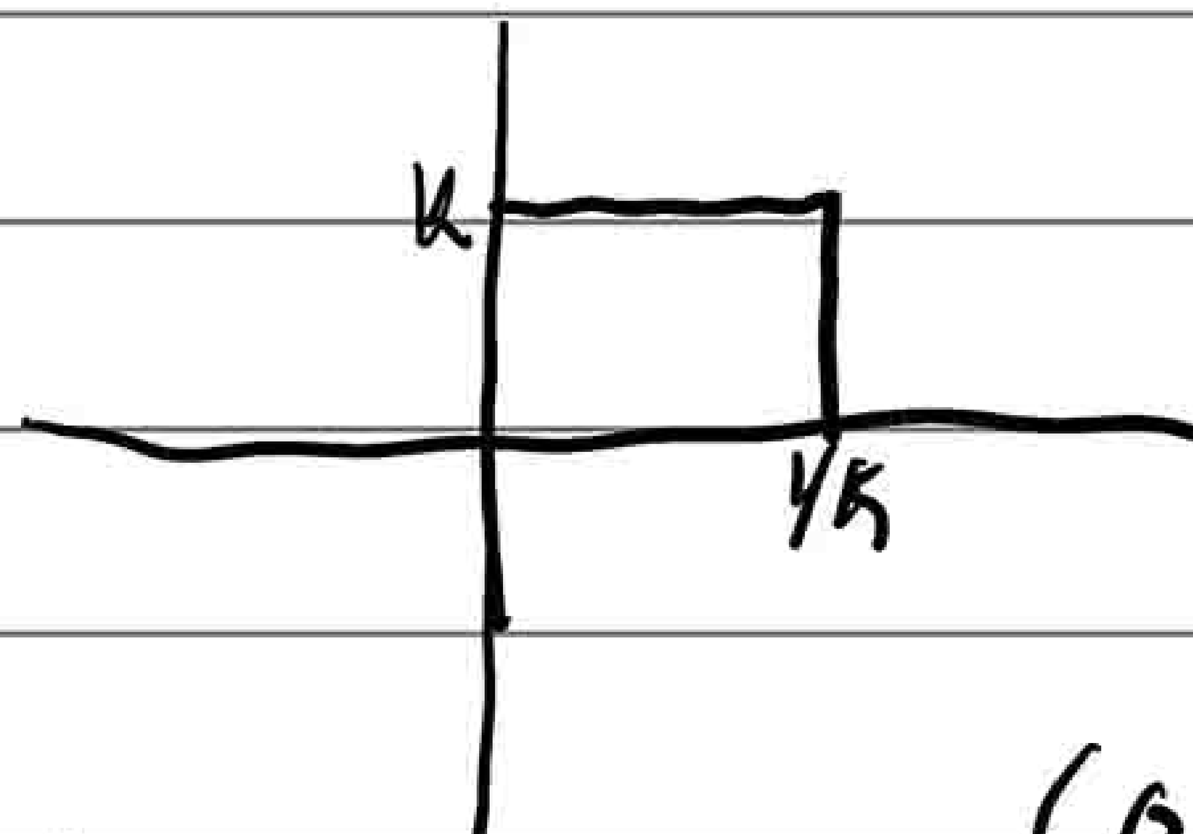
