

## A Experiments for the $U$ -feature

We generate datasets with instance sizes varying from 101 to 500, and both coordinates of each node in the instances are generated independently from a unit uniform distribution. The number of instances with size  $n$  is  $500000/n$ . We use solver Concorde to obtain the edge labels.

### A.1 Experiments for $\alpha$ -values on instances with different sizes

To analyze the relation between  $\alpha$ -value and the number of nodes in the instance, we collect datasets with sizes 101, 200, 300, 400, 500, and analyze the non-zero  $\alpha$ -values of edges in the optimal solution. We sort the non-zero  $\alpha$ -values in nondecreasing order, and calculate the 20th, 40th, 60th, 80th, 100th percentiles. Then, we count the number of edges in different range, and show the results of edges with  $\alpha$ -values larger than 0 in Table 4. It is easy to see that the  $\alpha$ -values in different instances vary a lot. Moreover, we calculate the average  $\alpha$ -value of each dataset. As shown in Table 5, the average  $\alpha$ -value decreases as the sizes of the instances become large.

Table 4: The  $\alpha$ -values on instances with different sizes

| Graph Size |                  | 0%-20%    | 20%-40%        | 40%-60%           | 60%-80%            | 80%-100%            |
|------------|------------------|-----------|----------------|-------------------|--------------------|---------------------|
| 101        | $\alpha$ -values | [1, 3400) | [3400, 235923) | [235923, 1436131) | [1436131, 5415824) | [5415824, 76258620] |
|            | number of edges  | 20574     | 20598          | 20586             | 20586              | 20584               |
| 200        | $\alpha$ -values | [1, 300)  | [300, 209240)  | [209240, 1092096) | [1092096, 3820962) | [3820962, 53092249] |
|            | number of edges  | 21020     | 21066          | 21044             | 21042              | 21044               |
| 300        | $\alpha$ -values | [1, 3)    | [3, 186881)    | [186881, 935513)  | [935513, 3126138)  | [3126138, 42741759] |
|            | number of edges  | 19284     | 20702          | 19992             | 19994              | 19991               |
| 400        | $\alpha$ -values | [1, 12)   | [12, 150340)   | [150340, 741981)  | [741981, 2538087)  | [2538087, 36610560] |
|            | number of edges  | 21030     | 21030          | 21034             | 21030              | 21031               |
| 500        | $\alpha$ -values | [1, 5)    | [5, 137752)    | [137752, 627791)  | [627791, 2117251)  | [2117251, 32658279] |
|            | number of edges  | 20990     | 21024          | 21006             | 21006              | 21006               |

Table 5: Average  $\alpha$ -values on instances with different sizes

| Graph Size              | 101       | 200       | 300       | 400       | 500       |
|-------------------------|-----------|-----------|-----------|-----------|-----------|
| Average $\alpha$ -value | 3,738,626 | 2,631,917 | 2,123,935 | 1,762,479 | 1,508,247 |

### A.2 Experiments for $M$ -scores on instances with different sizes

To analyze the relation between  $M$ -score and the number of nodes in the instance, we collect Training Set with sizes 101, 300, 500, and analyze the  $M$ -scores of edges in the optimal solution. We count the number of edges with  $M$ -scores in different range, and for better visualization, we show the results of edges with  $M$ -scores larger than  $10^{-5}$  in logarithmic coordinates. As shown in figure 2, for instances with different number of nodes, the  $M$ -score curves are almost similar. Besides, we collect datasets with sizes 101, 200, 300, 400, 500, and calculate the average  $M$ -score of each dataset. As shown in Table 6, the average  $M$ -scores are similar in instances with different number of nodes. Thus, the  $M$ -score distributions in different instances are approximately the same.

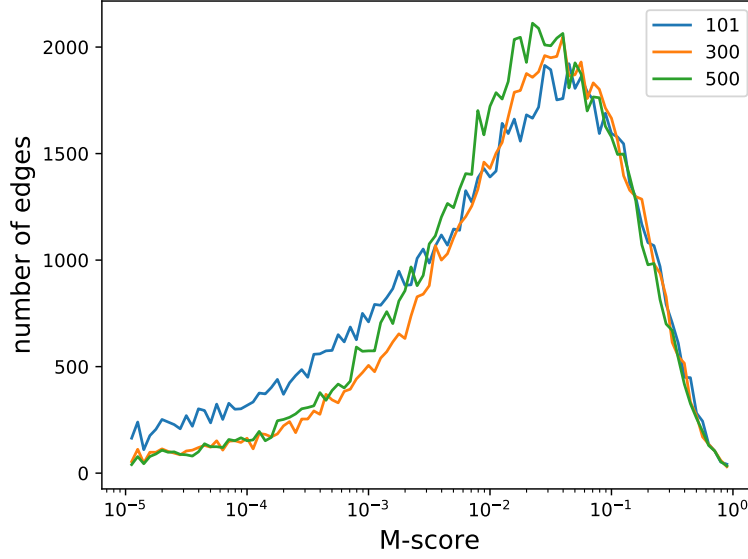


Figure 6: The  $M$ -scores on instances with different sizes

Table 6: Average  $M$ -score on instances with different sizes

| Graph Size         | 101     | 200     | 300     | 400     | 500     |
|--------------------|---------|---------|---------|---------|---------|
| Average $M$ -score | 0.05773 | 0.06102 | 0.06142 | 0.05997 | 0.05804 |

### A.3 Experiments for the logarithm operation on $M$ -score

We calculate  $M$ -scores of edges in datasets, and analyze the total number of edges in the optimal solution with different range of  $M$ -scores. As shown in Table 7, most  $M$ -scores of edges in the optimal solution are smaller than 0.05, and more precisely, for all instances in datasets, 385, 781, 250 edges are in the optimal solution, and  $M$ -scores of 97.3% (375, 410, 514) edges are smaller than 0.05. By imposing denary logarithm function on non-zero  $M$ -scores, we analyze the number of edges in the optimal solution with different range of values. As shown in Table 8, the edges in the optimal solution may not have large differences.

Table 7: The  $M$ -scores of the edges in the optimal solution

| $M$ -score      | 0           | $(0, 10^{-3}]$ | $(10^{-3}, 10^{-2}]$ | $(0.01, 0.05]$ | $(0.05, 0.1]$ | $(0.1, 1]$ | $(1, +\infty)$ |
|-----------------|-------------|----------------|----------------------|----------------|---------------|------------|----------------|
| Number of edges | 353,176,520 | 4,499,512      | 7,699,630            | 10,034,852     | 4,334,768     | 6,023,245  | 12,723         |

Table 8: The logarithm values of non-zero  $M$ -scores for the edges in the optimal solution

| $M$ -score with logarithm | $(-\infty, -5]$ | $(-5, -4]$ | $(-4, -3]$ | $(-3, -2]$ | $(-2, -1]$ | $(-1, 0]$ | $(0, +\infty)$ |
|---------------------------|-----------------|------------|------------|------------|------------|-----------|----------------|
| Number of edges           | 740,854         | 1,078,926  | 2,679,732  | 7,699,630  | 14,369,620 | 6,023,245 | 12,723         |

### A.4 Experiments for the constants in the $U$ -feature

In order to decide the constants in the  $U$ -feature, we add  $U$ -features with different constants to the graph convolutional network solving TSP in [18]. We train the networks with 10 graph convolution

layers and 100 hidden dimensions on training datasets containing instances with 50 nodes, and validate the loss of the networks on three datasets with instance sizes 50, 100 and 200, each of which contains 1000 instances. The results are given in Table 9 (a), (b), (c), respectively. For edges in the minimum 1-tree, we set  $C_1 = 0$  (other values also work). We set  $C_1 = 0$ ,  $C_2 = 0$ ,  $C_3 = C_4$ , and test the appropriate value for  $C_3 - C_2$ . It is easy to see that the network trained with  $C_3 - C_2 = 3$  shows better performance. Next, we set  $C_1 = 0$ ,  $C_3 - C_2 = 3$ ,  $C_3 = C_4$ , and test the appropriate value for  $C_2 - C_1$ . It is easy to see that the network trained with  $C_2 - C_1 = 1$  shows better performance. Then, we set  $C_1 = 0$ ,  $C_2 = 1$ ,  $C_3 = 4$ , and test the appropriate value for  $C_4 - C_3$ . It is easy to see that the network trained with  $C_4 - C_3 = 1$  shows better performance. Experiments show that  $U$ -feature is not sensitive to the constants, and we choose four appropriate constants 0, 1, 4 and 5 for  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$ , respectively.

