

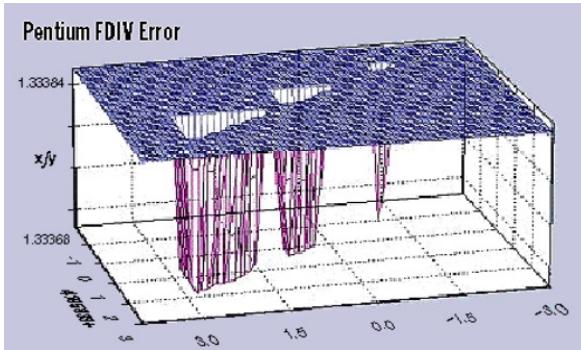


浙江大学  
ZHEJIANG UNIVERSITY

# 蜕变测试导引

rainoftime  
[rainoftime.github.io](https://rainoftime.github.io)  
pyaoaa@zju.edu.cn

# 保障软件质量至关重要



Intel Pentium漏洞导致声誉和巨额经济损失



Ariane5火箭升空数秒后爆炸  
(损失85亿美元)



软件漏洞导致丰田回收120万辆Prius汽车



纳斯达克OMX系统发生故障  
造成千万美元损失



软件数据竞争问题导致美国东北部大面积停电



数据竞争导致Therac25放疗仪使用超过量的放射物

# 程序分析: 软件质量保证的重要手段

## 静态分析



## 动态分析



SonarQube



Coverity



Cppcheck



PVS-Studio



AEL++



...



## 从测试预言问题到蜕变测试

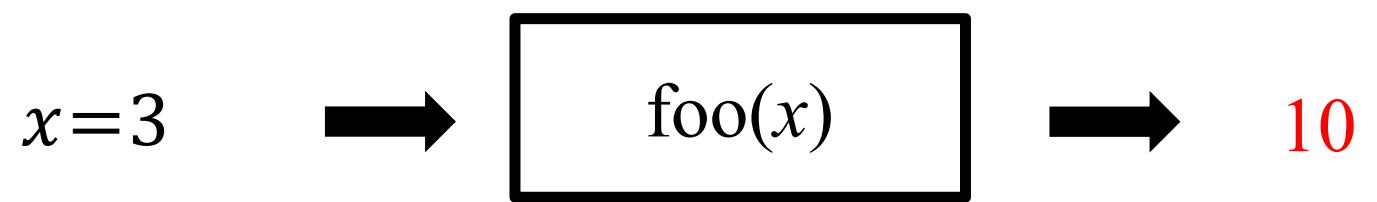
- 测试预言问题
- 蜕变测试概述
- 蜕变关系的属性

## 面向SMT求解器的蜕变测试

- SMT求解器及其逻辑缺陷
- 基于语义融合的方法
- 基于近似枚举的方法

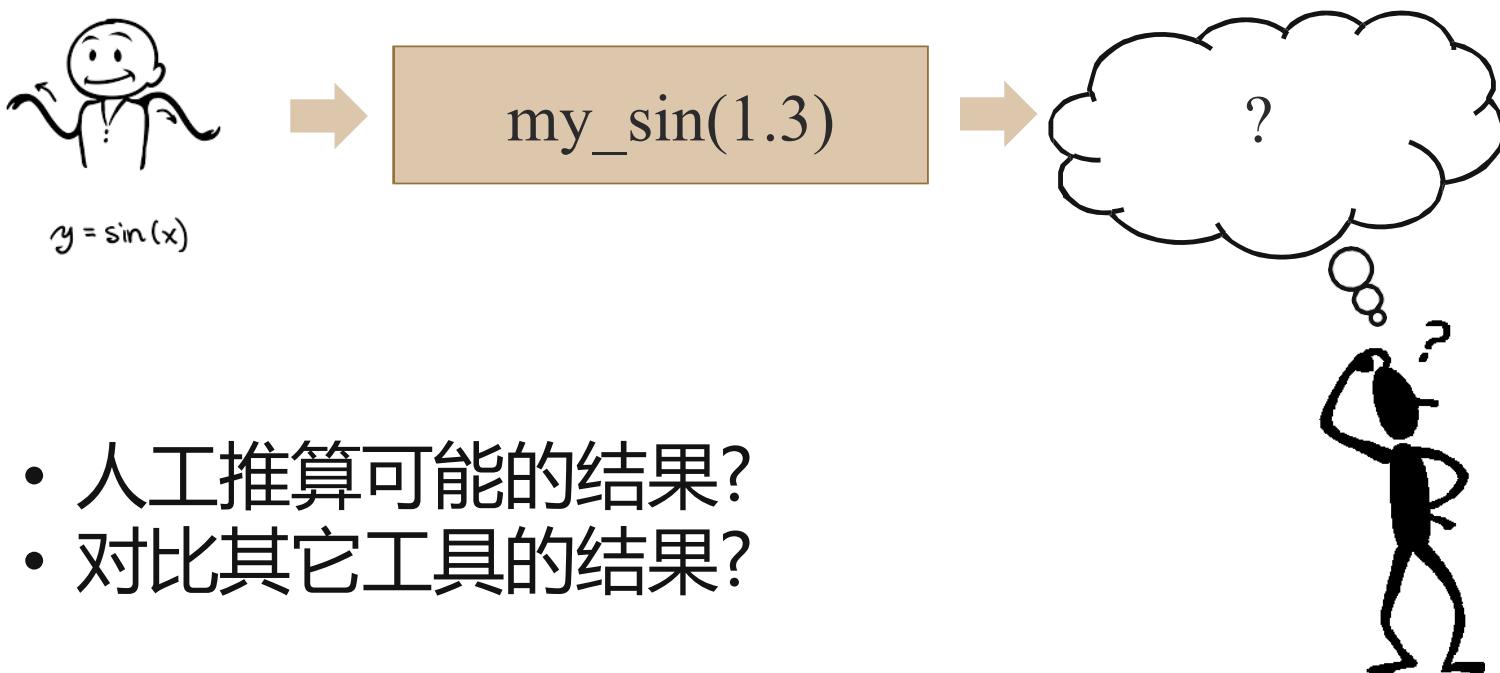
# 逻辑缺陷(正确性缺陷)

- 假设函数  $\text{foo}(x)$  的目的是“计算并返回  $x$  的平方”



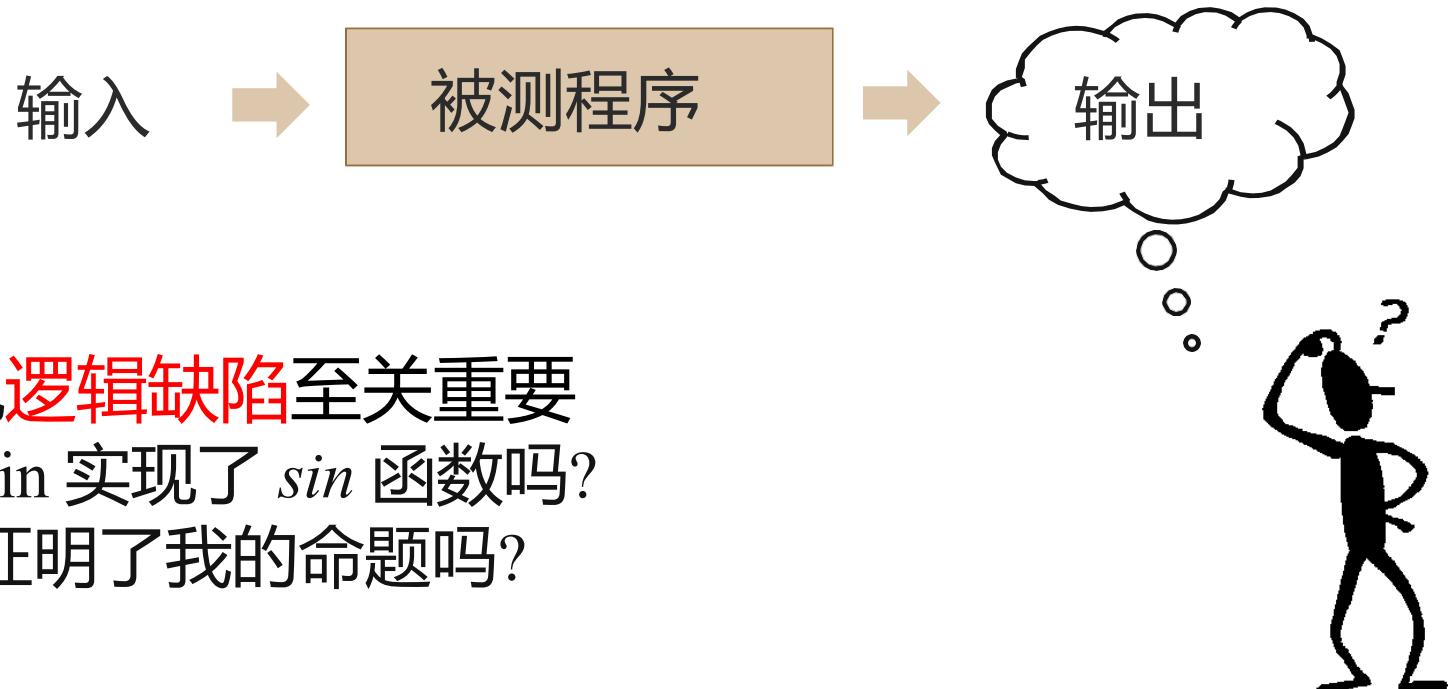
# 例：测试my\_sin的逻辑缺陷

- 假设程序 my\_sin 的目标是实现三角函数  $\sin$



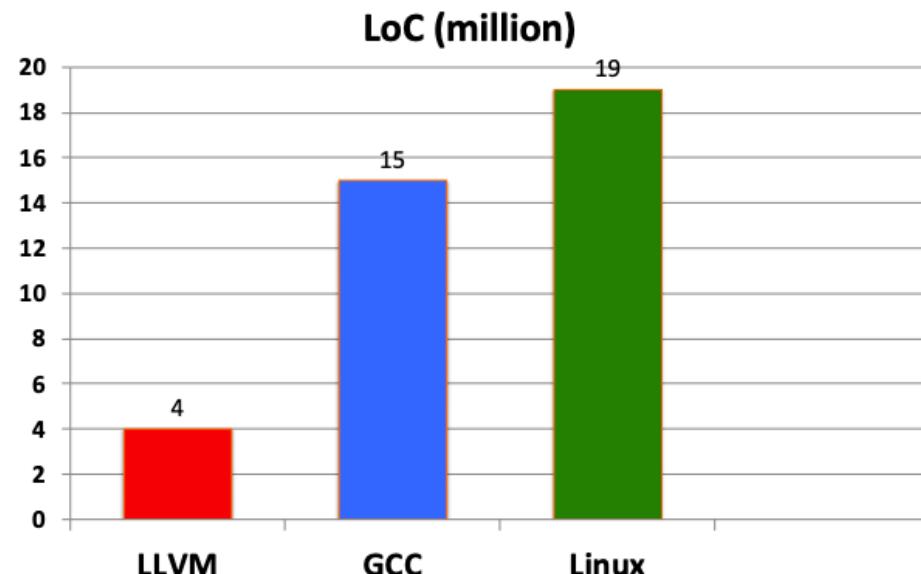
# 测试预言(Test Oracle)

给定测试输入, 判断输出正确与否的机制



# 测试预言问题(Oracle Problem)

- Elaine Weyuker: “On Testing Non-testable Programs”, 1982
  1. 不存在测试预言
  2. 理论上存在, 但实际上很难(自动)检查



- GCC把Linux编译对了么?