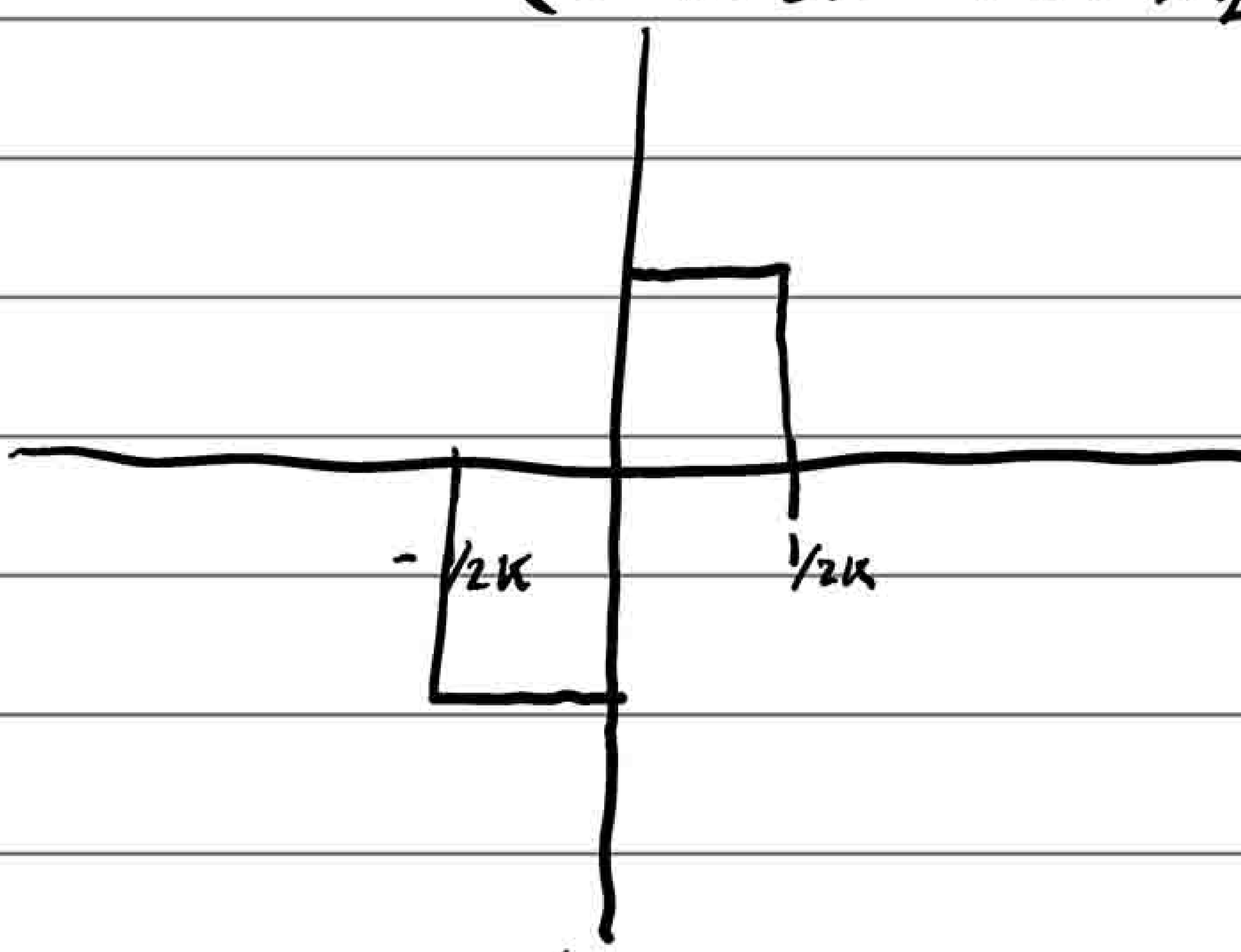
Sec. 3 Exercises

