Speed Always Wins: A Survey on Efficient Architectures for Large Language Models

Weigao Sun[*1] **Jiaxi Hu**[*2] **Yucheng Zhou**[*3] **Jusen Du**[*1] **Disen Lan**[*1] **Kexin Wang**[*4]
Tong Zhu[*5] **Xiaoye Qu**[*1] **Yu Zhang**[5] **Xiaoyu Mo**[6] **Daizong Liu**[7] **Yuxuan Liang**[2]
Wenliang Chen[5] **Guoqi Li**[4] **Yu Cheng**[8 ✉]

[1]Shanghai AI Laboratory   [2]HKUST (GZ)   [3]University of Macau
[4]Institute of Automation, Chinese Academy of Sciences   [5]Soochow University
[6]KTH Royal Institute of Technology   [7]Peking University   [8]The Chinese University of Hong Kong

Large Language Models (LLMs) have delivered impressive results in language understanding, generation, reasoning, and pushes the ability boundary of multimodal models. Transformer models, as the foundation of modern LLMs, offer a strong baseline with excellent scaling properties. However, the traditional transformer architecture requires substantial computations and poses significant obstacles for large-scale training and practical deployment. In this survey, we offer a systematic examination of innovative LLM architectures that address the inherent limitations of transformers and boost the efficiency. Starting from language modeling, this survey covers the background and technical details of linear and sparse sequence modeling methods, efficient full attention variants, sparse mixture-of-experts, hybrid model architectures incorporating the above techniques, and emerging diffusion LLMs. Additionally, we discuss applications of these techniques to other modalities and consider their wider implications for developing scalable, resource-aware foundation models. By grouping recent studies into the above category, this survey presents a blueprint of modern efficient LLM architectures, and we hope this could help motivate future research toward more efficient, versatile AI systems.
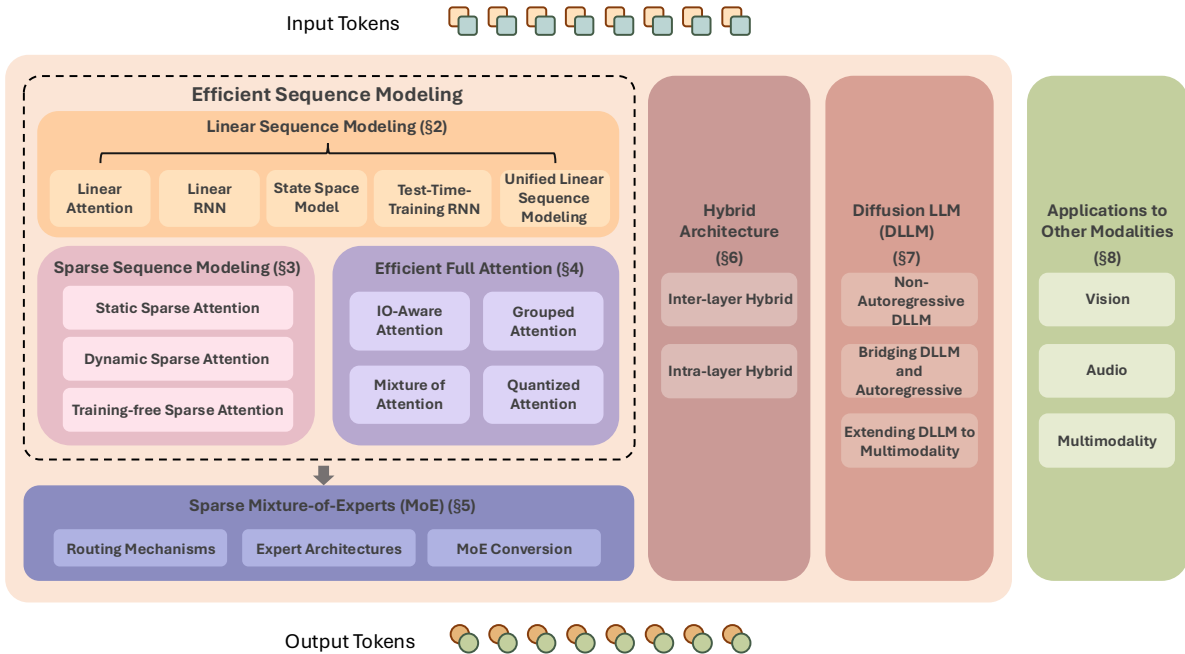
GitHub: https://github.com/weigao266/Awesome-Efficient-Arch

Figure 1: Overview of Efficient Architectures for Large Language Models.

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*Equal Contribution   ✉Corresponding Author (chengyu@cse.cuhk.edu.hk)

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1. Introduction

1.1. Background

In recent years, Large Language Models (LLMs) have emerged extraordinary capabilities in understanding and generating natural language have driven substantial progress across a wide range of tasks, including text generation [1, 2, 3], code generation [4, 5, 6], question answering [7, 8], and machine translation [3, 9]. Prominent LLM families such as ChatGPT [2, 10, 11, 12, 13, 14, 15, 16, 17], Claude [18, 19, 20, 21, 22], Gemini [23, 24, 25], DeepSeek [26, 27, 28, 29], Qwen [30, 31, 32, 33], LLaMA [34, 35, 36, 37], GLM [38], Minimax-Text [39], InternLM [40, 41], Hunyuan [42, 43] have continuously pushed the boundaries of performance, while also reshaping how people interact with machines in daily life. Beyond their initial role in language tasks, LLMs are increasingly being applied in two demanding areas: multimodality and complex reasoning. In multimodal applications, LLMs now play a key role in systems that integrate and generate information across multiple data types. Recent advances in Vision-Language Models (VLMs), such as Qwen-VL [44, 45, 46], InternVL [47, 48, 49, 50], Seed-VL [51], Kimi-VL [52], Minimax-VL [39], illustrate this shift, showcasing enhanced abilities in handling cross-modal understanding and generation by combining language skills with visual processing. At the same time, a growing line of work focuses on strengthening the reasoning capabilities of LLMs, often referred to as Large Reasoning Models (LRMs). Representative systems like OpenAI o1/o3 [14, 15], DeepSeek-R1 [29], Seed1.5-Thinking [53], Minimax-M1 [54], Kimi-k1.5/K2 [55, 56] incorporate strategies such as long-chain Chain-of-Thought (CoT) prompting [57] and Reinforcement Learning (RL) [58] to support multi-step reasoning and more deliberate cognitive behavior.

Although LLMs, VLMs, and LRMs have brought major advances in language understanding, multimodal processing, and complex reasoning, they also introduce considerable computational demands [59, 60, 61]. These increased requirements result in significantly higher development and deployment costs, which present practical barriers to widespread adoption. This challenge is shared across LLMs, VLMs, and LRMs, highlighting a common trade-off between model capability and efficiency. While such models offer a promising path toward intelligence, their high resource consumption raises important questions about the sustainability and practicality of pursuing even more powerful systems under current computational constraints. This keeps us thinking: **Have we paused to consider the immense hidden costs behind such unrivaled capabilities, and what is the true price of this "intelligence"?**

The core architecture behind many of latest breakthroughs is the Transformer [62], introduced in 2017. Its self-attention mechanism allows models to capture long-range dependencies more effectively than traditional Recurrent Neural Networks (RNNs) [63], enabling the scaling of LLMs to hundreds of billions or even trillions of parameters [2]. However, one major limitation of the Transformer lies in the quadratic complexity of its self-attention mechanism, which scales as $O(N^2)$ with the input sequence length N [64]. This computational inefficiency leads to extremely high training and inference costs, particularly in tasks that involve long-context inputs [65]. With the continued advancement of artificial intelligence (AI), long-sequence scenarios are becoming increasingly popular.

As shown in Figure 2, tasks such as Retrieval-Augmented Generation (RAG) [7] often requires LLMs to process entire documents. In the emerging era of AI agents [66], long sequences frequently arise from repeated generations and multiple tool invocations. When models are equipped with enhanced reasoning abilities [58], forming LRMs, they must handle lengthy chains of thought, which also result in long sequences. Similarly, in multimodal applications[67], high-resolution images, videos, and audio introduce additional long-sequence challenges. Another key component of the Transformer architecture, the Feed-Forward Network (FFN) [68], also faces challenges as model size increases. When the number of parameters grows beyond a certain scale, the training cost and inference efficiency of the FFN layer become increasingly difficult
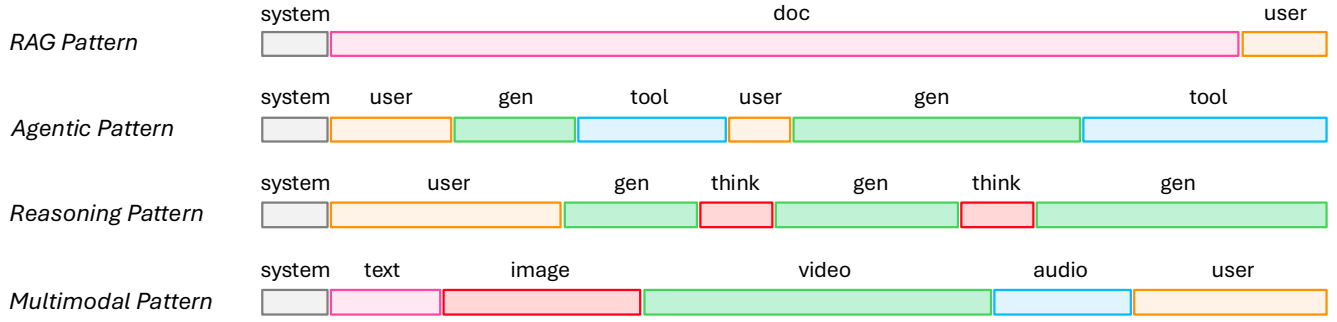
Figure 2: Long Context Patterns. We provide representative examples of long-context usage patterns across various scenarios, including retrieval-augmented generation (RAG), agentic, reasoning, and multimodal applications.

to manage. In this condition, another question arises: **How can we break through the Transformer's efficiency ceiling? Is costly "intelligence" our only path forward?**

To address these pressing challenges and unlock the full potential of LLMs, the research community has been actively exploring a spectrum of innovative architectural designs and optimization strategies. This survey delves into these innovative approaches, systematically categorizing them to provide a comprehensive overview. The specific methods encompassed within each category can be found in Figure 3. Here we summary each category as below:

- **Linear Sequence Modeling**: These methods aim to reduce the quadratic complexity of self-attention to linear complexity ($O(N)$) by reformulating the attention mechanism, often drawing inspiration from conventional attention, RNNs or state-space models (SSMs). These methods also eliminate the need to store Key-Value (KV) cache during inference, thereby lower the deployment cost.
- **Sparse Sequence Modeling**: Instead of computing attention over all token pairs, these methods selectively focus on a subset of interactions (i.e., the attention map), thereby reducing computational and memory requirements while striving to preserve performance.
- **Efficient Full Attention**: These methods enhance the standard softmax attention's efficiency while retaining its theoretical quadratic complexity, such as improving memory access efficiency through IO-aware attention mechanisms, and reducing the KV cache size through grouped query mechanisms.
- **Sparse Mixture of Experts**: This paradigm introduces a conditional computation approach where only a subset of a model's parameters (called experts) are activated for each input token, allowing for a massive increase in model capacity without a proportional increase in computational cost.
- **Hybrid Architectures**: These designs strategically combine linear sequence modeling components with traditional full attention layers. This can be achieved through intra-layer hybrid, where both types of operations co-exist within the same layer, or inter-layer hybrid, where different layers utilize distinct attention types, leveraging the strengths of each to trade-off both efficiency and model capacity.
- **Diffusion LLMs**: An emerging area that explores the non-autoregressive diffusion models for language generation, potentially offering new avenues for efficient and high-quality text synthesis.
- **Applications to Other Modalities**: Importantly, the architectural principles driving efficiency in LLMs are not confined to language; their adaptability is increasingly evident in other domains such as vision, audio, and multi-modality, a trend we will also explore.
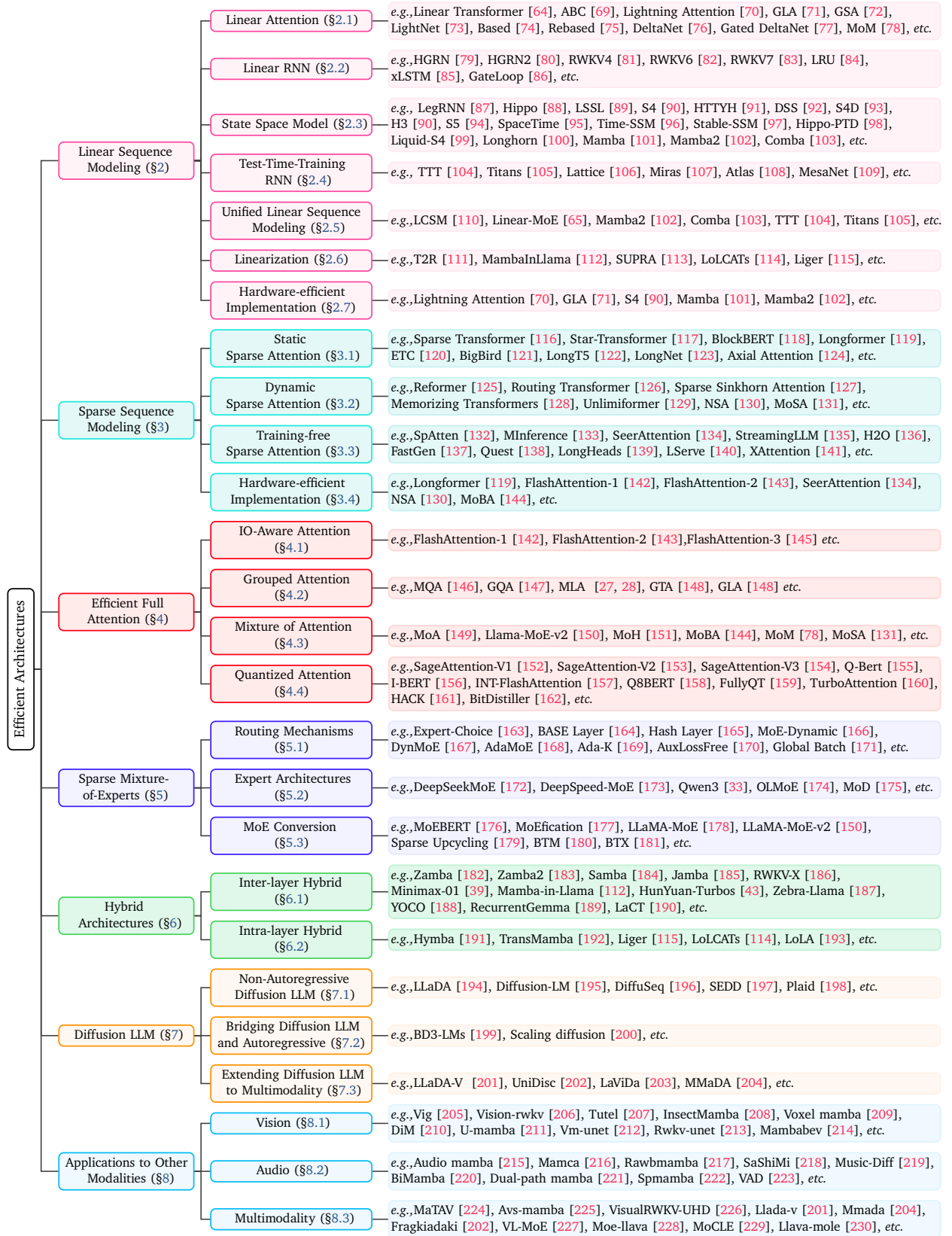
Efficient Architectures

Linear Sequence Modeling (§2)

- **Linear Attention (§2.1)**: *e.g.,* Linear Transformer [64], ABC [69], Lightning Attention [70], GLA [71], GSA [72], LightNet [73], Based [74], Rebased [75], DeltaNet [76], Gated DeltaNet [77], MoM [78], *etc.*

- **Linear RNN (§2.2)**: *e.g.,* HGRN [79], HGRN2 [80], RWKV4 [81], RWKV6 [82], RWKV7 [83], LRU [84], xLSTM [85], GateLoop [86], *etc.*

- **State Space Model (§2.3)**: *e.g.,* LegRNN [87], Hippo [88], LSSL [89], S4 [90], HTTYH [91], DSS [92], S4D [93], H3 [90], S5 [94], SpaceTime [95], Time-SSM [96], Stable-SSM [97], Hippo-PTD [98], Liquid-S4 [99], Longhorn [100], Mamba [101], Mamba2 [102], Comba [103], *etc.*

- **Test-Time-Training RNN (§2.4)**: *e.g.,* TTT [104], Titans [105], Lattice [106], Miras [107], Atlas [108], MesaNet [109], *etc.*

- **Unified Linear Sequence Modeling (§2.5)**: *e.g.,* LCSM [110], Linear-MoE [65], Mamba2 [102], Comba [103], TTT [104], Titans [105], *etc.*

- **Linearization (§2.6)**: *e.g.,* T2R [111], MambaInLlama [112], SUPRA [113], LoLCATs [114], Liger [115], *etc.*

- **Hardware-efficient Implementation (§2.7)**: *e.g.,* Lightning Attention [70], GLA [71], S4 [90], Mamba [101], Mamba2 [102], *etc.*

Sparse Sequence Modeling (§3)

- **Static Sparse Attention (§3.1)**: *e.g.,* Sparse Transformer [116], Star-Transformer [117], BlockBERT [118], Longformer [119], ETC [120], BigBird [121], LongT5 [122], LongNet [123], Axial Attention [124], *etc.*

- **Dynamic Sparse Attention (§3.2)**: *e.g.,* Reformer [125], Routing Transformer [126], Sparse Sinkhorn Attention [127], Memorizing Transformers [128], Unlimiformer [129], NSA [130], MoSA [131], *etc.*

- **Training-free Sparse Attention (§3.3)**: *e.g.,* SpAtten [132], MInference [133], SeerAttention [134], StreamingLLM [135], H2O [136], FastGen [137], Quest [138], LongHeads [139], LServe [140], XAttention [141], *etc.*

- **Hardware-efficient Implementation (§3.4)**: *e.g.,* Longformer [119], FlashAttention-1 [142], FlashAttention-2 [143], SeerAttention [134], NSA [130], MoBA [144], *etc.*

Efficient Full Attention (§4)

- **IO-Aware Attention (§4.1)**: *e.g.,* FlashAttention-1 [142], FlashAttention-2 [143], FlashAttention-3 [145] *etc.*

- **Grouped Attention (§4.2)**: *e.g.,* MQA [146], GQA [147], MLA [27, 28], GTA [148], GLA [148] *etc.*

- **Mixture of Attention (§4.3)**: *e.g.,* MoA [149], Llama-MoE-v2 [150], MoH [151], MoBA [144], MoM [78], MoSA [131], *etc.*

- **Quantized Attention (§4.4)**: *e.g.,* SageAttention-V1 [152], SageAttention-V2 [153], SageAttention-V3 [154], Q-Bert [155], I-BERT [156], INT-FlashAttention [157], Q8BERT [158], FullyQT [159], TurboAttention [160], HACK [161], BitDistiller [162], *etc.*

Sparse Mixture-of-Experts (§5)

- **Routing Mechanisms (§5.1)**: *e.g.,* Expert-Choice [163], BASE Layer [164], Hash Layer [165], MoE-Dynamic [166], DynMoE [167], AdaMoE [168], Ada-K [169], AuxLossFree [170], Global Batch [171], *etc.*

- **Expert Architectures (§5.2)**: *e.g.,* DeepSeekMoE [172], DeepSpeed-MoE [173], Qwen3 [33], OLMoE [174], MoD [175], *etc.*

- **MoE Conversion (§5.3)**: *e.g.,* MoEBERT [176], MoEfication [177], LLaMA-MoE [178], LLaMA-MoE-v2 [150], Sparse Upcycling [179], BTM [180], BTX [181], *etc.*

Hybrid Architectures (§6)

- **Inter-layer Hybrid (§6.1)**: *e.g.,* Zamba [182], Zamba2 [183], Samba [184], Jamba [185], RWKV-X [186], Minimax-01 [39], Mamba-in-Llama [112], HunYuan-Turbos [43], Zebra-Llama [187], YOCO [188], RecurrentGemma [189], LaCT [190], *etc.*

- **Intra-layer Hybrid (§6.2)**: *e.g.,* Hymba [191], TransMamba [192], Liger [115], LoLCATs [114], LoLA [193], *etc.*

Diffusion LLM (§7)

- **Non-Autoregressive Diffusion LLM (§7.1)**: *e.g.,* LLaDA [194], Diffusion-LM [195], DiffuSeq [196], SEDD [197], Plaid [198], *etc.*

- **Bridging Diffusion LLM and Autoregressive (§7.2)**: *e.g.,* BD3-LMs [199], Scaling diffusion [200], *etc.*

- **Extending Diffusion LLM to Multimodality (§7.3)**: *e.g.,* LLaDA-V [201], UniDisc [202], LaViDa [203], MMaDA [204], *etc.*

Applications to Other Modalities (§8)

- **Vision (§8.1)**: *e.g.,* Vig [205], Vision-rwkv [206], Tutel [207], InsectMamba [208], Voxel mamba [209], DiM [210], U-mamba [211], Vm-unet [212], Rwkv-unet [213], Mambabev [214], *etc.*

- **Audio (§8.2)**: *e.g.,* Audio mamba [215], Mamca [216], Rawbmamba [217], SaShiMi [218], Music-Diff [219], BiMamba [220], Dual-path mamba [221], Spmamba [222], VAD [223], *etc.*

- **Multimodality (§8.3)**: *e.g.,* MaTAV [224], Avs-mamba [225], VisualRWKV-UHD [226], Llada-v [201], Mmada [204], Fragkiadaki [202], VL-MoE [227], Moe-llava [228], MoCLE [229], Llava-mole [230], *etc.*

Figure 3: A Comprehensive Taxonomy of Efficient Architectures for Large Language Models.

