Homework 3

Ryan Coyne

February 11, 2024

Question 1

Let $(G, \circ)$ be a group, $g \in G$ and $n \in \mathbb{Z}$. We define the notation

$$g^0 := e$$
$$g^n := \underbrace{g \circ g \circ \cdots \circ g}_{n \text{ times}}$$
$$g^{n-1} := \underbrace{g^{-1} \circ g^{-1} \circ \cdots \circ g^{-1}}_{n \text{ times}}.$$

a) Show that
$$\langle g \rangle := \{g^n | n \in \mathbb{Z}\}$$
is a subgroup of G.

b) Consider the group $(S_5, 0)$. Determine the elements of $\langle (12)(345) \rangle$.

Question 2

Consider the group $(S_4, \circ)$. Find one subgroup of S_4 with:

a) 2 elements,

b) 3 elements,

c) 4 elements,

d) 6 elements,

e) 8 elements,

f) 12 elements.

You do not need to justify that your answers are subgroups.