Problem Set 7

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1. Prove: the product of an irrational number and a nonzero rational number is irrational. Proof: Suppose that $xy = z$, for some $x, z \in \mathbb{Q}$ and $y \in \mathbb{R} - \mathbb{Q}$. By definition, $x = \frac{a}{b}$ and $z = \frac{a'}{b'}$, for some $a, a', b, b' \in \mathbb{Z}$, with $a, a', b, b' \neq 0$. Then, $\frac{a}{b}y = \frac{a'}{b'}$, and so, $y = \frac{a'b}{b'a}$. Now, $a'b$ and $b'a$, are integers, and so y must be rational by definition. This contradicts the initial assumption that y is irrational, and thus, the product of an irrational and a nonzero rational number, cannot be rational. The product must therefore be rational. ■

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