1. (a) $w_k(x) = \begin{cases} k & x \le 1/k \\ 0 & 0 \ge x,\ x \ge 1/k \end{cases}$



$\int_{-\infty}^{\infty} w(x)\,dx = 1$

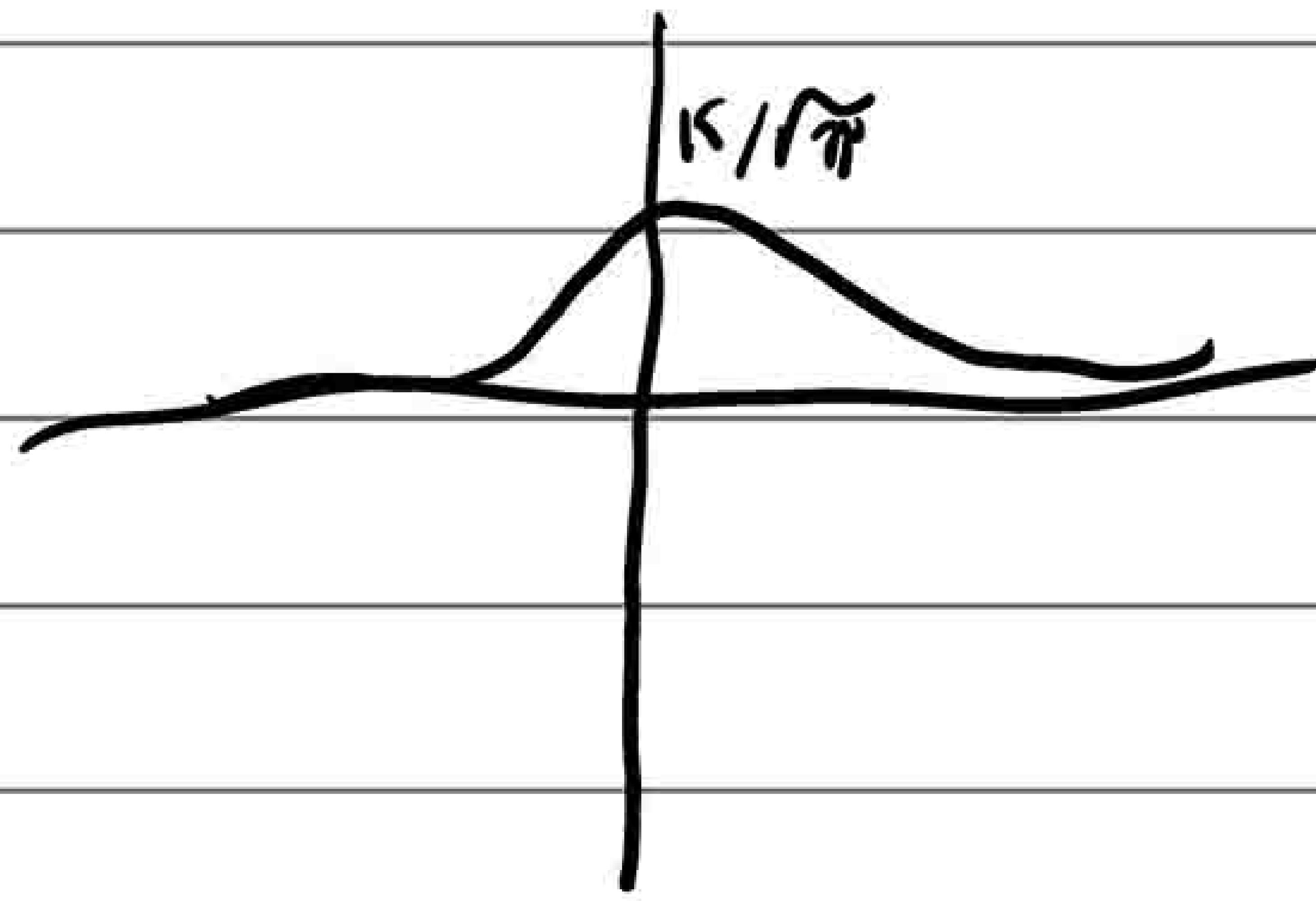
(b) $w_k(x) = \begin{cases} 0 & |x| > \frac{1}{2k} \\ 4k^2 x + 2k & -\frac{1}{2x} \le x \le 0 \\ -4k^2 x + 2k & 0 \le x \le \frac{1}{2k} \end{cases}$