1.2. Position and Contributions

The pursuit of more efficient model architectures has attracted substantial attention, resulting in a number of survey articles that chart the evolution of this research area. For example, Tay et al. [231] provides a detailed examination of Efficient Transformers, discussing a wide range of strategies designed to improve the self-attention mechanism, including early efforts in linear attention. Recent years, Patro et al. [232] offers an extensive survey of SSMs, positioning these architectures as promising alternatives to Transformer-based approaches for handling very long input sequences; their work systematically classifies different SSM designs and highlights applications that span language, vision, and other domains. More recently, Tiezzi et al. [233] reviews the renewed interest in recurrent processing, presenting models that blend core ideas from both Transformers and traditional recurrent networks, as well as the latest advances in state-space formulations. Sun et al. [234] summarizes recent advances in linear attention and sparse attention, and hybrid pretrained LLMs with these techniques. Although these surveys cover some of the efficient architectures like linear models, they only focus on a limit scope of methods to address Transformer's high computation problem.

Compared with the above reviews, this survey starts from key components of the Transformer model and provides a more comprehensive and organized overview of recent advances in efficient architectures of LLMs. We focus on summarizing their key design principles, performance benefits, known limitations, and possible future developments. Through this synthesis of current progress and emerging trends, we aim to support researchers and practitioners in developing more efficient and scalable LLMs and beyond.

The key contributions of this survey can be summarized as follows:

- **Comprehensive Survey of Efficient Sequence Modeling:** We present an in-depth review of recent progress in efficient sequence modeling. This includes methods that reduce the quadratic cost of attention through linear sequence models, strategies that selectively sparsify attention scores, and approaches that optimize the full softmax attention operation. By examining their design principles, performance trade-offs, and implementation details, we identify common themes and trace each technique back to its original motivation.
- **Broader Transformer Component Optimizations:** In addition to sequence modeling, we extend our analysis to other critical Transformer sub-modules. We cover advances in sparse MoE layers that enable conditional computation, hybrid architectures that blend linear and standard attention within or across layers, and the emerging area of diffusion-based language generation architectures. This wider perspective highlights how different parts of the Transformer can be re-imagined for efficiency.
- **Multi-Modal and Multi-Domain Applications:** Recognizing that efficient architectures are not limited to text processing, we explore their adaptation to other data modalities. We discuss applications in vision, audio, and multimodal settings, demonstrating how efficiency gains in token and channel mixing can benefit tasks ranging from high-resolution image understanding to multimodal sequence modeling.

2. Linear Sequence Modeling

In general, linear sequence modeling approaches can be broadly categorized into four groups: Linear Attention, Linear Recurrent Neural Network (Linear RNN), State Space Model (SSM), and Test-Time-Training (TTT) RNN, each originating from distinct motivations and mathematical formulations [65]. Following we first present these methods individually, and then describe how they can be unified under a common linear sequence modeling framework. We further explore a new direction called Linearization, which aims to convert standard Transformer models with softmax attention into linear sequence modeling architectures, allowing them to benefit from the efficiency of linear methods at a low cost [115]. In addition, this section
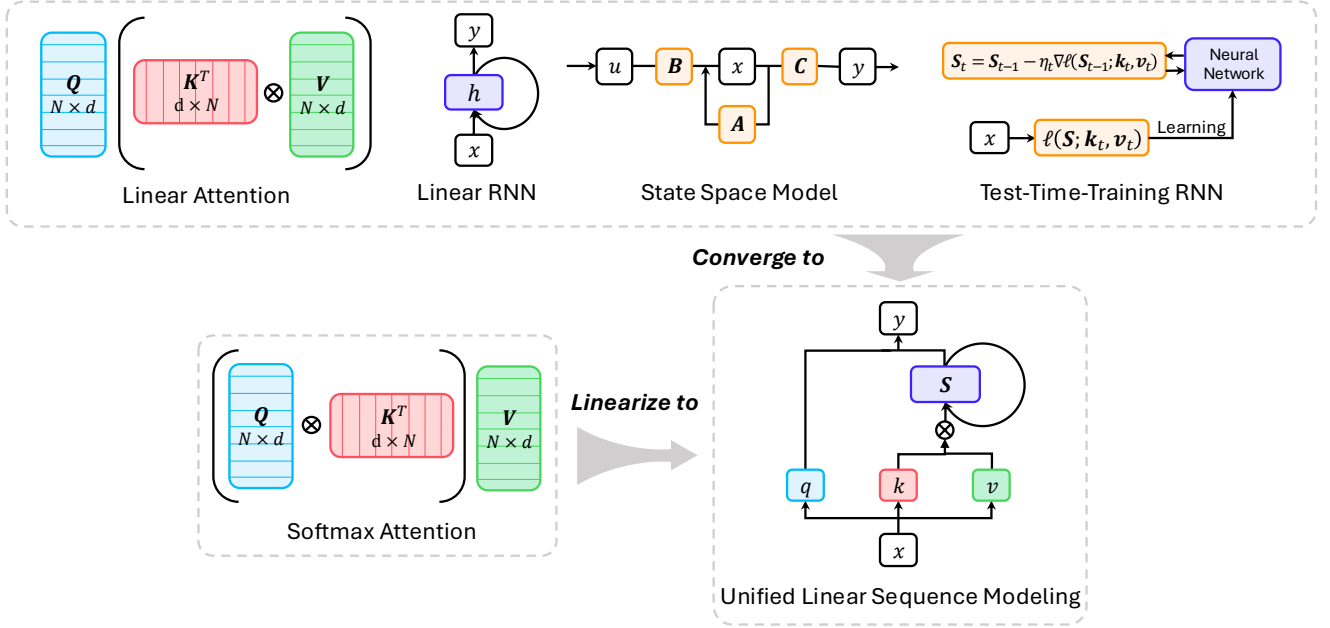
Figure 4: Linear Sequence Modeling Methods and Their Connections. The formulations of linear attention, linear RNNs, state space models, and test-time training RNNs have gradually converged toward a unified representation. Moreover, softmax attention can also be transformed into a linear sequence modeling form through the linearization techniques.

also covers implementation strategies that enhance the hardware efficiency of these linear sequence modeling approaches.

2.1. Linear Attention

Standard transformer [62] employs softmax attention mechanism which takes the input token x_t and computes the output o_t through:

$$q_t, k_t, v_t = x_t W_Q, x_t W_K, x_t W_V, \quad o_t = \frac{\sum_{i=1}^{t} \exp(q_t k_i^\top) v_i}{\sum_{i=1}^{t} \exp(q_t k_i^\top)} \tag{1}$$

Linear Attention was initially proposed in the Linear Transformer [64], which replaces the standard softmax attention with a linear approximation based on feature maps or kernel functions. This modification addresses the computational inefficiencies of conventional attention mechanisms and enables linear-time complexity. The generalized form of attention using an arbitrary similarity function can be expressed as:

$$o_t = \frac{\sum_{i=1}^{t} \text{sim}(q_t, k_i) v_i}{\sum_{i=1}^{t} \text{sim}(q_t, k_i)} \tag{2}$$

Note that Eq. (2) is equivalent to Eq. (1) if we substitute the similarity function with the specific implementation $\text{sim}(q, k) = \exp(q k^\top)$. Linear attention introduces kernel-based function to represent $\text{sim}(q, k)$ as $\phi(q)\phi(k)^\top$ with feature mapping $\phi(\cdot)$. Then Eq. (2) can be rewritten as:

$$o_t = \frac{\sum_{i=1}^{t} \phi(q_t) \phi(k_i) v_i}{\sum_{i=1}^{t} \phi(q_t) \phi(k_i)} \tag{3}$$

Eq. 3 can be further simplified by using the associative property of matrix multiplication as:

$$o_t = \frac{\phi(q_t) \sum_{i=1}^{t} \phi(k_i) v_i}{\phi(q_t) \sum_{i=1}^{t} \phi(k_i)} \tag{4}$$

Let $S_t = \sum_{i=1}^{t} \phi(k_i^\top) v_i$ and $z_t = \sum_{i=1}^{t} \phi(k_t)$, the above formulation in Eq. (4) can be written in a recurrent form [64] as:

$$\begin{cases} S_t = S_{t-1} + \phi(k_t)^\top v_t, \\ z_t = z_{t-1} + \phi(k_t)^\top, \end{cases} \quad o_t = \frac{\phi(q_t) S_t}{\phi(q_t) z_t} \tag{5}$$

By reconfiguring the order of operations $(QK^\top)V$ into $Q(K^\top V)$ using the associative property of matrix multiplication can reduce both computational and memory complexity from quadratic complexity $\mathcal{O}(N^2 d)$ to linear complexity $\mathcal{O}(Nd^2)$ over sequence length N.

Softmax Approximation via Feature Mapping. The pursuit of linear computational complexity in linear attention mechanisms fundamentally relies on decoupling attention weight computation from the sequential dependencies inherent in standard softmax attention. This is achieved by reformulating softmax$(QK^\top)V$ into $\phi(Q)(\phi(K^\top)V)$ through the associative property of matrix multiplication. Core to this approach is the approximation of exponential similarity $\exp(q^\top k)$ via feature mapping functions $\text{sim}(q, k) = \phi(q)\phi(k)^\top$. For example, Linear Transformer [64] employs a feature map $\phi(x) = \text{elu}(x) + 1$ ensuring non-negative similarity while mitigating gradient vanishing for negative inputs compared to relu$(\cdot)$. Random feature attention (RFA) [235] proposes to utilize random feature projection [236] as feature mapping function $\phi(\cdot)$ for efficient softmax function approximation, outperforming its baseline $\phi_{\text{elu}}(\cdot)$. Nystromformer [237] and Skyformer [238] adopt a similar idea to utilize low-rank kernel-based method for softmax attention approximation. Subsequent work by [239] points out that the distribution of attention weights in linear transformers is relatively smooth, limiting focus on informative features, therefore proposes focused function ϕ_p as query-key feature mapping to improve feature diversity in linear attention weights and a simple rank restoration module by using depthwise convolution (DWC) to the attention matrix, enhancing the expressiveness of linear attention while maintaining low computation complexity.

While linear attention models offer considerable gains in sequence modeling efficiency, the fidelity of their approximation to standard attention mechanisms remains deficient, especially when applied to long-context recall tasks. Based [74] introduces a simple hybrid architecture combining linear attention and sliding window attention, achieving better throughput and recall tradeoff. ReBased [75] improves long-context modeling performance by further developing Based architecture using polynomial kernels with learnable parameters and normalization operations. Concurrently, Zhang et al. [240] propose Hedgehog feature mapping for mimicking low-entropy ("spiky") weights and dot-product monotonicity properties in softmax attention, achieving better recall performance. However, Qin et al. [241] observes that kernel-based linear attention for softmax attention approximation suffer from unbounded gradients, which would cause unstable convergence during training, and proposes output normalization for gradients stability. ReGLA [242] addresses this problem by using the normalized exponential feature mapping function, and introduces variance reduction factor for training stability.

Gating Mechanism in Linear Attention. Linear attention mechanisms often exhibit suboptimal sequence modeling capability compared to softmax attention, with empirical observations indicating performance disparities exceeding marginal levels; this limitation is partly attributed to vanilla linear attention's reliance on cumulative memory updating, which induces memory conflicts and hinders effective sequence modeling [71]. Furthermore, this cumulative process intrinsically lacks mechanisms for dynamically regulating information flow over sequences. Consequently, the introduction of gating mechanism becomes essential to endow the model with adaptive forgetting and controlled updating capabilities, which allows the model to selectively attenuate or discard obsolete or irrelevant accumulated states while strategically integrating new inputs, mitigating memory interference and enabling more precise, contextually relevant representation learning essential for complex sequence tasks. Early work like TransNormerLLM [243] utilizes time-invariant memory decay and introduces Lightning Attention [70] for I/O-aware hardware-efficient optimization, and further introduces Lightning Attention-2 [244] by refining block-wise attention computations, which significantly improves computation efficiency. RetNet [245] adopts a retention mechanism by introducing a *data-independent* decay term into linear attention and gain improvement in sequence modeling.

While prior approaches utilized *data-independent* gating mechanisms for memory management, the ideal scenario involves models dynamically removing less important key-value associations based on the interaction between new inputs and existing memory content to accommodate new information. Recent advancements in linear attention variants demonstrate significant improvements by incorporating gating or decay factors for forgetting and updating. However, a more effective strategy is the adoption of *data-dependent* gating mechanisms, which dynamically control memory retention and updates through input-driven selection. For instance, Gated Linear Attention (GLA) [71] employs a data-dependent gating scheme to enhance the performance of sequence modeling and hardware efficiency, while Gated Slot Attention (GSA) [72] leverages context-aware gated linear attention with bounded-memory control (ABC) [69] to improve sequence modeling and long-context recall. To address the inefficiency of linear attention in multi-dimensional sequence modeling, where multiplicative decay requires multiple scans over the input, LightNet [73] introduces an additive linear recurrence that enables efficient single-pass processing of multi-dimensional data. Theoretical work in MetaLA [246] establishes the necessity of time-varying gating for optimal softmax attention approximation within linear attention frameworks. Further refinement is achieved by ReGLA [242], which introduces an additional refining gate to modify the forget gate when activations near saturation, thereby enhancing overall model performance and stability.

Delta Learning Rule in Linear Attention An effective sequence model should be able to remove less relevant key-value associations to accommodate new information, and this removal should depend on the interaction between incoming inputs and the memory state. From the perspective of fast weight programming, the recurrent hidden state or memory in linear attention mechanisms, such as in GLA, functions as a fast weight matrix that maps the input query q_t to the output o_t. This mapping is updated using a Hessian-like rule [247], which imposes limitations on memory capacity. To improve the expressiveness and adaptability of memory, recent work has explored the delta (Widrow-Hoff) learning rule [248, 249], enabling meta-learning or online adaptation during inference, which updates memory as follows:

$$S_t = S_{t-1} - \beta_t(S_{t-1}k_t - v_t)k_t^\top$$

where the update is driven by the difference of predicted output $S_{t-1}k_t$ and the target value v_t. This mechanism enhances memory capacity and gives rise to several delta-rule-based models for linear sequence learning. DeltaNet [76] is closely related to the Test-Time Training (TTT) framework [104], where memory is treated as a trainable component and updated through gradient descent steps. To better highlight the distinctions, we introduce the TTT family separately in Section 2.4. To overcome the limitation of DeltaNet,

which updates only a single key-value pair at each step, Gated DeltaNet [77] introduces a gating mechanism that enables more flexible memory control and allows rapid removal of outdated or irrelevant information. MesaNet [109] further advances this approach by incorporating the Mesa layer [250], which can dynamically adjust computational cost at inference time. It employs a recursive least squares loss for fast weight updates and can be viewed as a second-order online learner. MesaNet reduces the computational overhead of matrix inversion using gradient conjugation and supports hardware-efficient chunkwise parallelization. Additionally, models such as Comba [103] and RWKV7 [83] also fall under this category. However, we present them separately in the sections on state-space models and linear RNNs, respectively, to highlight their unique structural characteristics and connections to these model families.

Log-Linear Memory in Linear Attention. Although linear attention (or more generally, linear sequence modeling) has achieved performance comparable to or even surpassing that of Transformers in various downstream tasks, it still suffers from an inherent limitation: the existence of only a single, fixed-size state, which constrains its memory capacity. As a result, its performance degrades in long-context retrieval scenarios. A possible compromise is to design a model whose memory grows at a rate that is logarithmic plus linear in the sequence length (another potential solution is to expand multiple memories, as done in MoM [78]). Log-Linear Attention [251] introduce a general framework for linear attention models by replacing the fixed size hidden state in linear attention with a logarithmically growing set of hidden states enables achieving $O(N \log N)$ training and $O(\log N)$ inference complexity, thus balancing the efficiency of linear attention and the expressiveness of softmax attention. PSM [252] and Attraos [253] adopt a similar idea to extend linear recursion to a logarithmic scale through Blelloch's parallel scanning [254].

2.2. Linear RNN

RNNs are one of the most common methods for processing and modeling sequence data. At any given time step t, an RNN processes an input x_t and updates its hidden recurrent state h_t. This updated hidden state is subsequently used to generate an output y_t:

$$h_t = \sigma(W_{hh}h_{t-1} + W_{hx}x_t + b_h), \quad y_t = f(h_t) \tag{6}$$

where h_{t-1} represents the previous hidden state, $f(\cdot)$ is the projection function, and $\sigma(\cdot)$ is the activation function. Even over various time steps, an RNN maintains a fixed-size hidden state h_t containing historical sequence information. Consequently, the parametrization cost for an RNN remains constant regardless of the increase in time steps, enabling linear-time complexity sequence modeling in a recurrent form.

Traditional RNNs face the problem of inability to conduct parallel training and low efficiency due to its recurrence characteristics, limiting the capability in modeling long-term dependencies and difficulty in scaling up. The main reason is that the update of hidden state involves matrix multiplication and nonlinear activation functions which not only leads to gradient issues but also prevents parallel training. Linear RNNs are proposed to address these issues by removing nonlinearity. In some variants, the recurrent transformation is restricted to element-wise operations for efficiency, while others maintain structured or diagonalized matrices. LRU [84] leverages complex-valued diagonalization for efficient parallel computation and stable exponential parameterization of its recurrent dynamics to effectively capture long-range dependencies. Typical linear RNN such as Gated Linear Recurrent Unit (GLRU) [79, 80, 255] is formulated as follows:

$$
\begin{aligned}
g_t &= \sigma(W_g x_t + b_g), \quad i_t = \tau(W_i x_t + b_i), \quad o_t = \sigma(W_o x_t + b_o), \\
h_t &= g_t \odot h_{t-1} + (1 - g_t) \odot i_t, \quad y_t = h_t \odot o_t
\end{aligned}
\tag{7}
$$

where $\odot$ denotes element-wise product. Since linear RNN (in Eq. (7)) removes nonlinearity, it enables efficient parallelized training and achieve linear recurrent inference for sequence modeling.

Recent works have made effort to explore more expressive Linear RNN architecture with efficient recurrence or gating mechanisms. HGRN [79] maintains a similar RNN form as in Eq.(7), using dot products and accumulation to update memory. Additionally, it employs a novel forget gate mechanism featuring a learnable, layer-wise monotonically increasing lower bound across network layers. This allows lower layers to prioritize local, short-term information by forgetting more rapidly, while upper layers, constrained by a higher forget gate lower bound, effectively retain historical context for long-term dependency modeling. To enhance its expressive power within the linear recurrence framework, HGRN incorporates complex-valued hidden states and recurrence, where the gate's magnitude governs memory retention and its phase can encode relative positional information. RWKV4 [81] adopts a similar RNN structure but uses a channel-wise exponential decay mechanism inspired by relative position modeling in AFT [256]. Moreover, it employs token shifts to integrate the input tokens. These linear RNN models use d-dimensional vectors as memory, which limits their capacity. Research has increasingly shown that expanding memory capacity is crucial for enhancing the performance of linear RNNs. Consequently, subsequent approaches have started using the outer product of two d-dimensional vectors to form a $d \times d$ matrix as the memory representation.

HGRN2 [80] modifies the hidden state update process to Eq.(8). By using the outer product with the forget gate, it expands the memory to a $d \times d$ matrix, thereby increasing memory capacity. GateLoop [86] replaces their static state transition matrices with diagonal state transitions that dynamically adapt to the input. This approach allows for a matrix form of memory update within its recurrent mechanism, enabling more flexible state evolution. RWKV6 [82] changes the original element-wise vector product to a $k^T v$ formulation to achieve a multi-headed matrix-valued memory, thus expanding its capacity. RWKV6 introduces an advancement with its dynamic recurrence. This means the channel-wise decay rates within the linear state update are no longer static but become data-dependent and vary at each timestep, achieved efficiently through Low-Rank Adaptation (LoRA) to augment learned base decay vectors. Additionally, it applies similar data-dependency to its token-shift mechanism, allowing for more flexible and context-aware integration of past and current token information. Similarly, xLSTM [85] extends traditional LSTMs by introducing variants like mLSTM, which replaces the standard vector cell state with a matrix-valued memory to significantly expand storage capacity.

$$h_t = h_{t-1} \cdot \text{Diag}\{f_t\} + i_t \otimes (1 - f_t) \in \mathbb{R}^{d \times d} \tag{8}$$

At this stage, the memory structure of both linear RNNs and linear attention mechanisms has converged, utilizing matrix-based memory derived from the outer product of vectors. This allows for the application of Test-Time-Training on the memory as will be discussed in §2.4. RWKV7 [83] incorporates test-time gradient descent by introducing a "dynamic state evolution" powered by a generalized delta rule for its recurrent state updates. This mechanism allows the model to dynamically adapt its multi-headed matrix-valued state at each time step, effectively performing a form of test-time learning or in-context adaptation. This update rule, while more complex than prior RWKV iterations, endows RWKV7 with enhanced expressivity, further enhancing performance.

The motivation behind linear RNNs is to optimize the hidden state update for parallel training. Their recurrent formulations now closely resemble those of linear attention mechanisms. Although their starting points differ, they have evolved towards structurally similar designs, differing mainly in notation and specific architectural choices.

2.3. State Space Model

State Space Model (SSM) [257] is a historical mathematical framework used to describe dynamic systems that evolve over time in control systems. In this section, we systematically go through its development and applications on efficient sequence modeling.

From Hippo Theory to Continuous-Time SSM. Unlike the development path of traditional control theory [258], which progresses from state space models to spectral projections, deep state-space models follow a reverse trajectory. According to Legendre-RNN [87] and Hippo [88] theory, given an input function $u(s)$, a set of orthogonal polynomial basis $\phi_n(t,s)$ that $\int_{-\infty}^t \phi_m(t,s)\phi_n(t,s)\mathrm{d}s = \delta_{m,n}$, and an inner product probability measure $\mu(t,s)$. This enables us to project the $u(s)$ onto the polynomial basis along the time dimension:

$$\langle u, \phi_n \rangle_\mu = \int_{-\infty}^t u(s)\phi_n(t,s)\omega(t,s)\mathrm{d}s \tag{9}$$

This process can also be viewed as a spectral transformation $\mathcal{K}u(s) = \int_D K(t,s)u(s)\mathrm{d}s$ with kernel $K_n = \phi_n(t,s)\omega(t,s)$ [91, 96]. According to Time-SSM [96], by adjusting the polynomial basis $\phi_n(t,s)$ and inner product probability measure $\mu(t,s)$, various integral transforms, e.g., Gabor and Laplace transforms, can be realized. Moreover, When ϕ is the orthogonal basis in closed-recursive-form (e.g., Legendre, Chebyshev, Laguerre), by collecting all coefficients of order n, we can obtain a continuous ODE system. Together with the corresponding polynomial reconstruction process [96], this leads to the form of a time-invariant continuous-time SSM formulated in LSSL [89] and S4 [90]:

$$x'(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t) \tag{10}$$

This can also be regarded as parameterized maps that transform the input $u(t)$ into an N-dimensional latent space and project it onto the output $y(t)$. $Du(t)$ term is typically regarded as a residual connection [259] and can often be omitted.

