

# Problem Set 9

Ryan Coyne

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**Result 1:** Find a formula for

$$1 + 4 + 7 + \cdots (3n - 2)$$

for positive integers then verify your formula by mathematical induction.

PROOF: ■

**Result 2:** Prove the following inequality for every positive integer  $n$ :

$$2! \cdot 4! \cdot 6! \cdots (2n)! \geq ((n+1)!)^n.$$

PROOF: We proceed by induction. Since  $(2 \cdot 1)! = (2!)^1$ , the statement is true when  $n = 1$ . Assume that

$$2! \cdot 4! \cdot 6! \cdots (2k)! \geq ((k+1)!)^k$$

for some integer,  $k$ . We show,

$$2! \cdot 4! \cdot 6! \cdots (2k)! \cdot (2(k+1)) \geq ((k+2)!)^{k+1}.$$

Now observe that,

$$2! \cdot 4! \cdot 6! \cdots (2k)! \cdot (2(k+1))! \geq ((k+1)!)^k \cdot (2(k+1))!.$$

We divide by  $2! \cdot 4! \cdot 6! \cdots (2k)!$ ,

$$\begin{aligned} (2(k+1))! &\geq ((k+1)!)^k \frac{(2(k+1))!}{2! \cdot 4! \cdot 6! \cdots (2k)!} \\ &\geq ((k+1)!)^k (2k+2) \end{aligned}$$

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