Computational Assignment #2

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The multiplicity of two large interacting Einstein solids, with total energy q, can be approximated by the equation $\Omega = \left(\frac{e}{N}\right)^{2N}(q_A q_B)^N$, where e is Euler's number, N is the number of oscillators in the solid, q_A is the total energy of solid A and q_B is the total energy of solid B. The energies of each solid are related by the equation $q_A = q - q_B$. If we let $z = q_A/q$, then, neglecting a constant portion, the multiplicity function is $[4z(1-z)]^N$, which is plotted in Figure 1. The Gaussian function, $\Omega = \Omega_{max} \cdot e^{-N(2x/q)^2}$, is plotted for $\Omega_{max} = 1$ in Figure 2. Both functions are initially plotted for $N = 1, 10, 100, 1000, 10000$.

Figure 1: Multiplicity Function

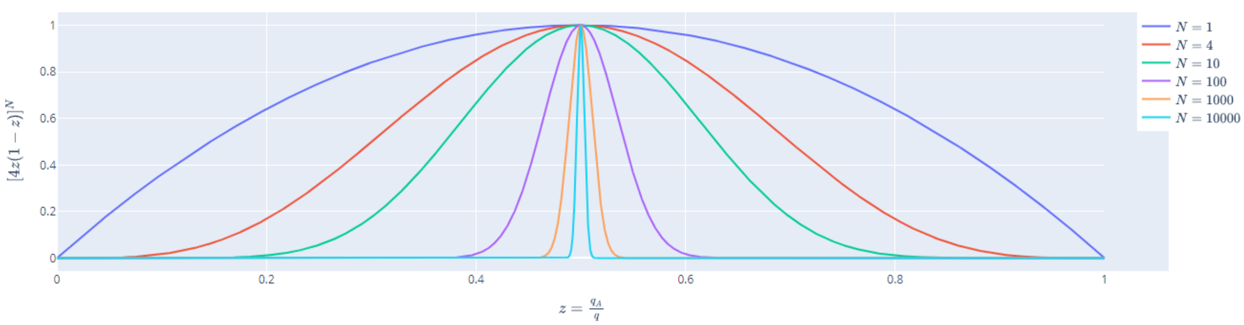
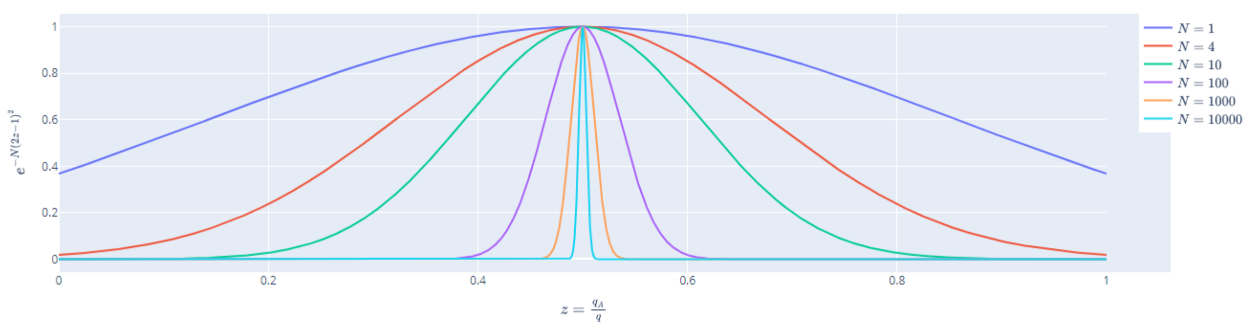


Figure 2: Gaussian Function



If the difference in the full width at half maximum of the two plots is less than 5%, then we say that the two distributions are the same. Of the given values of N, the plots are the same for $N = 10, 100, 1000, 10000$. The smallest integer value of N for which they are the same is $N = 4$. The normalized distribution, or probability density function, is of the multiplicity is $\frac{\Gamma(2+2N)}{4^N(\Gamma(N))^2}\left[4z(1-z)\right]^N$ and the probability density function for the gaussian is $\frac{2\sqrt{N}}{\sqrt{\pi}\mathrm{erf}(\sqrt{N})}e^{-N(2x/q)^2}$.