HW #7 Ryan Coyne

3.10) (a) $\Delta s = \frac{Q}{T}$

$Q = 30g \cdot 333\,J/g = 9990\,J$

$\Delta s = \frac{9990\,J}{273\,K}$

$= 36.6\,J/K$

(b) $ds = \frac{Q}{T} = \frac{c_v}{T}dT$

$\Delta s = \int_{T_i}^{T_f} \frac{c_v}{T} dT = \int_{273}^{298} 4.2\,J/g \cdot 30g \cdot \frac{1}{T} dt$

$= 126 \ln T \Big|_{273}^{298} = 11.0\,J/K$

(c) $9990\,J + 4.2 \cdot 30 \cdot 25 = 13140\,J$

$\Delta s = \frac{-13140\,J}{298\,K} = -44.1\,J/K$

(d) $\Delta s_{net} = 36.6 + 11.0 - 44.1 = 3.5\,J/K$

3.14) $C_V = aT + bT^3$

$\Delta s = \int_0^T (a + bT^2)\,dT$

$= \left(aT + \frac{b}{3}T^3\right)\Big|_0^T$

$= aT + \frac{b}{3}T^3$

$S(1) = 0.00135 + \frac{2.48 \times 10^{-5}}{3} = 0.00136\,J/K$

$S(10) = 0.00135 \cdot 10 + \frac{2.48 \times 10^{-5}}{3} \cdot 10^3 = 0.0218\,J/K$

3.25) $\Omega \approx \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N$

(a) $s = k \ln \Omega$

$= k \ln\left(\left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N\right)$

$= kq \ln\left(\frac{q+N}{q}\right) + kN \ln\left(\frac{q+N}{N}\right)$

(b) $\frac{1}{T} = \frac{\partial s}{\partial U} = \frac{\partial s}{\partial q}\frac{\partial q}{\partial U} = \frac{1}{\varepsilon}\frac{\partial s}{\partial q}$

$= \frac{1}{\varepsilon} k \ln\left(1 + \frac{N}{q}\right)$

$T = \frac{\varepsilon}{k \ln\left(1 + \frac{N}{q}\right)}$

(c) $\ln\left(1+\frac{N\varepsilon}{U}\right) = \frac{\varepsilon}{kT}$

$1+\frac{N\varepsilon}{U} = e^{\varepsilon/kT}$

$U = \frac{N\varepsilon}{e^{\varepsilon/kT}-1}$

$C = \frac{\partial U}{\partial T} = \frac{N\varepsilon}{\left(e^{\varepsilon/kT}-1\right)^2} e^{\varepsilon/kT} \cdot \frac{\varepsilon}{k} \cdot \left(-\frac{1}{T^2}\right)$

$= \frac{N\varepsilon^2}{kT^2} \frac{e^{\varepsilon/kT}}{\left(e^{\varepsilon/kT}-1\right)^2}$

(d) $\lim_{T\to\infty} \frac{\varepsilon}{kT} = 0$

$e^{\varepsilon/kT} \approx 1+\frac{\varepsilon}{kT}$

$C = \frac{N\varepsilon^2}{kT^2} \frac{1+\varepsilon/kT}{\varepsilon/kT} = Nk\left(1+\frac{\varepsilon}{kT}\right) \approx Nk$