Notes on "The Calabi-Yau Landscape"

Ryan Coyne

1 Prologue

- Σ - Surface
- $g(\Sigma)$ - Genus of the surface, i.e. the number of holes.
- $\chi(\Sigma) = 2 - 2g(\Sigma)$ - Euler characteristic
  - Signed alternating sum of the number of independent objects in each dimension. For example, a cube drawn on S^2 has 8 vertices, 12 edges and 6 faces, thus $\chi(\Sigma) = 8 - 12 + 6 = 2$
  - Betti number, b_i, counts the number of independent algebraic cycles in dimension i.
  - $\chi(\Sigma) = \sum_{i=0}^{2} b_i \ , \quad b_i = \text{rk}(H_i(\Sigma))$
  - $\chi(\Sigma) = \frac{1}{2\pi} \int_\Sigma R$
    - R - Ricci curvature
  - $\chi(\Sigma) > 0$: Spherical geometry. Positive curvature
  - $\chi(\Sigma) = 0$: flat/torus geometry. Ricci flat.
  - $\chi(\Sigma) < 0$: hyperbolic geometry. Negative curvature
-