Discretization. Since real-world data is typically in discrete form, it is necessary to convert continuous-time SSM parameters (AB) into their discrete counterparts $(\overline{A}, \overline{B})$ with step Δ. Early approaches like S4[89, 90] employ Zero-Order Hold (ZOH) discretization (use the general solution for Eq. (10)), while later studies like Mamba [101] found that using hybrid discretization methods (use forward Euler method [88] for B) could lead to more compact and efficient representations.

$$(\text{ZOH}): \quad \overline{A} = \exp(\Delta A), \quad \overline{B} = (\Delta A)^{-1}(\exp(\Delta A) - I) \cdot \Delta B \tag{11}$$

$$(\text{In Practice}): \quad \overline{A} = \exp(\Delta A), \quad \overline{B} = \Delta B \tag{12}$$

Diagonal SSMs. Early SSMs such as S4 and LSSL leveraged the HiPPO theory for initialization, while in practice, the naive recursive calculation of SSM Kernel can be quite computationally intensive. Ideally, when matrix A is diagonal, the computation simplifies to exponentiating the diagonal elements only. S4 [90] initialize the state matrix as a diagonal plus low-rank structure. Later, DSS [92] expressed it as a negative diagonal matrix (since any real skew-symmetric matrix derived from Hippo can be conjugate-diagonalized into a complex matrix, and the process $A = V^{-1}\Lambda V$ equal to a coordinate transform [96]), and in S4D [93], this is empirically simplified to a real diagonal matrix, which can be regarded as a rough approximation of the HiPPO-LegS matrix, and S4D introduces a method known as the left half-plane control to ensure that the diagonal elements of matrix $A^{(D)}$ remain negative. Subsequently, HTTYH [91] provided a theoretical

extension to the HiPPO framework to encompass the initialization scheme used in S4D. S5 [94] expands the single-input/output (**SISO** [90]) SSM to multi-input/output (**MIMO**) systems and introduces Blelloch parallel scan [254] to faster recurrent computation. SpaceTime [95] enhances model expressiveness by augmenting a diagonal state matrix with full-rank column vectors, effectively transforming it into a companion matrix. In contrast, H3 [260] adopts a two-layer SSM design: it initializes the state matrix of the first SSM layer as a shift matrix, and computes the diagonal parameters of the second SSM via key-value dot products. This design realizes token shift operations and thereby strengthens the model's ability to recall relevant information. StableSSM [97] and Hipo-PTD [98] further discuss how to carry out more stable initialization. Formally, these models can all be expressed as convolution operations during the recursive computation process:

$$\overline{K} = (C\overline{B}, C\overline{AB}, \dots, C\overline{A}^{L}\overline{B}), \quad y = x * \overline{K} \tag{13}$$

Time-variant (Selective) SSM. Although initialization based on the HiPPO framework is theoretically well-grounded, it results in system dynamics that remain the same at every time step. This uniformity prevents the model from selectively attending to important information, as attention mechanisms do, thereby limiting its expressive capacity. Based on the original formulation of S4 (with low-rank correction), Liquid-S4 [99] combines with a liquid time-constant network to obtain a new linear time-varying system. Mamba [101], on the other hand, abandons HiPPO-based initialization and instead directly derives data-dependent SSM parameter matrices through a linear projection layer. Moreover, it simplifies the computation into element-wise multiplications to accelerate inference. Building upon this, Attraos [253] initializes the diagonal matrix A with all ones values to approximate a Lebesgue measure, and further proposes a multi-resolution SSM based on piecewise orthogonal polynomial projection. Mamba-2 [102] further simplifies the state matrix to a scalar and introduces a hardware-efficient algorithm for blockwise parallel computation.

Recently, inspired by closed-loop control theory, Comba [103] introduced two key improvements to enhance the effectiveness of recurrent sequence modeling. First, it replaces the scalar state transition in Mamba2 with a scalar-plus-low-rank (SPLR) matrix, updated via a delta learning mechanism [248]. This matrix-based formulation improves the expressiveness of the transition dynamics and, interpreted as a form of Householder transformation [261, 262], introduces supervised memory management. This mitigates memory conflicts typically found in recurrent models that rely on fixed-size hidden states. Furthermore, the SPLR structure naturally supports negative eigenvalues, which further increases representational capacity. A similar formulation also arises in Longhorn [100], which extends Mamba to L2-based objectives. Second, Comba incorporates an output correction mechanism that introduces a learnable scalar to connect queries and keys. From the perspective of neural memory, this design ensures that the value is stored with high fidelity and can be accurately retrieved by the corresponding query. The output correction mechanism plays a central role in enabling this functionality. Moreover, Comba improves computational efficiency by eliminating unnecessary matrix inversions, which allows for the use of a highly optimized Triton kernel, resulting in significantly faster execution.

2.4. Test-Time-Training RNN

Although models like Comba [103], (Gated)-DeltaNet [76, 77], and RWKV7 [83] can also be formulated in an SGD-like manner for test-time training, a range of models such as TTT [104] offer a more explicit formulation. These models treat the model's state matrices directly as fast-adapting weights, which are updated through a learnable optimizer. From this perspective, the model is no longer restricted to a fixed linear or bilinear kernel but can instead leverage more advanced optimization algorithms to gain stronger

expressive power.

In terms of formulation, this class of models abandons closed-form recurrent representations and instead expresses the model directly in an online learning paradigm, similar to early meta-learning [263, 264] and fast weight programming [265, 266]. For the general form:

$$S_t = \alpha_t S_{t-1} - \eta_t \nabla_S \ell\left(S_{t-1}; k_t, v_t\right) \tag{14}$$

Initially, TTT [104] still employed an SGD-based optimizer for updates and introduced a deep state based on a two-layer MLP, which has been widely adopted by subsequent models. It is worth noting that this type of model typically applies LayerNorm to the state or its output to stabilize gradients and the distribution of the memory stored in the state. Titans [105] introduces a first-order momentum term to represent long-term memory and momentary surprise. Lattice [106] introduced a mechanism designed for linear models that can compress information into a limited number of memory slots efficiently and can update exclusively with information that is orthogonal to its current state, ensuring that only new and non-redundant data is written into memory and reducing the memory interference. Miras [107] presents a general framework for explaining the role of standard model architectural choices, including: (1) associative memory architecture, (2) attentional bias objective, (3) retention gate, and (4) memory learning algorithm, and introduces three novel sequence model variants based on the guidance of this framework, outperforming traditional Transformers and other modern linear recurrent models on language modeling and recall intensive tasks. Atlas [108] adopts higher-order feature mappings for memory capacity enhancing, and proposes the Omega rule and Muon optimizer [267] for memory updating, which can be demonstrated effective in language modeling downstream tasks. LaCT [190] further replaces the loss function with a dot product operation and employs large chunks of gradient descent to improve computational efficiency.

2.5. Unified Linear Sequence Modeling

While linear attention, linear RNNs, state space models, and test-time-training RNNs have traditionally followed separate lines of development, recent studies [65, 110] have begun to integrate these methods within a unified theoretical framework. In this paper, we present a consolidated view of these approaches, focusing on their memory update rules and optimization strategies.

2.5.1. A Memory Perspective

In this part, we adopt the conceptual framework proposed in Comba [103], wherein "linear" specifically refers to models whose dynamics with respect to key-value associative memory are linear.

Linear Update Rule. In contrast to autoregressive Transformers [62], which preserve all contextual information explicitly in a key-value (KV) cache, linear RNNs distill higher-level representations into a fixed-size hidden state to enhance generalization. From a metaphysical perspective, this transition can be viewed as a shift from conductive to inductive attention [103, 270], reflecting the notion that the compression is the intelligence [271]. Structurally, they bear similarities to energy-based models [272] such as Hopfield networks [273, 274] and neural systems employing Hebbian learning principles [247]. Early instances, including Linformer [275], S4 [90], and RetNet [245], did not incorporate sufficiently adaptive, data-driven memory management, which limited their performance relative to models utilizing softmax attention. More recent approaches, exemplified by Mamba [101] and GLA [71], have mitigated this limitation by adopting dynamic, projection-based gating mechanisms, thereby achieving notable gains. From a formal perspective, these architectures can be regarded as linear register systems equipped with key-value associative memory:

Table 1: A comparative overview of various linear sequence modeling approaches in terms of their memory update rules and optimization objectives. For reference, we also include the principles of softmax attention and sliding-window attention mechanisms. Broadly speaking, linear attention, linear RNNs, and state-space models follow a convergent developmental trajectory, evolving through mutual influence. This trajectory reflects a shift from data-independent to data-dependent gating mechanisms, and from L1-based to L2-based optimization objectives. In contrast, the development of TTT models has been primarily guided by advances in modern optimization algorithms.

Type	Model	Memorizing with Gate	Optimization Objective $\mathcal{L}_{\min}$
Softmax Attn	SA [62]	$S_t = S_{t-1}.\mathrm{append}(k_t, v_t)$	$\sum_{i=1}^{t} \exp(q_t^\mathsf{T} k_i) \|v - v_i\|^2$ [268]
	SWA [119]	$S_t = S_{t-1}.\mathrm{append}(k_t, v_t).\mathrm{drop}(k_{t-M}, v_{t-M})$	$\sum_{i=t-M}^{t} \exp(q_t^\mathsf{T} k_i) \|v - v_i\|^2$
Linear Attn	LA [64]	$S_t = S_{t-1} + v_t k_t^\mathsf{T}$	$\langle S_t k_t, v_t \rangle$
	Lightning Attn [70]	$S_t = \alpha S_{t-1} + v_t k_t^\mathsf{T}$	$\langle S_t k_t, v_t \rangle + \frac{\alpha}{2} \|S_t\|_2^2$
	RetNet [245]	$S_t = \alpha S_{t-1} + v_t k_t^\mathsf{T}$	$\langle S_t k_t, v_t \rangle + \frac{\alpha}{2} \|S_t\|_2^2$
	GLA [71]	$S_t = S_{t-1}\mathrm{diag}(\alpha_t) + v_t k_t^\mathsf{T}$	$\langle S_t k_t, v_t \rangle + \frac{\alpha_t}{2} \|S_t\|_2^2$
	MetaLA [246]	$S_t = S_{t-1}\mathrm{diag}(\alpha_t) + v_t(1 - \alpha_t)^\mathsf{T}$	$\langle S_t(1 - \alpha_t), v_t \rangle + \frac{\mathrm{diag}(\alpha_t)}{2} \|S_t\|_2^2$
	GSA [72]	$K_t = K_{t-1}\mathrm{diag}(\alpha_t) + k_t(1 - \alpha_t)^\mathsf{T}$ (same as v)	$\langle K_t(1 - \alpha_t), k_t \rangle + \frac{\mathrm{diag}(\alpha_t)}{2} \|K_t\|_2^2$
	DeltaNet [76]	$S_t = S_{t-1}(I - \beta_t k_t k_t^\mathsf{T}) + \beta_t v_t k_t^\mathsf{T}$	$\beta_t \|v_t - S_t k_t\|^2$
	Gated-DeltaNet [77]	$S_t = S_{t-1}\alpha_t^{\sim 1}(I - \beta_t k_t k_t^\mathsf{T}) + \beta_t v_t k_t^\mathsf{T}$	$\beta_t \left\|\frac{1}{\alpha_t} v_t - \alpha_t S_t k_t\right\|^2 + \frac{\alpha_t}{2} \|S_t\|_2^2$
	Delta-Product [269]	$S_t = S_{t-1}\prod_{i=1}^{n}(I - \beta_{ti} k_{ti} k_{ti}^\mathsf{T}) + \sum_{i=1}^{n}\prod_{i=1}^{n}(I - \beta_{ti} k_{ti} k_{ti}^\mathsf{T})\beta_{ti} v_{ti} k_{ti}^\mathsf{T}$	$\sum_{i=1}^{n} \beta_{ti} \|v_{ti} - S_{ti} k_{ti}\|^2$
	MesaNet [109]	$S_t = \alpha_t S_{t-1} + \beta_t v_t k_t^\mathsf{T}, \ H_t = \alpha_t H_{t-1} + \beta_t k_t k_t^\mathsf{T}$	$\sum_{i=1}^{t} \beta_t \|v_t - S_t k_t\|^2$
Linear RNN	HGRN2 [80]	$S_t = S_{t-1}\mathrm{diag}(\alpha_t) + v_t(1 - \alpha_t)^\mathsf{T}$	$\langle S_t(1 - \alpha_t), v_t \rangle + \frac{\alpha_t}{2} \|S_t\|_2^2$
	RWKV6 [82]	$S_t = S_{t-1}\mathrm{diag}(\alpha_t) + v_t k_t^\mathsf{T}$	$\langle S_t k_t, v_t \rangle + \frac{\alpha_t}{2} \|S_t\|_2^2$
	RWKV7 [83]	$S_t = S_{t-1}(\mathrm{diag}(\alpha_t) - \beta_t \hat{k}_t \hat{k}_t^\mathsf{T}) + v_t \tilde{k}_t^\mathsf{T}$	$\beta_t \left\|\frac{1}{\beta_t} v_t - S_t k_t\right\|^2 + \frac{\alpha_t}{2} \|S_t\|_2^2$
SSM	S4 [90]	$S_t = S_{t-1}\mathrm{diag}(\alpha) + v_t b^\mathsf{T} \in \mathbb{C}$	$\langle S_t b, v_t \rangle + \frac{\alpha}{2} \|S_t\|_2^2$
	Mamba [101]	$S_t = S_{t-1}\mathrm{diag}(\alpha) + \beta_t^{\sim 0} v_t k_t^\mathsf{T}$	$\beta_t \langle S_t k_t, v_t \rangle + \frac{\alpha_t}{2} \|S_t\|_2^2$
	Mamba2 [102]	$S_t = \alpha_t^{\sim 1} S_{t-1} + \beta_t^{\sim 0} v_t k_t^\mathsf{T}$	$\beta_t \langle S_t k_t, v_t \rangle + \frac{\alpha_t}{2} \|S_t\|_2^2$
	Longhorn [100]	$S_t = S_{t-1}\left(I - \frac{\beta_t}{1 + \beta_t k_t^\mathsf{T} k_t} k_t k_t^\mathsf{T}\right) + \beta_t v_t k_t^\mathsf{T}$	$\beta_t \|v_t - S_t k_t\|^2$
	Comba [103]	$S_t = S_{t-1}\left(\alpha_t^{\sim 1} - \beta_t^{\downarrow} k_t k_t^\mathsf{T}\right) + \beta_t^{\uparrow} v_t k_t^\mathsf{T}$	$\beta_t \|v_t - S_t k_t\|^2 + \frac{\alpha_t}{2} \|S_t\|_2^2 + \langle q_t, d k_t \rangle$
TTT	TTT-MLP [104]	$S_t(\cdot) = S_{t-B}(\cdot) - \sum_{i=1}^{B} \beta_i \nabla_S \mathcal{L}(S_{t-1}, k_t, v_t)$	$\beta_i \|v_i - \psi(S_j(k_i))\|^2$
	MIRAS [107]	$S_t = \alpha_t S_{t-1} - \beta_t \nabla_S \mathcal{L}(g, M_{t-1}, k_t, v_t)$	$\beta_t \|v_t - g(\psi(S_t), k_t)\|_p^p + \frac{\alpha_t}{2} \|S_t\|_2^2$
	Titans [105]	$M_t = \gamma_t M_{t-1} + S_t, \ S_t = \alpha_t S_{t-1} - \beta_t \nabla_M \mathcal{L}(M_{t-1}, k_t, v_t)$	$\beta_t \|v_t - \psi(S_t), k_t\|_2^2 + \frac{\alpha_t}{2} \|S_t\|_2^2$
	Atlas [108]	$M_t = \gamma_t M_{t-1} - \eta_t \mathrm{Muon}(S_t), \ S_t = \alpha_t S_{t-1} - \beta_t \nabla_M \mathcal{L}(M_{t-1}, k_t, v_t)$	$\beta_t \|v_t - \psi(S_t), k_t\|_2^2 + \frac{\alpha_t}{2} \|S_t\|_2^2$

information is written through learnable forgetting and input gates (denoted as α and β) and subsequently accessed by query-based retrieval:

$$S_t = (\alpha_t, \beta_t)@(S_{t-1}, k_t^\top v_t)^\top \quad \text{(Write)}, \quad o_t = S_t q_t \quad \text{(Read)} \tag{15}$$

Bilinear Update Rule. From the perspective of neural memory systems [276], achieving effective memory regulation remains a fundamental challenge. In contrast to the Hebbian learning principle [247], which depends on reinforcement-driven updates, the Delta learning rule [248] emphasizes supervised control of memory traces. These systems are linear with respect to state and input individually, but nonlinear overall due to the product term (e.g., Sk). They are regarded as a special class of nonlinear systems that preserve controllability [277, 278, 279, 280]. For a general formulation:

$$S_t = (\alpha_t, \beta_t)@(S_{t-1}, k_t^\top v_t)^\top + \gamma_t S_{t-1} k_t \quad \text{(Write)}, \quad o_t = S_t q_t \quad \text{(Read)} \tag{16}$$

Where Sk is the bilinear term in models such as Comba [103], (Gated)-DeltaNet [76, 77], and RWKV7 [83]. Alternatively, under the constraint that the spectral radius of the state transition matrix remains less than 1, it can also adopt an interaction between S and v, similar to that in Lattice [106]. Under the bilinear update rule, these models can be computed efficiently on GPUs through chunk-wise parallelism.

Nonlinear Update Rule. Apart from early models such as LSTM [281] and GRU [282], the series of models discussed in §2.4 (e.g., TTT) can be regarded as modern versions of nonlinear RNNs. These models employ nonlinear operations for state memory (such as nonlinear activations in a two-layer MLP memory), resulting in inherently nonlinear dynamics. While this provides theoretically stronger expressiveness, these models are constrained by block-wise parallel computation and can only be updated via minibatch gradient descent, which leads to very low hardware utilization (less than 5%). A potential solution is to follow LaCT [190] by adopting large-batch gradient descent, combined with sliding-window attention for intra-batch information learning as a hybrid architecture.

2.5.2. An Optimizer Perspective

Another perspective for unified linear sequence modeling is from an optimizer angle, which was initially proposed in TTT [104], Titans [105] and Test-time Regression [268]. We further classify the methods according to different forms of objective loss as below.

Local L1 Loss. Early models like linear attention [275] used an L1 loss to optimize the model in a single step, but this update scheme is unbounded, making it easy for S to suffer from memory conflicts. Subsequently, models such as RetNet [245], Mamba [101], and GLA [71] introduced memory gating mechanisms, which can be seen as a form of L2 regularization.

$$\hat{S}_t = \arg\min_S \left[\beta_t |v_t - S_t k_t| + \frac{1}{2} \text{Tr} \left(S_t^\top \Lambda S_t \right) \right] \tag{17}$$

Local L2 Loss. However, an L1 loss typically cannot directly enforce $v = Sk$, and thus cannot achieve precise key-value associative memory retrieval. A better choice is the squared loss, which provides stronger memory management capabilities. Such models also include an L2 regularization term on the memory states.

$$\hat{S}_t = \arg\min_S \frac{1}{2} \left[\beta_t \|v_t - S_t k_t\|^2 + \text{Tr} \left(S_t^\top \Lambda S_t \right) \right] \tag{18}$$

Multi-step L2 Loss. Models such as Delta-Product [269] apply a sequence of Householder transformations within a single timestep, effectively treating the latent update as a composition of structured orthogonal operators. This allows the training objective to be interpreted as a form of multi-step L2 loss, where the model is supervised not just on immediate transitions but on future latent states over multiple horizons.

In this formulation, a single-step L2 loss encourages the model to approximate simple reflections, which is the most basic form of orthogonal transformations [103]. In contrast, a multi-step L2 loss allows the model to approximate general orthogonal matrices through compositions of reflections, thereby capturing richer linear dynamics such as rotations, permutations, or shears. This property significantly improves performance on state-tracking tasks like S5 [283]. Specifically, Delta-Product formalizes the following objective:

$$\hat{S}_t = \arg\min_{S} \frac{1}{2} \sum_{i=1}^{n} \left[\beta_{ti} \left\| v_{ti} - S_{ti} k_{ti} \right\|^2 \right] \quad (\textit{Step by Step}) \tag{19}$$

Global L2 Loss. Inspired by Recursive Least Squares (RLS), MesaNet [109] defines a global L2 optimization for memory state, enabling the model to take the global optimum into account at each update step.

$$\hat{S}_t = \arg\min_{S} \frac{1}{2} \left[\sum_{i=1}^{t} \left(\gamma_i \left\| v_i - S k_i \right\|^2 \right) + \text{Tr}\left(S^\top \Lambda S \right) \right] \tag{20}$$

By deriving the analytical solution of this formula, two linear recursions can be used for fast computation. To further improve computational efficiency, gradient conjugation is applied for correction at the model output instead of directly computing the matrix inverse.

2.6. Linearization

The Transformer-based architecture used in modern LLMs suffers from quadratic computational complexity and glowing memory usage. In contrast, the linear recurrent model architecture benefits from linear computational complexity and constant memory usage, solving the inefficiency challenge of the Transformer architecture. Despite this, the pretraining process for LLMs from inception necessitates substantial computational and financial resources, which remains a significant barrier to their widespread adoption and real-world implementation. In this case, **linearization** is a more ideal choice, whose purpose is to convert the pretrained transformer-based LLM architecture into a linear recurrent structure using less training and fine-tuning costs, and restore the capabilities of the original model on natural language understanding and generation scenarios.