你这是在为难我胖虎



## 从测试预言问题到蜕变测试

- 测试预言问题
- **蜕变测试概述**
- 蜕变关系的属性

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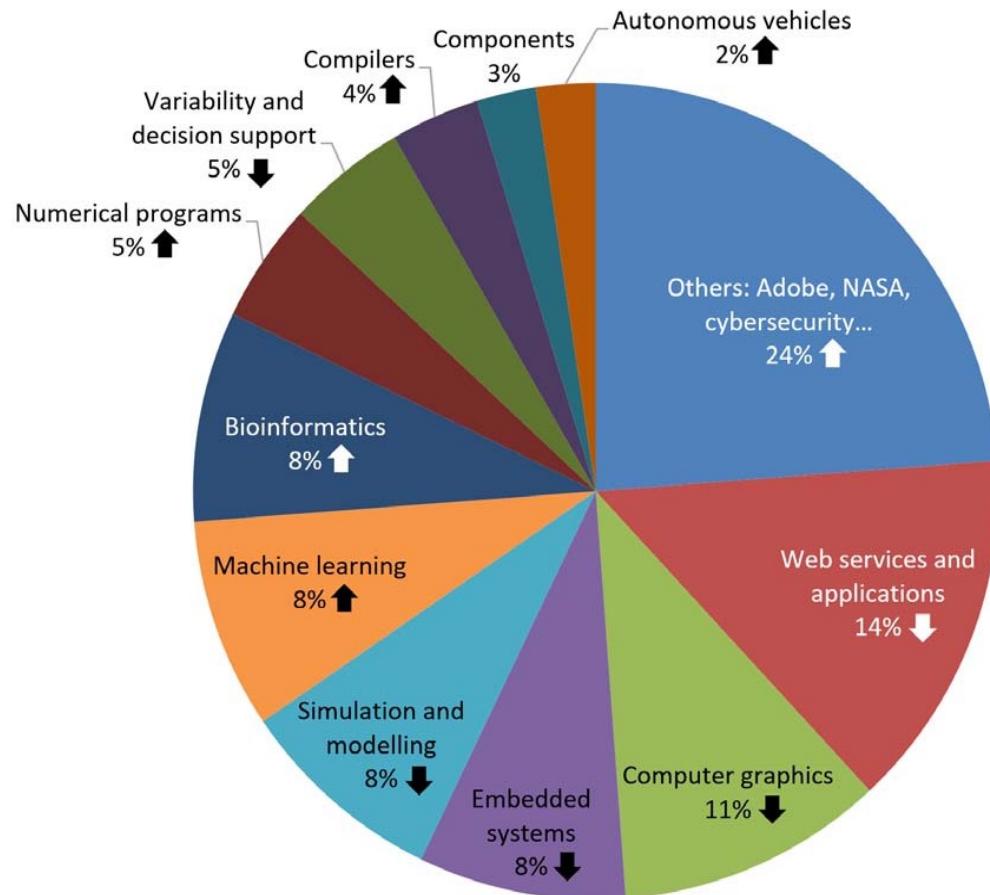
# 蜕变测试(Metamorphic Testing)

- 一类解决/缓解测试预言问题的方法 [T.Y. Chen et al., 1998]
- **主要想法:** 利用相关**领域知识**、交叉检查多组输入及其输出



虽然很难知道各个具体输入的输出应该是什么,  
但可以利用**多组输入/输出之间的关系**

# 蜕变测试的应用



Google 学术搜索  搜索

找到约 11,400 条结果 (用时 0.04 秒)

时间不限  
2022以来  
**2021以来**  
2018以来  
自定义范围...

按相关性排序  
按日期排序

不限语言  
中文网页  
简体中文网页

类型不限  
评论性文章

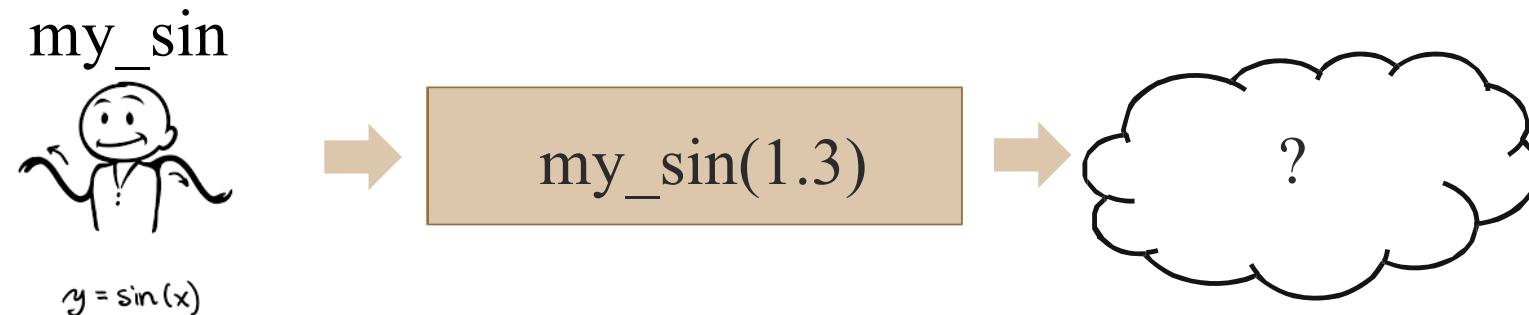
包括专利  
 包含引用

创建快讯

- Perception matters: Detecting perception failures of vqa models using metamorphic testing**  
Y Yuan, S Wang, M Jiang... - Proceedings of the IEEE ..., 2021 - openaccess.thecvf.com  
Visual question answering (VQA) takes an image and a natural-language question as input and returns a natural-language answer. To date, VQA models are primarily assessed by ...  
☆ 保存 99 引用 被引用次数: 13 相关文章 所有 6 个版本
- Testing web enabled simulation at scale using metamorphic testing**  
J Ahlgren, M Berezin, K Bojarczuk... - 2021 IEEE/ACM ..., 2021 - ieexpt.ieee.org  
We report on Facebook's deployment of MIA (**Metamorphic** Interaction Automaton). MIA is used to test Facebook's Web Enabled Simulation, built on a web infrastructure of hundreds ...  
☆ 保存 99 引用 被引用次数: 20 相关文章 所有 5 个版本
- [HTML] Testing multiple linear regression systems with metamorphic testing**  
QH Luu, MF Lau, SPH Ng, TY Chen - Journal of Systems and Software, 2021 - Elsevier  
Regression is one of the most commonly used statistical techniques. However, testing regression systems is a great challenge because of the absence of test oracle in general. In ...  
☆ 保存 99 引用 被引用次数: 9 相关文章 所有 5 个版本 Web of Science: 1
- [HTML] DeepBackground: Metamorphic testing for Deep-Learning-driven image recognition systems accompanied by Background-Relevance**  
Z Zhang, P Wang, H Guo, Z Wang, Y Zhou... - Information and Software ..., 2021 - Elsevier  
Abstract Context: Recently, advances in Deep Learning (DL) have promoted the development of DL-driven image recognition systems in various fields, such as medical ...  
☆ 保存 99 引用 被引用次数: 5 相关文章 所有 2 个版本 Web of Science: 1

# 例：测试my\_sin函数

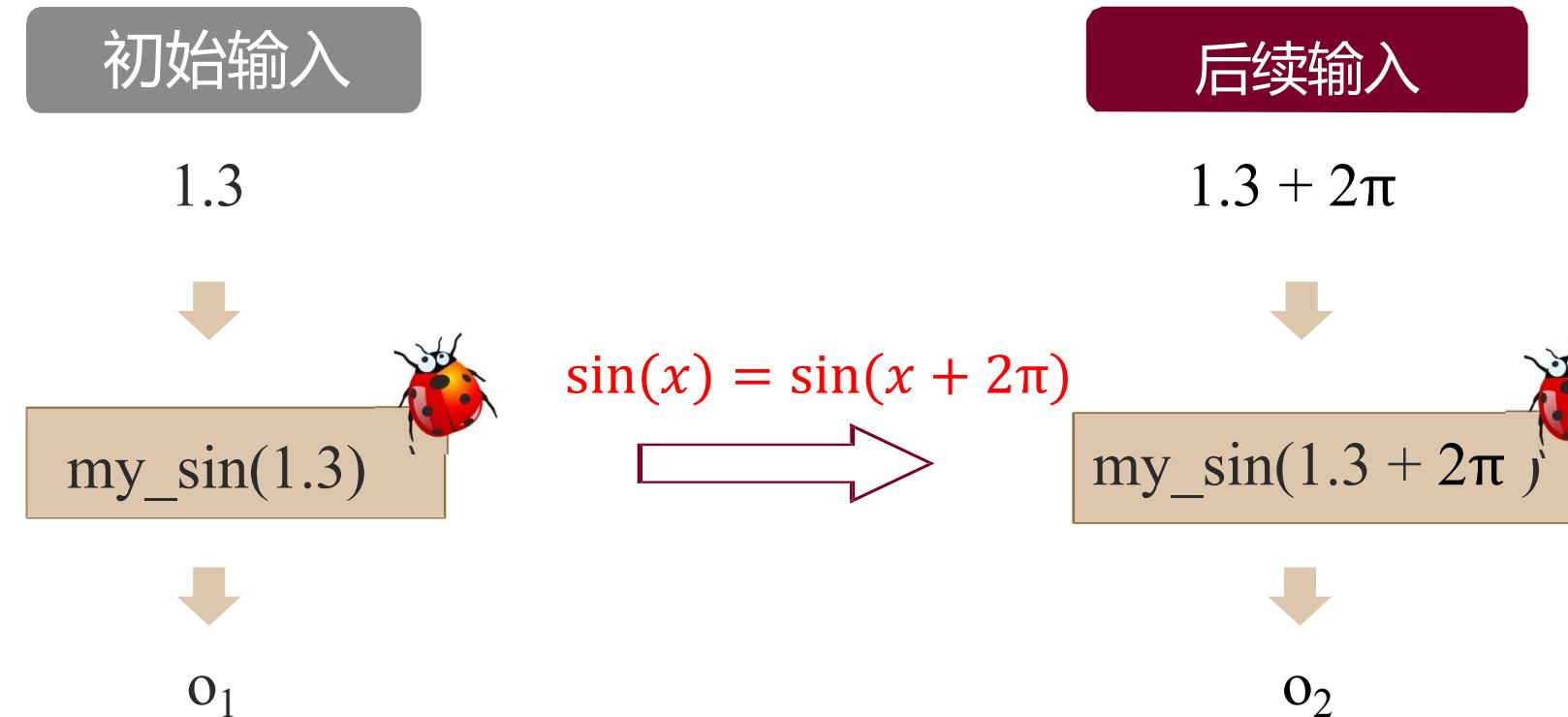
- 假设程序 my\_sin 的目标是实现三角函数  $\sin$



- 不知道:  $\text{my\_sin}(1.3)$  应该返回什么?
- 但是知道:**  $\sin(x) = \sin(x + 2\pi)$



# 例：测试my\_sin函数

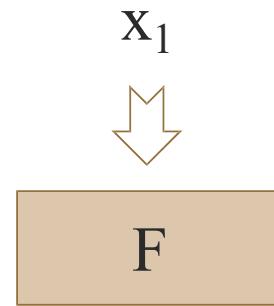


检查:  $o_1 = o_2?$



# 蜕变测试的基本要素

初始测试集



$o_1$

后续测试集



$o_2$

蜕变关系  
 $R(x_1, x_2, o_1, o_2)$

→

检查R是否被违背



# 蜕变测试工作流程

- 1 发现/设计蜕变关系
- 2 收集并运行初始测试集
- 3 根据蜕变关系构造后续测试集
- 4 检查是否违背蜕变关系



# 练习: 为my\_sin填空

1

蜕变关系

$$\sin(x) = \sin(x + 2\pi)$$

2

起始测试集

$$1.2$$

3

构造后续测试集

$$1.2 + 2\pi$$

4

检查蜕变关系

$$\text{my\_sin}(1.2) = \text{my\_sin}(1.2 + 2\pi)?$$



## 从测试预言问题到蜕变测试

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- 蜕变测试概述
- **蜕变关系的属性**

## 面向SMT求解器的蜕变测试

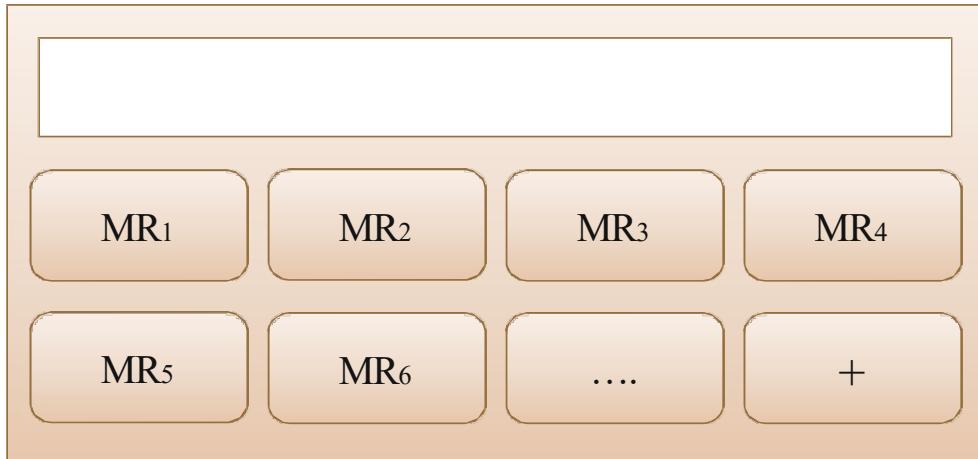
- SMT求解器及其逻辑缺陷
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- 基于近似枚举的方法