Table 9: Experiments for the constants in the  $U$ -feature

| (a) Experiments for $C_3 - C_2$ ( $C_1 = C_2 = 0$ , $C_3 = C_4$ ) |                 |                 |                 |                 |                 |                 |
|---|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|   | $C_3 - C_2 = 2$ | $C_3 - C_2 = 3$ | $C_3 - C_2 = 4$ | $C_3 - C_2 = 5$ | $C_3 - C_2 = 6$ | $C_3 - C_2 = 7$ |
| TSP50   | 0.0413          | 0.0382          | 0.0387          | 0.0378          | 0.0384          | 0.0385          |
| TSP100  | 0.0455          | <b>0.0434</b>   | 0.0441          | 0.0468          | 0.0573          | 0.0975          |
| TSP200  | 0.0542          | <b>0.0454</b>   | 0.0474          | 0.0506          | 0.1098          | 0.2513          |

| (b) Experiments for $C_2 - C_1$ ( $C_1 = 0$ , $C_3 - C_2 = 3$ , $C_3 = C_4$ ) |                   |                   |                   |                 |                 |                 |
|---|-------------------|-------------------|-------------------|-----------------|-----------------|-----------------|
|   | $C_2 - C_1 = 0.2$ | $C_2 - C_1 = 0.4$ | $C_2 - C_1 = 0.8$ | $C_2 - C_1 = 1$ | $C_2 - C_1 = 2$ | $C_2 - C_1 = 3$ |
| TSP50   | 0.0381            | 0.0377            | 0.0383            | 0.0378          | 0.0389          | 0.0381          |
| TSP100  | 0.0451            | 0.0441            | 0.0412            | <b>0.0382</b>   | 0.0391          | 0.0589          |
| TSP200  | 0.0465            | 0.0455            | 0.0437            | <b>0.0394</b>   | 0.0418          | 0.1015          |

| (c) Experiments for $C_4 - C_3$ ( $C_1 = 0$ , $C_2 = 1$ , $C_3 = 4$ ) |                 |                 |                 |                 |                 |                 |
|---|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|   | $C_4 - C_3 = 0$ | $C_4 - C_3 = 1$ | $C_4 - C_3 = 2$ | $C_4 - C_3 = 3$ | $C_4 - C_3 = 4$ | $C_4 - C_3 = 5$ |
| TSP50   | 0.0378          | 0.0385          | 0.0378          | 0.0382          | 0.0377          | 0.0372          |
| TSP100  | 0.0382          | <b>0.0374</b>   | 0.0386          | 0.0401          | 0.0413          | 0.0403          |
| TSP200  | 0.0394          | <b>0.0372</b>   | 0.0391          | 0.0418          | 0.0435          | 0.0413          |

### A.5 Comparison results of $\alpha$ -value, $M$ -score, and $U$ -feature

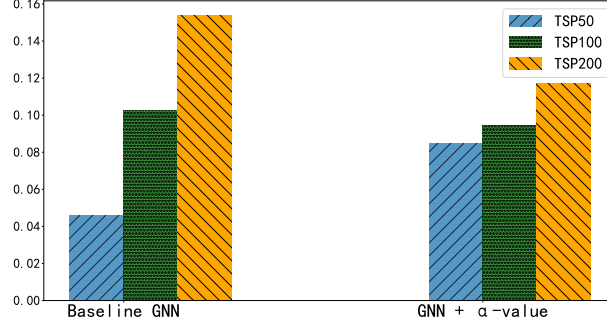
In order to illustrate the advantage of  $U$ -feature, we add  $\alpha$ -value,  $\alpha$ -value with logarithm function,  $M$ -score, and  $U$ -feature separately to the graph convolutional network (GNN) solving TSP in [18], which is our baseline for the comparison. We train the networks with 10 graph convolution layers and 100 hidden dimensions on training datasets containing instances with 50 nodes, and validate the loss of the networks on three datasets with instance sizes 50, 100 and 200, each of which contains 1000 instances.

We first compare baseline GNN and GNN with  $\alpha$ -value (denoted as GNN +  $\alpha$ -value), and the results are given in Figure 7 (a). It is easy to see that for instances with 50 nodes, the loss of GNN with  $\alpha$ -value is larger than baseline GNN. For instances with 100 and 200 nodes, the loss of GNN with  $\alpha$ -value is slightly smaller than baseline GNN. Thus, directly using  $\alpha$ -value as a training feature cannot significantly improve the model performance.

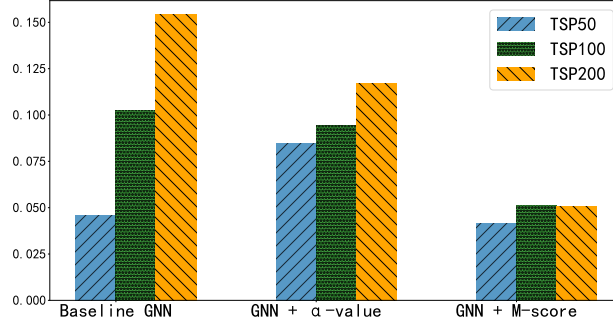
Next, we compare baseline GNN, GNN with  $\alpha$ -value, and GNN with  $M$ -score (denoted as GNN +  $M$ -score), and the results are given in Figure 7 (b). It is easy to see that, the loss of GNN with  $M$ -score is much smaller than others in all instances. Thus, directly adding  $M$ -score to training model can obviously improve the model performance.

Finally, we compare the performance of GNN with  $\alpha$ -value, GNN with logarithm function on  $\alpha$ -value (denoted as GNN +  $\alpha$ -value + log), GNN with  $M$ -score, and GNN with  $U$ -feature (denoted as GNN

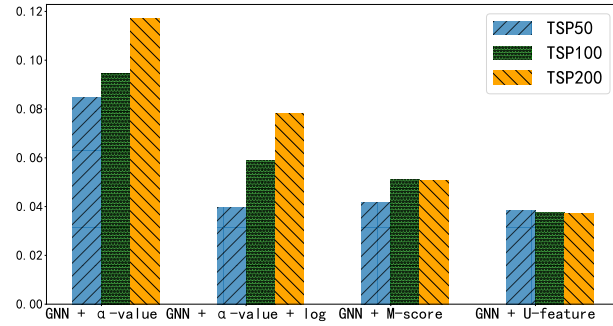
+  $U$ -feature), and the results are given in Figure 7 (c). It is easy to see that adding  $U$ -feature to the training model shows better performance than  $M$ -score. As shown in Figure 7 (c), after logarithm operation, the loss of GNN with  $\alpha$ -value becomes smaller. However, the loss increases obviously as the sizes of the instances become large.



(a)



(b)



(c)

Figure 7: Comparison results of  $\alpha$ -value,  $M$ -score,  $U$ -feature on different instances

## B Detailed comparisons on Test Set

In this section, we give the detailed comparison results. We take instances with 500 nodes to illustrate our experiments. For all the 1000 instances, we run LKHM on them with 500 trials of exchanges in LKH, and 1000 tours can be obtained, such that the average cost of LKHM on the instances with 500 nodes can be obtained, denoted by  $C'$ . Similarly, by running NeuroLKH and LKH on the 1000 instances, the average cost of NeuroLKH and LKH, denoted by  $C''$  and  $C'''$ , can be obtained. For each number of trials  $i$  ( $1 \leq i \leq 500$ ), we run LKHM, NeuroLKH and LKH separately on the 1000 instances, and three average costs can be computed, denoted as  $C'_i$ ,  $C''_i$ ,  $C'''_i$ , respectively. We use  $C = \min\{C', C'', C'''\}$  as a standard cost for our experiments. For each  $i$  ( $1 \leq i \leq 500$ ), three values  $C'_i - C$ ,  $C''_i - C$  and  $C'''_i - C$  can be computed for LKHM, NeuroLKH and LKH, respectively. Then, for LKHM, a list of values can be obtained by considering all the number of trials from 1 to 500. Similarly, for NeuroLKH and LKH, two list of values can also be obtained. Based on the three lists, the comparison results of LKHM, NeuroLKH and LKH for the average tour costs are given in Figure 8. We demonstrate the differences of LKHM, NeuroLKH and LKH with number of trials from 1 to 50 on Figure 8 (a), and differences of LKHM, NeuroLKH and LKH with number of trials from 51 to 500 on Figure 8 (b). The comparison results of LKHM, NeuroLKH and LKH on the instances with sizes 1000, 2000, 5000, 10000, 20000 are given in Figure 9, 10, 11, 12, 13, respectively.