Finetuning-based Linearization. Finetuning-based linearization directly replaces standard softmax attention with linear sequence modeling in a pre-trained transformer, then finetune the modified model for architecture adaption without relying on knowledge from external models. Transformer-to-RNN (T2R) [111] introduces a method to convert a pretrained transformer to an RNN with linear-time computation and constant memory complexity. T2R follows a swap-then-finetune procedure to align the attention computation of a pretrained transformer, then finetunes the converted RNN for model architecture adaption. Mao [284] investigates various update rule configurations to finetune pretrained autoregressive Transformers into RNNs and propose a simple element-wise decay fast weights update rule with no feature map for linearizing transformers. DiJiang [285] leverages linear attention with the weighted Quasi-Monte Carlo method for efficient sampling. Based on the Discrete Cosine Transform (DCT) kernelization process, DiJiang significantly reduce the training cost of the transformation of a pre-trained vanilla Transformer into a linear complexity
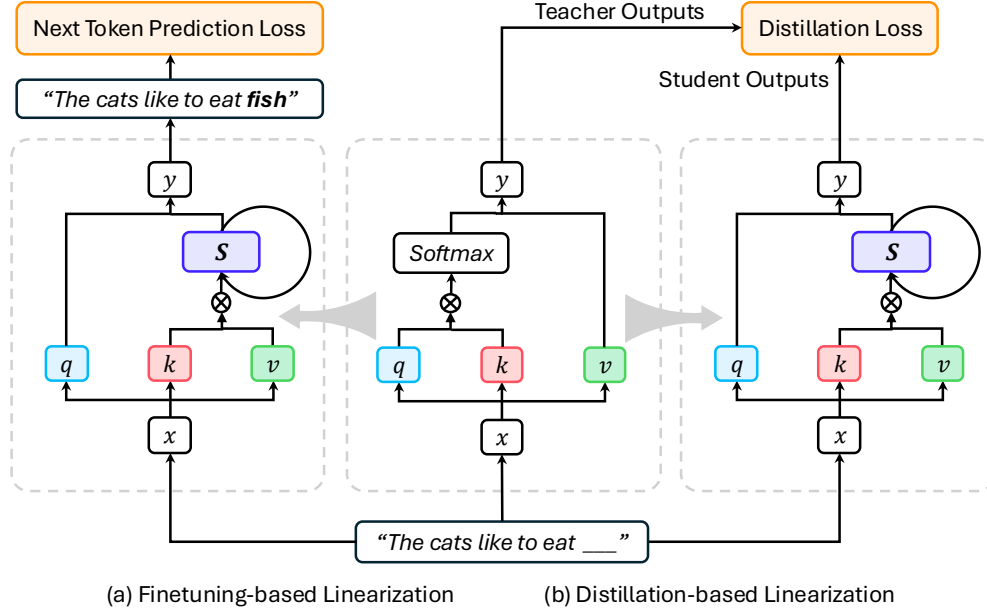
Figure 5: Mechanism Comparison of Finetuning-based and Distillation-based Linearization Procedures.

model. SUPRA [113] introduces a linearization technique for converting large-scale pretrained softmax-attention transformers into RNNs by replacing the softmax operation with GroupNorm and using a small MLP for queries and keys feature mapping. Liger [115] proposes a novel linearization technique for converting Transformer-based LLM to gated recurrent structures by fully reuse original pretrained model weights without introducing any extra learnable parameters, thus simplifying the linearization process to a single-stage end-to-end training without relying on distillation and significantly reducing the linearization cost.

Distillation-based Linearization. Distillation-based linearization utilizes knowledge distillation [286] to transfer the capabilities of a pre-trained transformer with standard softmax attention as a teacher model to a student model with linear sequence modeling. LoLCATs [114] proposes a simple two-step linearization process via Low-rank Linear Conversion via Attention Transfer, significantly improves linearizing performance, training efficiency, and scalability with low-rank adaption (LoRA). Llamba [287] demonstrates the effectiveness and scalability of cross-architecture distillation based on Llama-3.x series using MOHAWK [288]. LightTransfer [289] transforms standard transformers into hybrid model architecture by identifing lazy layers which are and replacing their full attention layers with efficient streaming attention, demonstrating throughput efficiency and effectiveness on long-context tasks with minimal performance loss. MOHAWK [288] proposes a distillation procedure for converting Transformer to Mamba. By incorporating three-stage progressive training: 1) matrix orientation, 2) hidden state alignment, and 3) weight transfer and knowledge distillation, MOHAWK can distill the Transformer architecture by matching different degrees of granularity in the SSM. MambaInLlama [112] also adopts a similar multi-stage training procedure to obtain Mamba model initialized from large Transformer model by combining cross-architecture distillation and hardware-aware speculative decoding algorithm, further accelerate the inference speed of Mamba and its hybrid models. Lizard [290], similar as Liger [115], also considers introducing gating mechanism into linearization process combining with sliding window attention with meta memory, and designed a hardware-efficient factorization algorithm that incorporates log-gating values into the query and key projections for training stability. LoLA [193] integrates sparse caching technique into linear attention, further improving language modeling and long-context ability
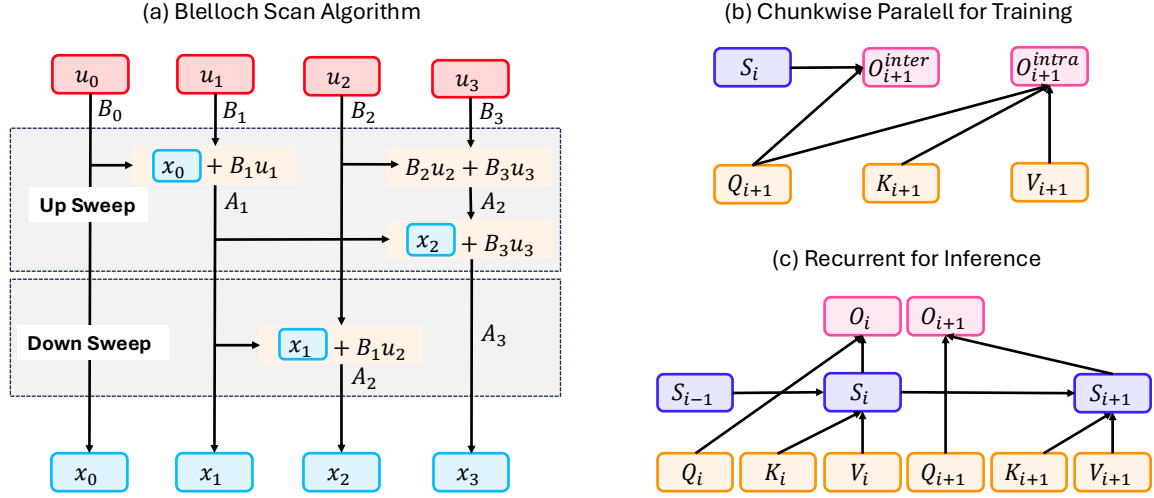
Figure 6: Hardware-efficient Implementation Algorithms for Linear Sequence Modeling. (a) In the Blelloch parallel scan, the upward sweep computes and stores the outputs at even indices; the downward sweep then reuses these stored values to compute the outputs at odd indices. (b) To better exploit tensor-core matmul acceleration during training and prefilling, linear-recurrence models can adopt intra-block parallel computation combined with inter-block recursion. (c) During inference, the recurrent formulation supports decoding with constant memory and $\mathcal{O}(1)$ per-step computational complexity.

after model architecture linearization. Yeonju et al. [291] proposes a dual-state linear attention (DSLA) for maintaining long-term historical information and tracking short-term recency-thereby, and introduced an online adaptive distillation system for linearizing quadratic complexity self-attention layers to linear-cost DSLA layers.

Linearization in the Era of RL Scaling. As the development of R1-like large reasoning models [29, 58, 292, 293] have been demonstrated their supreme advantages on reasoning tasks, one significant question is prompted: *Can we boost linear recurrent models' reasoning performance by scaling test-time computation through long CoT?*

A promising direction involves leveraging existing transformer-based reasoning models to create **linearized** LLMs—models that retain strong reasoning capabilities while benefiting from the efficiency of linear sequence modeling. This transformation is typically achieved through architectural linearization followed by SFT and RL to enhance test-time reasoning performance. Daniele et al. [294] explore this by distilling both pure and hybrid Mamba models from large-scale LLaMA models, investigating their reasoning scalability at test time. Similarly, M1 [295] introduces a hybrid large reasoning model built on the Mamba architecture, achieved by distilling and fine-tuning a linearized transformer. Their results highlight the potential of linear recurrent architectures in supporting efficient and scalable reasoning.

2.7. Hardware-efficient Implementation

One of the key reasons for the rapid development and adoption of linear sequence modeling methods is the availability of hardware-efficient implementations. Many approaches provide optimized code using tools such as Triton or CUDA to achieve efficient computation on Modern GPUs. In the following, we review and summarize these hardware-efficient implementation methods.
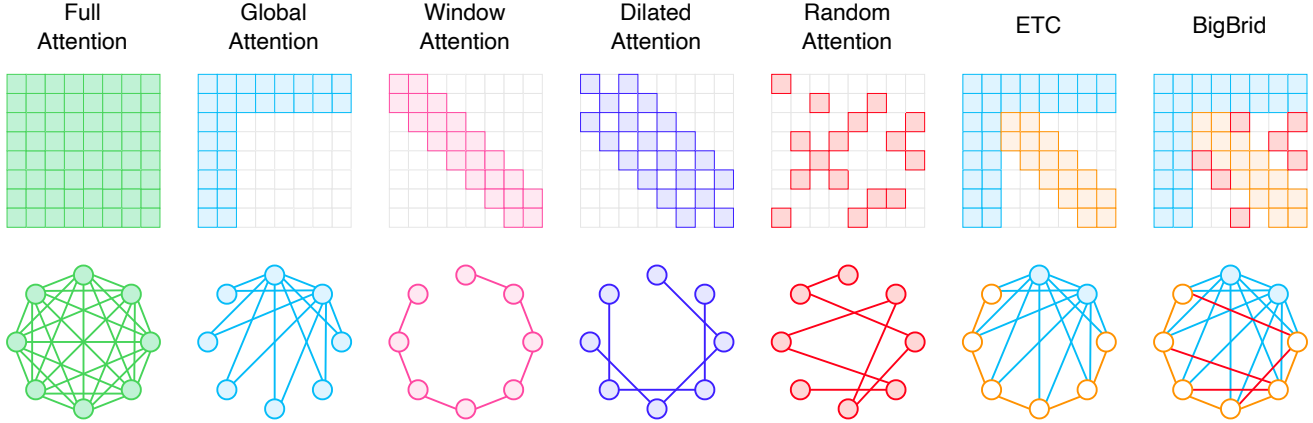
Figure 7: Example Patterns of Static Sparse Attention. ETC and BigBird are representative examples of mixed static sparse attention mechanisms, which combine multiple fixed sparsity patterns within a single attention framework.

Fast Recurrent. Early linear sequence modeling methods often leverage the properties of structured matrices for acceleration. For example, state-space models like S4 [90] can be written in the form of convolutions to enable fast computation. Models like Mamba [101] use the Blelloch scan algorithm [94, 254] to achieve fast recursion. Specifically, the Blelloch scan operates as a tree-like structure. During the computation, the even positions in the sequence are first processed via an upward scan, which serves as an intermediate step. Then, during the downward scan, the odd positions of the sequence are computed. This computational approach has recently inspired works such as multi-scale state-space models [253] and log-linear attention mechanisms [251, 252].

Chunk-wise Parallel. Although Linear RNNs achieve a favorable pretraining time complexity of $\mathcal{O}(Nd^2)$, they are often slower than softmax attention, which has a complexity of $\mathcal{O}(N^2d)$, on shorter sequences. This is primarily due to the fact that current hardware is highly optimized for `matmul` operations, which limits the efficiency of linear recurrence and necessitates additional training optimizations. Recent methods inspired by FlashAttention [142], including Lightning Attention [39, 70, 244], GLA [71], and Mamba2 [102], introduce inter-chunk recurrence combined with intra-chunk parallelism to fully leverage matrix compute throughput. These techniques optimize computation graphs and algorithms, ensuring efficient parallel execution across data chunks, which significantly enhances model performance on modern hardware. A basic formulation using chunk size C can be expressed as:

$$S_{[t+1]} = S_{[t]} + V_{[t]}^{\mathsf{T}} K_{[t]} \ \in \mathbb{R}^{D \times D}, \quad O_{[t]} = Q_{[t]} S_{[t]}^{\mathsf{T}} + (Q_{[t]} K_{[t]}^{\mathsf{T}} \odot \mathrm{Mask}_{[t]}) V_{[t]} \ \in \mathbb{R}^{C \times D} \tag{21}$$

The open-source framework *Flash Linear Attention* [296] offers a collection of chunk-wise parallel Triton kernels specifically designed for linear sequence modeling, with a strong emphasis on hardware efficiency. These implementations not only accelerate the computation of linear attention mechanisms but also enable easy integration into existing model training and inference pipelines. By making such optimized kernels publicly available, this framework lowers the barrier for researchers and practitioners to experiment with and deploy efficient linear attention variants, promoting further innovation within the community.

3. Sparse Sequence Modeling

Sparse sequence modeling is a paradigm designed to enhance the efficiency of processing sequential data by limiting interactions between elements to a strategically chosen subset. A typical example is sparse attention mechanism in transformer models, which addresses the computational bottlenecks of traditional full attention methods while aiming to preserve modeling performance.

3.1. Static Sparse Attention

Unlike full self-attention, which scales quadratically with sequence length, static sparse attention reduces computational complexity by restricting each token to attend to a predefined, fixed subset of other tokens. These static patterns are designed before training and remain unchanged during inference, making them highly efficient and easy to deploy. Common structural partterns include global, window, strided, dilated, random and blockwise sparsity. Such approaches have been widely adopted in natural language, vision, and multimodal models due to their strong inductive biases and scalability [297].

Early work on Sparse Transformer [116] pioneered this idea by introducing fixed strided and dilated attention patterns that ensure each token connects to both nearby and distant tokens in a deterministic manner. This structure reduces computation to near-linear complexity while preserving the model's ability to capture long-range dependencies, establishing a foundation for later sparse models. Building on this foundation, Star-Transformer [117] proposed a radial topology with a central relay node that connects to all tokens, while peripheral tokens form a ring with local connections. This architecture maintains global communication with linear-time attention and performs well on tasks requiring global structure, making it both simple and efficient. BlockBERT [118] adapts BERT for longer sequences using blockwise sparse attention. It divides input into fixed-size blocks, allowing dense intra-block attention and sparse inter-block communication through selected key tokens. This enables effective long-document modeling while lowering GPU memory requirements, proving useful for document classification and QA tasks.

To enhance static sparse models with richer contextual representations, Longformer [119] extends this idea with a combination of sliding window attention and a small set of global tokens. The window handles local context, while global tokens enable information flow across distant positions. This hybrid setup allows linear complexity and proves effective in tasks like summarization and long-context QA. GMAT [298] injects global memory tokens that act as information hubs, enabling interactions across segments of long sequences. Compatible with other sparse structures like Longformer and Sparse Transformer, GMAT demonstrates improved performance in long-context understanding tasks. A similar dual-attention structure is introduced in ETC [120], which separates tokens into local and global streams. Local tokens attend within sliding windows, while global tokens attend to the full sequence, allowing hierarchical representations. ETC also incorporates segment-aware attention and relative position encodings, boosting performance in document-level comprehension. BigBird [121] generalizes static sparse patterns by integrating local, global, and random connections, forming a sparse attention graph with small-world properties. This design ensures all tokens remain connected through short paths, and provides theoretical guarantees of universal approximation. BigBird scales effectively to long sequences in both encoder-only and encoder-decoder setups, supporting tasks across NLP and bioinformatics. LongT5 [122] builds upon T5 for long-text generation using transient global attention. It summarizes local contexts into temporary global tokens, which adaptively aggregate information layer by layer. This allows encoder-decoder models to scale to longer inputs without relying on dense attention, improving performance in summarization and long-form QA. Pushing the boundary further, LongNet [123] introduces exponentially dilated attention, where each layer increases the attention span in powers of two. This hierarchical scheme enables efficient modeling of sequences up to 1 billion tokens with

logarithmic complexity.

Beyond text, static sparse attention has also proven effective in vision. Axial Attention [124] factorizes 2D attention into independent operations along height and width axes, dramatically reducing complexity while maintaining expressiveness. This technique underpins many high-resolution vision transformers and has seen wide adoption in image and video models. In multimodal settings, sparse spatiotemporal attention mechanisms, such as those used in Open-Sora [299], decouple attention along spatial and temporal dimensions for efficient video modeling. By applying windowed attention in space and strided or pooled attention over time, these methods significantly reduce compute cost while preserving temporal coherence, enabling scalable video understanding and generation in large multimodal models.

3.2. Dynamic Sparse Attention

Unlike static sparse attention, dynamic sparse mechanisms determine attention patterns adaptively based on the input content. These models aim to approximate the expressiveness of full attention by focusing computation on a dynamically selected subset of token interactions, thereby retaining task-relevant information over long contexts while minimizing computational overhead. The evolution of these methods can be seen as a progress from heuristic grouping techniques to more sophisticated, learnable retrieval systems.

Early approaches focused on grouping or clustering semantically similar tokens to restrict the scope of attention. Transformer-XL [300], proposes to create a recurrent memory by reusing hidden states from previous segments, breaking the fixed-length barrier. Compressive Transformer [301], refines this memory by actively compressing older states into a more efficient long-term store. Adaptive Attention Span [302] was proposed to tackle the problem from an efficiency angle, allowing each attention head to learn its optimal context size, which dramatically extended the manageable sequence length without a proportional increase in computational cost. Furthermore, Reformer [125] pioneered using locality-sensitive hashing (LSH) to bucket tokens, allowing each query to attend only to keys within the same hash bucket. This created content-dependent sparsity with near-linear complexity. Similarly, the Routing Transformer [126] applies online k-means clustering to partition tokens, with attention confined to these dynamically formed clusters. A related idea, Sparse Sinkhorn Attention [127], learns a differentiable permutation to reorder tokens, enabling efficient block-wise attention on soft-sorted, semantically coherent segments.

These heuristic-based grouping methods were later generalized under the unified framework of Attention with Bounded-Memory Control (ABC) [69]. This work frames the context as a memory of a fixed, constant size and demonstrates that many prior methods are specific instances of managing this memory. Its key innovation, ABC-MLP, moves beyond fixed heuristics by introducing a learned, contextualized controller to manage the memory, achieving a better trade-off between efficiency and accuracy. An alternative to compressing the immediate context is to augment the model with an external memory and use retrieval mechanisms for sparsity. Memorizing Transformers [128] implements this by using a k-nearest-neighbor (kNN) index to retrieve relevant past token representations, enabling sparse access to long-term memory. CoLT5 [303] introduced a conditional routing approach where lightweight routers select a small subset of "important" tokens for global attention, while most tokens undergo cheaper local attention. Unlimiformer [129] refined this concept for cross-attention, allowing pre-trained models to attend over vastly longer inputs by retrieving relevant keys from an index without any weight modification. The focus then shifted towards learning explicit routing and gating mechanisms to prune the attention matrix directly.

The latest advancements aim not only for theoretical efficiency but also for practical, hardware-aligned speedups and architectural specialization. Native Sparse Attention (NSA) [130] is designed for this purpose, employing a hardware-aligned hierarchical strategy that combines coarse-grained token compression for

global context with fine-grained selection for local precision, delivering significant real-world speedups. Concurrently, Mixture-of-Sparse-Attention (MoSA) [131] adapts the popular Mixture-of-Experts (MoE) paradigm to dynamic sparse attention. Each head acts as an "expert" that dynamically selects a small subset of tokens, and the computational savings are reinvested to train a larger number of specialized heads, achieving superior performance within the same FLOP budget.

3.3. Training-free Sparse Attention

While many sparse attention methods are designed for training, a significant body of work focuses on accelerating inference, which is composed of two distinct phases: the computationally-intensive prefill of processing the initial prompt, and the memory-bandwidth-bound decoding stage of generating subsequent tokens. Different techniques have emerged to tackle the specific bottlenecks in each phase.

Accelerating Prefill Stage. Optimizing the prefill stage involves reducing the massive computation of the initial self-attention pass over a long prompt. Early approaches impose structured sparsity based on offline analysis. For instance, LongLoRA [304] fine-tunes a model for longer contexts by using shifted sparse attention (S2-Attn). Instead of full attention, S2-Attn performs attention within local token groups and then shifts the groups to allow information to flow across the entire context, approximating global attention with significantly less computational cost. MInference [133] observes that attention maps often conform to a few prototype shapes (e.g., diagonal, vertical stripes) and leverages specialized GPU kernels for these fixed patterns. Similarly, MoA (Mixture of Attention) [149] uses gradient-based profiling to assign a static, heterogeneous sliding-window size to each attention head, effectively creating a fixed sparse mask that reduces computation without retraining.

A more adaptive approach involves learning the sparsity pattern dynamically based on the input content. SeerAttention [134] exemplifies this by augmenting each attention layer with a lightweight gating module. After a brief self-distillation phase, this gate learns to predict which blocks of the attention matrix are most important for a given input, generating a dynamic mask that prunes irrelevant computations on-the-fly. This allows the model to achieve significant prefill speedups while maintaining high accuracy by adapting the sparsity pattern to the specific context. The follow-up work SeerAttention-R [305] builds on self-distilled attention sparsity and introduces key changes for efficient auto-regressive decoding. It removes sequence-level query pooling and adopts a GQA-style shared sparsity pattern for better hardware efficiency. SeerAttention-R can be applied to any pretrained Transformer by inserting a learnable gate into the attention layer, without modifying the original model weights.