# 讨论: 脱变关系是唯一的么?

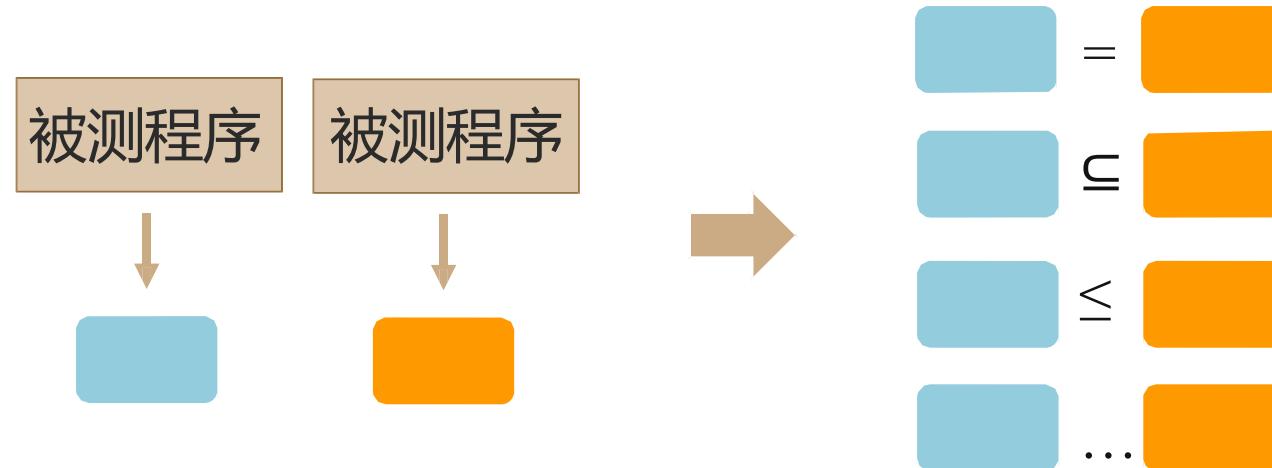
一个被测程序可能有多个(甚至无穷个)脱变关系

- $\sin(x) = \sin(x + 2\pi)$
- $\sin(x+\pi) = -\sin(x)$
- $\sin(\pi-x) = \sin(x)$



# 讨论: 脱变关系 = 相等关系?

脱变关系不限于相等关系, 也不限于数值关系



- 若  $|x| \leq \pi/2$ , 则  $x < y \Rightarrow \sin(x) < \sin(y)$

# 讨论: 每个蜕变关系都能找到Bug?

不同蜕变关系可能有不同的缺陷查找效果

蜕变关系1



蜕变关系2



蜕变关系3

# 小结: 从测试预言问题到蜕变测试

## 测试预言和测试预言问题

### On Testing Non-testable Programs

Elaine J. Weyuker

Department of Computer Science, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012, USA

A frequently invoked assumption in program testing is that there is an oracle (i.e. the tester or an external mechanism can accurately decide whether or not the output produced by a program is correct). A program is non-testable if either an oracle does not exist or the tester must expend some extraordinary amount of time to determine whether or not the output is correct. The reasonableness of the oracle assumption is examined and the conclusion is reached that in many cases this is not a realistic assumption. The consequences of assuming the availability of an oracle are examined and alternatives investigated.

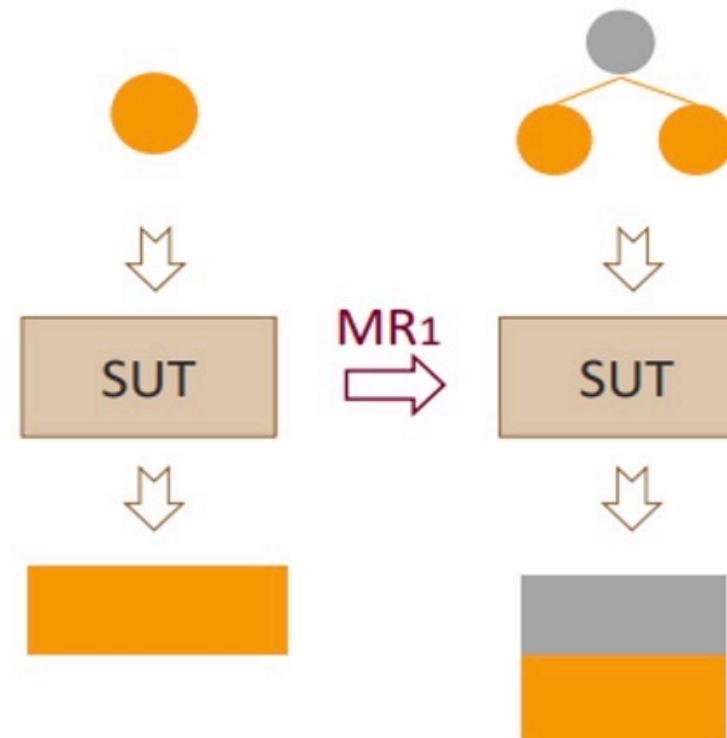
#### 1. INTRODUCTION

It is widely accepted that the fundamental limitation of using program testing techniques to determine the correctness of a program is the inability to extrapolate from the correctness of results for a proper subset of the input domain to the program's correctness for all elements of the domain. In particular, for any proper subset of the domain there are infinitely many programs which produce the correct output on those elements, but produce an incorrect output for some other domain element

tics of programs for which such assumptions are not valid. Section 3 considers how to test such programs. Section 4 looks at techniques which are particularly applicable to numerical and scientific computations, and Section 5 discusses the consequences of accepting the oracle assumption. Section 6 concludes with suggestions for software users and procurers.

#### 2. THE ORACLE ASSUMPTION AND NON-TESTABLE PROGRAMS

## 蜕变测试与蜕变关系





## 从测试预言问题到蜕变测试

- 测试预言和预言问题
- 蜕变测试概述
- 蜕变关系的属性

## 面向SMT求解器的蜕变测试

- **SMT求解器及其逻辑缺陷**
- 基于语义融合的方法
- 基于近似枚举的方法



# Satisfiability Module Theories (SMT) 问题

$$\varphi : x > 0 \wedge x < 0$$

**UNSAT**



# Satisfiability Module Theories (SMT) 问题

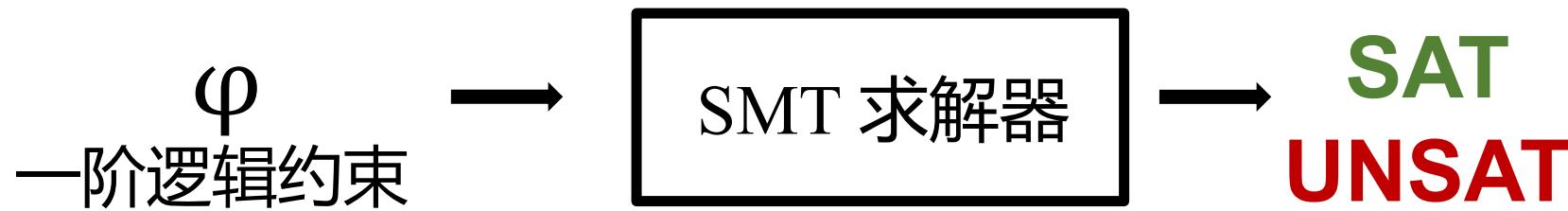
$$\varphi : x > 0 \wedge x < 1$$

**SAT**



# Satisfiability Module Theories (SMT) 求解器

- 如果 $x$ 是整数:  $x * x = 3$  UNSAT
- 如果 $x$ 是实数:  $x * x = 3$  SAT





# Satisfiability Module Theories (SMT) 求解器

- **Z3求解器**: 获得多个领域的重要奖项
  - 形式化方法
    - TACAS most influential paper award
    - ETAPS test-of-the-time award
    - CAV award
  - 编程语言
    - SIGPLAN software award
  - 自动推理
    - Skolem award
    - Herbrand award





# SMT求解器的一些应用

- 寻找可满足解
  - 生成测试用例 [OSDI'08 最佳论文]
  - 生成线程调度 [PLDI'13 杰出论文(亚洲首次)]
- 证明不可满足
  - 检查验证条件 [POPL'02, 软件模型检验“开山之作”]
  - 检查类型签名 [ICFP'14, “自带SMT求解器的编译器”]



亚马逊每天调用数千万次Z3和CVC4 SMT求解器![1]

# 例：基于SMT求解的程序分析

```

1 int main() {
2     int x, y = input();
3     if(y != 0){
4         int w = x / y;
5         print(10 / (w + 1));
6     }
7 }
```



第5行可能有除零错误吗？



$$\varphi : y \neq 0 \wedge w = \frac{x}{y} \wedge w + 1 = 0$$



SMT求解器



$x = 2, y = -2, w = -1$

港科大近期相关成果：

Guo et al., Precise Divide-By-Zero Detection with Affirmative Evidence, ICSE'22



# SMT求解器的逻辑缺陷



```
% cat formula.smt2
(declare-fun a () Int)
(declare-fun b () Int)
(assert (= (div a b) (- 1)))
(check-sat)
```

```
% z3 formula.smt2
sat
```

```
% cvc4 formula.smt2
unsat
```