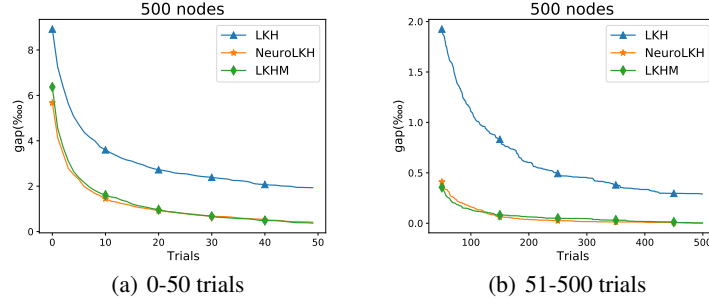


Figure 8: Comparison results of LKHM, NeuroLKH and LKH for optimal gap vs. trials on instances with 500 nodes

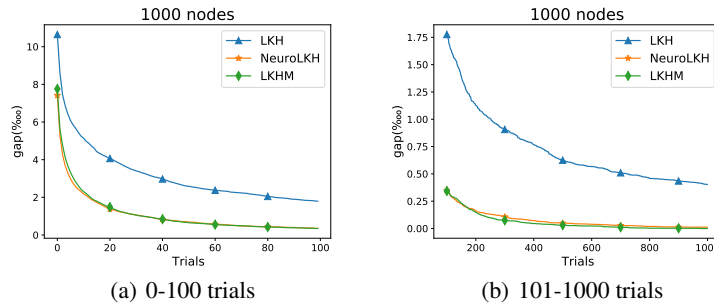


Figure 9: Comparison results of LKHM, NeuroLKH and LKH for optimal gap vs. trials on instances with 1000 nodes

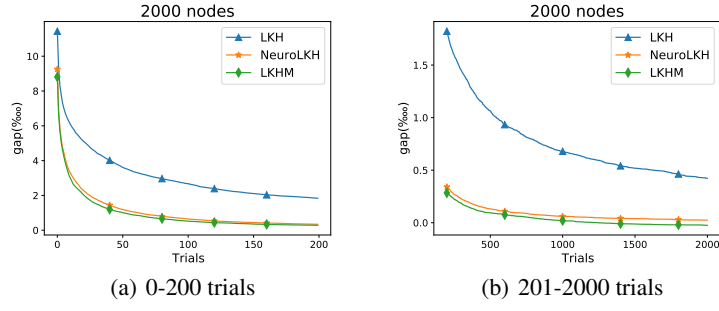


Figure 10: Comparison results of LKHM, NeuroLKH and LKH for optimal gap vs. trials on instances with 2000 nodes

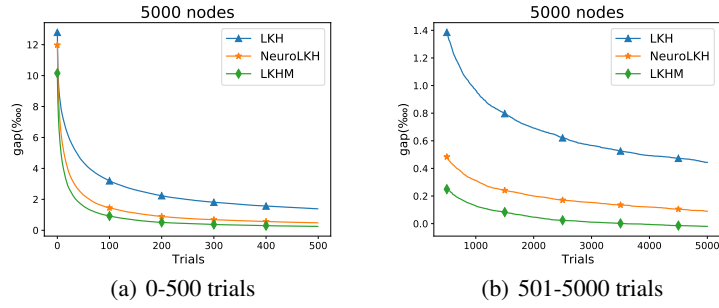


Figure 11: Comparison results of LKHM, NeuroLKH and LKH for optimal gap vs. trials on instances with 5000 nodes

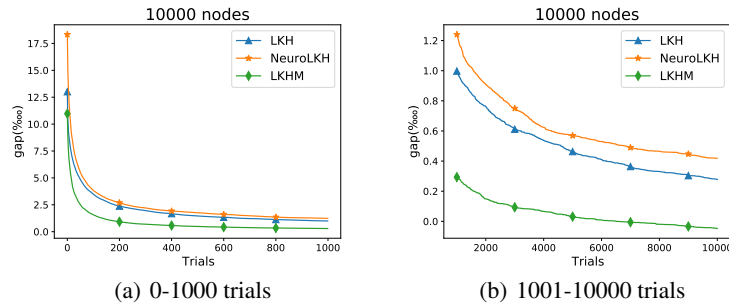


Figure 12: Comparison results of LKHM, NeuroLKH and LKH for optimal gap vs. trials on instances with 10000 nodes

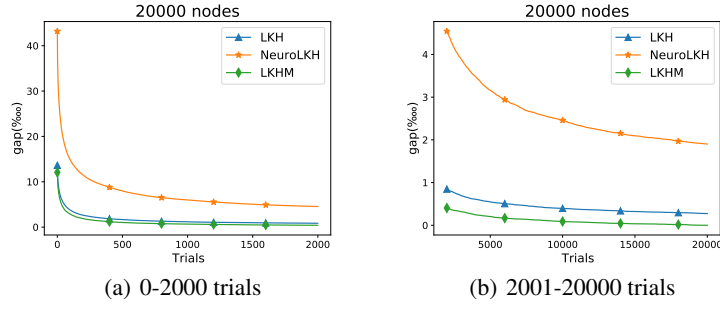


Figure 13: Comparison results of LKH, NeuroLKH and LKHM for optimal gap vs. trials on instances with 20000 nodes

## C Corresponding IDs for all 79 instances on Euclidean space in TSPLIB

Table 10: Corresponding IDs for all 79 instances on Euclidean space in TSPLIB

| ID | Instance | ID | Instance | ID | Instance | ID | Instance |
|----|----------|----|----------|----|----------|----|----------|
| 1  | eil51    | 21 | ch150    | 41 | pcb442   | 61 | vm1748   |
| 2  | berlin52 | 22 | kroA150  | 42 | d493     | 62 | u1817    |
| 3  | st70     | 23 | kroB150  | 43 | u574     | 63 | rl1889   |
| 4  | eil76    | 24 | pr152    | 44 | rat575   | 64 | d2103    |
| 5  | pr76     | 25 | u159     | 45 | p654     | 65 | u2152    |
| 6  | rat99    | 26 | rat195   | 46 | d657     | 66 | u2319    |
| 7  | kroA100  | 27 | d198     | 47 | u724     | 67 | pr2392   |
| 8  | kroB100  | 28 | kroA200  | 48 | rat783   | 68 | pcb3038  |
| 9  | kroC100  | 29 | kroB200  | 49 | pr1002   | 69 | fl3795   |
| 10 | kroD100  | 30 | ts225    | 50 | u1060    | 70 | fnl4461  |
| 11 | kroE100  | 31 | tsp225   | 51 | vm1084   | 71 | rl5915   |
| 12 | rd100    | 32 | pr226    | 52 | pcb1173  | 72 | rl5934   |
| 13 | eil101   | 33 | gil262   | 53 | d1291    | 73 | rl11849  |
| 14 | lin105   | 34 | pr264    | 54 | rl1304   | 74 | usa13509 |
| 15 | pr107    | 35 | a280     | 55 | rl1323   | 75 | brd14051 |
| 16 | pr124    | 36 | pr299    | 56 | nrw1379  | 76 | d15112   |
| 17 | bier127  | 37 | lin318   | 57 | fl1400   | 77 | d18512   |
| 18 | ch130    | 38 | rd400    | 58 | u1432    | 78 | pla33810 |
| 19 | pr136    | 39 | fl417    | 59 | fl1577   | 79 | pla85900 |
| 20 | pr144    | 40 | pr439    | 60 | d1655    |    |          |

## D Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB

Table 11: Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB

| Name                   | Opt.   | Method   | Success | Best_cost | Avg_cost | Trials | Time |
|------------------------|--------|----------|---------|-----------|----------|--------|------|
| eil51                  | 426    | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
| berlin52               | 7542   | LKH      | 10/10   | Opt.      | Opt.     | 0      | 0    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 0      | 0.01 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 0      | 0    |
| st70                   | 675    | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| eil76                  | 538    | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
| pr76                   | 108159 | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
| rat99                  | 1211   | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
| kroA100                | 21282  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| kroB100                | 22141  | LKH      | 10/10   | Opt.      | Opt.     | 1.2    | 0.03 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1.8    | 0.03 |
| kroC100                | 20749  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| kroD100                | 21294  | LKH      | 10/10   | Opt.      | Opt.     | 1.8    | 0.02 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| kroE100                | 22068  | LKH      | 10/10   | Opt.      | Opt.     | 3.2    | 0.03 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
| rd100                  | 7910   | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| eil101                 | 629    | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
| Continued on next page |        |          |         |           |          |        |      |



Table 12: Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB (continued)

| Name                   | Opt.   | Method   | Success | Best_cost | Avg_cost | Trials | Time |
|------------------------|--------|----------|---------|-----------|----------|--------|------|
| lin105                 | 14379  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
| pr107                  | 44303  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.09 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1.1    | 0.09 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.18 |
| pr124                  | 59030  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.03 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0.04 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.05 |
| bier127                | 118282 | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | NeuroLKH | 4/10    | Opt.      | 118300.6 | 102.5  | 0.09 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
| ch130                  | 6110   | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1.1    | 0.02 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
| pr136                  | 96772  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.06 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 4.5    | 0.11 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.06 |
| pr144                  | 58537  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.33 |
|                        |        | NeuroLKH | 1/10    | Opt.      | 58584.7  | 131.8  | 2.71 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.55 |
| ch150                  | 6528   | LKH      | 10/10   | Opt.      | Opt.     | 1.7    | 0.03 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1.1    | 0.02 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 3.9    | 0.04 |
| kroA150                | 26524  | LKH      | 10/10   | Opt.      | Opt.     | 3.8    | 0.05 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 2.6    | 0.03 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1.7    | 0.03 |
| kroB150                | 26130  | LKH      | 2/10    | Opt.      | 26131.6  | 128.4  | 0.28 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 9.8    | 0.05 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 17.3   | 0.1  |
| pr152                  | 73682  | LKH      | 10/10   | Opt.      | Opt.     | 29.4   | 0.59 |
|                        |        | NeuroLKH | 8/10    | Opt.      | 73709.2  | 59.6   | 1.08 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 17.7   | 0.52 |
| u159                   | 42080  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 1      | 0    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| rat195                 | 2323   | LKH      | 9/10    | Opt.      | 2323.5   | 55     | 0.18 |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 8.4    | 0.07 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 2.8    | 0.06 |
| d198                   | 15780  | LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.49 |
|                        |        | NeuroLKH | 0/10    | 15789     | 15825    | 198    | 2.24 |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 1.8    | 0.53 |
| Continued on next page |        |          |         |           |          |        |      |