Accelerating Decoding Stage. In the auto-regressive decoding stage, the primary bottleneck is not computation but the memory bandwidth required to load the ever-growing Key-Value (KV) cache at each step. Research in this area focuses on intelligently pruning this cache to retain only the most critical information. SpAtten [132] proposes a sparse attention architecture that improves efficiency through cascade token and head pruning, removing unimportant elements layer by layer using a Top-K engine. It also introduces progressive quantization, loading low-precision bits only when needed. Together, these methods reduce memory and computation without harming accuracy, enabling faster long-context inference. Another simple yet powerful heuristic is to exploit the "attention sink" phenomenon, as demonstrated by StreamingLLM [135]. This work found that initial tokens consistently attract high attention throughout generation and are crucial for maintaining stability. By caching only these sink tokens alongside a sliding window of recent tokens, models can handle infinitely long streams with a small, fixed-size cache. Other methods adopt dynamic eviction policies. TOVA [306] continuously evicts the token with the lowest attention score to make room for new ones, while H2O [136] formulates eviction as a submodular optimization problem to retain a set

of "heavy hitter" tokens that have the greatest influence on the attention output. RetrievalAttention [307] accelerates inference by building an approximate nearest neighbor index of the KV cache in CPU memory. During generation, it uses an attention-aware vector search to retrieve only a small subset of the most relevant KV pairs for the current query, thus avoiding the computational and memory costs of the full attention mechanism.

More structured approaches introduce sparsity at a coarser granularity. FastGen [137] profiles each attention head to classify its behavior (e.g., local vs. global) and applies a tailored eviction policy to each, compressing the KV cache more aggressively for heads with localized attention patterns. Taking this further, query-aware methods select relevant context blocks on-the-fly. Quest [138] divides the KV cache into pages and uses a lightweight scoring function to retrieve only the most relevant pages for the current query. Similarly, LongHeads [139] re-purposes multi-head attention by allowing each head to independently select and attend to a small number of context chunks, effectively parallelizing the search for relevant information across the sequence. DuoAttention [308] separates attention heads into retrieval heads for long-term context and streaming heads for local context. Only retrieval heads maintain full KV caches, while streaming heads use sliding windows, reducing memory and latency. A data-driven method is used to identify retrieval heads, enabling efficient long-context decoding with minimal performance loss. ShadowKV [309] offloads the KV cache of a large target model to the CPU, while a smaller draft model maintains a compact "shadow" KV cache on the GPU. The draft model generates tokens using speculative decoding, and the target model only retrieves the necessary KV data from the CPU to verify the draft, minimizing slow memory access. LServe [140] introduces a unified sparse attention design for efficient long-sequence serving. It combines block-level token skipping with query-driven KV pruning, retaining only relevant memory pages. By converting half of the heads into lightweight streaming heads, LServe accelerates both prefill and decoding stages while preserving accuracy. PQCache [310] compresses the KV cache using Product Quantization (PQ) to group and quantize vector dimensions into low-bit representations. To maintain accuracy, it introduces a lightweight, learnable "dequantization helper" module that is trained to reconstruct the high-fidelity vectors from their compressed form during inference. XAttention [141] proposes an efficient block-sparse attention framework using the sum of antidiagonals as a lightweight block importance metric. This enables accurate pruning with minimal overhead, achieving up to $13.5\times$ speedup while maintaining accuracy across long-context benchmarks.

3.4. Hardware-efficient Implementation

Efficient hardware implementation plays a key role in scaling sparse attention to support longer sequences without compromising runtime or memory efficiency. This section reviews recent approaches aimed at enhancing the hardware efficiency of sparse attention computation.

Except exact attention optimization, FlashAttention-1 [142] also incorporates block-sparse FlashAttention by introducing structured sparsity to further reduce memory access overhead and accelerate computation for long sequences. By applying a predefined block-wise sparsity mask, it skips unnecessary attention blocks during computation:

$$\boldsymbol{S} = \boldsymbol{Q}\boldsymbol{K}^T \in \mathbb{R}^{N \times N}, \quad \boldsymbol{P} = \text{softmax}(\boldsymbol{S} \odot \tilde{\boldsymbol{M}}) \in \mathbb{R}^{N \times N}, \quad \boldsymbol{O} = \boldsymbol{P}\boldsymbol{V} \in \mathbb{R}^{N \times d} \tag{22}$$

where $\tilde{\boldsymbol{M}}$ denotes the block mask and aligns with the block structure of $\boldsymbol{Q}$ and $\boldsymbol{KV}$ in the FlashAttention, specifically expressed as follows:

$$S_{kl} = \begin{cases} S_{kl}, & if \ \tilde{\boldsymbol{M}}_{kl} = \boldsymbol{M}_{ij} = 1 \\ -inf, & if \ \tilde{\boldsymbol{M}}_{kl} = \boldsymbol{M}_{ij} = 0 \end{cases} , \quad i = \lfloor k/B_r \rfloor, \ j = \lfloor l/B_c \rfloor, \ \boldsymbol{M} \in \{0,1\}^{N/B_r \times N/B_c} \tag{23}$$

where B_r and B_c denote the block sizes of Q and KV, respectively. Empirically, block-sparse FlashAttention achieves $2 - 4\times$ speedups over dense FlashAttention and scales transformers to sequence lengths up to 64K. FlashAttention-2 [143] extends FlashAttention-1's block-sparse attention to support structured sparsity patterns such as local, dilated, and general block-sparse attention.

Recent work NSA [130] integrates three different sparse attention mechanisms and improves hardware efficiency through three strategies. First, *Group-Centric Data Loading* ensures that all query vectors $Q \in \mathbb{R}^{[h,d_k]}$ in a GQA group and their corresponding sparse key/value indices I_t are loaded together in each inner loop. Second, *Shared KV Fetching* minimizes memory access by sequentially loading continuous key/value blocks $K \in \mathbb{R}^{[B_k,d_k]}$, $V \in \mathbb{R}^{[B_k,d_v]}$ into SRAM, where B_k is a kernel-aligned block size. Finally, by leveraging the uniform inner-loop length across blocks, the *Outer Loop on Grid* utilizes Triton's grid scheduler for efficient parallelization of query and output computation. This design gives $9.0\times/6.0\times$ speedups in forward/backward passes for 64k sequences compared to FlashAttention-2, showing strong performance for long-context tasks in GQA/MQA. Furthermore, MoBA [144] implements dynamic sparse attention efficiently through blockwise variable-length computation. By partitioning sequences into fixed-size chunks and encoding block selections via segment length tensors, it transforms irregular sparsity into hardware-friendly memory access patterns. This varlen formulation maintains FlashAttention's optimization benefits while enabling adaptive routing, achieving $16\times$ speedup for 10M-token sequences at 95% sparsity.

Besides block sparse attention, other sparse patterns also benefit from efficient kernels. Longformer's [119] CUDA kernel implements optimized sparse attention with three key patterns: sliding window attention for local context, dilated window attention for wider receptive fields, and task-specific global attention for special tokens. SeerAttention [134] introduces a CUDA kernel supporting dynamic sparsity shapes like A-shape, vertical-slash, and diagonal blocks. It integrates FlashAttention's tiling scheme to skip inactive blocks, reducing computation and memory overhead.

4. Efficient Full Attention

4.1. IO-Aware Attention

Transformer self-attention scales quadratically in both compute and memory with the sequence length N, which quickly becomes the dominant bottleneck in large-context language models. A series of *FlashAttention* kernels eliminates most of the memory traffic while preserving exact softmax attention, delivering large wall-clock speed-ups on modern GPUs.

Given matrices $Q, K, V \in \mathbb{R}^{N \times d}$, the standard softmax attention computation on GPU can be written as:

1. Load Q and K by blocks from HBM, compute $S = QK^\top$, write S to HBM.
2. Read S from HBM, compute $P = \text{softmax}(S)$, write P to HBM.
3. Load P and V by blocks from HBM, compute $O = PV$, write O to HBM.

It suffers from several issues: 1) **High memory usage.** When the hidden dimension is large, it becomes difficult to load both Q and K into SRAM simultaneously for fast computation. 2) **Excessive HBM and SRAM access.** This leads to a major performance bottleneck during training. Specifically, standard softmax attention computation:

1. Requires reading Q and K from HBM twice, and writing the intermediate result S once. Total: 3 memory operations.

2. Reads S once and writes O once. Total: 2 memory operations.
3. Reads P and V twice, and writes O once. Total: 3 memory operations.

4.1.1. FlashAttention-1

Exist algorithms aim to reduce attention computation complexity, but often do not result in significant speed-ups because GPU runtimes are mainly limited by global memory traffic, not computation intensity. FlashAttention-1 [142] overcomes this by optimizing memory transfers between global and on-chip memory. This optimization greatly reduces memory traffic and memory usage, resulting in faster computations and enabling longer sequence lengths without needing approximations. FlashAttention-1 improves standard attention computation by dividing the sequence into smaller query and key-value tiles. The sequence is processed in blocks to minimize memory access. The query and key-value pairs are loaded into shared memory in one operation, and the score block is computed efficiently using matrix multiplication on specialized hardware. A partial softmax operation is applied across the tiles, maintaining running maximums and sums for numerical stability. The final step accumulates partial outputs with the computed probabilities.

The key contributions of FlashAttention-1 can be summarized as:

Online Softmax. FlashAttention-1 reduces memory usage by computing softmax normalization incrementally, keeping track of the running maximum and cumulative weights. This avoids storing all intermediate scores and ensures efficient memory usage.

Fused Attention Computation. Instead of performing separate kernel invocations for computing attention scores, softmax, and weighted sums, FlashAttention-1 combines these steps into a single kernel. This reduces redundant memory transfers and takes full advantage of GPU parallelism, improving computation efficiency.

Recomputation in Backward Pass. During backpropagation, FlashAttention-1 saves memory by recomputing attention weights instead of storing all intermediate results. It uses previously saved normalization statistics to recompute these values, allowing the method to handle longer sequences and larger batch sizes without exceeding memory limits.

4.1.2. FlashAttention-2

FlashAttention-1 significantly outperforms standard attention methods, but its forward pass still achieves only a fraction of the device's theoretical peak throughput. The backward pass performs even worse, reaching just a small portion of the peak throughput. The bottleneck arises due to suboptimal work partitioning across thread blocks and warps, as well as numerous non-matmul operations, which result in low occupancy and unnecessary memory accesses.

FlashAttention-2 [143] addresses these issues and optimizes performance from the following aspects:

Figure 8: Attention Map Computation in FlashAttention-2.

More Matmul Operations. FlashAttention-2 reduces softmax book-keeping by storing the Intermediate terms and fusing epilogue operations directly within the Tensor-Core pipeline.

Query-Outer, Key-Value-Inner Loop Structure. In FlashAttention-2, the structure of the computation is optimized by processing the query and key-value pairs in a more efficient loop format, improving parallelism and reducing overhead.
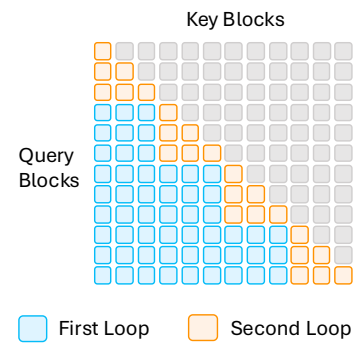
Row-wise Computation. For each query block, attention is computed with every key and value block in the same row and the results are summed to produce the output; with causal attention, this is usually done in two passes: first, the outputs for all blocks strictly below the main diagonal are computed without any masking, and then a mask is applied to compute the lower-triangular part of each diagonal block:

$$
A_{[i],[j]} \propto \begin{cases} \exp\left(Q_{[i]}^\top K_{[j]}\right), & \text{if } i < j \\ \exp\left(\text{lower}\left(Q_{[i]}^\top K_{[i]}\right)\right), & \text{if } i = j \end{cases}
\tag{24}
$$

4.1.3. FlashAttention-3

FlashAttention-3 [145] brings substantial improvements tailored for Hopper-class GPUs, integrating two pivotal hardware features: the Tensor Memory Accelerator (TMA) and the WGMMA Tensor-Core instructions. These innovations optimize performance by enabling warp-asynchronous memory operations and enhancing the efficiency of matrix multiplications, which are critical for accelerating attention mechanisms.

The key contributions FlashAttention-3 can be summarized as:

Producer-Consumer Asynchrony. This technique restructures the processing pipeline by assigning warp groups to distinct roles—*producers* and *consumers*. The producers are tasked with prefetching data from memory using TMA, while the consumers focus on the computationally intensive tasks of matrix multiplication and softmax operations. This separation of responsibilities allows for a two-stage pipeline that concurrently handles memory transfers and computation. By overlapping these operations, FlashAttention-3 minimizes idle time, improving resource utilization and increasing throughput. The result is more efficient hardware utilization, especially in high-latency stages of computation such as memory access.

Interleaved Matmul+Softmax. In this method, blocks are processed in a double-buffered fashion, where the softmax operation for a block is computed while the next block's scores are calculated using WGMMA. This interleaving of the matrix multiplication and softmax operations ensures that the GPU's compute resources are kept fully occupied, reducing the time spent waiting between these two stages. This technique exploits the parallelism of modern GPUs to maintain high throughput and reduce latency, thus significantly accelerating the attention computation pipeline.

Block-Wise FP8 Quantization with Incoherent Processing. In FlashAttention-3, all numerical operations, including GEMM and softmax, are performed using FP8 precision, which reduces memory footprint and improves computational efficiency. However, to mitigate the precision loss typically associated with FP8, each block independently selects a per-tile scaling factor, allowing it to adjust for cumulative errors that would otherwise degrade performance. This approach significantly reduces numerical errors compared to traditional FP8 kernels, ensuring that the results remain robust while maintaining the memory and computation benefits of reduced precision.

4.2. Grouped Attention

Grouped attention techniques, including Multi-Query Attention (MQA), Grouped-Query Attention (GQA), and Multi-Head Latent Attention (MLA), have been widely adopted in the training of large language models (LLMs). These methods have been extensively validated for their ability to reduce key-value (KV) cache sizes during inference, leading to improved memory efficiency without compromising model performance.

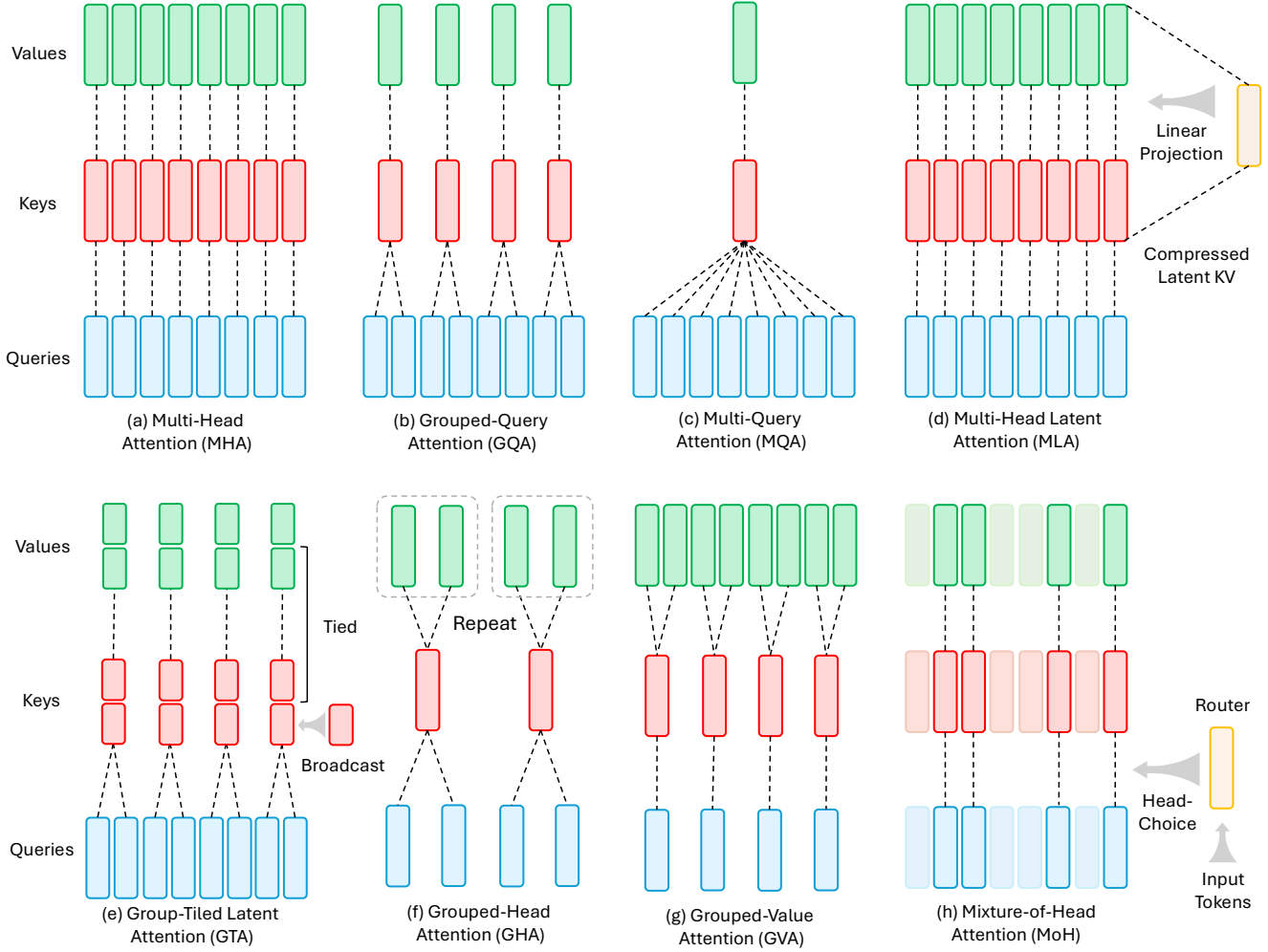Specifically, MQA [146] was designed to address the significant memory bandwidth bottleneck encoun-

Figure 9: Mechanism Comparison of Primary Grouped Attention Methods.

tered during the incremental decoding process in autoregressive models. By allowing multiple query heads to share a single key and value head, MQA dramatically reduces the size of the KV cache, leading to substantial improvements in inference speed. However, this efficiency sometimes comes at the cost of model quality. To mitigate the performance degradation observed in MQA while retaining its speed advantages, Grouped-Query Attention (GQA) was proposed [147]. GQA serves as a middle ground between traditional Multi-Head Attention (MHA) and MQA. It works by dividing query heads into groups, with each group sharing a single key and value head. This approach offers a favorable trade-off, achieving quality comparable to MHA while maintaining inference speeds close to those of MQA. A key innovation in the GQA paper is the concept of "uptraining," a cost-effective method for converting existing MHA models into GQA or MQA models with only a small fraction of the original pre-training compute [147].

Building on these efforts to optimize attention mechanisms, Multi-head Latent Attention (MLA) was introduced as part of the DeepSeek-V2 [27] and V3 models [28]. MLA further enhances inference efficiency by compressing the KV cache into a low-rank latent vector. This technique significantly reduces the KV cache size, even more so than MQA, while maintaining strong performance [27]. MLA's design decouples the positional information from the compressed keys and values, which is a key difference from standard

attention mechanisms and allows for its aggressive compression.

Group Tied Attention (GTA) and Group Latent Attention (GLA) [148] are proposed as hardware-friendly and memory-efficient attention mechanisms aimed at addressing key inefficiencies in LLM inference, particularly those related to the KV cache. GTA builds upon the foundation of GQA by tying the Key and Value representations into a single shared projection, which is then reused across small groups of query heads. This design not only reduces the KV cache size by approximately 2× but also improves arithmetic intensity by minimizing memory movement relative to compute. A critical innovation in GTA lies in its careful handling of Rotary Position Embeddings (RoPE): only half of the tied KV state is used for the non-RoPE part of the key, while a separate smaller head computes the RoPE-associated component, which is broadcast across the group. Experimental results show that GTA preserves or even improves perplexity and task performance compared to GQA, offering a drop-in replacement with both memory and computational efficiency gains.

GLA extends the idea of latent compression in MLA by improving its compatibility with tensor parallelism. While MLA compresses the KV cache into a single latent representation, this compression limits its ability to parallelize efficiently, often requiring cache duplication across devices. GLA resolves this by sharding the latent KV cache into multiple segments, such as two halves, and assigning separate groups of query heads to each shard. Local attention is independently computed within each group and later merged, eliminating the need for full cache replication and enabling better performance in distributed and imbalanced workloads. In addition to maintaining or improving model quality over MLA, GLA demonstrates substantial speedups during inference. Specifically, its custom kernel achieves up to 2× speedup over FlashMLA in speculative decoding. Together, GTA and GLA provide highly efficient, scalable alternatives to existing attention variants, offering strong trade-offs between memory usage, compute efficiency, and parallelization capability.

4.3. Mixture of Attention

The idea of combining multiple attention strategies into a unified attention layer began with MoA (Mixture-of-Attention) [149], which proposed assigning different sparse attention patterns to each head and layer within a Transformer. By automatically selecting head-wise sparsity from a pool of patterns, MoA significantly extends the effective context length while improving throughput and reducing memory consumption. This early work emphasized the benefits of heterogeneous attention structures, highlighting that not all tokens or heads require uniform computation. Expanding on this notion, SwitchHead [311] treated attention heads as experts and introduced routing mechanisms to activate only a subset of them per token, reducing attention computation by up to 8× without sacrificing performance. In parallel, MoH (Mixture-of-Heads) [151] adopted a similar perspective, but instead of discrete routing, it used soft selection: each token computes a weighted combination of attention heads, enabling partial pruning while improving or maintaining accuracy. These works collectively mark a shift toward dynamic head-level specialization, bringing MoE-style routing into the self-attention mechanism.

Building on this head-wise mixture foundation, LLaMA-MoE v2 [150] scaled the concept to full LLMs by sparsifying both the attention and feedforward layers through structured expert partitioning. The model achieves performance on par with dense LLaMA variants across multiple domains, demonstrating that MoE-based sparsity is viable for large-scale, instruction-tuned transformers. Further pushing flexibility, MoBA (Mixture of Block Attention) [144] introduced block-level routing where each chunk of tokens can choose between full or sparse attention dynamically. Rather than relying on predefined patterns, MoBA allows the model to adaptively allocate computation, improving long-context performance and aligning efficiency with task demands. These advancements show the trend of increasing granularity and adaptivity in MoE routing, from heads to full blocks, making attention computation more efficient and task-aware.

