Homework 2

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Theorem 1: Let $(G, \circ)$ be a group, and let $g, h \in G$. Then

$$(g \circ h)^{-1} = h^{-1} \circ g^{-1}.$$

PROOF: First, we compose $(g \circ h)^{-1}$ with $(g \circ h)$. This gives us,

$$(g \circ h)^{-1} \circ (g \circ h) = e.$$

Next compose $g^{-1} \circ h^{-1}$ with $(g \circ h)$. This gives us

$$
\begin{aligned}
(h^{-1} \circ g^{-1}) \circ (g \circ h) &= h^{-1} \circ (g^{-1} \circ g) \circ h \\
&= h^{-1} \circ h \\
&= e.
\end{aligned}
$$

Now, since $(g \circ h)^{-1} \circ (g \circ h) = (h^{-1} \circ g^{-1}) \circ (g \circ h)$, it follows that $(g \circ h)^{-1} = h^{-1} \circ g^{-1}$. ∎