Table 13: Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB (continued)

| Name    | Opt.   | Method   | Success | Best_cost   | Avg_cost    | Trials | Time  |
|---------|--------|----------|---------|-------------|-------------|--------|-------|
| kroA200 | 29368  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1.7    | 0.05  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.03  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04  |
| kroB200 | 29437  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.02  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.01  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.03  |
| ts225   | 126643 | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04  |
| tsp225  | 3916   | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.05  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.05  |
| pr226   | 80369  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.07  |
|         |        | NeuroLKH | 6/10    | <b>Opt.</b> | 80381.7     | 146.2  | 0.91  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1.5    | 0.16  |
| gil262  | 2378   | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 10.6   | 0.12  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 8      | 0.09  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 2.2    | 0.07  |
| pr264   | 49135  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 14.4   | 0.2   |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 6.2    | 0.08  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 2.7    | 0.11  |
| a280    | 2579   | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.02  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.02  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.02  |
| pr299   | 48191  | LKH      | 9/10    | <b>Opt.</b> | 48194.3     | 51.7   | 0.41  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 10.1   | 0.17  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.3    | 0.17  |
| lin318  | 42029  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 27.9   | 0.26  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.6    | 0.12  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.1    | 0.1   |
| rd400   | 15281  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 33     | 0.25  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.9    | 0.07  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1.4    | 0.06  |
| fl417   | 11861  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.3    | 3.34  |
|         |        | NeuroLKH | 5/10    | <b>Opt.</b> | 11867.6     | 337.2  | 18.15 |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 48.2   | 7.37  |
| pr439   | 107217 | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 39.5   | 0.59  |
|         |        | NeuroLKH | 3/10    | <b>Opt.</b> | 107267.4    | 320.1  | 1.73  |
|         |        | LKHM-LKH | 6/10    | <b>Opt.</b> | 107245.8    | 240.1  | 2.48  |
| pcb442  | 50778  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 8.2    | 0.14  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.8    | 0.07  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 5.4    | 0.12  |

Continued on next page

Table 14: Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB (continued)

| Name    | Opt.   | Method   | Success | Best_cost   | Avg_cost    | Trials | Time  |
|---------|--------|----------|---------|-------------|-------------|--------|-------|
| d493    | 35002  | LKH      | 6/10    | <b>Opt.</b> | 35002.8     | 219.6  | 3.93  |
|         |        | NeuroLKH | 6/10    | <b>Opt.</b> | 35032.2     | 320.5  | 6.37  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 6.7    | 0.31  |
| u574    | 36905  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 149.9  | 0.88  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.8    | 0.13  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.6    | 0.22  |
| rat575  | 6773   | LKH      | 2/10    | <b>Opt.</b> | 6773.8      | 526.9  | 2.76  |
|         |        | NeuroLKH | 9/10    | <b>Opt.</b> | 6773.1      | 179    | 1.67  |
|         |        | LKHM-LKH | 8/10    | <b>Opt.</b> | 6773.2      | 380.8  | 4.05  |
| p654    | 34643  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 22.9   | 9.64  |
|         |        | NeuroLKH | 1/10    | <b>Opt.</b> | 34765.8     | 619    | 44.42 |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 32.4   | 14.02 |
| d657    | 48912  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 33.5   | 0.46  |
|         |        | NeuroLKH | 5/10    | <b>Opt.</b> | 48912.5     | 511.5  | 7.47  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 23.6   | 0.53  |
| u724    | 41910  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 125.4  | 2.03  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 46.6   | 0.96  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 25.4   | 0.73  |
| rat783  | 8806   | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.2    | 0.07  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.2    | 0.1   |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 10.1   | 0.25  |
| pr1002  | 259045 | LKH      | 8/10    | <b>Opt.</b> | 259045.6    | 549    | 3.55  |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 330.6  | 5.83  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 45.9   | 1.63  |
| u1060   | 224094 | LKH      | 5/10    | <b>Opt.</b> | 224107.5    | 663.3  | 97.59 |
|         |        | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 206.9  | 30.74 |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 95.9   | 12.74 |
| vm1084  | 239297 | LKH      | 3/10    | <b>Opt.</b> | 239372.6    | 824.1  | 37.02 |
|         |        | NeuroLKH | 1/10    | <b>Opt.</b> | 239379.5    | 1028.9 | 22.48 |
|         |        | LKHM-LKH | 7/10    | <b>Opt.</b> | 239312.6    | 559.6  | 21.45 |
| pcb1173 | 56892  | LKH      | 4/10    | <b>Opt.</b> | 56895       | 844    | 4.64  |
|         |        | NeuroLKH | 9/10    | <b>Opt.</b> | 56892.5     | 410.4  | 4.81  |
|         |        | LKHM-LKH | 8/10    | <b>Opt.</b> | 56893       | 370    | 5.48  |
| d1291   | 50801  | LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 192.1  | 8.95  |
|         |        | NeuroLKH | 9/10    | <b>Opt.</b> | 50803.4     | 274.4  | 6.34  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 74.3   | 5.89  |
| rl1304  | 252948 | LKH      | 3/10    | <b>Opt.</b> | 253156.4    | 1170   | 14.59 |
|         |        | NeuroLKH | 9/10    | <b>Opt.</b> | 252953.1    | 370.8  | 6.24  |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 237.2  | 6.62  |
| rl1323  | 270199 | LKH      | 6/10    | <b>Opt.</b> | 270219.6    | 718.8  | 11.58 |
|         |        | NeuroLKH | 7/10    | <b>Opt.</b> | 270247.9    | 742.2  | 11.93 |
|         |        | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 100.8  | 4.03  |

Continued on next page

Table 15: Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB (continued)

| Name                   | Opt.   | Method   | Success | Best_cost | Avg_cost | Trials | Time     |
|------------------------|--------|----------|---------|-----------|----------|--------|----------|
| nrw1379                | 56638  | LKH      | 6/10    | Opt.      | 56640    | 759.3  | 10.54    |
|                        |        | NeuroLKH | 9/10    | Opt.      | 56638.5  | 372.4  | 11.46    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 194.7  | 7.62     |
| fl1400                 | 20127  | LKH      | 1/10    | Opt.      | 20160.3  | 1372.9 | 3739.8   |
|                        |        | NeuroLKH | 0/10    | 20165     | 20235.5  | 1400   | 257.31   |
|                        |        | LKHM-LKH | 7/10    | Opt.      | 20138.1  | 668.9  | 2743.98  |
| u1432                  | 152970 | LKH      | 10/10   | Opt.      | Opt.     | 5.3    | 0.54     |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 7.1    | 0.69     |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 3.3    | 0.67     |
| fl1577                 | 22249  | LKH      | 0/10    | 22254     | 22260.6  | 1577   | 1336.67  |
|                        |        | NeuroLKH | 1/10    | Opt.      | 22256.6  | 1445.8 | 468.94   |
|                        |        | LKHM-LKH | 3/10    | 22254     | 22253.1  | 1381.2 | 2736.35  |
| d1655                  | 62128  | LKH      | 10/10   | Opt.      | Opt.     | 176    | 7.73     |
|                        |        | NeuroLKH | 8/10    | Opt.      | 62128.2  | 870.4  | 24.75    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 29.7   | 2.92     |
| vm1748                 | 336556 | LKH      | 9/10    | Opt.      | 336557.3 | 1007.9 | 14.06    |
|                        |        | NeuroLKH | 5/10    | Opt.      | 336628   | 1282.9 | 28.08    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 156.5  | 7.5      |
| u1817                  | 57201  | LKH      | 1/10    | Opt.      | 57251.1  | 1817   | 78.82    |
|                        |        | NeuroLKH | 2/10    | Opt.      | 57227.3  | 1803.4 | 149.38   |
|                        |        | LKHM-LKH | 4/10    | Opt.      | 57222.2  | 1447.4 | 161.84   |
| rl1889                 | 316536 | LKH      | 0/10    | 316549    | 316549.8 | 1889   | 70.84    |
|                        |        | NeuroLKH | 0/10    | 316638    | 316648.7 | 1889   | 86.85    |
|                        |        | LKHM-LKH | 8/10    | Opt.      | 316556.4 | 917.2  | 56.87    |
| d2103                  | 80450  | LKH      | 0/10    | 80454     | 80462    | 2103   | 136.44   |
|                        |        | NeuroLKH | 4/10    | Opt.      | 80452.1  | 1508   | 240.74   |
|                        |        | LKHM-LKH | 2/10    | Opt.      | 80455.4  | 1915.5 | 458.3    |
| u2152                  | 64253  | LKH      | 3/10    | Opt.      | 64287.7  | 1614   | 86.68    |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 386    | 31.22    |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 295.1  | 38.73    |
| u2319                  | 234256 | LKH      | 10/10   | Opt.      | Opt.     | 3.1    | 0.65     |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 5.2    | 0.98     |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 5.1    | 1.22     |
| pr2392                 | 378032 | LKH      | 10/10   | Opt.      | Opt.     | 5.8    | 0.57     |
|                        |        | NeuroLKH | 10/10   | Opt.      | Opt.     | 25.5   | 1.34     |
|                        |        | LKHM-LKH | 10/10   | Opt.      | Opt.     | 14.1   | 1.41     |
| pcb3038                | 137694 | LKH      | 4/10    | Opt.      | 137701.2 | 2078.6 | 78.84    |
|                        |        | NeuroLKH | 8/10    | Opt.      | 137695   | 1104   | 102.34   |
|                        |        | LKHM-LKH | 9/10    | Opt.      | 137694.5 | 758.9  | 91.04    |
| fl3795                 | 28772  | LKH      | 0/10    | 28813     | 28813.7  | 3795   | 34045.95 |
|                        |        | NeuroLKH | 0/10    | 28999     | 29010.6  | 3795   | 80797.24 |
|                        |        | LKHM-LKH | 0/10    | 28813     | 28813.4  | 3795   | 34265.47 |
| Continued on next page |        |          |         |           |          |        |          |