$\int_{-\infty}^{\infty} w(x)\,dx = \left(4x + 2 - 4x + 2\right)\frac{1}{2}$

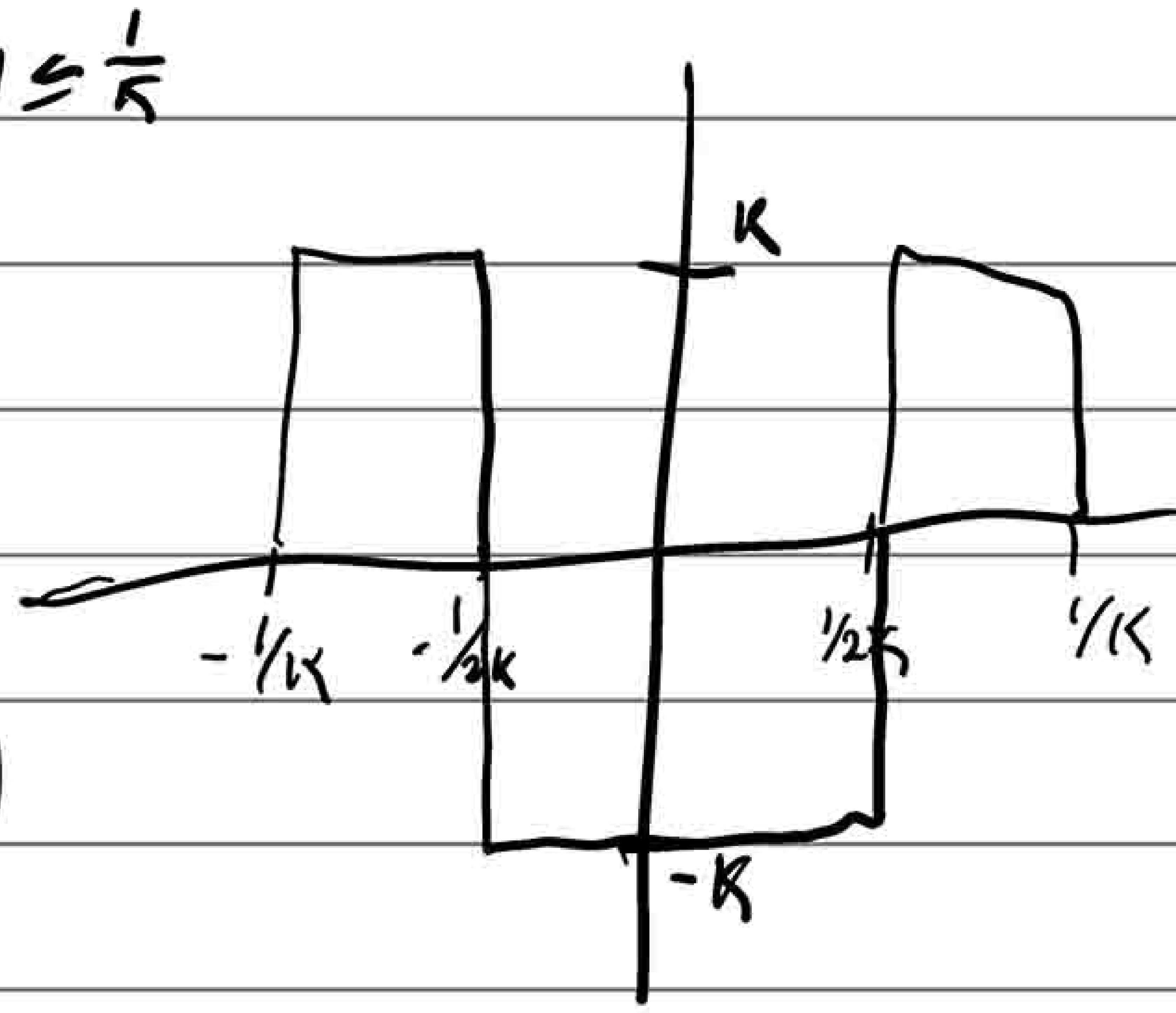
$= 2$

(c) $w_K(x) = Ke^{-K^2x^2}/\sqrt{\pi}$



$$\int_{-\infty}^{\infty} w(x)\,dx = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-x^2}\,dx$$
$$= \frac{1}{\sqrt{\pi}}\cdot\sqrt{\pi}$$
$$= 1$$

(d) $$w_K(x) = \begin{cases} -K, & |x| < \frac{1}{2K} \\ 2K, & \frac{1}{2K} \le |x| \le \frac{1}{K} \\ 0, & |x| > \frac{1}{K} \end{cases}$$



$$\lim_{K\to\infty}\int_{-\infty}^{\infty} w_K(x)\,dx = \lim_{K\to\infty}\left(\frac{3}{K}-1\right)$$
$$= 0$$

2.

6.(a) $\int_{-\infty}^{\infty} e^{x}\delta(x) f(x)\, dx = \int \delta(x) \overbrace{g(x)}^{e^{x}f(x)}\, dx$

$= g(0)$

$= e^{0} f(0)$

$= f(0)$

$= \int_{-\infty}^{\infty} \delta(x) f(x)\, dx$

$\therefore e^{x}\delta(x) = \delta(x)$

(b) let $g(x) = x f(x)$

$\int_{-\infty}^{\infty} x\delta'(x) f(x)\, dx = \int_{-\infty}^{\infty} \delta'(x) g(x)\, dx$

$= g'(0)$

$= 0 \cdot f'(0) + f(0)$

$= f(0)$

Part I. Sec 4 Exercises.

1. Not clear on what this means.

2. verify by direct diff.

$$u(x) = \int_0^1 G\phi\, d\xi = (x-1)\int_0^x \xi\phi\, d\xi + x\int_x^1 (\xi-1)\phi(\xi)\, d\xi$$

does satisfy:

$$u''(x) = \phi(x) \qquad u(0) = u(1) = 0$$

$$u'(x) = \frac{d}{dx}\left((x-1)\int_0^x \xi\phi(\xi)\, d\xi\right) + \frac{d}{dx}\left(x\int_x^1 (\xi-1)\phi(\xi)\, d\xi\right)$$

$$= \int_0^x \xi\phi(\xi)\, d\xi + \cancel{x(x-1)\phi(x)} + \int_x^1 (\xi-1)\phi(\xi)\, d\xi \cancel{- x(x-1)\phi(x)}$$

$$= \int_0^x \xi\phi(\xi)\, d\xi + \int_x^1 (\xi-1)\phi(\xi)\, d\xi$$

$$= x\phi(x) - (x-1)\phi(x)$$

$$= \phi(x)$$