Extending MoE principles beyond traditional attention layers, MoM (Mixture-of-Memories) [78] applies expert routing to memories with linear sequence modeling. Instead of maintaining a single shared state, MoM uses multiple sparsely activated memory slots, each acting as an expert updated via token-wise routing, enhancing long-term recall and reducing interference. Finally, MoSA (Mixture of Sparse Attention) [131] brings MoE routing to a fine-grained level by allowing each attention head to dynamically select the top-k relevant tokens based on content. This results in a token-wise sparse structure that outperforms dense attention under equal compute budgets while significantly reducing memory usage. Across these works, the MoA paradigm evolves from fixed sparsity patterns to fully dynamic, learnable routing strategies at the head, block, memory, and token levels, offering a unified path toward more scalable, efficient, and adaptable attention mechanisms in modern large models.

4.4. Quantized Attention

To address the trade-off between computational efficiency and task accuracy in low-precision quantization, numerous hardware acceleration platforms and quantization techniques have been developed in recent years. Quantized Attention mechanisms have been proposed and studied to improve computational efficiency and reduce memory requirements, while maintaining as much of the original model accuracy as possible.

Post-training Quantized Attention. Post-training quantization (PTQ) methods convert the Transformer attention operators to low-bit arithmetic without any retraining. For example, SageAttention [152] quantizes the QK^T product to INT8 format (with per-channel *smoothing* of outliers) while keeping the $(\text{softmax}(QK^T), V)$ matmul in FP16. This mixed-precision approach (INT8 for the first matmul, FP16 for the second matmul) doubles the matmul computation speed on GPUs with negligible accuracy loss. SageAttention's Triton kernel even fuses RoPE [312] and quantization, using NVIDIA Tensor Cores to reach high efficiency. Similarly, INT-FlashAttention [157] builds a FlashAttention-compatible INT8 kernel: it quantizes Q, K, V and the softmax input into 8-bit and performs the entire attention in INT8. INT-FlashAttention achieves faster inference and smaller quantization error than the FP16 baseline, with only minor quality degradation. Q-BERT [155] uses Hessian-based analysis to quantize attention to very low bits; e.g., uniform 4-bit Q-BERT shows only 0.5% accuracy drop on SQuAD versus over 11% for naive quantization. In summary, PTQ attention methods quantize the main matmuls (often QK^T or $(\text{softmax}(QK^T), V)$) and introduce scaling or smoothing to preserve fidelity, enabling plug-and-play INT8 inference.

Quantization-Aware Training for Attention. Quantization-aware training (QAT) embeds low precision into training so the model learns to cope with reduced bit-width. For example, Q8BERT [158] finetunes BERT under 8-bit constraints (8-bit weights and activations) and can compress BERT by $4\times$ with minimal loss. I-BERT [156] goes further by training the model so that every operation (including GELU and softmax) can be performed in integer arithmetic: it learns simple integer approximations for the non-linearities, allowing a RoBERTa model to run end-to-end in INT8 with near-identical accuracy. FullyQT [159] quantizes all matrix multiplies, including softmax inputs/outputs, during training, and even approximates the softmax exponent via bit-shifts. It shows that an 8-bit Transformer can match FP32 BLEU scores when properly trained.

Mixed-Precision Attention. These schemes mix precisions within the attention computation to balance speed and accuracy. The SageAttention example again illustrates this: it uses INT8 for the QK^T multiplication but keeps the V-matmul in FP16. More generally, one might quantize Q and K aggressively (e.g., 8-bit) but perform the softmax/V multiplication in higher precision so that the final output remains accurate. TurboAttention [160]'s FlashQ is also a form of hybrid attention: it applies separate quantization to each head (one scale per head) so that most matmuls run in low precision, while still using sufficient precision to compress the KV cache. Other schemes similarly choose mixed precision by layer or token: for example,

some layers might be run in INT8 while others (or certain operations like softmax normalization) remain FP16. These mixed-precision designs leverage the fact that modern GPUs can execute INT8 and FP16 ops on different tensor cores; by assigning the most sensitive sub-operations to a higher precision, they recover accuracy without sacrificing the overall throughput boost of quantization.

INT8 Fused Attention Kernels. Beyond algorithmic tricks, specialized INT8 attention kernels have been developed. INT-FlashAttention [157] is one example: it provides a fused CUDA kernel that takes INT8 Q, K, V and softmax inputs, performs all matmuls and softmax in 8-bit, and outputs FP16 or FP32 results. Similarly, SageAttention's implementation fuses quantization and FlashAttention-style tiling in a Triton kernel, exploiting INT8 TensorCore MMA operations for both matmuls. HACK [161]'s attention prefill is another fused kernel: it quantizes Q, K, V on-the-fly and feeds them into the attention computation without separate quantize/dequantize steps.

Ultra-Low-Bit (Sub-4-Bit) Attention. The most aggressive quantizers push below 4 bits. SageAttention2 [153] does this by using warp-wise INT4 for the QK^T and FP8 for the V-matmul, combined with outlier smoothing on Q and V. SageAttention3 [154] goes further: it uses FP4 "microscaling" for both matmuls. Specifically, it quantizes blocks of the matrices to FP4 with per-block scale factors and per-token normalization, which contains quantization error and enables $5\times$ speedup. On the training side, BitDistiller [162] is a QAT framework for sub-4-bit LLMs: it employs asymmetric quantization and a self-distillation KL objective so that 3-bit and 2-bit models can approach full-precision performance. Even earlier, Q-BERT showed that carefully assigning 2-3 bits per layer (with some layers at higher precision) can yield reasonable accuracy; for example, uniform 2-bit Q-BERT retained 70% accuracy on SQuAD versus 5% for naive 2-bit [155].

5. Sparse Mixture-of-Experts

Sparsely-activated Mixture-of-Experts (MoE) is a cost-effective approach to enlarge the model capacity while maintaining the computational consumption [313, 314, 315]. Unlike conventional model ensembling, MoE is comprised of a gate and many specialized experts. The gate is a router that accepts input tokens and adaptively selects the most relevant experts, which brings better robustness [316]. Besides, the gate induces specialized experts and the routing preference may be used for dataset-level representations [317, 318]. Through such a sparse activation mechanism, MoE could be significantly scaled with greater model capacity, leading to better task performance [319, 320, 321, 322, 323]. Based on these advantages, the MoE paradigm has been widely applied in existing LLMs [33, 42, 172, 174, 324, 325, 326, 327, 328].

This section introduces *gate* and *experts*, which are two core components of the MoE paradigm. Besides, we also introduce modern strategies to build MoE from dense models, which alleviate significant cost to train MoE models from scratch.

5.1. Routing Mechanisms

The gate is a crucial component to bring sparsity in MoE models. For a batch of input token representations $X \in \mathbb{R}^{T \times D}$, the gate function G determines the probabilities of dispatching token x_i to each expert e_i:

$$P = G(X), \ P \in \mathbb{R}^{T \times N}, \quad G(X) = \text{Softmax}(XW_g + b_g) \tag{25}$$

where N is the number of experts, $W_g \in \mathbb{R}^{D \times N}$ and $b_g \in \mathbb{R}^N$ are trainable parameters in each MoE layer.

As shown in Eq. (25), the most common case of gate function $G(\cdot)$ is the softmax function after a linear projection, and the calculation needs to be cast into FP32 for numerical stability especially when the model
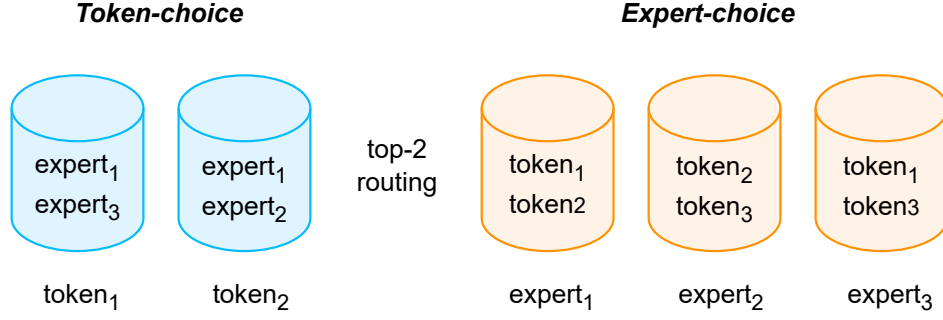
Figure 10: MoE Routing Strategies.

parameter is in lower precision formats. There are some variants of G to replace the linear projection with a feed-forward network (FFN) with nonlinear activations [178] for better performance, or substituting softmax with sigmoid [28, 329] or cosine similarity function [330] to avoid representation collapse and expert competition problems [331].

After obtaining the routing probabilities, MoE introduces sparsity by only selecting the top-k experts. It is also a common practice to normalize the top-k routing probabilities and make the summation to be 1.0 for better convergence [327].

$$Y = \sum_i^k \text{Top-}k\left(G(\boldsymbol{X})_i\right) \cdot E_i(\boldsymbol{X}) \tag{26}$$

Routing Strategies. Routing mechanisms in modern LLMs are usually **token-choice**-based, while a token would selects corresponding k experts to calculate new representations, as illustrated in Figure 10. A significant pitfall of this token-choice strategy is its tendency to lead poor load balancing across experts. This often results in some experts being over-utilized and others under-utilized, leading to wasted capacity and potentially unprocessed tokens due to expert capacity limits (i.e. token dropping).

Besides token-choice, each expert could also selects top-k tokens in an **expert-choice** manner. Expert-choice [163] routing simply changes the softmax operation from the N dimension to T dimension in Eq. (25). Furthermore, the top-k operation in Eq. (26) would select k tokens for each expert, leading to perfect load balancing. Expert-choice is effective in encoder-based models, such as T5 encoder [3] and ViT [332, 333]. However, for autoregressive language modeling, experts could not see the whole sequence at each step, and an expert may accepts too many tokens to fill all its capacity, remaining limit space for future tokens. Therefore, modern LLMs with expert-choice has to pair an additional estimator or a loss constraint to make capacities for future tokens [174].

BASE (Balanced Assignment of Experts) Layer [164] discards the auxiliary load balancing loss in token-choice by formulating token routing as a linear assignment problem, reaching an equal number of tokens for each expert during training. To avoid the potential future token leakage problem that is similar to expert-choice in autoregressive language modeling, a simpler greedy assignment is used when inference.

Instead of training a gate function with learnable parameters, Hash Layer [165] proposes to use a fixed predefined hash function (lookup table) to route tokens into specific experts. To build a balanced hash function, the authors first analyze the training corpus and assign the most frequent token in the vocabulary into the emptiest bucket until all tokens are assigned. Based on this setup, Hash Layer achieves balanced expert loads and surpasses BASE Layer. However, due to the nature of Zipf's law in word distribution, perfect

balance is nearly impossible.

Adaptive Top-k Routing. Besides top-k routing with fixed k values, recent studies also focus on dynamic routing to adaptive k experts, where the number of selected experts is determined by the task difficulties or computational requirements. Based on this mechanism, models could be both efficient and effective on complex tasks. There are three types of adaptive routing: *(1)* differentiable activation; *(2)* expert activation estimation; and *(3)* zero computational experts.

Differentiable activation replaces the traditional top-k & softmax router with differentiable activation functions, such as ReLU and sigmoid. ReMoE [334] utilizes ReLU as the router's activation function, and experts with zero activation scores would be discarded to process the input token. Since the router is totally differentiable, it could be optimized adaptively based on the task complexities. However, training from scratch may induce all experts to be activated. Therefore, ReMoE applies a regularization to ensure MoE could reach certain sparsity levels. BlockFFN [335] also uses ReLU to make adaptive routing. Instead of regularization to obtain target sparsity, it introduces chunk-level sparsity training objectives to encourage the model to be sparse in both the token-level and the chunk-level. DynMoE [167] regards the expert selection process as a multi-label classification problem where each expert is an individual class. For an input token, DynMoE calculates the cosine similarities with all expert representations and select experts with a predefined threshold.

Expert activation estimation reforms the top-k routing to be directly adaptive with dynamic k estimation. MoE-Dynamic [166] selects experts by a predefined confidence score p, and the gate would incorporate experts in a descending order of their selection probabilities until the cumulative score of the chosen experts exceeds p. Therefore, MoE-Dynamic converts the fixed-size top-k routing to dynamic top-p routing, thereby dynamically allocating more computational resources to more complex inputs while conserving them for simpler ones. Ada-K [169] dynamically adjusts the number of activated experts for each token in MoE-based LLMs through a learnable allocator. This allocator module is similar to the gate and takes the input token and outputs a probability distribution over the number of experts, from which an optimal count is sampled.

Zero computational experts introduces heterogeneous experts with zero or few computational costs as placeholders. MoE++ [336] keeps the number of activated experts k unchanged, and devises zero computational experts. By introducing heterogeneous load balancing across normal experts and these zero computational experts, some tokens would skip the computation process and therefore improve the overall efficiency. AdaMoE [168] achieves token-adaptive routing by introducing *null* experts alongside the standard experts in MoE, where these null experts perform no computations. Similar to MoE++, the router still selects a fixed number of k experts, where it is larger than the k in a comparable vanilla MoE. This allows different tokens to dynamically use a different number of true experts based on their needs. For instance, semantically rich tokens might engage more true experts, while less significant tokens might be routed to more null experts, thus achieving the adaptive routing.

Load Balancing. *Gate load* is the number of tokens routed to each expert. As discussed in **Routing Strategies**, token-choice routing is prone to be unbalanced due to biased gates, where an expert may be selected by many tokens and other experts may process less tokens. This phenomenon would make MoE collapses where some experts are not sufficiently trained. Furthermore, unbalanced routing would result in slow training because experts with less tokens may *wait* for other experts to finish processing. Given the fact that modern language models prefer token-choice routing for effective autoregressive token prediction, it is crucial for MoE to reach balancing.

A common practice for addressing unbalanced routing is to add an auxiliary load balancing loss $\mathcal{L}_{\text{aux}}$ as a soft balance constraint, and the final optimization object of LLM training would be the summation of a cross-

entropy loss $\mathcal{L}_{\text{ce}}$ for language modeling and the auxiliary loss $\mathcal{L}_{\text{aux}}$ for load balancing. Shazeer et al. [313] proposes to apply the coefficient of variance (CV) of gate importance scores (i.e. routing probabilities) as an auxiliary loss, where a more balanced training would lead to lower CV scores. For a stricter balance constraint, the gate load $D(\boldsymbol{X})$ is also estimated for the back-propagation compatibility:

$$\mathcal{L}_{\text{aux}} = \alpha \, \text{CV}(\sum G(\boldsymbol{X}))^2 + \beta \, \text{CV}(\sum D(\boldsymbol{X}))^2, \tag{27}$$

where α and β are hyper-parameters.

GShard [314] provides a simplified version of auxiliary loss that considers both the gate load and the gate importance scores:

$$\mathcal{L}_{\text{aux}} = \frac{1}{N} \sum_{i=1}^{N} D_i(\boldsymbol{X}) \cdot G_i(\boldsymbol{X}) \tag{28}$$

This design is prevalent in MoE and has become a default setting in many popular LLMs [172, 327].

Although such auxiliary losses are effective for reaching load balancing, it may violate the optimization direction due to the interference gradient. To push the language modeling's performance upper bound, Wang et al. [170] introduces a method to discard the auxiliary loss while maintaining the load balancing. The implementation involves applying an expert-wise bias to the routing scores of each expert before the top-k routing decision is made. This bias is not static and it is dynamically updated for each expert based on its recent load. This iterative process of token routing and bias updating allows the model to maintain a balanced distribution of expert load consistently throughout training by directly adjusting gating scores rather than introducing an auxiliary loss term, thereby avoiding interference with the primary language modeling objective.

Qiu et al. [171] addresses the issue of inhibited expert specialization in MoE models when using traditional load balancing loss (LBL) calculated at the micro-batch level. The authors argue that micro-batch load balancing loss forces tokens, even domain-specific ones, to be uniformly routed to all experts within each small sequence, hindering the ability of experts to specialize. To solve this, they propose a global-batch load balancing loss by introducing an extra communication step to synchronize the expert load $D(\boldsymbol{X})$ across all parallel micro-batches that form a global batch. The load balancing loss is then calculated using these globally aggregated gate load. This approach aims to loosen the strict constraint of micro-batch LBL, encouraging load balance at the broader corpus level, thereby fostering better domain specialization among experts and improving overall model performance in both pretraining perplexity and downstream tasks.

5.2. Expert Architectures

Traditional MoE introduces sparsity in FFNs and the experts are composed of multiple FFN blocks [315]. Although FFN experts are effective in most cases, there are still rooms for improving the efficiency, robustness, and training stability [337]. Besides, experts are specialized in a pretrained MoE model, where each expert may process tokens in certain field or have similar patterns. This also innovates new Parameter-Efficient Fine-Tuning (PEFT) paradigms to only finetune task specific experts for lower computational cost [338].

As shown in Figure 11, there are multiple expert architectures in MoE layers, including fine-grained experts, shared experts, MoD experts, and other heterogeneous experts (e.g. zero experts). In this section, we introduce different types of expert architectures in FFN blocks. For attention experts, please refer to § 4.3.

Fine-grained Experts. For an MoE layer with N experts and each expert has a intermediate size of M, there are NM neurons in total. While keeping the number of total parameters unchanged, the fine-grained
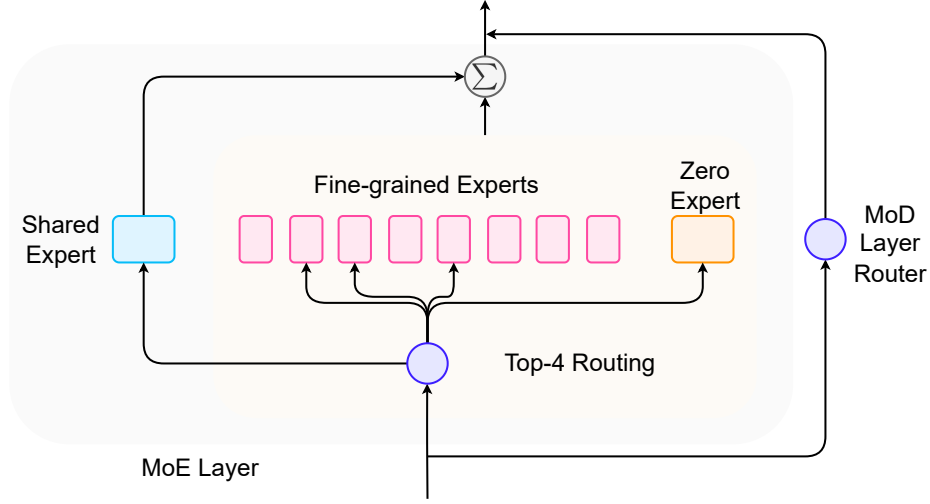
Figure 11: MoE Expert Architectures.

MoE has smaller M values, leading to larger N (i.e. more small experts). Fine-grained experts bring more combinations and better performance with a clear scaling property [174]. For instance, a 16-top-2 setting only has $\binom{16}{2} = 120$ expert selection combinations, while 64-top-8 has $\binom{64}{8} = 4.4 \times 10^9$ combinations. Such a fine-grained setting has been verified to be effective in modern LLMs like DeepSeekMoE [172], Qwen-MoE [33], and OLMoE [174]. However, more experts would bring additional overhead in routing and bring new challenges in efficient training/inference framework designs [339].

Shared Experts. Shared experts (or residual experts) are fixed experts where tokens are always routed to. DeepSpeed-MoE [173] utilizes another linear projection and softmax to make a weighted sum of the shared experts and the vanilla experts. Qwen2-MoE [32, 340] applies a similar linear projection with sigmoid to calcualte the contribution weight of shared experts and sum two types of experts' outputs as the final output. DeepSeekMoE [172] directly adds the shared experts' and the vanilla experts' outputs without complex re-weighting. These implementations all show performance improvements in the language modeling perplexities or downstream task performance.

Mixture-of-Depths. Besides the above horizontally distributed experts, experts could also be vertically distributed from the layer perspective. Mixture-of-Depths (MoD) [175] regards transformer layers as experts and select top-k tokens for each layer with a corresponding router. Thus it significantly reduces the computational cost by consuming less tokens on each layer. Different from LayerSkip [341], tokens dropped from former layers may be processed by latter layers. MoD is orthogonal with MoE and could be combined with MoE as Mixture-of-Depths-and-Experts (MoDE).

Other Special Experts. Besides the conventional modules that could be constructed as experts, there are also special variants that integrate the idea of sparse MoE. SoftMoE [342] mixes tokens into soft slots, applies experts to process such slots, and utilizes output projectio to reveal token representations. MoE++ [336] introduces zero, copy, and constant experts in MoE to adaptively control the top-k routing and the overall computational cost. ModuleFormer [343] provides a scheduled expert expansion method to add new specialized experts while maintaining the original knowledge. Inspired by the parameter-efficient fine-tuning methods, experts could also be LoRA weights for lightweight design [344, 345, 346, 347], and the number of experts could be scaled to 1 million with a specially designed expert retrieval strategy [348].
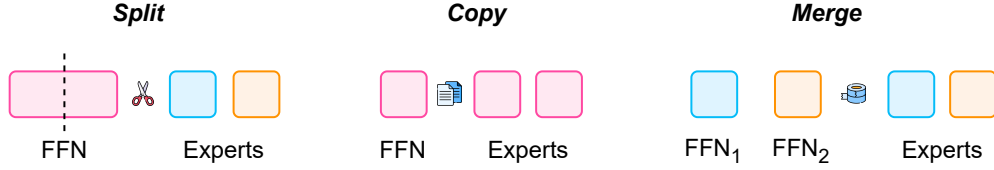
Split **Copy** **Merge**

FFN Experts FFN Experts FFN$_1$ FFN$_2$ Experts

Figure 12: MoE Conversion Strategies.

5.3. MoE Conversion

Training MoE models from scratch costs a mass of computational resources. Therefore, it is worth constructing MoE models from existing dense models to efficiently scale the model capacity or reduce the inference cost. As shown in Figure 12, there are three main strategies: *(1)* Splitting existing FFN to construct experts (MoEfication [177]); *(2)* Copying FFNs and aggregate as an expert (Sparse Upcycling [179, 349]); *(3)* Merging two existing dense models as an MoE model (BTX [181]). Here we introduce these strategies in two aspects: from existing "dense to MoE", and "sparse model routing" to aggregate two dense models as an MoE model.