Table 16: Comparison results of LKH, NeuroLKH and LKHM-LKH on all 79 instances in TSPLIB (continued)

| Name     | Opt.      | Method   | Success | Best_cost   | Avg_cost    | Trials  | Time     |
|----------|-----------|----------|---------|-------------|-------------|---------|----------|
| fnl4461  | 182566    | LKH      | 9/10    | <b>Opt.</b> | 182566.5    | 923.1   | 32.47    |
|          |           | NeuroLKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 171.5   | 18.96    |
|          |           | LKHM-LKH | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 107.1   | 16.41    |
| rl5915   | 565530    | LKH      | 0/10    | 565544      | 565581.2    | 5915    | 267.26   |
|          |           | NeuroLKH | 0/10    | 565585      | 565969.9    | 5915    | 439.82   |
|          |           | LKHM-LKH | 5/10    | <b>Opt.</b> | 565548.6    | 3856    | 380.18   |
| rl5934   | 556045    | LKH      | 0/10    | 556136      | 556309.8    | 5934    | 429.12   |
|          |           | NeuroLKH | 8/10    | <b>Opt.</b> | 556059.5    | 3470.2  | 238.35   |
|          |           | LKHM-LKH | 7/10    | <b>Opt.</b> | 556072.3    | 4156    | 526.21   |
| rl11849  | 923288    | LKH      | 2/10    | <b>Opt.</b> | 923362.7    | 10933.4 | 2560.73  |
|          |           | NeuroLKH | 0/10    | 923416      | 923578.7    | 11849   | 3369.16  |
|          |           | LKHM-LKH | 5/10    | <b>Opt.</b> | 923303.5    | 10444.2 | 5711.95  |
| usa13509 | 19982859  | LKH      | 1/10    | <b>Opt.</b> | 19983103.4  | 13509   | 3337.58  |
|          |           | NeuroLKH | 0/10    | 19991781    | 19999655.1  | 13509   | 4990.12  |
|          |           | LKHM-LKH | 2/10    | <b>Opt.</b> | 19983080.3  | 12250.1 | 6520.91  |
| brd14051 | 469385    | LKH      | 0/10    | 469393      | 469398.3    | 14051   | 4242.63  |
|          |           | NeuroLKH | 0/10    | 469485      | 469530.5    | 14051   | 8025.63  |
|          |           | LKHM-LKH | 1/10    | <b>Opt.</b> | 469401.3    | 14051   | 12514.46 |
| d15112   | 1573084   | LKH      | 0/10    | 1573085     | 1573142.7   | 15112   | 4875.2   |
|          |           | NeuroLKH | 0/10    | 1573245     | 1573374.9   | 15112   | 10765.47 |
|          |           | LKHM-LKH | 0/10    | 1573087     | 1573112.8   | 15112   | 15225.6  |
| d18512   | 645238    | LKH      | 0/10    | 645239      | 645260.6    | 18512   | 7280.07  |
|          |           | NeuroLKH | 0/10    | 645409      | 645483      | 18512   | 14218.28 |
|          |           | LKHM-LKH | 0/10    | 645243      | 645262.8    | 18512   | 22157.14 |
| pla33810 | 66048945  | LKH      | 0/10    | 66062117    | 66062876.33 | 4736    | 100000   |
|          |           | NeuroLKH | 0/10    | 66061689    | 66062779.33 | 4236    | 100000   |
|          |           | LKHM-LKH | 0/10    | 66061689    | 66062779.33 | 4369    | 100000   |
| pla85900 | 142382641 | LKH      | 0/10    | 142455685   | 142456427.7 | 18231   | 100000   |
|          |           | NeuroLKH | 0/10    | 142418786   | 142419423.7 | 17129   | 100000   |
|          |           | LKHM-LKH | 0/10    | 142418786   | 142419423.7 | 17436   | 100000   |

## E Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB

Table 17: Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB

| Name     | Opt.   | Method       | Success | Best_cost | Avg_cost | Trials | Time |
|----------|--------|--------------|---------|-----------|----------|--------|------|
| eil51    | 426    | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0    |
| berlin52 | 7542   | VSR-LKH      | 10/10   | Opt.      | Opt.     | 0      | 0.01 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 0      | 0    |
| st70     | 675    | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0    |
| eil76    | 538    | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0    |
| pr76     | 108159 | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.03 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
| rat99    | 1211   | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0    |
| kroA100  | 21282  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.04 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| kroB100  | 22141  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1.5    | 0.03 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 2.6    | 0.05 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.03 |
| kroC100  | 20749  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.03 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| kroD100  | 21294  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.02 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
| kroE100  | 22068  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 17.3   | 0.07 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 7.4    | 0.06 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1.5    | 0.01 |
| rd100    | 7910   | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.01 |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0    |
| eil101   | 629    | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0    |
|          |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0    |

Continued on next page

Table 18: Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB (continued)

| Name                   | Opt.   | Method       | Success | Best_cost | Avg_cost | Trials | Time  |
|------------------------|--------|--------------|---------|-----------|----------|--------|-------|
| lin105                 | 14379  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0     |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 0      | 0.01  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0     |
| pr107                  | 44303  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.1   |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1.7    | 2.18  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1.6    | 0.98  |
| pr124                  | 59030  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.03  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.06  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.05  |
| bier127                | 118282 | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02  |
|                        |        | NeuroLKH-VSR | 1/10    | Opt.      | 118350.4 | 120.6  | 0.35  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.01  |
| ch130                  | 6110   | VSR-LKH      | 10/10   | Opt.      | Opt.     | 7.7    | 0.04  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 2.1    | 0.04  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.01  |
| pr136                  | 96772  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.06  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.13  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.04  |
| pr144                  | 58537  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.26  |
|                        |        | NeuroLKH-VSR | 0/10    | 58590     | 58590    | 144    | 5.12  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.29  |
| ch150                  | 6528   | VSR-LKH      | 10/10   | Opt.      | Opt.     | 4.7    | 0.03  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 10.6   | 0.13  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 30.6   | 0.11  |
| kroA150                | 26524  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.04  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.03  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1.5    | 0.03  |
| kroB150                | 26130  | VSR-LKH      | 3/10    | Opt.      | 26131.4  | 119.3  | 0.17  |
|                        |        | NeuroLKH-VSR | 9/10    | Opt.      | 26130.2  | 48.3   | 0.23  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 24.2   | 0.26  |
| pr152                  | 73682  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 20.4   | 0.35  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 36.3   | 1.05  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 32.4   | 0.44  |
| u159                   | 42080  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.01  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 1      | 0.01  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 1      | 0.01  |
| rat195                 | 2323   | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.02  |
|                        |        | NeuroLKH-VSR | 10/10   | Opt.      | Opt.     | 7.2    | 0.13  |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 12.7   | 0.08  |
| d198                   | 15780  | VSR-LKH      | 10/10   | Opt.      | Opt.     | 1      | 0.37  |
|                        |        | NeuroLKH-VSR | 1/10    | Opt.      | 15797.9  | 197.4  | 11.95 |
|                        |        | LKHM-VSR     | 10/10   | Opt.      | Opt.     | 38.7   | 1.8   |
| Continued on next page |        |              |         |           |          |        |       |