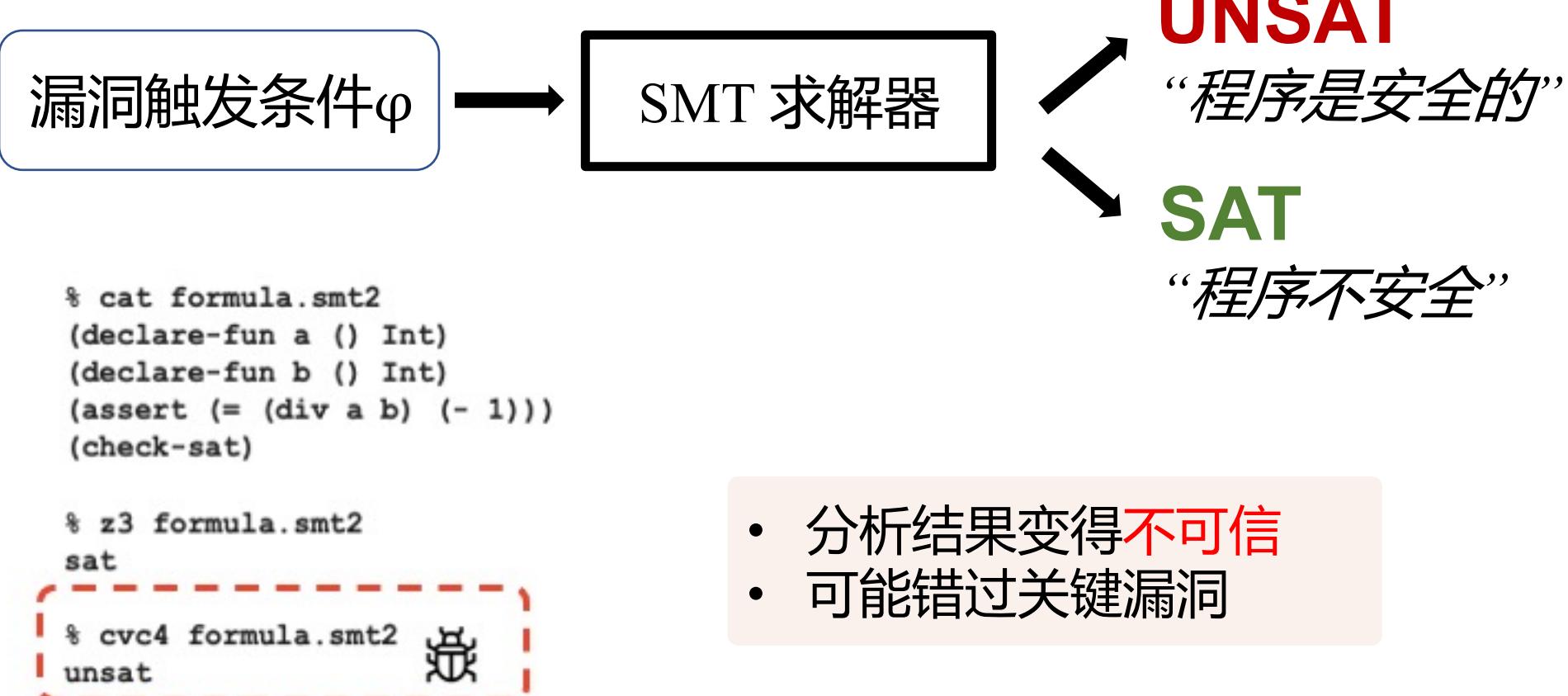
4tXJ7f commented on 28 Oct 2019 Member ...  
I can reproduce this issue and will look into it.  
1 like

4tXJ7f self-assigned this on 28 Oct 2019

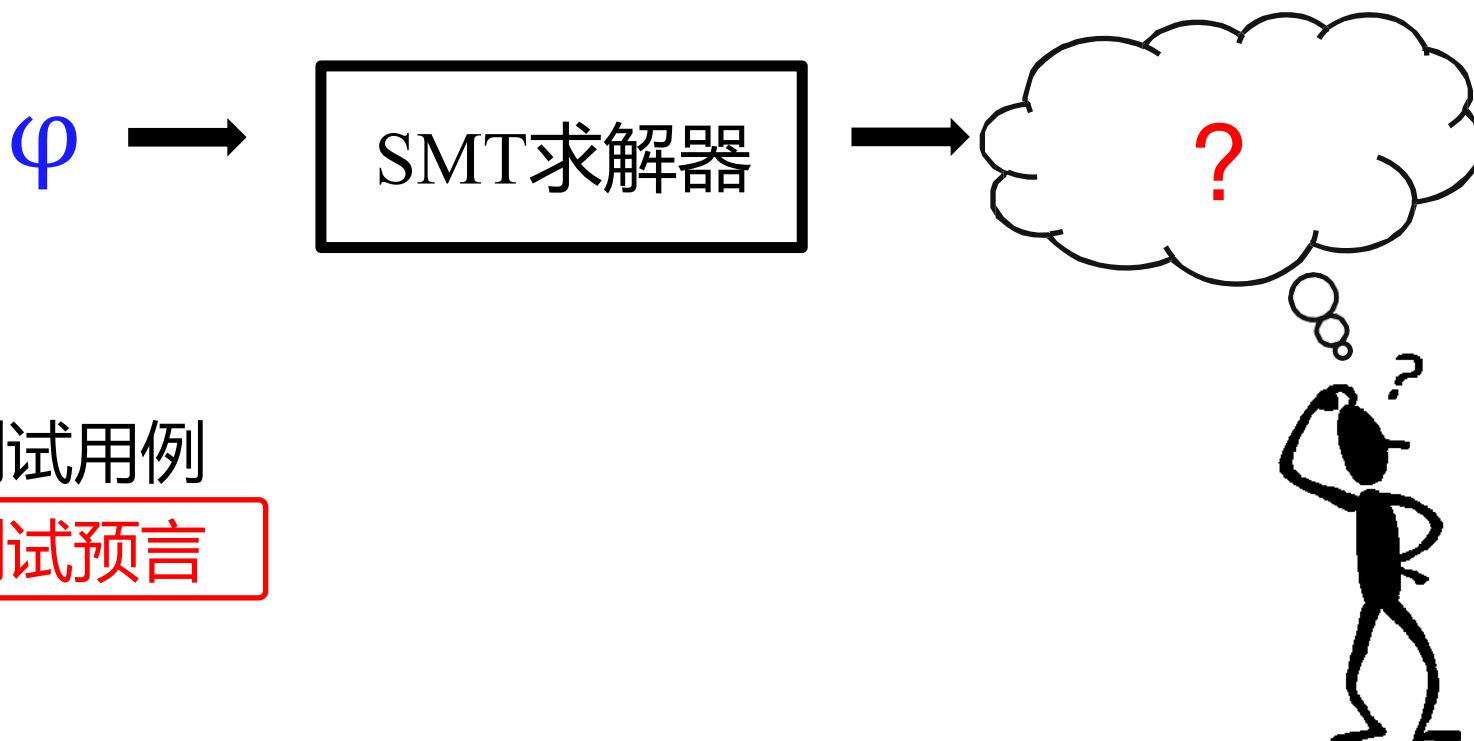
4tXJ7f added bug major labels on 28 Oct 2019