Dense to MoE. MoEBERT [176] builds MoE by splitting encoder-based BERT-family models and proposes to use importance scores [350] for guiding the model adaptation. MoEfication [177] construct MoE by splitting T5 models from a co-activation graph and the results are very close to the original T5 model. LLaMA-MoE [150, 178] partition FFNs in LLaMA [351, 352] to build MoE models with continual pretraining and supervised fine-tuning. Besides splitting dense FFNs to sparse experts, Komatsuzaki et al. [179] propose to scale the model capacity by copying current FFNs and initialize new gates for each MoE layer, namely sparse upcycling. For vertical moe conversion, DLO [353] expands layers and build MoD experts. MoDification [354] converts layers in an existing dense model to MoD-style experts with a threshold-p gate, where tokens exceed a predefined threshold are processed.

Sparse Model Routing. Besides module-based MoE, models could be merged or aggregated to build MoE-like models for better comprehending user instructions and generating responses with specialized experts. Branch-Train-Merge (BTM) [180] proposes to train several expert language models (LMs) with domain specific datasets. During inference, these expert models are weighted averaged by the input query's domain category and a new model would generate the response. Instead of model merging, Branch-Train-Mix (BTX) [181] directly aggregates expert LMs' FFNs into MoE layers and train a new gate for each layer to dynamically select specialized experts.

6. Hybrid Architectures

As the core component of the Transformer, softmax attention has demonstrated its superior performance across a wide range of tasks. However, its quadratic computational complexity, coupled with the growing KV-cache overhead, has become a critical barrier to efficiency in long-sequence processing. In contrast, linear models have achieved significant breakthroughs in computational efficiency. Nevertheless, extensive studies have shown that these linear models often fall short of standard softmax attention in tasks requiring precise recall, sparse information retrieval, and long sequence modeling. In response to these challenges, researchers have been developing hybrid models to achieve a better balance between efficiency and performance.

The core challenge of hybrid models lies in efficiently and effectively integrating the efficiency benefits of linear models and the expressive power of standard softmax attention. As shown in Figure 13, current
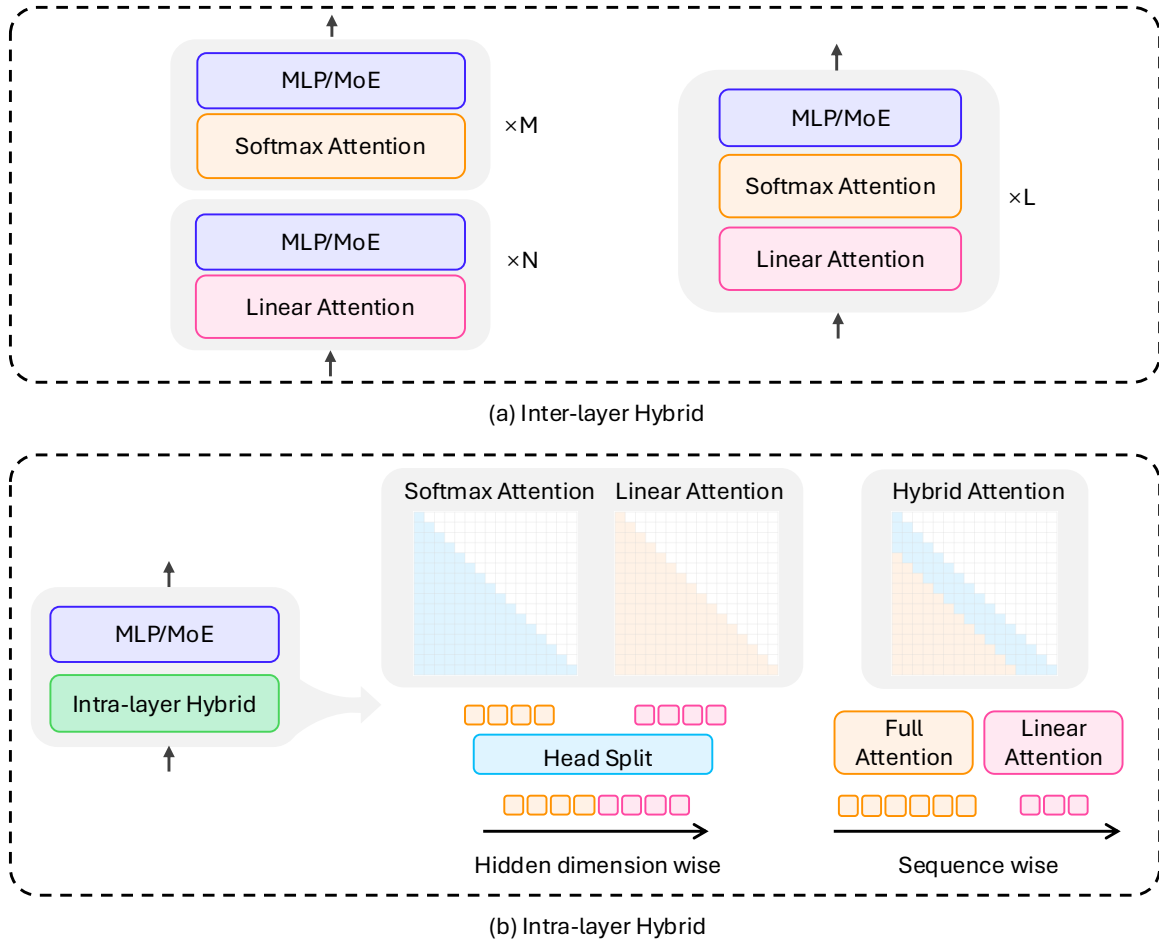
Figure 13: Hybrid Model Architectures. (a) illustrates the classical paradigm of inter-layer hybrid approach. (b) demonstrates the classical paradigm of intra-layer hybrid approach. The left side of (b) represents a pattern similar to Hymba [191], which employs head-wise partitioning with either softmax attention or linear attention. The right side, depicts a pattern analogous to LoLCATs [114], featuring sequence-wise partitioning where local regions utilize softmax attention while distant regions employ linear attention.

hybrid models can be categorized into two main categories based on the integration approach:

- **Inter-layer Hybrid** employs a hierarchical alternating approach, where softmax attention layers are inserted at specific intervals between consecutive linear sequence modeling layers. This approach ensures that the majority of layers retain linear computational complexity while still benefiting from the high representational capability of softmax attention layers.
- **Intra-layer Hybrid** achieves fine-grained fusion within individual layers by strategically blending linear sequence modeling layers with softmax attention transformer layers.

6.1. Inter-layer Hybrid

The inter-layer hybrid model typically combines softmax attention layers and linear sequence modeling layers in a predefined proportion. Due to its practical effectiveness and straightforward implementation, it

has become a dominant approach in the development of hybrid architectures.

Many notable hybrid models have been built on the Mamba [101] architecture. Zamba [182] is a 7B-parameter hybrid language model built primarily on the Mamba with periodic insertions of a single global shared self-attention block. Zamba demonstrates competitive accuracy on general language understanding benchmarks, approaching leading models like Mistral [355] despite using significantly fewer training tokens. Compared to Zamba, Zamba2 [183] introduces key improvements including a switch to Mamba2 [102], two alternating shared attention blocks and non-shared LoRAs. These changes lead to better performance and efficiency. Jamba [185] introduces a more expressive and scalable 52B hybrid architecture. Jamba adopts a hybrid architecture combining Mamba, standard attention, and MoE, interleaved at a 7:1 ratio. This design enables Jamba's strong in-context learning, and efficient inference, achieving $3\times$ higher throughput than Mixtral and supporting 256K context with only 4GB KV cache. Samba [184] combines Mamba and Sliding Window Attention. Samba avoids full attention and adopts linear-time mechanisms, enabling it to extrapolate up to 1 million tokens in zero-shot and recall up to 256K tokens with minimal fine-tuning. Mamba-in-Llama [112] distills pretrained Transformers into Mamba, and proposes a hybrid version of standard attention and Mamba. The method leverages weight reuse from Transformer attention layers to initialize Mamba blocks, followed by a multi-stage distillation pipeline to preserve the capabilities of standard attention layer. Hunyuan-TurboS [43] is a 56B activated (560B total) hybrid model that combines Mamba2, standard attention, and MoE layers to balance long-sequence efficiency and contextual reasoning. It uses an interleaved Attention-Mamba2-FFN/Mamba2-FFN block structure supports a 256K context length. Zebra-Llama [187] proposes an efficient hybrid architecture combining Mamba2 and MLA [27] to achieve Transformer-level performance with minimal overhead. The method employs SVD-based initialization and layer distillation to transfer knowledge from pre-trained models, along with a sensitivity-aware layer replacement strategy for optimal efficiency.

In addition to Mamba, numerous works combine standard attention with linear attention, linear RNNs, and sliding window attention. RWKV-X [186] strategically interleaves sparse attention MoBA [144] within RWKV-7 [83] layers, with about 25% of layers using sparse attention, achieving strong performance across both short- and long-context tasks. It demonstrates superior decoding stability and lower latency than full-attention models at long sequence lengths. YOCO [188] combines sliding-window attention with standard attention. It features shared KV cache across layers, where the self-decoder generates a single KV cache that is reused by the cross-decoder layers. YOCO reduces memory usage and improves efficiency, especially for long-context tasks, by caching only once for all layers. RecurrentGemma [189] combines Griffin [356] with sliding window attention, avoiding global attention entirely. This design enables RecurrentGemma to match the performance of similarly sized transformer models like Gemma, while offering significantly better memory and inference efficiency. MiniMax-01 [39] is a 456B-parameter MoE hybrid Model that integrates Lightning Attention with standard softmax attention to handle ultra-long sequences up to 4M tokens efficiently. LaCT [190] combines large-chunk TTT with local window attention to efficiently model long sequences while optimizing hardware utilization. This hybrid design enables linear-complexity global context modeling with quadratic costs only locally, supporting tasks like novel view synthesis with 1M tokens and autoregressive video diffusion.

6.2. Intra-layer Hybrid

Intra-layer hybrid architectures combine linear and standard attention within a single layer. Typical designs include: (1) head-wise split, where different heads use either linear or standard attention and (2) sequence-wise split, applying different attention types to different input segments.

Head-wise split. Hymba [191] represents a classic head-wise hybrid architecture, where the attention heads are partitioned into two subsets: one employing Mamba and the other utilizing standard softmax attention. Additionally, it incorporates innovations such as learnable meta tokens for memory initialization, sliding window attention with selective global attention, and cross-layer KV cache sharing. 1.5B Hymba outperforms Llama-3.2-3B across reasoning/recall tasks while achieving $3.49\times$ higher throughput and $19\times$ smaller cache size. WuNeng [357] combines RWKV-7 state-driven mechanisms with Transformer attention, balancing high-resolution recall and efficiency through cross-head interaction. WuNeng outperforms LLaMA and Hymba, achieving better results in complex reasoning and long-context tasks.

Sequence-wise split. LoLCATs [114] is a sequence-level hybrid model that replaces softmax attention with a combination of linear attention and SWA. It uses linear attention for all earlier tokens and local softmax for the most recent tokens, capturing local dependencies. The model is trained in two steps: (1) attention transfer to match softmax outputs, and (2) LoRA to fine-tune the replaced layers. With less than 0.2% parameter updates and just 0.04B tokens used, LoLCATs successfully linearizes models up to 405B parameters, outperforming previous hybrid and subquadratic models in both efficiency and quality. Liger [115] transforms pretrained Transformer-based LLMs into gated linear recurrent structures by reusing key projection weights to build gate matrix—avoiding added parameters and preserving efficiency. Liger blends Linear Attention with Sliding Window Attention, enabling softmax-like expressivity with linear-time inference and constant memory. Supporting models from 1B to 8B parameters, Liger recovers up to 93% of original model performance using only 0.02B fine-tuning tokens via LoRA. TransMamba [192] is a sequence-level hybrid model that dynamically combines standard softmax attention and Mamba state space modeling using shared parameters and a learned TransPoint to switch between them. Early tokens are processed with standard softmax attention for precision, while later tokens use Mamba for efficient long-range modeling, facilitated by a lossless Memory Converter. Supporting models up to 1.5B parameters, TransMamba achieves up to 25% faster training than Transformers and excels in long-context tasks. LoLA [193] is developed to overcome the limitations of conventional linear attention models in long-context tasks. By integrating three complementary memory systems, low-rank linear attention for efficient global token storage, sliding window attention for precise local context modeling, and a sparse global cache for retaining high-fidelity representations of interference-prone key-value pairs.

7. Diffusion Large Language Models

In the preceding sections, we discussed various Transformer-based efficient architectures for LLMs, focusing primarily on attention mechanisms and MoE modules. These approaches improve efficiency by optimizing computation within the standard autoregressive (AR) framework. However, despite their advantages, they remain constrained by the sequential nature of AR generation, where tokens are produced one at a time. This token-wise dependency requires as many forward passes as the sequence length, representing a fundamental bottleneck for inference latency.

In this section, we introduce recent advances in diffusion large language models (Diffusion LLMs) [358, 359]. In contrast to autoregressive models, Diffusion LLMs generate text by progressively denoising a sequence from a noisy or masked state to a coherent output. This fundamental shift in generation enables several unique advantages. Most notably, Diffusion LLMs support parallel decoding, allowing multiple tokens to be produced at each refinement step, which significantly reduces inference latency by avoiding sequential token generation. Moreover, the formulation of text generation within Diffusion LLMs as a denoising or infilling process over a fixed-length canvas inherently provides superior controllability. This allows the model to better adhere to specific output constraints, such as length, format, or structure, challenges that
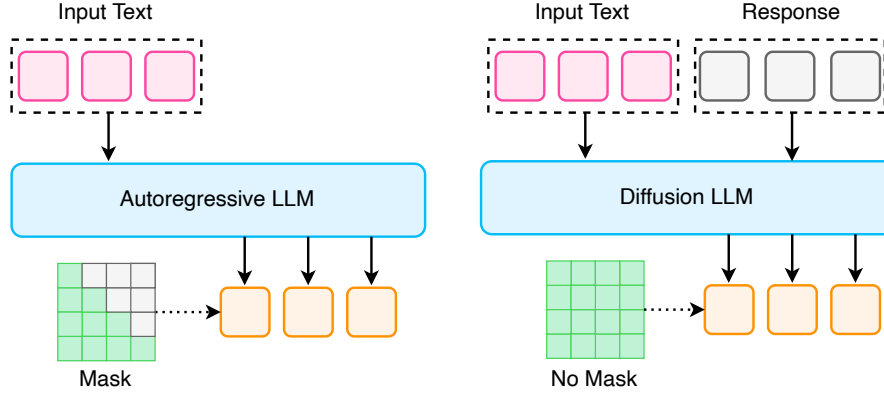
Figure 14: Mechanism Comparison of Autoregressive Models and Diffusion LLMs.

are difficult to address with standard autoregressive methods. Finally, Diffusion LLMs utilize bidirectional attention, enabling the model to access and revise context across the entire sequence at every step. This global view helps mitigate issues such as the reversal curse, which arise from the unidirectional nature of autoregressive decoding.

7.1. Non-Autoregressive Diffusion LLM

LLaDA [194] is a non-autoregressive Diffusion LLM with 8 billion parameters, trained from scratch. It models the data distribution through a forward process that progressively masks tokens and a reverse denoising process that simultaneously predicts all masked tokens per step. Empirically, LLaDA exhibits strong scalability and achieves performance on par with leading AR models such as Llama3-8B across diverse benchmarks, including in-context learning and instruction-following after supervised fine-tuning (SFT). Furthermore, it overcomes known failure modes of AR models, such as the "reversal curse", attributed to their unidirectional processing.

In this section, we formalize the LLaDA's probabilistic framework for better understanding. The model distribution $p_\theta(x_0)$ is characterized by a predefined forward corruption process and a learned reverse generative process. The forward process systematically transforms a clean sequence x_0 into a corrupted intermediate x_t by masking tokens with an independent probability dictated by a timestep $t \in [0, 1]$. The reverse process is trained to recover the original data by iteratively denoising x_t, progressively predicting the masked tokens as t is annealed from 1 to 0.

Central to this framework is a parametric mask predictor, $p_\theta(\cdot|x_t)$, which operates on a corrupted sequence x_t to jointly infer the entire set of masked (M) tokens. Its parameters θ are optimized by minimizing the following cross-entropy objective, computed exclusively over the masked positions:

$$\mathcal{L}(\theta) \triangleq -\mathbb{E}_{t,x_0,x_t} \left[\frac{1}{t} \sum_{i=1}^{L} \mathbf{1}[x_t^i = \mathrm{M}] \log p_\theta(x_0^i|x_t) \right], \tag{29}$$

where x_0 is a sample drawn from the data distribution, t is a timestep uniformly sampled from the interval $[0, 1]$, and x_t denotes the resulting corrupted sequence.

Upon training, sampling from $p_\theta(x_0)$ is performed by simulating the reverse process. Crucially, the

training objective $\mathcal{L}(\theta)$ is a variational upper bound on the model's negative log-likelihood:

$$-\mathbb{E}_{p_{\text{data}}(x_0)}\left[\log p_\theta(x_0)\right] \leq \mathcal{L}(\theta), \tag{30}$$

making it a principled objective for density estimation.

Unlike masked language models such as BERT [360] that employ a fixed masking ratio, LLaDA's use of a random ratio is fundamental to its design. This distinction, theoretically grounded by the negative log-likelihood bound in Eq. (30), establishes LLaDA as a true generative model. This principled foundation enables emergent capabilities like *in-context learning*, positioning LLaDA as a viable, non-autoregressive alternative to mainstream LLMs.

Whether Diffusion LLMs can match the reasoning prowess of Reinforcement Learning (RL)-enhanced AR models has been a significant open problem. The d1 framework [361] provides a compelling affirmative answer. It adapts pre-trained masked Diffusion LLMs for complex reasoning via a two-stage process: first, SFT on reasoning traces, followed by a novel RL algorithm, diffu-GRPO. This policy-gradient algorithm is tailored for the non-autoregressive nature of Diffusion LLMs, featuring an efficient one-step log-probability estimation regularized via random masking prompt. Applied to the LLaDA-8B-Instruct model, d1 achieves substantial gains on mathematical and logical reasoning benchmarks, surpassing not only the base model but also rigorous SFT- and RL-only ablations. This work provides strong evidence that Diffusion LLMs can be potent reasoners when augmented with RL, establishing them as a competitive architecture in a domain previously dominated by AR models.

7.2. Bridging Diffusion LLM and Autoregressive

Diffusion LLMs offer significant advantages over their AR counterparts, namely parallelized generation and greater controllability. Nevertheless, they face challenges in likelihood modeling and are inherently constrained to fixed-length outputs. A significant thrust in recent Diffusion LLM research is to combine the strengths of both AR and diffusion methods. BD3-LMs [199] interpolates between discrete denoising diffusion and AR models by defining an autoregressive distribution over blocks of tokens, while performing diffusion within each block. By synergizing diffusion and autoregressive paradigms, this hybrid approach remedies the fixed-length constraint of the former and the high inference latency of the latter. Specifically, BD3-LMs integrate KV caching from AR models with parallel in-block token sampling, thereby enabling flexible-length generation and substantially accelerating inference. They further propose an efficient training algorithm to tackle the key challenge of gradient variance estimation in Diffusion LLMs, complemented by data-driven noise schedules engineered to minimize this variance.

Formally, BD3-LMs operates on a sequence $x = (x_1, \ldots, x_L)$ by partitioning it into B non-overlapping blocks of length L', denoted $x = (x^{(1)}, \ldots, x^{(B)})$. The model defines the joint log-likelihood through an autoregressive factorization over these blocks:

$$\log p_\theta(x) = \sum_{b=1}^{B} \log p_\theta(x^{(b)} \mid x^{(<b)}), \tag{31}$$

where each conditional distribution $p_\theta(x^{(b)} \mid x^{(<b)})$ is modeled by a discrete diffusion process. This process learns a denoising model $p_\theta(x^{(b)} \mid x_t^{(b)}, x^{(<b)})$ to reverse a fixed forward noising process $q(\cdot \mid \cdot)$, according to the reverse-time Markov chain:

$$p_\theta(x_s^{(b)} \mid x_t^{(b)}, x^{(<b)}) = \sum_{x^{(b)}} q(x_s^{(b)} \mid x_t^{(b)}, x^{(b)}) \, p_\theta(x^{(b)} \mid x_t^{(b)}, x^{(<b)}) \tag{32}$$

The denoising function is parameterized by a single transformer, f_θ, which employs a block-causal attention mask to enforce the autoregressive dependency between blocks. For a given corrupted block $x_t^{(b)}$ and its causal context $x^{(<b)}$, the model predicts the original clean block $\hat{x}_0^{(b)}$:

$$f_\theta\left(x_t^{(b)}, x^{(<b)}\right) \rightarrow \hat{x}_0^{(b)} \tag{33}$$

Inference proceeds autoregressively across blocks while leveraging parallel decoding within each block. This structure naturally accommodates KV caching for all preceding blocks $x^{(<b)}$, significantly improving efficiency.

The training objective is a variational upper bound on the negative log-likelihood, constructed by aggregating the diffusion loss from each individual block:

$$-\log p_\theta(x) \leq \mathcal{L}_{\text{BD}}(x;\theta) := \sum_{b=1}^{B} \mathcal{L}(x^{(b)}, x^{(<b)};\theta), \tag{34}$$

where each term $\mathcal{L}(x^{(b)}, x^{(<b)};\theta)$ is a standard objective for discrete diffusion, adaptable to various formulations such as continuous-time or masking-based processes.

From another perspective, DiffuLLaMA [200] pioneers an alternative approach that leverages the abundance of pre-trained AR models. Their method adapts foundational AR models, such as GPT-2 and LLaMA (spanning 127M to 7B parameters), into text diffusion architectures named DiffuGPT and DiffuLLaMA. Grounded in a demonstrated connection between AR and diffusion objectives, this conversion is achieved via a simple continual pre-training strategy. The resulting models outperform prior Diffusion LLMs and achieve performance competitive with their AR counterparts, despite requiring significantly less training data than training from scratch. This adaptation also enables the Diffusion LLMs to inherit capabilities like in-context learning and instruction following, and they naturally excel at non-sequential tasks such as "fill-in-the-middle" without prompt reordering, leveraging the inherent nature of the diffusion process.

