Homework #3

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1) Storing information requires energy that energy is not itself stored. For example if I use the location of an object on my desk to convey information, I could move it from one location to another to store information. That movement requires energy, but the object has no more energy now than it did before. However, you laptop is heavier when it is fully charged. When charged there is energy stored in the potential difference of the battery.

2) Based on the equation

$$v^2 = \frac{c^2}{E^2}\left(E^2 - m_0^2 c^4\right)$$

as derived in my answer for problem 7. If rest mass, m_0, is zero, then $v^2 = \frac{c^2}{E^2}(E^2)$ and therefore $v = c$.

3) Einstein made the assumptions that interial mass and gravitation mass are equivalent, and that the laws of nature have the same form for any observer in any frame of reference regardless of acceleration.

4. (a) $p = 1.9\, p_c$

$p = \gamma m_0 v$

$p_c = m_0 v$

$\gamma = 1.9$

$1.9 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$1.9^2(1 - v^2/c^2) = 1$

$1 - \frac{v^2}{c^2} = \frac{1}{1.9^2}$

$1 - 1.9^{-2} = \frac{v^2}{c^2}$

$v = c\sqrt{1-1.9^{-2}}$

$= 2.59 \times 10^8\, m/s$

(b) The result would not change becuase there is no dependance on mass or charge.

5. $q = e$

$[q][B][R] = e\, T\, m$

$= e \cdot \frac{kg\, m}{s^2 A}$

$[p] = \frac{MeV}{c}$

$= \frac{MeV \cdot s}{3 \times 10^8\, m}$

$= e \cdot \frac{V \cdot s}{300\, m}$

$= e \cdot \frac{kg\, m}{300\, s^2 A}$

6. $v_p = 0.95c$

(a) $E_0 = m_0 c^2$

$= 938\, MeV$

(b) $p = \gamma m_0 c$

$\gamma = (1 - 0.95^2)^{-1/2}$

$= 3.2$

$$E^2 = p^2c^2 + m_0^2c^4$$
$$= \gamma^2 m_0^2 c^4 + m_0^2 c^4$$
$$= (1+\gamma^2)\, m_0^2 c^4$$
$$= (1 + 10.3) \cdot E_0^2$$
$$= 11.3 \cdot (938\,MeV)^2$$
$$E = 3004\,MeV$$

(c) $m = \gamma m_0$
$$= 3.2 \cdot 938\,MeV/c^2$$
$$= 3004\,MeV/c^2$$

(d) $E = K + m_0c^2$
$$K = 3004\,MeV - 938\,MeV$$
$$= 2066$$

7. (a) $K = 5\,E_0$
$$= 5\,m_0c^2$$
$$= 5 \cdot 0.510\,MeV$$
$$= 2.555\,MeV$$
$$E = (2.555 + 0.511)\,MeV$$
$$= 3.066\,MeV$$

(b) $E = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}$
$$E^2 = \frac{m_0^2c^4}{1 - v^2/c^2}$$
$$E^2 - \frac{v^2}{c^2}E^2 = m_0^2c^4$$
$$\frac{v^2}{c^2}E^2 = E^2 - m_0^2c^4$$
$$v^2 = \frac{c^2}{E^2}\left(E^2 - m_0^2c^4\right)$$
$$v = \frac{c}{E}\sqrt{E^2 - m_0c^4}$$
$$= \frac{c}{E}\sqrt{(3.066\,MeV)^2 - (0.511\,MeV)^2}$$
$$= \frac{3.023}{3.066} \cdot c$$
$$= 0.986\,c$$

8. $K = 50\ GeV$

$$E = K + m_0c^2$$

$$= 50\ GeV + 938\ MeV$$

$$= 50.938\ GeV$$

$$E^2 = p^2c^2 + m_0^2c^4$$

$$p^2 = \frac{E^2 - m_0^2c^4}{c^2}$$

$$= \frac{(50.934\ GeV)^2 + (0.938\ MeV)^3}{c^2}$$

$$p^2 = 2595.56\ GeV/c^2$$

$$p = 50.947\ GeV/c$$

$$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p^2 - \frac{v^2}{c^2}p^2 = m_0^2 v^2$$

$$p^2 = v^2\left(\frac{p^2}{c^2} + m_0^2\right)$$

$$v^2 = \frac{p^2}{m_0^2 + \frac{p^2}{c^2}}$$

$$= 0.9997c^2$$

$$v = 0.9998c$$

9. (a) $v^2 = \frac{p^2}{m_0^2 + p^2/c^2}$

$$= \frac{E^2 - m_0^2c^4}{m_0^2c^2 + \frac{E^2 - m_0^2c^4}{c^2}}$$

$$= \frac{1.6\times10^5 E_0^2 - E_0^2}{m_0E_0 + \frac{1.6\times10^5 E_0^2 - E_0^2}{c^2}}$$