# SMT求解器逻辑缺陷的影响



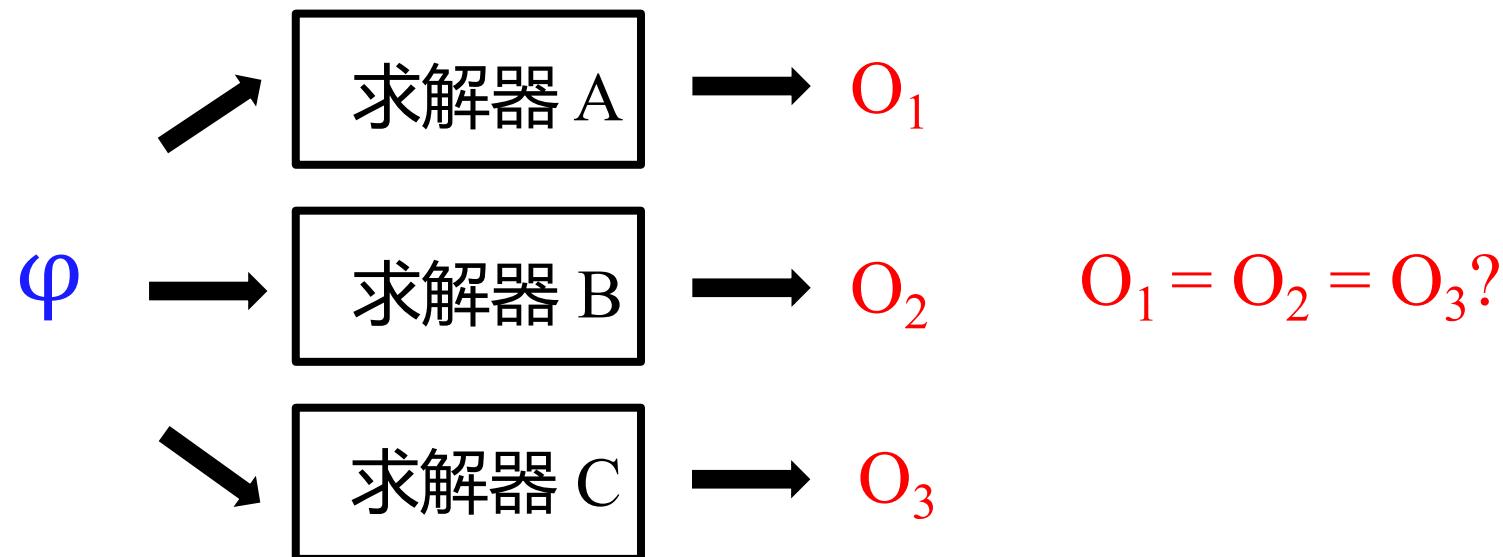
# SMT求解器自动测试的挑战



1. 如何获得测试用例
2. 如何获得测试预言

# 获得测试预言的可能方案

**差分测试(differential testing)** e.g., [SMT'09, CAV'18, OOPSLA'20, ...]



**局限:** 不能处理特殊类型的约束(比如只有求解器A支持)



## 蜕变测试方法简介

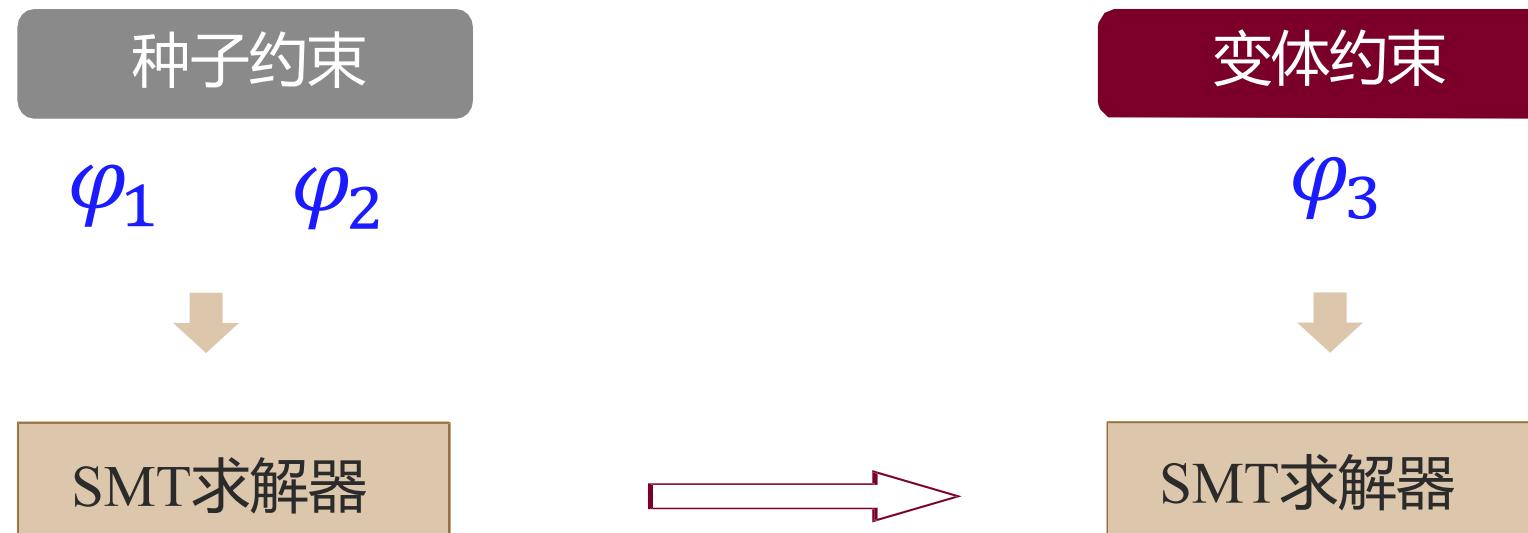
- 测试预言问题
- 蜕变测试概述
- 蜕变关系的属性

## 面向SMT求解器的蜕变测试

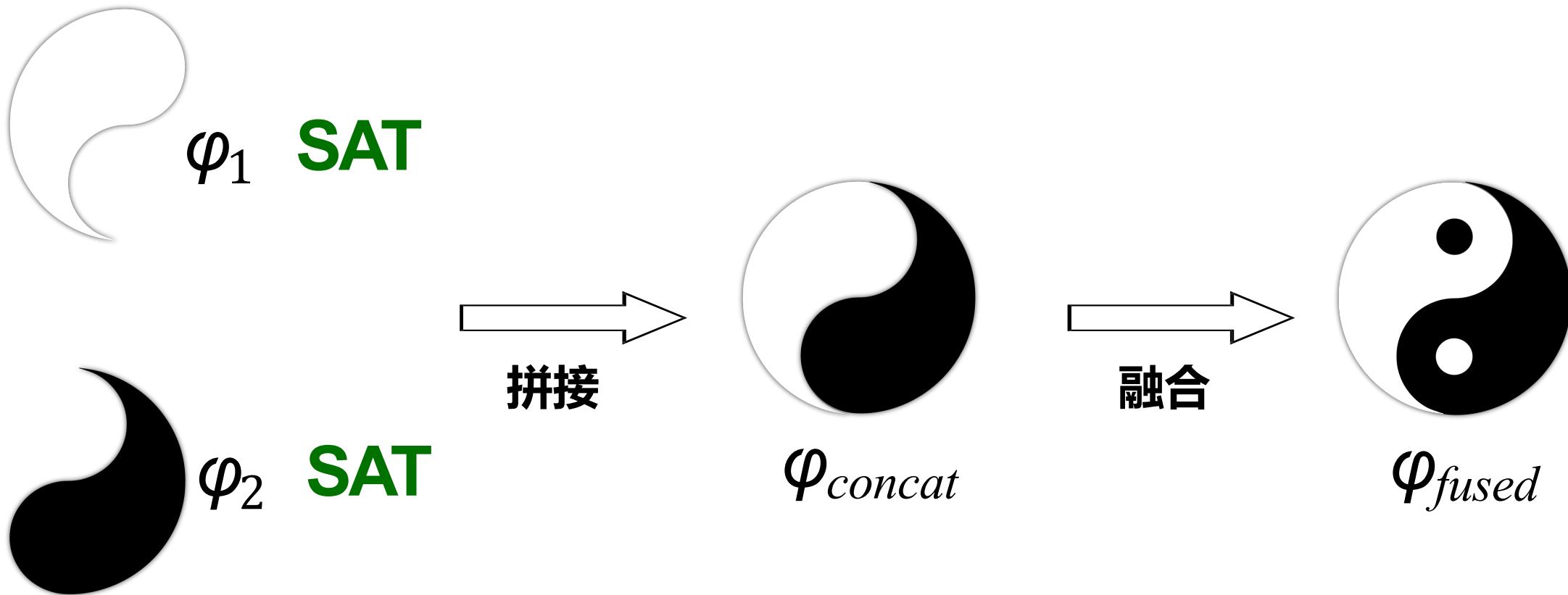
- SMT求解器及其逻辑缺陷
- **基于语义融合的方法**
- 基于近似枚举的方法



- “融合”已知可满足性的种子约束, 得到带预期结果的新约束



# 语义融合: 方法流程



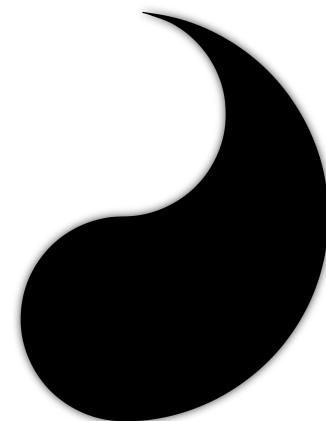


## 例：拼接两个约束

$$\varphi_1 = x > 0 \wedge x > 1 \text{ SAT}$$



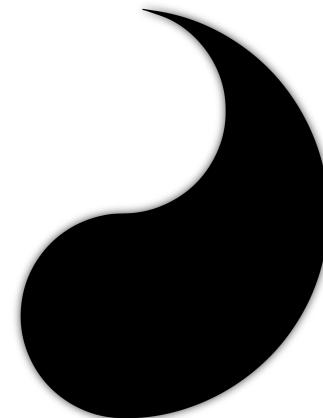
$$\varphi_2 = y < 0 \wedge y < 1 \text{ SAT}$$



# 例：拼接两个约束

 $\varphi_1$  $\varphi_2$ 

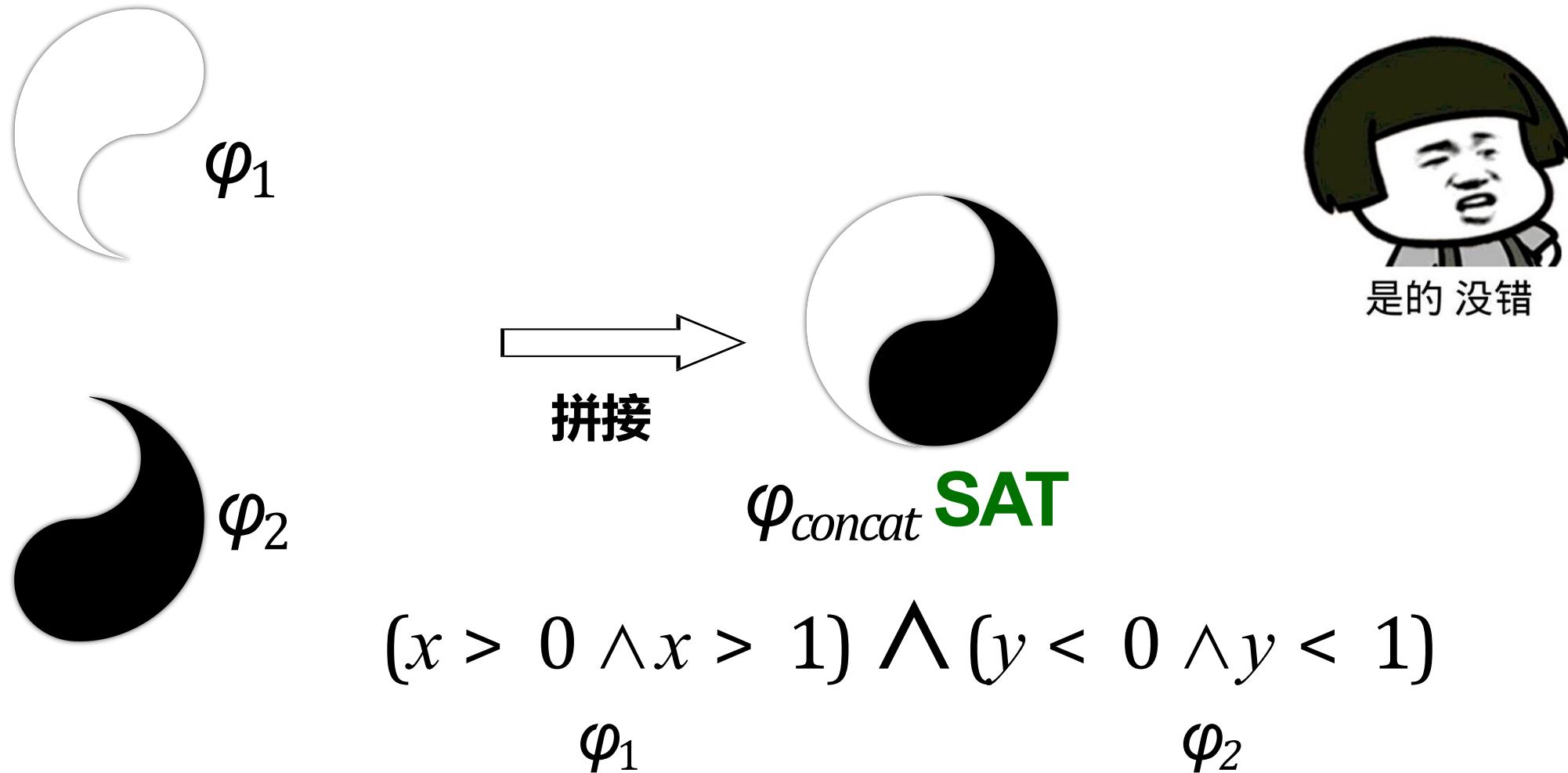
$$\varphi_{concat} = (x > 0 \wedge x > 1) \quad \textcolor{blue}{\wedge} \quad (y < 0 \wedge y < 1) \quad \textcolor{green}{SAT}$$



