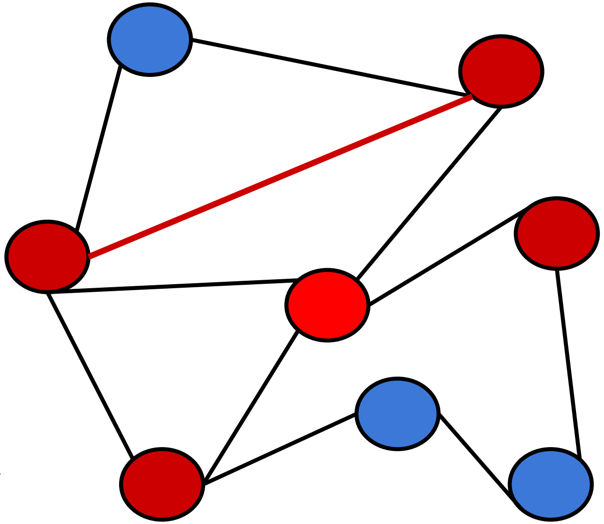
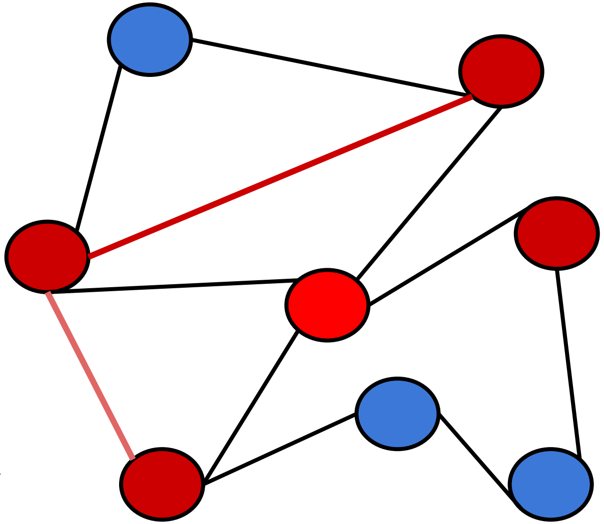


CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

Links between neighbours = 2

$$C_i = \frac{2\,|\{e_{jk}\}|}{k_i(k_i - 1)} : v_j, v_k \in N_i, e_{j,k} \in E$$



CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

Links between neighbours = 2

$$C_i = \frac{2\,|\{e_{jk}\}|}{k_i(k_i - 1)} : v_j, v_k \in N_i, e_{j,k} \in E$$

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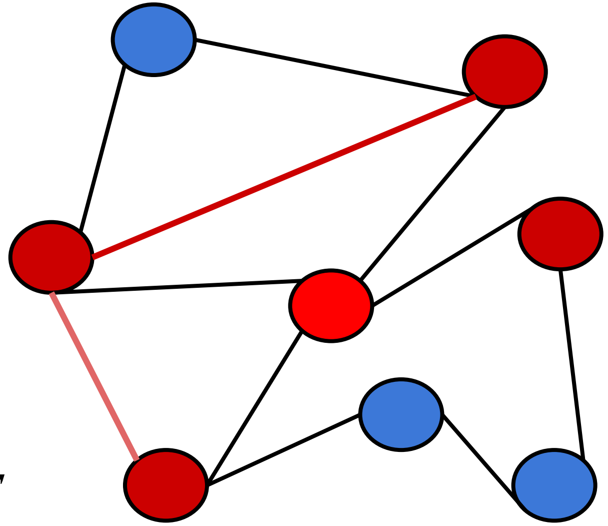

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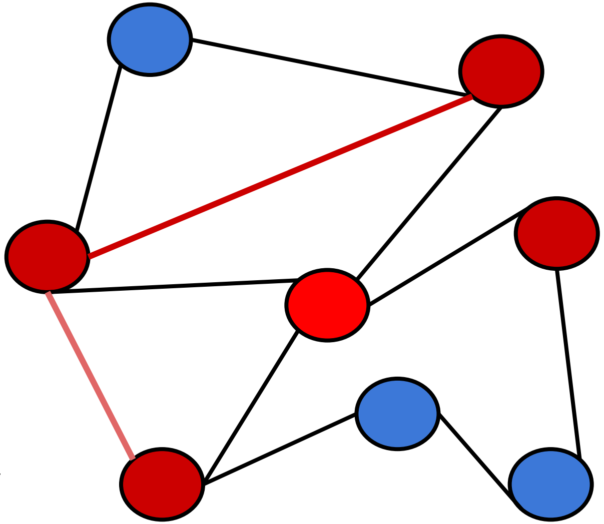

$$C_i = \frac{4}{12}$$

CLUSTERING COEFFICIENT: EXAMPLE

Degree = 4

Links between neighbours = 2

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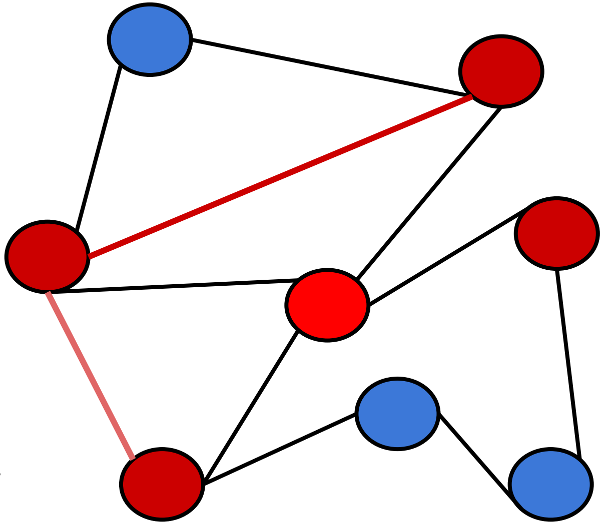

$$C_i = \frac{1}{3}$$

CLUSTERING COEFFICIENT: EXAMPLE

Average clustering coefficient for degree = 5 is about **0.4** in the Facebook graph

compared to Twitter mutual graph **0.23**