Table 19: Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB (continued)

| Name    | Opt.   | Method       | Success | Best_cost   | Avg_cost    | Trials | Time    |
|---------|--------|--------------|---------|-------------|-------------|--------|---------|
| kroA200 | 29368  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.08    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04    |
| kroB200 | 29437  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1.8    | 0.03    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.04    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.02    |
| ts225   | 126643 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.01    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.05    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.02    |
| tsp225  | 3916   | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.05    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.08    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.05    |
| pr226   | 80369  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.07    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 35.2   | 1.07    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.3    | 0.13    |
| gil262  | 2378   | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 5.8    | 0.09    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.1    | 0.14    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3      | 0.06    |
| pr264   | 49135  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.8    | 0.09    |
|         |        | NeuroLKH-VSR | 9/10    | <b>Opt.</b> | 49159.3     | 66.4   | 1.21    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 44     | 0.26    |
| a280    | 2579   | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.01    |
|         |        | NeuroLKH-VSR | 0/10    | 2619        | 2619        | 280    | 0.22    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.03    |
| pr299   | 48191  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 18.3   | 0.21    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1      | 0.21    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1.6    | 0.12    |
| lin318  | 42029  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.4    | 0.14    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.9    | 0.15    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.6    | 0.33    |
| rd400   | 15281  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 11.9   | 0.12    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 5.1    | 0.16    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.7    | 0.11    |
| fl417   | 11861  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 6.1    | 1.86    |
|         |        | NeuroLKH-VSR | 0/10    | 11862       | 11865.2     | 417    | 1169.24 |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 29.8   | 23.78   |
| pr439   | 107217 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 12.3   | 0.22    |
|         |        | NeuroLKH-VSR | 7/10    | <b>Opt.</b> | 107238.6    | 236    | 2.25    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 9.6    | 0.14    |
| pcb442  | 50778  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 2.6    | 0.04    |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 12.8   | 0.23    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.1    | 0.07    |

Continued on next page



Table 20: Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB (continued)

| Name    | Opt.   | Method       | Success | Best_cost   | Avg_cost    | Trials | Time   |
|---------|--------|--------------|---------|-------------|-------------|--------|--------|
| d493    | 35002  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 17.8   | 0.55   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 30     | 1.68   |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.9    | 0.46   |
| u574    | 36905  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.1    | 0.15   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 14.8   | 0.5    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.7    | 0.21   |
| rat575  | 6773   | VSR-LKH      | 7/10    | <b>Opt.</b> | 6773.3      | 379.6  | 4.07   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 125.5  | 2.85   |
|         |        | LKHM-VSR     | 3/10    | <b>Opt.</b> | 6773.7      | 473.2  | 5.88   |
| p654    | 34643  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 9.3    | 3.68   |
|         |        | NeuroLKH-VSR | 9/10    | <b>Opt.</b> | 34644.2     | 129.5  | 312.85 |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 90.9   | 33.25  |
| d657    | 48912  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 7.3    | 0.24   |
|         |        | NeuroLKH-VSR | 5/10    | <b>Opt.</b> | 48912.5     | 405.4  | 12.51  |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 28.9   | 0.57   |
| u724    | 41910  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 35.6   | 1.13   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 20     | 1.47   |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 27.6   | 0.91   |
| rat783  | 8806   | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1.7    | 0.04   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 6.9    | 0.22   |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 5.5    | 0.11   |
| pr1002  | 259045 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 14.3   | 0.57   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 16.7   | 0.89   |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.8    | 0.22   |
| u1060   | 224094 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 36.3   | 6.07   |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 94.9   | 25.53  |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 17.9   | 4.65   |
| vm1084  | 239297 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 106.5  | 7.42   |
|         |        | NeuroLKH-VSR | 7/10    | <b>Opt.</b> | 239312.6    | 505.8  | 64.89  |
|         |        | LKHM-VSR     | 9/10    | <b>Opt.</b> | 239302.2    | 134.1  | 12.53  |
| pcb1173 | 56892  | VSR-LKH      | 8/10    | <b>Opt.</b> | 56893       | 549.8  | 6.1    |
|         |        | NeuroLKH-VSR | 9/10    | <b>Opt.</b> | 56892.5     | 333.2  | 8.64   |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 287.9  | 4.89   |
| d1291   | 50801  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 76.6   | 5.86   |
|         |        | NeuroLKH-VSR | 9/10    | <b>Opt.</b> | 50803.4     | 150.8  | 21.41  |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 307.6  | 3.32   |
| rl1304  | 252948 | VSR-LKH      | 1/10    | <b>Opt.</b> | 252993.9    | 1231.1 | 42.97  |
|         |        | NeuroLKH-VSR | 9/10    | <b>Opt.</b> | 252953.1    | 621.5  | 58.23  |
|         |        | LKHM-VSR     | 5/10    | <b>Opt.</b> | 252973.5    | 958.6  | 39.08  |
| rl1323  | 270199 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 48.4   | 1.5    |
|         |        | NeuroLKH-VSR | 2/10    | <b>Opt.</b> | 270528      | 1211.6 | 32.37  |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 34     | 2.03   |

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Table 21: Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB (continued)

| Name    | Opt.   | Method       | Success | Best_cost   | Avg_cost    | Trials | Time      |
|---------|--------|--------------|---------|-------------|-------------|--------|-----------|
| nrw1379 | 56638  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 203.9  | 7.56      |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 247.2  | 20.06     |
|         |        | LKHM-VSR     | 8/10    | <b>Opt.</b> | 56638.2     | 529.1  | 25.77     |
| fl1400  | 20127  | VSR-LKH      | 8/10    | <b>Opt.</b> | 20134.4     | 779.5  | 2878.26   |
|         |        | NeuroLKH-VSR | 0/10    | 20174       | 20175.9     | 1400   | 4013.54   |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 137    | 203.09    |
| u1432   | 152970 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 3.6    | 0.36      |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.7    | 1.06      |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 2.5    | 0.57      |
| fl1577  | 22249  | VSR-LKH      | 0/10    | 22254       | 22256.1     | 1577   | 5378.59   |
|         |        | NeuroLKH-VSR | 5/10    | <b>Opt.</b> | 22251.9     | 924.5  | 2531.16   |
|         |        | LKHM-VSR     | 9/10    | <b>Opt.</b> | 22249.1     | 337.5  | 1218.02   |
| d1655   | 62128  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 11.9   | 1.99      |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 33.7   | 12.04     |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 14.4   | 1.8       |
| vm1748  | 336556 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 73.9   | 6.54      |
|         |        | NeuroLKH-VSR | 0/10    | 336684      | 336690.4    | 1748   | 376.88    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 72.3   | 5.88      |
| u1817   | 57201  | VSR-LKH      | 4/10    | <b>Opt.</b> | 57233       | 1354.9 | 176.76    |
|         |        | NeuroLKH-VSR | 0/10    | 57216       | 57242.2     | 1817   | 276.22    |
|         |        | LKHM-VSR     | 7/10    | <b>Opt.</b> | 57214.7     | 1085.6 | 196.3     |
| rl1889  | 316536 | VSR-LKH      | 6/10    | <b>Opt.</b> | 316541.2    | 1191.2 | 98.53     |
|         |        | NeuroLKH-VSR | 9/10    | <b>Opt.</b> | 316540      | 523.7  | 107.45    |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 78.9   | 9.97      |
| d2103   | 80450  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 466.8  | 119.26    |
|         |        | NeuroLKH-VSR | 8/10    | <b>Opt.</b> | 80450.6     | 562    | 312.74    |
|         |        | LKHM-VSR     | 9/10    | <b>Opt.</b> | 80451.2     | 537.5  | 257.91    |
| u2152   | 64253  | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 256    | 42.23     |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 73.8   | 24.36     |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 444.2  | 75.35     |
| u2319   | 234256 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.7    | 1.24      |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 2.9    | 1.44      |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 4.7    | 1.27      |
| pr2392  | 378032 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 11.9   | 1.41      |
|         |        | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 12.9   | 2.68      |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 8.4    | 1.42      |
| pcb3038 | 137694 | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 355.4  | 48.39     |
|         |        | NeuroLKH-VSR | 8/10    | <b>Opt.</b> | 137694.9    | 901.2  | 281.83    |
|         |        | LKHM-VSR     | 7/10    | <b>Opt.</b> | 137695.4    | 1458.3 | 272.46    |
| fl3795  | 28772  | VSR-LKH      | 5/10    | <b>Opt.</b> | 28793       | 2215.6 | 47206.85  |
|         |        | NeuroLKH-VSR | 0/10    | 28784       | 28923.2     | 3795   | 160134.04 |
|         |        | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 198.9  | 2609.6    |