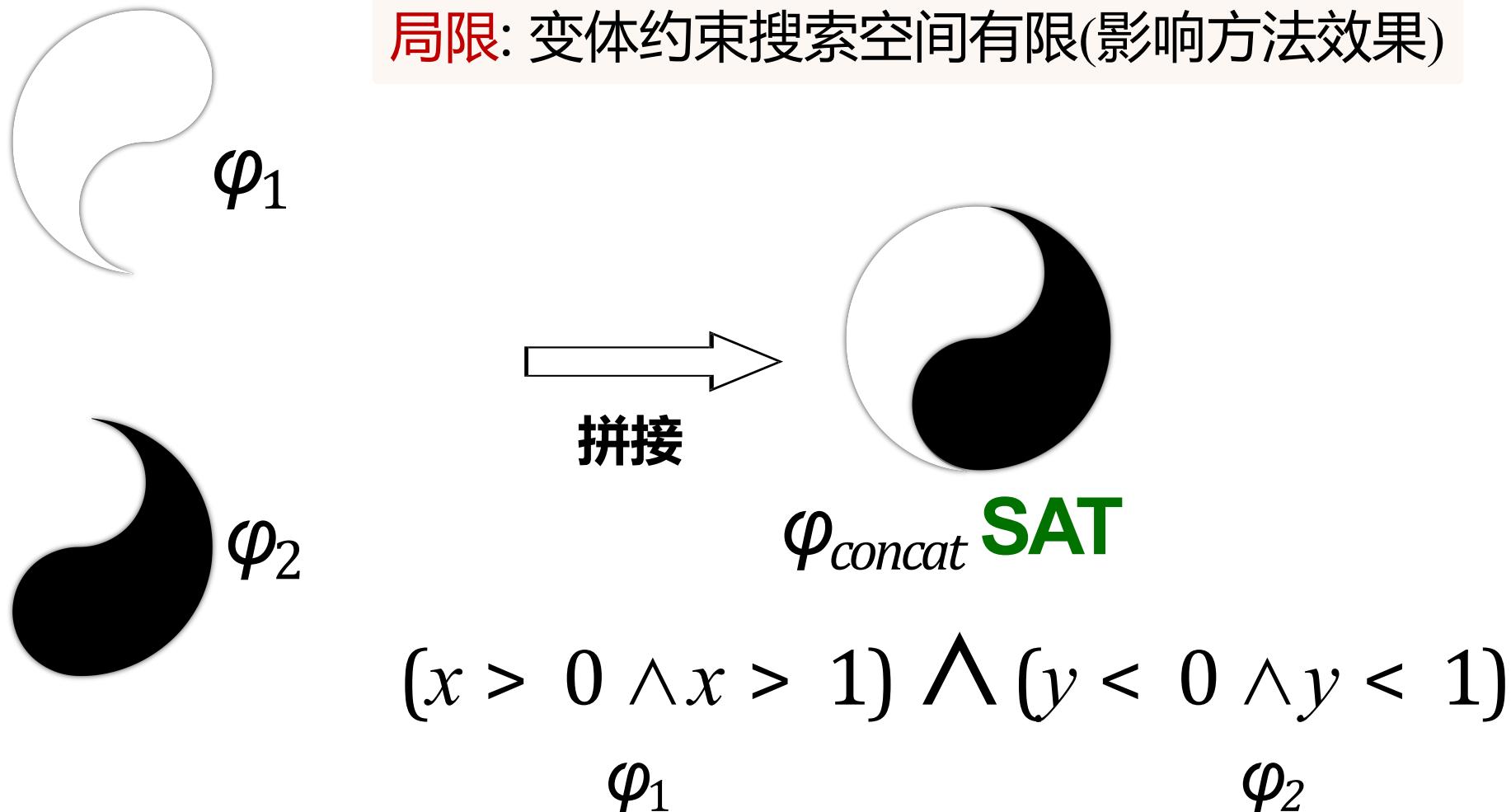
$$x = 2$$

$$y = -2$$

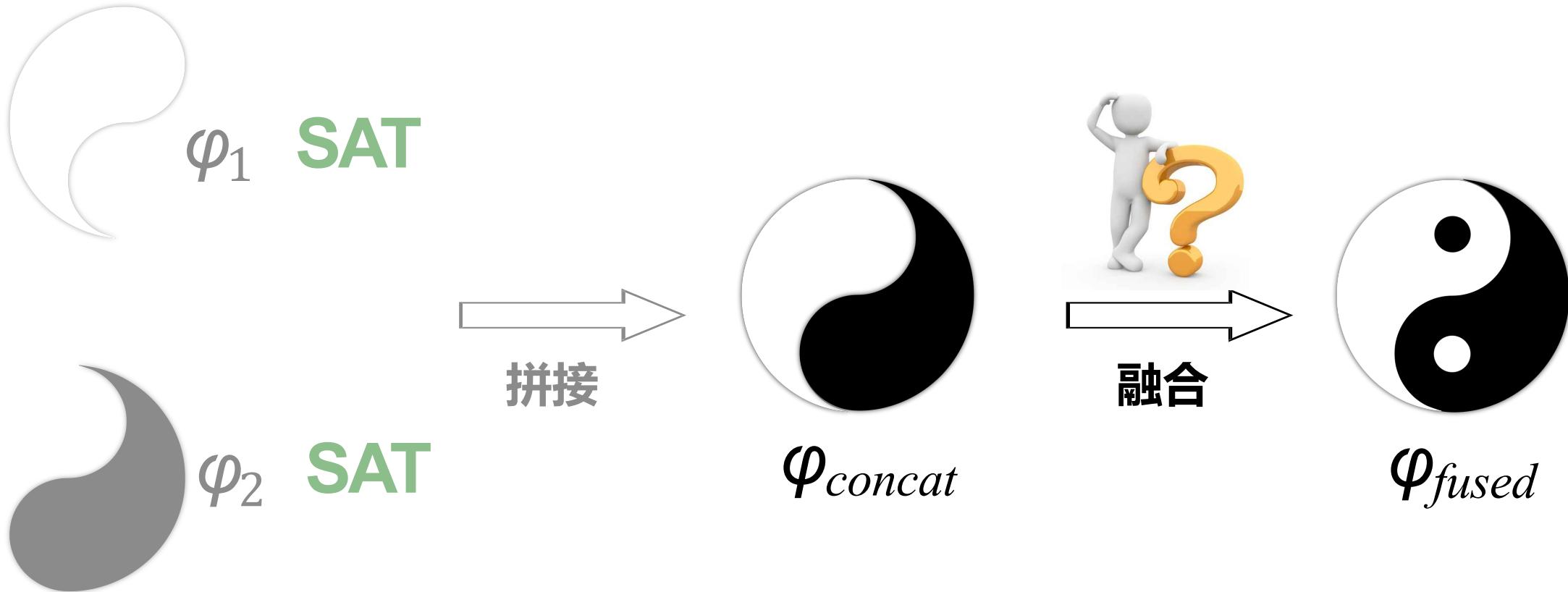
# 讨论: $\varphi_1, \varphi_2, \varphi_{concat}$ 是否构成某种蜕变关系?



# 讨论: 直接把 $\varphi_{concat}$ 作为变体约束?



# 关键挑战: 如何进一步做融合?



**目标** : 形式多样性; 可满足性明确



# 解决方案: 融合函数(Fusion Function)

$$\varphi_{concat} = (\varphi_1 \wedge \varphi_2) \text{ SAT}$$

$$\varphi_1 \qquad \qquad \qquad \varphi_2$$



$$z = x + y$$

Fusion Function



# 构造融合函数的逆函数

$$\varphi_{concat} = (x > 0 \wedge \textcolor{red}{x} > 1) \wedge (\textcolor{red}{y} < 0 \wedge y < 1) \quad \text{SAT}$$
$$\begin{array}{ccc} & \varphi_1 & \varphi_2 \\ & \swarrow & \searrow \\ z = x + y & & \\ \searrow & & \swarrow \\ \textcolor{red}{x} = z - y & & y = z - x \end{array}$$

Inversion Function

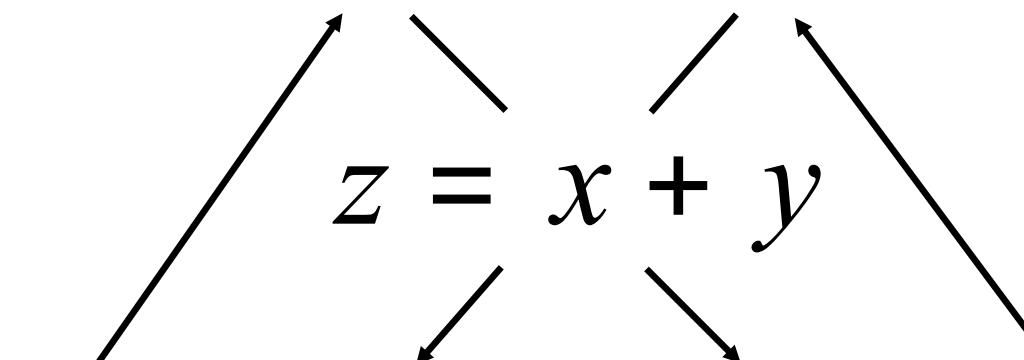
# 利用逆函数做变量替换

$$\varphi_{concat} = (x > 0 \wedge x > 1) \wedge (y < 0 \wedge y < 1) \text{ SAT}$$

$$\varphi_1 \qquad \qquad \varphi_2$$

$$z = x + y$$

$$x = z - y \qquad y = z - x$$



# 利用逆函数做变量替换

$$\varphi_{fused} = (x > 0 \wedge (z - y) > 1) \wedge ((z - x) < 0 \wedge y < 1) \quad \text{SAT}$$

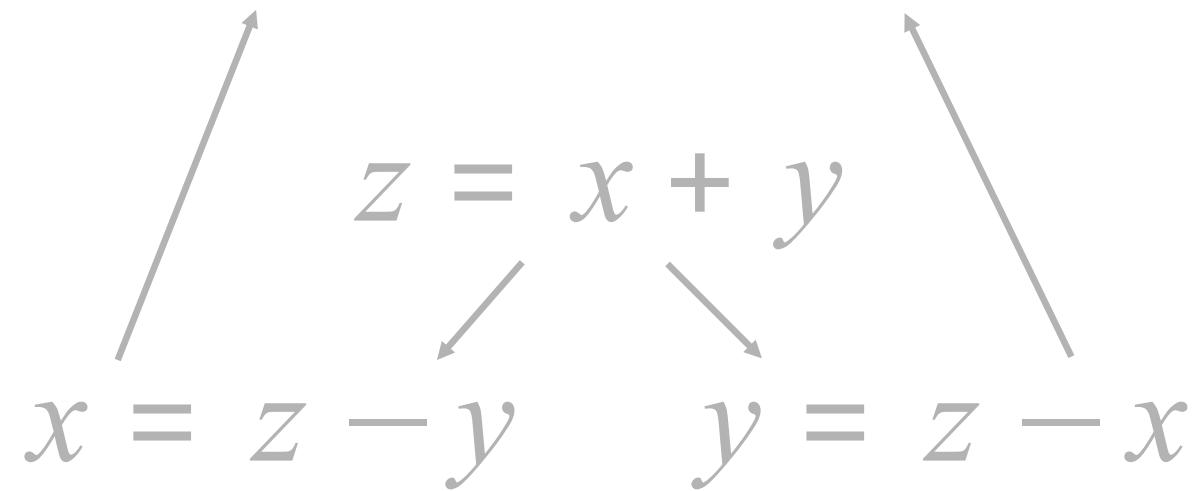
$$\begin{array}{c} \varphi_1 \qquad \qquad \qquad \varphi_2 \\ z = x + y \\ \swarrow \qquad \qquad \qquad \searrow \\ x = z - y \qquad \qquad \qquad y = z - x \end{array}$$

# $\Phi_{fused}$ 保留了 $\Phi_{concat}$ 的可满足性

$$\Phi_{concat} = (x > 0 \wedge x > 1) \wedge (y < 0 \wedge y < 1)$$

$x = 2$        $y = -2$

$$\Phi_{fused} = (x > 0 \wedge (z - y) > 1) \wedge ((z - x) < 0 \wedge y < 1) \quad \text{SAT}$$



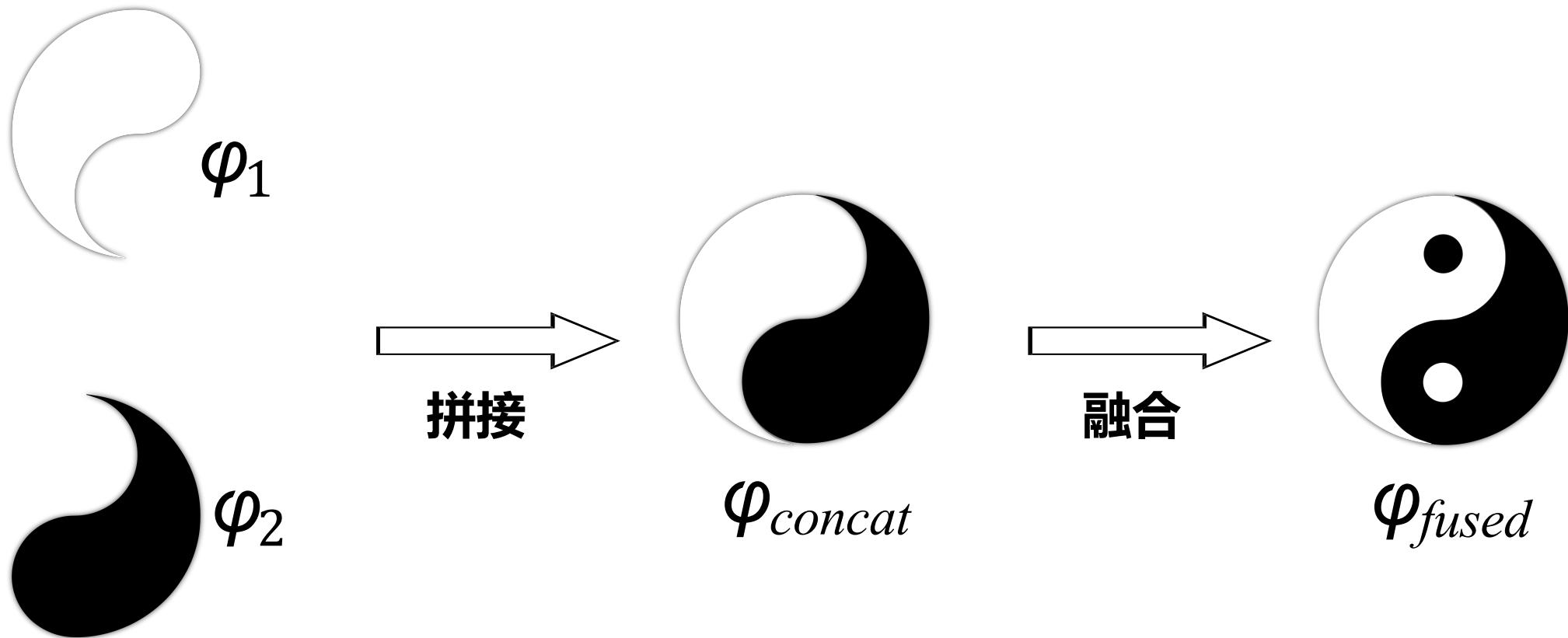
$$\varphi_{concat} = (x > 0 \wedge x > 1) \wedge (y < 0 \wedge y < 1)$$

$$x = 2 \quad z = x + y = 0 \quad y = -2$$

$$\varphi_{fused} = (x > 0 \wedge (z - y) > 1) \wedge ((z - x) < 0 \wedge y < 1) \quad \text{SAT}$$

$$\begin{array}{ccc} & z = x + y & \\ / & & \backslash \\ x = z - y & & y = z - x \end{array}$$