$$= \frac{1.6\times10^5 m_0^2c^4 - m_0^2c^4}{\cancel{m_0^2c^2} + 1.6\times10^5 m_0^2c^2 - \cancel{m_0^2c^2}}$$

$$= \frac{1.6\times10^5 - 1}{1.6\times10^5}c^2$$

$$= 0.999994c^2$$

$$v = 0.999997c$$

(b) $E = K + m_0c^2$

$$K = E - m_0c^2$$

$$= 400m_0c^2 - m_0c^2$$

$$= 399.938\ MeV$$

$$= 374\ GeV$$

(c) $m = \frac{E}{c^2}$

$= 400 m_0$

$= 400 \cdot 934 \text{ MeV}$

$= 375 \text{ GeV}$

10) (a) $E = 4.0 \times 10^{26} \text{ J/s} = mc^2/s$

$m/s = \frac{4 \times 10^{26} \text{ J/s}}{(3 \times 10^8 \text{ m/s})^2}$

$= 4.4 \times 10^9 \text{ kg/s}$

(b) $t = 2 \times 10^{30} \text{ kg} / 4.4 \times 10^9 \text{ kg/s}$

$= 4.5 \times 10^{20} \text{ s}$

$= 1.4 \times 10^{13}$ years

11) (a) $K = 50 \text{ KeV}$

$\gamma m_0 c^2 = K + m_0 c^2$

$\gamma = \frac{K + m_0 c^2}{m_0 c^2}$

$= \frac{50 \text{ keV} + 511 \text{ keV}}{511 \text{ keV}}$

$= 1.098$

$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2}$

$v = c\sqrt{1 - \frac{1}{\gamma^2}}$

$= 0.41c$

(b) $K = 50 \text{ keV} = \frac{1}{2} mv^2$

$v = \sqrt{\frac{100 \text{ keV}}{511 \text{ keV}/c^2}}$

$= 0.44c$

(c) $\frac{0.03}{0.41} = 0.073$

The relative error is 7.3% and given that CRT televisions were invented shortly after Einstein discovered special relativity it seems unlikely that it was taken into account. Additionally it seems likely that such a small difference wouldn't be noticed by the naked eye.

12. $^{226}Ra \rightarrow {}^{222}Rn + {}^{4}He$

$m = 226.0254\,u - 222.0175\,u - 4.0026\,u = 0.0053\,u$

$= 0.0053\,u \cdot \frac{931.494102\ MeV}{u\,c^2}$

$= 4.94\ MeV/c^2$

13. $n \rightarrow p + e^- + \bar{\nu}$

$c^2 m_n = 939.565\ MeV$

$c^2 m_p + c^2 m_e + K = 938.272\,MeV + 0.511\ MeV + (0.781 \pm 0.005)\ MeV$

$= (939.564 \pm 0.005)\ MeV$

$(939.565 - 939.564)\ MeV = 0.001\ MeV$

$0.001\ MeV < 0.005\ MeV$

14. $a = 9.8\ m/s^2$

$v(t) = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}}$

$x(t) = \int v\,dt = \int \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}}\,dt$

$u = \frac{a}{c}t$, $du = \frac{a}{c}dt$

$dt = \frac{c}{a}du$, $t = \frac{c}{a}u$

$x(t) = \int \frac{at}{\sqrt{1+u^2}} \frac{c}{a}du$

$= \frac{c^2}{a} \int \frac{u}{\sqrt{1+u^2}} du$

$= \frac{c^2}{a}\sqrt{1+u^2}$

$= \frac{c^2}{a}\sqrt{1 + \frac{a^2t^2}{c^2}}$

$x_{1/2} = 2.1\ l_y$

$x_{1/2}^2 = \frac{c^4}{a^2} + c^2 t_{1/2}^2$

$t_{1/2}^2 = \left(x_{1/2}^2 - \frac{c^4}{a^2}\right)/c^2$

$t_{1/2}^2 = \left(\frac{2.1 l_y}{c}\right)^2 - \left(\frac{c}{9.81 m/s^2}\right)^2$

$= (2.1y)^2 - (3.058 \times 10^7\ s)^2$

$= (2.1y)^2 - (0.9697y)^2$

$= 3.4697\,y^2$

$t_{1/2} = 1.86\,y$

$t_f = 3.73\ y$

15. $v_f = 0.95c$

$E = \gamma m_0 c^2 = K + m_0 c^2$

$K = m_0 c^2 (\gamma - 1)$

$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

$= (1 - 0.95^2)^{-1/2}$

$= 3.20$

$K = 2.88 \times 10^{17}\, m_0$

Elon would never reach Alpha Centauri because it would require 3.20 times the mass of the rocket to be perfectly converted to kinetic energy. to reach 0.95c.