Continued on next page

Table 22: Comparison results of VSR-LKH, NeuroLKH-VSR and LKHM-VSR on all 79 instances in TSPLIB (continued)

| Name     | Opt.      | Method       | Success | Best_cost   | Avg_cost    | Trials  | Time      |
|----------|-----------|--------------|---------|-------------|-------------|---------|-----------|
| fnl4461  | 182566    | VSR-LKH      | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 87.1    | 23.41     |
|          |           | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 124.2   | 78.13     |
|          |           | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 88.9    | 26.04     |
| rl5915   | 565530    | VSR-LKH      | 0/10    | 565585      | 565585.4    | 5915    | 1240.9    |
|          |           | NeuroLKH-VSR | 1/10    | <b>Opt.</b> | 565590.4    | 5800.2  | 2953.02   |
|          |           | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1113.5  | 310.91    |
| rl5934   | 556045    | VSR-LKH      | 1/10    | <b>Opt.</b> | 556126.9    | 5584.5  | 1249.18   |
|          |           | NeuroLKH-VSR | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 407.7   | 411.41    |
|          |           | LKHM-VSR     | 9/10    | <b>Opt.</b> | 556054.1    | 2488.7  | 707.6     |
| rl11849  | 923288    | VSR-LKH      | 2/10    | <b>Opt.</b> | 923307.2    | 10544.2 | 11008.81  |
|          |           | NeuroLKH-VSR | 0/10    | 924815      | 925125.5    | 11849   | 25291.79  |
|          |           | LKHM-VSR     | 10/10   | <b>Opt.</b> | <b>Opt.</b> | 1351.9  | 1494.6    |
| usa13509 | 19982859  | VSR-LKH      | 3/10    | <b>Opt.</b> | 19983003.2  | 12852.9 | 18675.7   |
|          |           | NeuroLKH-VSR | 0/10    | 20015177    | 20030157    | 13509   | 122922.6  |
|          |           | LKHM-VSR     | 4/10    | <b>Opt.</b> | 19983018.1  | 11954.2 | 17292.74  |
| brd14051 | 469385    | VSR-LKH      | 2/10    | <b>Opt.</b> | 469391.9    | 12525.8 | 21630.97  |
|          |           | NeuroLKH-VSR | 0/10    | 471030      | 471419.7    | 14051   | 122857.21 |
|          |           | LKHM-VSR     | 1/10    | <b>Opt.</b> | 469390.9    | 13370.6 | 20896.09  |
| d15112   | 1573084   | VSR-LKH      | 1/10    | <b>Opt.</b> | 1573138     | 13617.3 | 26782.39  |
|          |           | NeuroLKH-VSR | 0/10    | 1579449     | 1580567.4   | 15112   | 71212.68  |
|          |           | LKHM-VSR     | 1/10    | <b>Opt.</b> | 1573109.4   | 13877.9 | 26521.56  |
| d18512   | 645238    | VSR-LKH      | 0/10    | 645257      | 645287      | 18512   | 44001.78  |
|          |           | NeuroLKH-VSR | 0/10    | 646926      | 647205.2    | 18512   | 105856.02 |
|          |           | LKHM-VSR     | 0/10    | 645253      | 645268.8    | 18512   | 39519.93  |
| pla33810 | 66048945  | VSR-LKH      | 0/10    | 66051736    | 66057108.33 | 2362    | 100000    |
|          |           | NeuroLKH-VSR | 0/10    | 66154253    | 66173899    | 1426    | 100000    |
|          |           | LKHM-VSR     | 0/10    | 66055720    | 66062396.67 | 2168    | 100000    |
| pla85900 | 142382641 | VSR-LKH      | 0/10    | 142416286   | 142422640   | 3936    | 100000    |
|          |           | NeuroLKH-VSR | 0/10    | 143485693   | 143610444   | 1063    | 100000    |
|          |           | LKHM-VSR     | 0/10    | 142408732   | 142413104.7 | 4046    | 100000    |

## F Visualization of instances on TSPLIB benchmark

Figure 14 visualizes 23 instances that LKHM algorithm has at least two more success times than NeuroLKH algorithm, and other 56 instances are given in Figure 15, 16, 17.

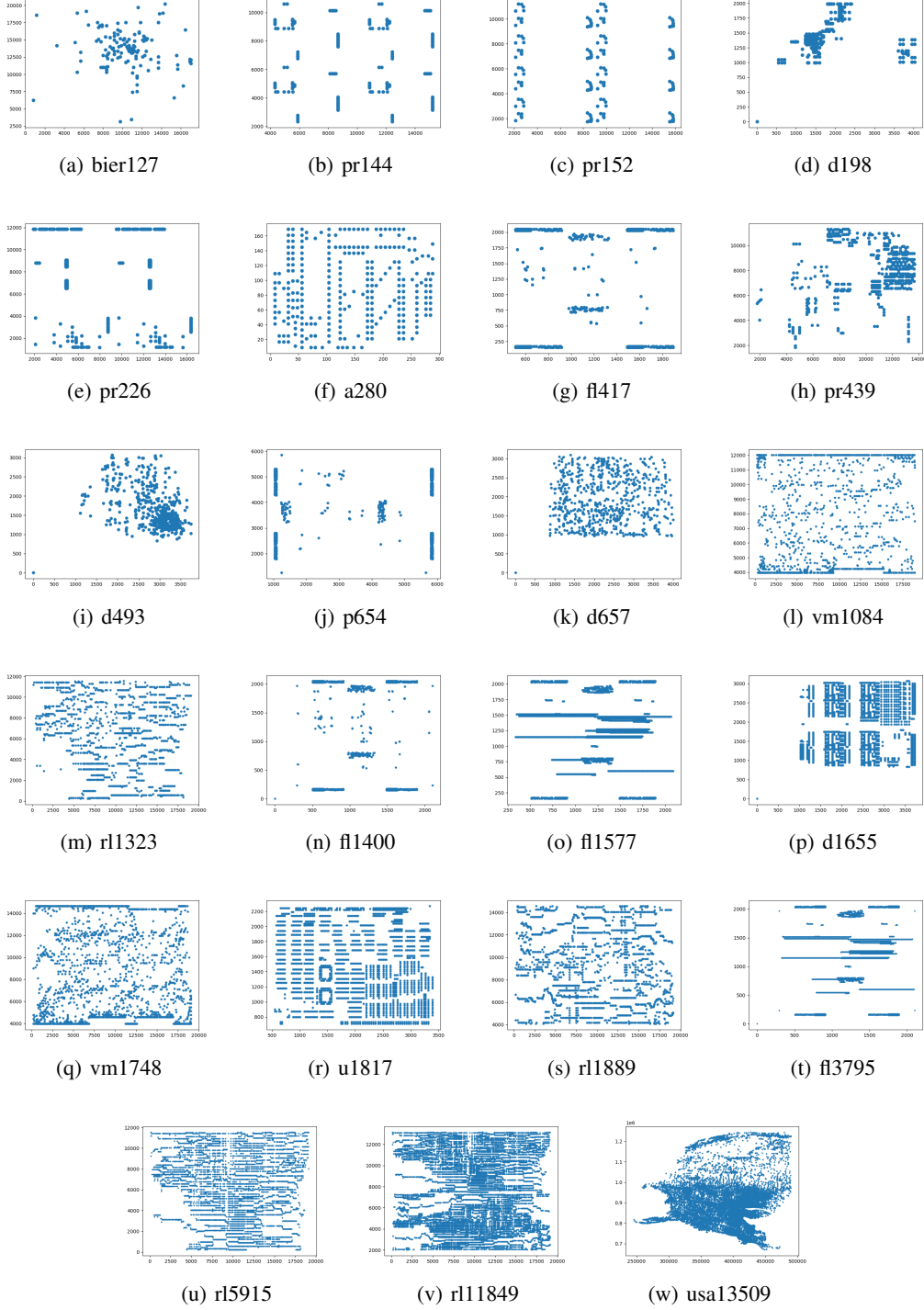
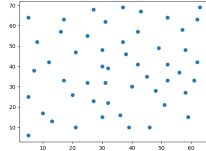
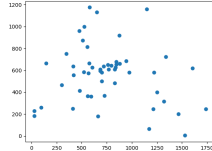


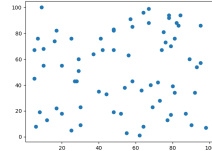
Figure 14: Visualization of 23 instances in TSPLIB benchmark