# 基于语义融合的方法





# YinYang部分实验结果

- 测试时间: 2019.07-2019.10

| Status    | Z3 | CVC4 | Total |
|-----------|----|------|-------|
| Reported  | 45 | 13   | 58    |
| Confirmed | 38 | 8    | 46    |
| Fixed     | 36 | 6    | 42    |
| Duplicate | 4  | 1    | 5     |
| Won't fix | 2  | 0    | 2     |

| Type        | Z3 | CVC4 | Total |
|-------------|----|------|-------|
| Soundness   | 24 | 6    | 30    |
| Crash       | 11 | 1    | 12    |
| Performance | 1  | 2    | 3     |
| Unknown     | 1  | 0    | 1     |

| Logic   | Z3 | CVC4 | Total |
|---------|----|------|-------|
| NIA     | 2  | 1    | 3     |
| NRA     | 15 | 1    | 16    |
| QF_NIA  | 0  | 1    | 1     |
| QF_NRA  | 2  | 0    | 2     |
| QF_S    | 16 | 4    | 20    |
| QF_SLIA | 3  | 1    | 4     |



# 语义融合方法的一些特征

需要知道种子约束的可满足性

## Semantic Fusion

$$\varphi_{concat} = (x > 0 \wedge x > 1) \wedge (y < 0 \wedge y < 1)$$

|         |          |
|---------|----------|
| $x = 2$ | $y = -2$ |
|---------|----------|

$$\varphi_{fused} = (x > 0 \wedge (z - y) > 1) \wedge ((z - x) < 0 \wedge y < 1) \quad \text{SAT}$$

$$\begin{array}{ccc} z = x + y & & \\ \diagup & & \diagdown \\ x = z - y & & y = z - x \end{array}$$

融合函数限制了变体约束搜索空间

| Type   | Fusion Function                       | Variable Inversion Functions                     |  |
|--------|---------------------------------------|--|--|
|        |                                       | $r_x$  | $r_y$  |
| Int    | $x + y$                               | $z - y$  | $z - x$  |
|        | $x + c + y$                           | $z - c - y$                                      | $z - c - x$  |
|        | $x * y$                               | $z \text{ div } y$                               | $z \text{ div } x$   |
|        | $c_1 * x + c_2 * y + c_3$             | $(z - c_2 * y - c_3) \text{ div } c_1$           | $(z - c_1 * x - c_3) \text{ div } c_2$                             |
| Real   | $x + y$                               | $z - y$  | $z - x$  |
|        | $x + c + y$                           | $z - c - y$                                      | $z - c - x$  |
|        | $x * y$                               | $z/y$  | $z/x$  |
|        | $c_1 * x + c_2 * y + c_3$             | $(z - c_2 * y - c_3)/c_1$                        | $(z - c_1 * x - c_3)/c_2$  |
|        |                                       |  |  |
| String | $x \text{ str++ } y$                  | $\text{str.substr } z \ 0 \ (\text{str.len } x)$ | $\text{str.substr } z \ (\text{str.len } x) \ (\text{str.len } y)$ |
|        | $x \text{ str++ } y$                  | $\text{str.substr } z \ 0 \ (\text{str.len } x)$ | $\text{str.replace } z \ x \ ""$                                   |
|        | $x \text{ str++ } c \text{ str++ } y$ | $\text{str.substr } z \ 0 \ (\text{str.len } x)$ | $\text{str.replace } (\text{str.replace } z \ x \ "") \ c \ ""$    |