7.3. Extending Diffusion LLM to Multimodality

Recent advancements have extended Diffusion LLMs into the more complex multimodal domain [45, 362, 363, 364, 365]. These multimodal Diffusion LLMs are engineered to operate on joint textual and visual data by incorporating a vision encoder and a projection layer, which maps extracted visual features into the language model's latent embedding space.

In this emerging area, several distinct approaches have emerged. LLaDA-V [201] introduces a purely diffusion-based Multimodal LLM architecture, utilizing a vision encoder and an MLP connector that maps the resulting visual representations into the language model's latent space. This strategy enables effective multimodal alignment, primarily achieved through visual instruction tuning. The model demonstrates notable multimodal performance and substantial data scalability. Similarly, UniDisc [202] advocates for a unified generative paradigm across both textual and visual domains, using purely discrete diffusion. It builds upon a shared architecture that jointly tokenizes text and images into a common vocabulary and uses full self-attention. UniDisc demonstrates strong joint multimodal inpainting and zero-shot editing capabilities without explicit optimization for these tasks. In a similar vein, LaViDa [203] equips discrete diffusion models with a vision encoder but specifically focuses on overcoming practical challenges in adapting diffusion models for vision-language tasks. It introduces novel techniques such as "complementary masking" to improve training data efficiency by ensuring all output tokens contribute to learning, "Prefix-Diffusion LLM decoding"

to enable KV caching for efficient inference with long multimodal prompts, and "timestep shifting" to enhance sample quality, particularly at reduced generation steps, thereby offering the unique advantages of diffusion models like speed-quality tradeoff and controllability.

To address the inherent instabilities and suboptimal performance associated with purely discrete diffusion training in multimodal settings, Dimple [366] proposes an innovative hybrid "Autoregressive-then-Diffusion" training paradigm. This approach first utilizes an autoregressive phase for robust vision-language alignment and instruction following. It is then succeeded by a transition to a diffusion-based masked language modeling stage, devised to reinstate parallel decoding capabilities. For inference, Dimple introduces "Confident Decoding" for dynamic modulation of tokens generated per step, and "Structure Priors", enabling fine-grained control over response length and format. Consequently, it achieves performance levels on par with prominent autoregressive baselines like LLaVA-NEXT [367]. Further pushing the boundaries of unified modeling, MMaDA [204] pioneers a novel category of multimodal diffusion foundation models, distinguished by a common probabilistic framework and a modality-agnostic design. This architecture inherently eliminates the requirement for discrete modality-specific components and is synergistically enhanced by a "mixed long chain-of-thought (CoT)" fine-tuning methodology. This methodology establishes a consistent CoT structure across diverse modalities, thereby enabling cold-start RL. Moreover, MMaDA introduces UniGRPO, a novel unified RL algorithm optimized for diffusion models, which exhibits remarkable generalization across tasks, including textual reasoning, multimodal comprehension, and even text-to-image synthesis.

8. Applications to Other Modalities

Originally developed and popularized for their success in the text domain, efficient architectures are now being broadly adapted to other domains. This section surveys the transfer of these powerful paradigms, namely linear-time sequence models like SSMs and RWKV-like architectures, and sparse computation strategies like MoE, to non-textual data. We explore their impact across several key modalities, beginning with their extensive applications in computer vision (§8.1). We then examine their growing influence in the audio domain (§8.2). Finally, we discuss their crucial role in multimodal learning (§8.3), where efficiency is paramount for integrating diverse data streams. This expansion beyond text demonstrates the versatility of these models, unlocking new capabilities and enabling scaling to previously intractable problem sizes across the broader landscape of artificial intelligence.

8.1. Vision

While ViTs excel in many tasks, their quadratic computational complexity has spurred the development of more efficient architectures. This section reviews the burgeoning application of linear-time sequence models, such as SSMs, and sparse computation strategies like MoE. We explore their impact across the vision landscape, beginning with foundational tasks including classification, detection, and segmentation (§8.1.1). We then cover their role in image enhancement, restoration, and generation (§8.1.2). Finally, we highlight their growing importance in domain-specific applications such as medicine, autonomous driving, and remote sensing (§8.1.3). Collectively, these advancements mark a shift towards models that achieve high performance while remaining computationally tractable, enabling new capabilities at scale.

8.1.1. Image Classification, Detection, and Segmentation

In image classification, recent efforts have focused on adapting linear-time sequence models, particularly Mamba-based SSMs, to various vision tasks. A key trend is the creation of hybrid architectures that integrate

Table 2: Overview of Applications of Efficient Architectures Across Modalities and Categories.

Modality	Category	Approach
Vision	Classification	InsectMamba [208], V-MoE [332], MoE-CNN [368], pMoE [369], Res-vmamba [370], Mammil [371], Spectralmamb [372], Memorymamba [373]
	Detection	Vig [205], Vision-rwkv [206], Tutel [207], Mamba yolo [374], Voxel mamba [209], Mimistd [375], Htd-mamba [376], Soar [377]
	Segmentation	RWKV-SAM [378], Segman [379], VM-Unet [380], Pyramidmamba [381], Vig [205], Vision-rwkv [206]
	Enhancement & Restoration	Restore-rwkv [382], Dvmsr [383], Q-mambar [384], RWKV-IR [385], Pixmamba [386], Watermamba [387], Retinexmamba [388], Llemamba [389], Fouriermamba [390], Vmambair [391], Serpent [392], Matir [393], Cu-mamba [394], Lfmamba [395]
	Generation	DiS [396], Dim [397], DiM [210], Zigma [398], AiM [399], Maskmamba [400], Dimba [401], DiM-3D [402], Gamba [403], Diffusion-rwkv [404], Sdit [405]
	Medicine	U-mamba [211], Vm-unet [212], Rwkv-unet [213], Segmamba [406], Zig-rir [407], Bsbp-rwkv [408], Mambamil [409], Mmr-mamba [410], Mambamir [411], Vmambamorph [412], I2i-mamba [413], Delta-wkv [414]
	Autonomous Driving	Mambabev [214], Occrwkv [415], H-mba [416], Trajectory mamba [417], Drama [418], Salm^2 [419]
	Remote Sensing	RS-Mamba [420], Samba [421], Changemamba [422], Rs3mamba [423], Hsimamba [424], Pan-mamba [425], LE-Mamba [426], Rsdehamba [427], FMSR [428], Rscama [429]
Audio	Understanding	Audio mamba [215, 430, 431], Mamca [216], Rawbmamba [217], BiMamba [220], Ssamba [432], Dual-path mamba [221], Spmamba [222], VAD [223]
	Enhancement & Generation	SEMamba [433], Tramba [434], SaShiMi [218], Music-Diff [219], oSpatialNet-Mamba [435]
Multimodality	Understanding	MaTAV [224], Avs-mamba [225], Av-mamba [436], VisualRWKV-UHD [226], Rwkv-clip [437], Lavida [203], LIMoE [438], Uni-MoE [439], VL-MoE [227], Moe-llava [228], MoCLE [229], Llava-mole [230], PaCE [440], Llada-v [201], Dimple [366]
	Unified	Fragkiadaki [202], Mmada [204]

SSMs with established modules, such as CNNs or residual connections, to effectively capture both local features and long-range dependencies [208, 370, 441]. To overcome the inherent 1D nature of SSMs for 2D image processing, researchers have proposed innovative data scanning strategies, such as multi-path scanning for remote sensing images [442] and topology-aware scanning for graph-structured whole-slide images in medical pathology [371]. These models are also being tailored for highly specialized domains by addressing unique data challenges, including the high dimensionality of hyperspectral images [372] and data scarcity in industrial defect recognition [373]. In a parallel pursuit of efficiency, sparse MoE models have been successfully applied to vision. The V-MoE demonstrated that sparsely activating "expert" sub-networks can scale Vision Transformers to billions of parameters while reducing computational costs [332]. Subsequent work has further matured this paradigm by enhancing its adversarial robustness [368] and providing theoretical guarantees for its sample efficiency [369].

Building on these foundational applications, the same principles are being extended to object detection, where linear-time models replace or augment traditional backbones to enhance efficiency and capability. A prominent trend is the direct integration of SSMs into established detector frameworks, such as in Mamba-YOLO, which leverages an SSM-based backbone to achieve real-time performance without pre-training [374]. The adaptability of SSMs is particularly evident in specialized and challenging domains. For instance, Voxel-Mamba introduces a novel group-free paradigm for 3D object detection from point clouds by processing entire voxel spaces as a single sequence [209]. In multimodal and niche applications, models like Fusion-Mamba pioneer cross-modal fusion in the hidden state space for RGB-Infrared detection [443], while others

employ hierarchical structures to efficiently detect small or specialized targets [375, 376, 377]. Beyond SSMs, other general-purpose linear-complexity backbones, such as Gated Linear Attention (ViG) and RWKV-based models (Vision-RWKV), are also emerging, demonstrating competitive performance with significantly reduced computational overhead [205, 206]. Complementing these architectural innovations, systems-level optimizations like Tutel are proving critical for enabling the efficient, large-scale training of sparse MoE models, paving the way for future scalable detection systems [207].

The demand for efficiency is further amplified in semantic segmentation, a dense prediction task that requires processing high-resolution imagery while maintaining a global receptive field. Here again, linear-time sequence models like Mamba and RWKV have emerged as strong backbones. A key strategy involves creating hybrid encoder-decoder architectures that pair the global context-capturing ability of these models with local mechanisms, such as convolutional blocks or local attention, to preserve fine-grained details for accurate boundary delineation [378, 379]. This approach is also adapted for specialized domains, from lightweight crack segmentation [380] to remote sensing, where innovations in the decoder use SSMs for efficient pyramid feature fusion [381]. Confirming their versatility, general-purpose backbones like ViG and Vision-RWKV also prove highly effective for dense prediction, offering significant speed and memory advantages over traditional Transformers without sacrificing performance [205, 206].

8.1.2. Image Enhancement, Restoration, and Generation

Efficient architectures are transforming image enhancement and restoration by modeling long-range dependencies with linear complexity. Mamba and RWKV-based models, often embedded in U-Net frameworks, are being tailored for specific tasks. In low-light and underwater enhancement, for example, they are often combined with physical principles like Retinex theory [386, 387, 388, 389]. For broader restoration challenges such as dehazing and super-resolution, key innovations focus on adapting these models to 2D data. These include sophisticated scanning strategies, hybrid Mamba-Transformer designs, and even modeling dependencies across feature channels [390, 391, 392, 393, 394]. The efficiency of these models is particularly advantageous for high-dimensional data like 4D light fields and medical images [382, 395, 444]. To ensure practical deployment, significant research also focuses on techniques like knowledge distillation, low-bit quantization, and the creation of balanced benchmark datasets [383, 384, 385].

Building on their success in restoration, these architectures are now being applied to the even more computationally demanding field of generative modeling. Here, a dominant trend is replacing expensive Transformer backbones in diffusion models with scalable architectures like Mamba [210, 396, 397]. A central challenge is adapting the 1D nature of SSMs for 2D image processing. This has spurred innovations in scanning patterns, such as zigzag scans and bidirectional processing, to better capture spatial context [398]. The architectural shift extends beyond diffusion, with Mamba also being adapted for autoregressive generation to achieve substantial inference speed-ups [399]. To leverage complementary strengths, hybrid models combining Mamba and Transformers are also being explored for tasks like masked image modeling and text-to-image synthesis [400, 401]. This new class of efficient backbones is also enabling advances in high-dimensional outputs like 3D shape and scene generation [402, 403]. Finally, other linear-time models like RWKV are emerging as a viable alternative, further diversifying the architectural landscape of generative AI [404, 405].

8.1.3. Domain-specific Applications

Medicine. The medical imaging field has rapidly adopted efficient sequence models like Mamba and RWKV, leveraging their ability to model long-range dependencies in high-resolution data. In semantic segmentation,

a dominant trend is the integration of these models into U-Net-like architectures to combine global context awareness with local feature extraction. This has been successfully applied to a wide range of tasks, from 2D biomedical and skin lesion segmentation [211, 212, 213] to challenging 3D volumetric data [406], with some approaches using innovative nested or boundary-preserving RWKV structures for enhanced efficiency and precision [407, 408]. This architectural flexibility also extends to classification, where these models excel in the data-limited settings common to medicine [445] and are particularly powerful in computational pathology for modeling interactions across vast sequences of patches in whole-slide images [409, 446]. Furthermore, these models are proving indispensable for complex image reconstruction, restoration, and synthesis tasks. They are being used for multi-contrast MRI reconstruction [410], joint reconstruction and uncertainty estimation [411], 3D deformable registration [412], and multi-modal image synthesis [413], often outperforming established methods. The RWKV architecture and its variants have shown similar promise in general medical image restoration and super-resolution [382, 414]. The versatility of these models is even enabling novel applications beyond static images, such as motion-guided tracking of endoscopic instruments in dynamic environments [447].

Autonomous Driving. In autonomous driving, efficient sequence modeling methods are being applied across the entire pipeline, from perception to prediction and planning. In the critical Bird's-Eye View (BEV) space, these models serve as powerful backbones for tasks like 3D object detection [214] and semantic occupancy prediction [415], where they efficiently process temporal and spatial information with linear complexity. Their capabilities extend to other fundamental perception tasks, including multi-modal video understanding for risk detection [416] and self-supervised depth estimation [448]. Beyond perception, these models are enhancing downstream modules by replacing computationally intensive attention mechanisms. For instance, they are used to build highly efficient trajectory forecasting models [417] and to enable lightweight, end-to-end motion planners that fuse multi-modal sensor data [418]. The extreme efficiency of these architectures also enables novel in-cabin applications, such as ultra-lightweight models for real-time driver attention monitoring [419].

Remote Sensing. The field of remote sensing, characterized by very high-resolution (VHR) and hyperspectral imagery, has become a fertile ground for efficient sequence modeling methods due to their linear complexity. For dense prediction tasks like semantic segmentation and change detection, a key innovation has been the development of specialized scanning mechanisms, such as omnidirectional scans, to effectively capture the complex spatial layouts of ground features from a global perspective [420, 421, 422, 423]. These models are also being tailored for specific data modalities, most notably for hyperspectral image classification and fusion (pansharpening), where they excel at modeling intricate spectral dependencies [424, 425, 426]. Similarly, in restoration tasks like dehazing and super-resolution, hybrid approaches that combine SSMs with convolutional or frequency-domain modules are proving effective for enhancing both spatial and spectral quality [427, 428]. The application of these architectures extends beyond pixel-level tasks to higher-level semantic understanding, such as generating descriptive captions for detected changes between images, demonstrating their versatility across the remote sensing pipeline [429].

8.2. Audio

The audio domain has widely adopted efficient sequence models as a powerful alternative to self-attention, leveraging their linear complexity for processing long audio signals. These architectures, particularly Mamba and its variants, are rapidly establishing new baselines across a range of fundamental audio tasks. They have

been successfully applied to audio tagging [215], classification [430], and specialized tasks like automatic modulation classification [216]. A key innovation for audio has been the development of bidirectional Mamba models, which are crucial for capturing the non-causal context inherent in many audio signals and have proven effective for tasks like deepfake detection and general speech processing [217, 220]. Furthermore, these models serve as a powerful backbone for self-supervised pre-training, learning robust general-purpose audio representations that outperform prior Transformer-based methods on a wide array of downstream tasks [431, 432].

Beyond general audio understanding, these models are setting new performance records in complex signal processing domains. In speech separation, a prominent trend is to replace the recurrent or attention modules in state-of-the-art frameworks like DPRNN and TF-GridNet with Mamba blocks. This strategy has led to new SOTA results on benchmark datasets while significantly reducing computational complexity [221, 222]. Similarly, in speech enhancement, Mamba-based systems have achieved top performance on standard benchmarks [433]. Hybrid architectures are also emerging, such as combining Mamba with Transformers to create highly efficient enhancement models tailored for resource-constrained mobile and wearable platforms [434].

The inherent recurrent nature of these models makes them exceptionally well-suited for generation and streaming applications. State Space Models were early pioneers in generating raw audio waveforms directly, outperforming classic autoregressive models in both quality and speed [218], and they continue to be relevant in advanced tasks like symbolic music generation [219]. For real-time processing, both Mamba and RWKV have demonstrated strong capabilities. They are being used to build low-latency streaming systems for multi-channel speech enhancement [435], voice activity detection (VAD) [223], and automatic speech recognition (ASR), where RWKV-based transducers match or exceed the performance of traditional chunk-based models with significantly less memory usage [449].

8.3. Multimodality

In multi-modality, efficient architectures are critical for aligning diverse data streams and scaling large models. Linear-time sequence models like Mamba and RWKV have proven effective for this purpose. Mamba, for instance, is used to build sophisticated alignment and fusion modules. In conversational emotion recognition, a Mamba-aligner synchronizes text, audio, and video features [224]. For audio-visual segmentation and question answering, specialized modules like a Temporal Mamba Block and a Cross-Modality Mamba enable efficient temporal modeling and selective cross-modal attention, respectively [225, 436]. Similarly, the RWKV architecture is being adapted for robust vision-language learning. Models like VisualRWKV-HD use techniques such as lossless downsampling to process high-resolution images without increasing sequence length, while RWKV-CLIP integrates the efficient RWKV into a contrastive learning framework, achieving strong performance in zero-shot tasks [226, 437].

In the generative space, an emerging trend is the use of diffusion models as an alternative to autoregressive systems for multimodal tasks. Researchers have developed large-scale diffusion language models like LaViDa and MMaDA, which are combined with visual encoders to achieve strong performance in multimodal understanding and generation [203, 204]. A key focus is on visual instruction tuning, where models like LLaDA-V demonstrate that a pure diffusion framework can be competitive with top-tier autoregressive models [201]. A notable sub-trend is the development of discrete diffusion models, such as Dimple and UniDisc. These models map all modalities to discrete token sequences, enabling parallel decoding and novel capabilities like unified cross-modal editing [202, 366].

To scale these large multimodal models efficiently, sparse MoE has become a dominant paradigm. MoE

architectures are used to build foundational models from scratch, such as LIMoE for contrastive learning and Uni-MoE for unified systems that handle text, image, audio, and video within a single framework [227, 438, 439]. Sparsity is also a key strategy for efficient fine-tuning. For example, MoE-LLaVA introduces a "MoE-Tuning" strategy to increase model capacity with constant computational cost [228]. Furthermore, a mixture of specialized LoRA experts is being used to mitigate task and data conflicts during instruction tuning, where different experts can focus on specific instruction clusters or data domains [229, 230]. Finally, the concept of compositional experts, as seen in PaCE, breaks down complex tasks like multimodal dialogue into sub-skills handled by different expert modules, which are trained progressively [440].

9. Conclusion and Future Directions

In this survey, we have reviewed the key architectural innovations and optimization strategies developed to overcome the efficiency bottlenecks of Transformer-based models. We highlighted how the quadratic cost of self-attention and the growth of FFN layers drive up both computation and memory demands, especially in long-sequence, multimodal, and multi-step reasoning scenarios. We categorized recent solutions into seven main areas: linear sequence modeling, sparse sequence modeling, efficient full attention, sparse mixture of experts, hybrid architectures, diffusion LLMs, and cross-modal applications. For each category, we examine the core ideas and underlying technical details, summarize representative works, and analyze the strengths and limitations of them. By organizing these approaches systematically, we aim to provide a clear picture of the current landscape and the common challenges they address.

Looking forward, we identify several promising directions for future exploration:

Efficient Architectures Design. As models continue to grow in scale and are expected to operate across a wide range of environments, from cloud to edge, there is a pressing need to rethink core design principles. The following research directions highlight key architectural innovations driving these goals forward:

- **Algorithm-System-Hardware Co-Design:** Jointly co-designing algorithm, system and hardware can improve efficiency for linear, sparse, or full attention, especially on edge devices and specialized chips.
- **Adaptive Attention Mechanisms:** Attention modules that dynamically adjust sparsity or computation based on input or hardware conditions can better balance efficiency and flexibility.
- **Enhanced MoE Routing:** Smarter MoE routing can improve expert utilization, reduce communication overhead, and lower latency during inference.
- **Efficient Large Models with Far More Parameters:** Scaling models to even larger sizes requires innovations in memory layout, sparse activation, and communication-efficient designs.
- **Hierarchical Memory Architectures:** Multi-tiered memory modules (local, short-term, long-term) integrated into the model to efficiently store and retrieve past computation results and world knowledge.
- **Efficient Small Models on Edge Devices:** Designing efficient small-scale LLMs or VLMs for edge deployment calls for quantization, pruning, and compact architecture design.
- **Non-Autoregressive Diffusion LLMs:** Diffusion-based LLMs offer parallel generation and faster inference, with potential to match autoregressive quality in tasks like dialogue and summarization.

Applications of Efficient Architectures. Beyond improving core architectural efficiency, an equally important frontier lies in applying these advancements to broaden the functional capabilities of language and multimodal models. As models are increasingly expected to operate in real-time, dynamic, and multimodal

environments, new design priorities emerge, ranging from infinite context handling and agentic behavior to lifelong learning and multimodal reasoning. The following directions outline practical applications of efficient design:

- **Infinite Long Context:** Efficient models facilitate handling of extremely long or even unbounded contexts, enhancing RAG, agents, reasoning, and multimodal tasks over extended inputs.
- **Efficient Agentic LLMs:** Models optimized for efficiency enable real-time tool usage, planning, and multimodal reasoning, supporting agile agent behaviors with minimal latency in interactive applications.
- **Efficient Large Reasoning Models:** Efficient reasoning models reduce redundant computation and leverage lightweight logic or memory components, improving scalability in tasks.
- **Efficient Vision-Language-Action (VLA) Models:** Efficient multi-modal fusion and rapid visual reasoning empower VLA models to perform real-time control in robotics and interactive systems.
- **Efficient Omni-modal Models:** Unified efficient models seamlessly process diverse modalities, including text, vision, audio, and 3D data.
- **Efficient Unified Multimodal Models for Understanding and Generation:** Combining multimodal perception with generation supports more coherent and context-aware outputs in applications.
- **Continual Adaptation and Lifelong Learning:** Architectures that support on-the-fly adaptation to new data streams without catastrophic forgetting, enabling LLMs to evolve continually in long-term changing environments.

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