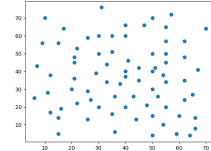
(a) eil51



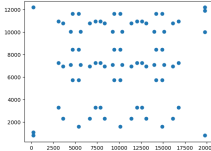
(b) berlin52



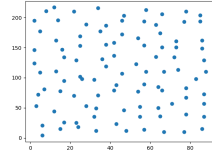
(c) st70



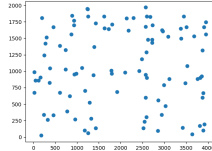
(d) eil76



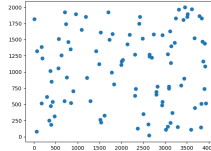
(e) pr76



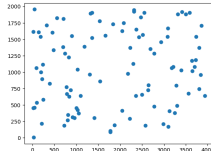
(f) rat99



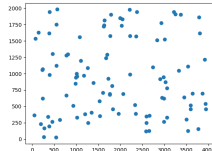
(g) kroA100



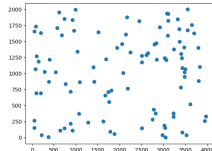
(h) kroB100



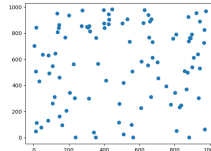
(i) kroC100



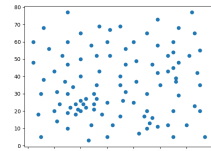
(j) kroD100



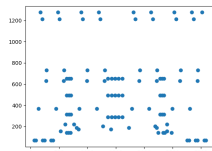
(k) kroE100



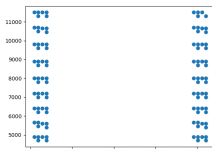
(l) rd100



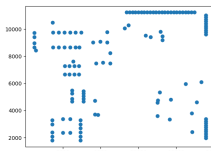
(m) eil101



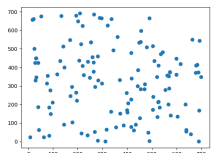
(n) lin105



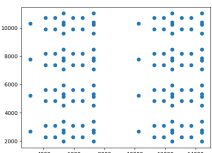
(o) pr107



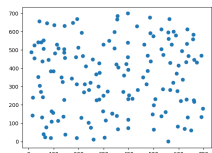
(p) pr124



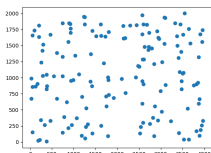
(q) ch130



(r) pr136



(s) ch150



(t) kroA150

Figure 15: Visualization of 20 instances in TSPLIB benchmark

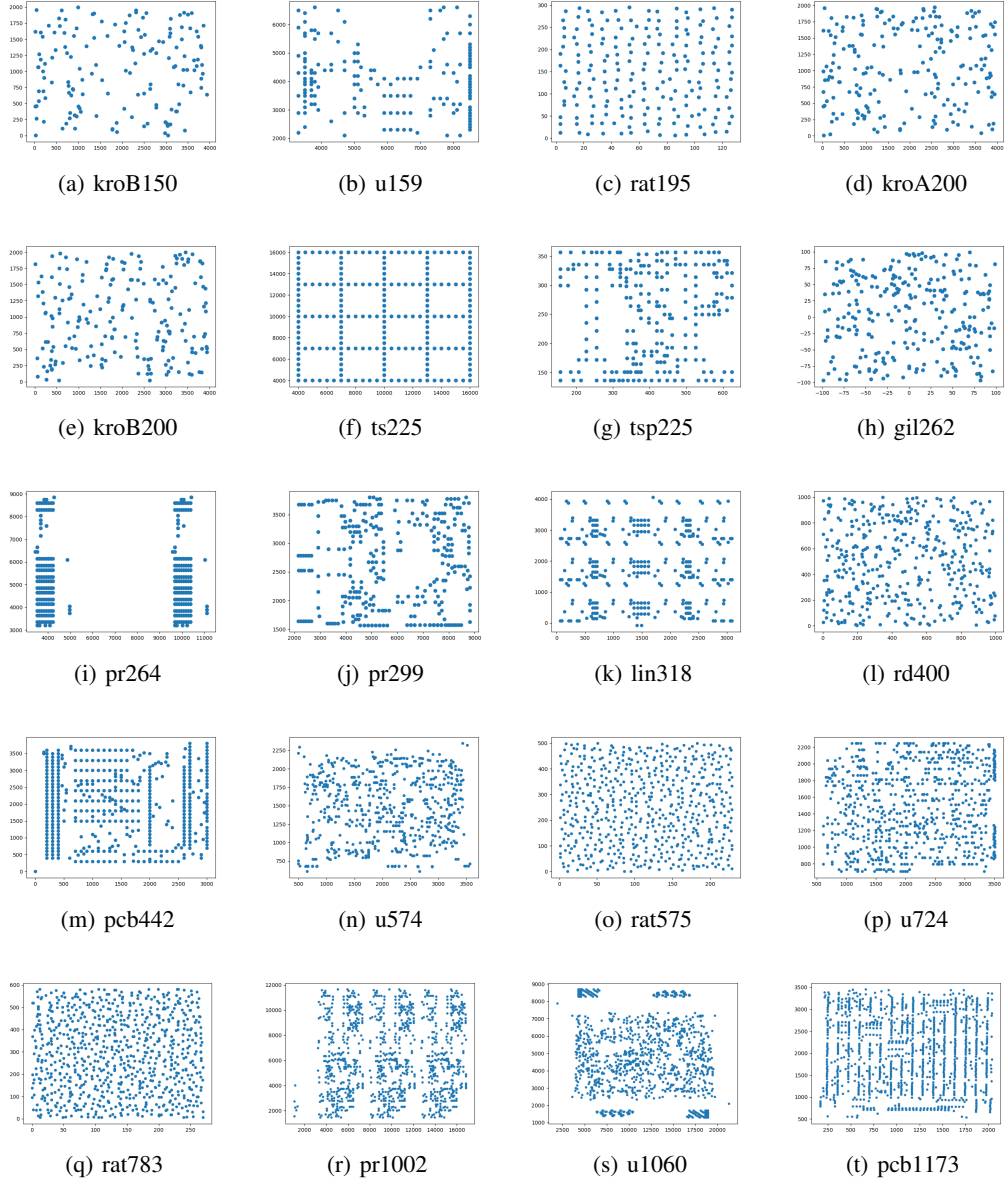


Figure 16: Visualization of 20 instances in TSPLIB benchmark

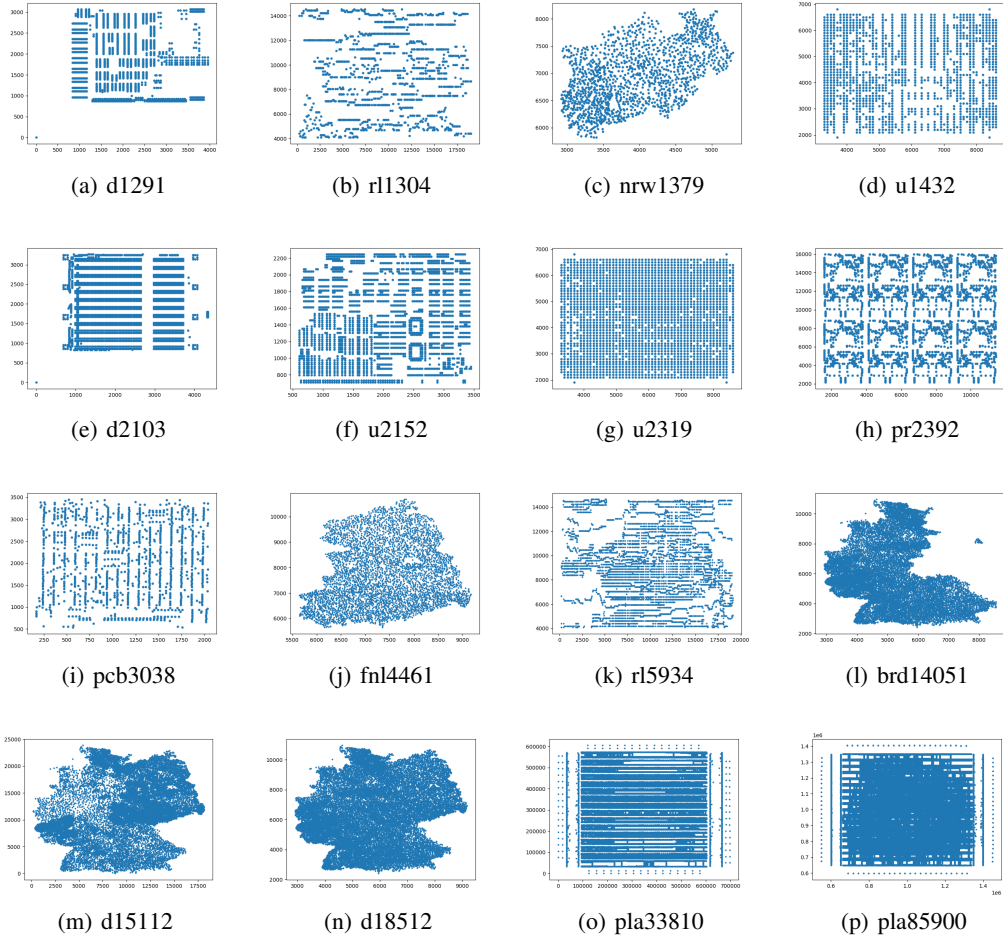


Figure 17: Visualization of 16 instances in TSPLIB benchmark