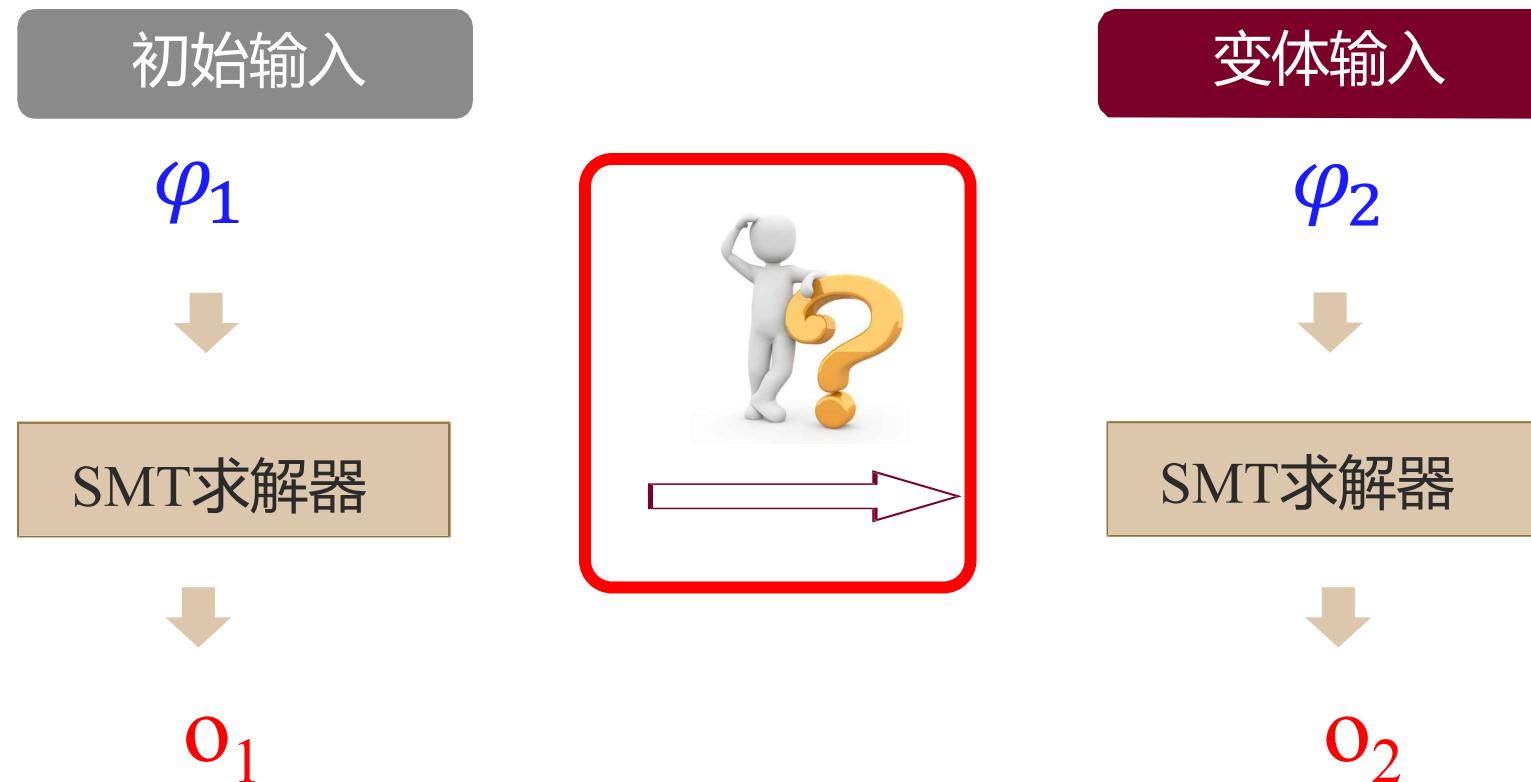
## 从测试预言问题到蜕变测试

- 测试预言问题
- 蜕变测试概述
- 蜕变关系的属性

## 面向SMT求解器的蜕变测试

- SMT求解器及其逻辑缺陷
- 基于语义融合的方法
- **基于近似枚举的方法**

- 对任意种子约束做等可满足性变换



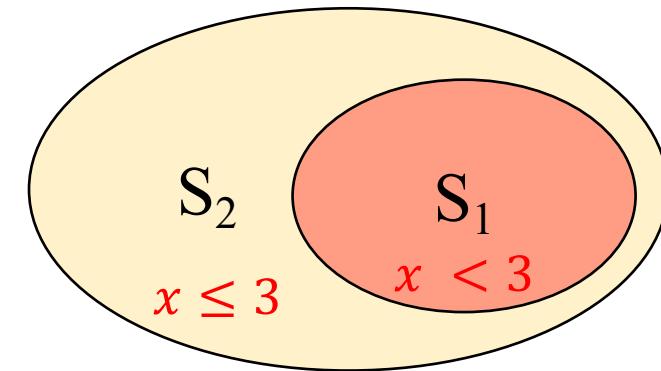
# 核心想法：逻辑近似指导约束变换

**下近似**  $S_2 \rightarrow S_1$

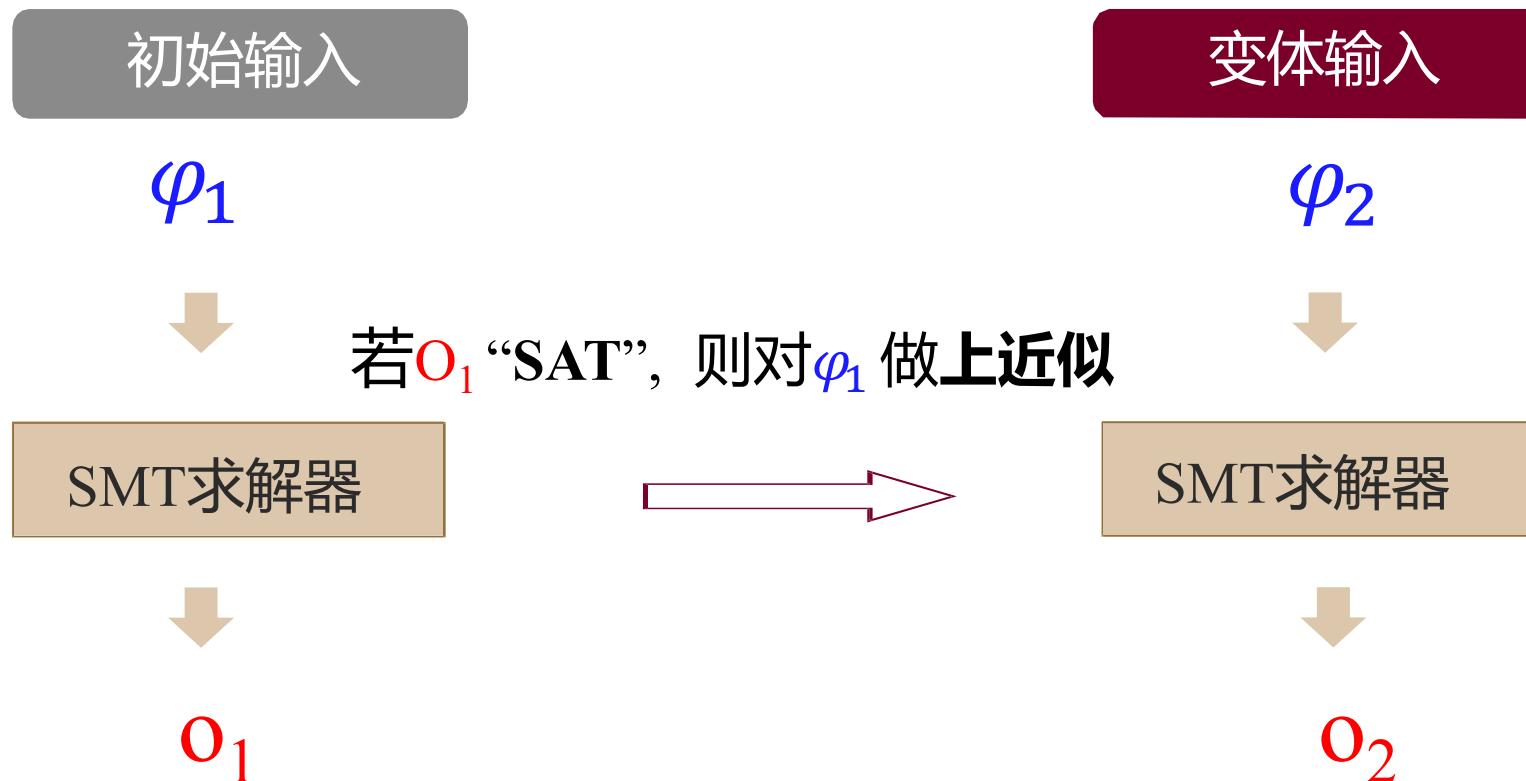
若  $S_2$  UNSAT, 则  $S_1$  一定也 UNSAT

**上近似**  $S_1 \rightarrow S_2$

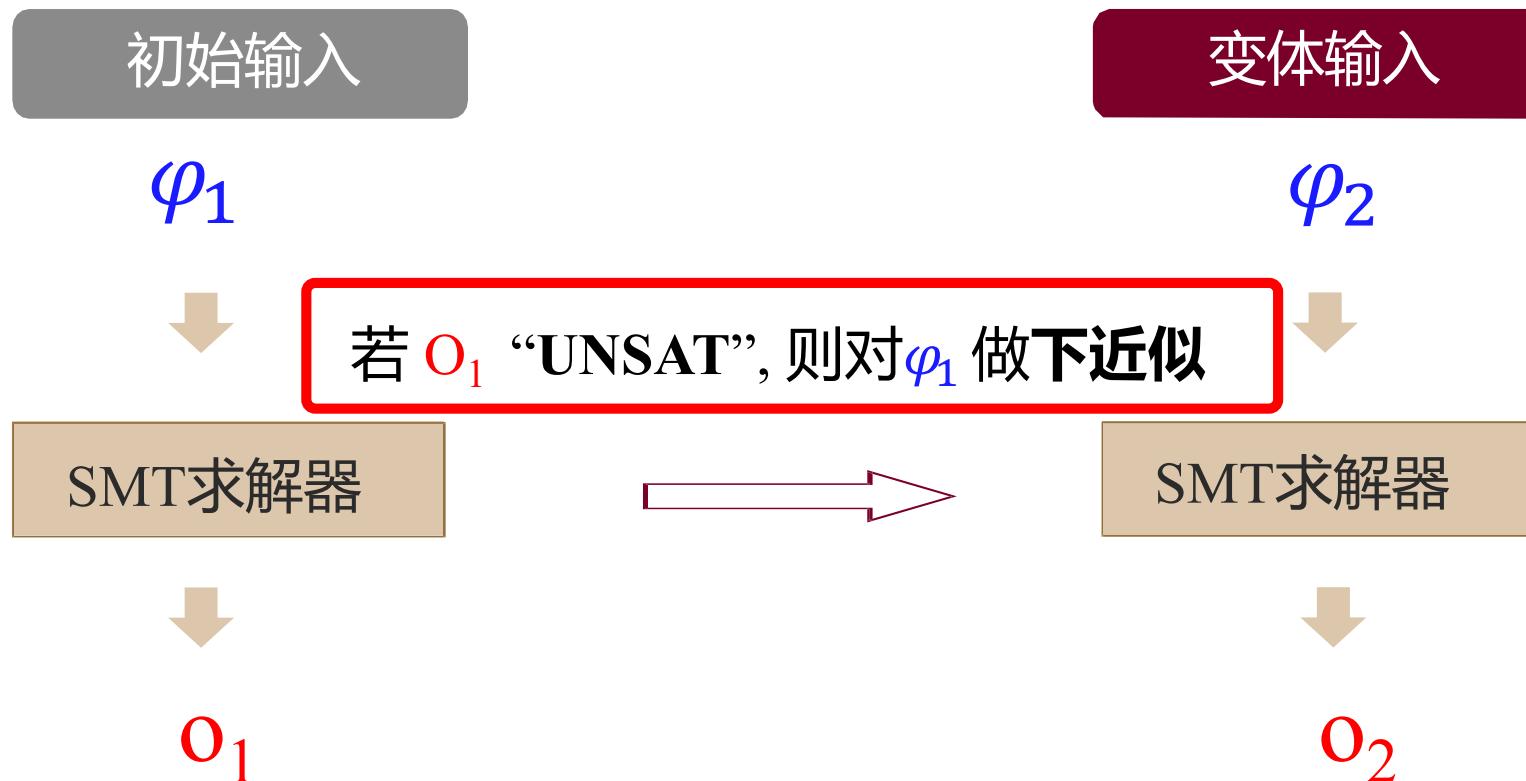
若  $S_1$  SAT, 则  $S_2$  一定也 SAT



# 逻辑近似指导的变换



# 逻辑近似指导的变换



# 关键挑战: 如何对约束做上/下近似?

- 在形式化方法领域有广泛研究
- 大部分现有的算法
  - 只支持特定类型的约束, e.g., [Bryant et al. TACAS'07]
  - 涉及重量级的逻辑推理, e.g., [McMillan et al. CAV'06]

**目标** : 通用且轻量级的逻辑近似方法?



# 解决方案: 基于骨架的近似枚举

$$\varphi_1 \longrightarrow (\boxed{\quad} \vee \boxed{p_1} \vee \boxed{\quad}) \wedge (\boxed{\quad} \vee \boxed{\quad} \vee \boxed{q_1}) \wedge$$

1. 变换到 Conjunctive Normal Form (CNF)形式



**定理:** 任意局部的上/下近似可以得到全局的上/下近似

$$\varphi_2 \longleftarrow (\boxed{\quad} \vee \boxed{p_2} \vee \boxed{\quad}) \wedge (\boxed{\quad} \vee \boxed{\quad} \vee \boxed{q_2}) \wedge$$

2. 枚举局部/原子层面的近似



# 原子层面的近似策略

## 1. 谓词符号变换

- E.g.,  $x < y + 3$  是对  $x \leq y$  的上近似(3为随机生成)

## 2. 约束片段植入

- “ $p \vee f$ ” 是对  $p$  的上近似
- “ $p \wedge f$ ” 是对  $p$  的下近似

$f$  是任意随机生成的约束片段



# Sparrow部分实验结果

- 测试时间: 2020.11-2022.02

Table 3: Status of the bugs found by Sparrow.

| Status    | Z3 | CVC4 | Total |
|-----------|----|------|-------|
| Reported  | 38 | 46   | 84    |
| Confirmed | 30 | 42   | 72    |
| Fixed     | 28 | 40   | 68    |
| Duplicate | 1  | 2    | 3     |
| Invalid   | 7  | 2    | 9     |

Table 4: Bug type of the confirmed bugs.

| Type          | Z3 | CVC4 | Total | Fixed |
|---------------|----|------|-------|-------|
| Soundness     | 4  | 4    | 8     | 8     |
| Invalid model | 10 | 12   | 22    | 20    |
| Crash         | 16 | 26   | 42    | 40    |

# 小结: 面向SMT求解器的蜕变测试

基于语义融合的蜕变测试

$$\varphi_{concat} = (x > 0 \wedge x > 1) \wedge (y < 0 \wedge y < 1)$$

|         |          |
|---------|----------|
| $x = 2$ | $y = -2$ |
|---------|----------|

$$\varphi_{fused} = (x > 0 \wedge (z - y) > 1) \wedge ((z - x) < 0 \wedge y < 1) \text{ SAT}$$

$$z = x + y$$

$$x = z - y \quad y = z - x$$

基于近似枚举的蜕变测试

$$\varphi_1 \rightarrow ( \boxed{\phantom{0}} \vee \boxed{p_1} \vee \boxed{\phantom{0}} ) \wedge ( \boxed{\phantom{0}} \vee \boxed{\phantom{0}} \vee \boxed{q_1} ) \wedge \downarrow \varphi_2 \leftarrow ( \boxed{\phantom{0}} \vee \boxed{p_2} \vee \boxed{\phantom{0}} ) \wedge ( \boxed{\phantom{0}} \vee \boxed{\phantom{0}} \vee \boxed{q_2} ) \wedge$$



# 蜕变测试导引: 课程小结

从测试预言问题到蜕变测试

面向SMT求解器的蜕变测试

开放问题: 蜕变关系在测试用例生成中的作用?



# 课后练习：调研其它应用

## Finding Bugs in Database Systems via Query Partitioning

MANUEL RIGGER, Department of Computer Science, ETH Zurich, Switzerland  
ZHENDONG SU, Department of Computer Science, ETH Zurich, Switzerland

## Metamorphic Testing of Deep Learning Compilers

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ZHIBO LIU, The Hong Kong University of Science and Technology, China  
YUANYUAN YUAN, The Hong Kong University of Science and Technology, China  
QI PANG, The Hong Kong University of Science and Technology, China  
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• • •

## Metamorphic Testing: A Review of Challenges and Opportunities

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HUAI LIU, Victoria University of Wellington,  
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DAVE TOWEY, The University of Sheffield,  
T. H. TSE, The University of Sheffield,  
ZHI QUAN ZHENG, The University of Sheffield

Metamorphic testing is a test oracle determines whether a test execution reveals a fault, often by comparing the observed program output to the expected output. This is not always practical, for example when a program's input-output relation is complex and difficult to capture formally. Metamorphic testing provides an alternative, where correctness is not determined by checking an individual concrete output, but by applying a transformation to a test input and observing how the program output "morphs" into a different one as a result. Since the introduction of such metamorphic relations in 1998, many contributions on metamorphic testing have been made, and the technique has seen successful applications in a variety of domains, ranging from web services to computer graphics. This article provides a comprehensive survey on metamorphic testing. It summarizes the research results and application areas, and analyses common practice in empirical studies of metamorphic testing as well as the main open challenges.

Index Terms—Metamorphic testing, oracle problem, survey

### 1 INTRODUCTION

SOFTWARE testing is an essential but costly activity applied during software development to detect faults in programs. Testing consists of executing a program with test inputs, and to detect faults there needs to be some procedure by which testers can decide whether the output of the program is correct or not, a so-called *test oracle* [1]. Often, the test oracle consists of comparing an expected output value with the observed output, but this may not always be feasible. For example, consider programs that produce complex output, like complicated numerical simulations, or code generated by a compiler—predicting the correct output for a given input and then comparing it with the observed output may be non-trivial and error-prone. This problem is referred to as the *oracle problem* and it is recognised as one of the fundamental challenges of software testing [1], [2], [3], [4].

Metamorphic testing [5] is a technique conceived to alleviate the oracle problem. It is based on the idea that often it is simpler to reason about relations between outputs of a program, than it is to fully understand or formalise its input-output behaviour. The prototypical example is that of a program that computes the sine function: What is the exact value of  $\sin(12)$ ? Is an observed output of  $-0.5365$  correct? A mathematical property of the sine function states that  $\sin(x) = \sin(\pi - x)$ , and we can use this to test whether  $\sin(12) = \sin(\pi - 12)$  without knowing the concrete values of either sine calculation. This is an example of a *metamorphic relation*: an input transformation that can be used to generate new test cases from existing test data, and an output relation, that compares the outputs produced by a pair

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1. Note that 86 out of the 119 papers reviewed in our survey were published in 2009 or later.

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