Homework #6

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3.1) $T = \left(\frac{\partial S}{\partial U}\right)^{-1}$

$= \frac{\partial U}{\partial S}$

$\approx \frac{2\varepsilon - 0\varepsilon}{10.7 - 0}$

$= \frac{2}{10.7}\frac{\varepsilon}{k}$

$= 0.186\frac{\varepsilon}{k}$

3.3) Energy is transferred from object B to object A until the slope of of the graph of each is equal at the current energy of each.

3.5) $\Omega(N, q) \approx \frac{(q+N)!}{q!\,N!}$

Let $q \ll N$.

$S = k \ln \Omega$

$= k\left(\ln(q+N)! - \ln q! - \ln N!\right)$

$\approx k\left((q+N)\ln(q+N) - q - N - q\ln q + q - N\ln N + N\right)$

$= k\left((q+N)\ln(q+N) - q\ln q - N\ln N\right)$

$= k\left((q+N)\ln\left(N\left(1+\frac{q}{N}\right)\right) - q\ln q - N\ln N\right)$

$\approx k\left((q+N)\ln N + (q+N)\frac{q}{N} - q\ln q - N\ln N\right)$

$= k\left(q\ln N + N\ln N + \frac{q^2}{N} + q - q\ln q - N\ln N\right)$

$= k\left(q + q\ln\frac{N}{q}\right)$

$= kq\left(\ln N - \ln q + 1\right)$

$= \frac{kU}{\varepsilon}\left(\ln N - \ln U + \ln\varepsilon + 1\right)$

$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{k}{\varepsilon}\left(\ln N - \ln U + \ln\varepsilon + 1\right) + \frac{kU}{\varepsilon}\left(-\frac{1}{U}\right)$

$= \frac{k}{\varepsilon}\ln\frac{N\varepsilon}{U} + \frac{k}{\varepsilon} - \frac{k}{\varepsilon}$

$= \frac{k}{\varepsilon}\ln\frac{N\varepsilon}{U}$

$$\frac{\varepsilon}{kT} = \ln \frac{N\varepsilon}{U}$$

$$\frac{N\varepsilon}{U} = e^{\varepsilon/kT}$$

$$U = N\varepsilon e^{-\varepsilon/kT}$$

3.8) $$C_V = \frac{\partial U}{\partial T}$$

$$= \frac{N\varepsilon^2}{kT^2} e^{-\varepsilon/kT}$$