

Experiment Activity 2: Measuring the Electron Charge-to-Mass Ratio

Data & Measurements

Problem 1. Data Collection

Helmholtz Coil Measurements	
Internal Coil Diameter D_{int}	External Coil Diameter D_{ext}
28.5 cm	31.5 cm
28.5 cm	31.0 cm
28.2 cm	31.3 cm
28.3 cm	31.0 cm

Average all of your coil diameter measurements together and determine the total coil diameter uncertainty (from the Type-A and Type-B uncertainties in your measurements above). Use this average coil diameter, D , and total coil diameter uncertainty, δD , to compute the coil radius, $R = D/2$, and the coil radius uncertainty, $\delta R = \delta D/2$. Give your result for $R \pm \delta R$ below. You will need this value in calculations to follow.

Helmholtz Coil Radius: $R \pm \delta R =$

Suggestion: It will probably be most efficient to first measure and record the coil currents and beam loop diameters (with uncertainties), then type up these measurements into a `*.txt` or `*.csv` file, then load those values into a Python program and write code to compute the magnetic field and charge-to-mass ratio values (with uncertainties).

Accelerating Voltage:
 $V \pm \delta V = (200 \pm 2) \text{ V}$

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Accelerating Voltage:
 $V \pm \delta V = (300 \pm 2) \text{ V}$

Coil Current $I \pm \delta I$	Beam Loop Diameter $d \pm \delta d$	Magnetic Field $B \pm \delta B$	Charge-to-Mass Ratio $e/m_e \pm \delta (e/m_e)$
1.30	11		
1.35	10.7		
1.40	10.2		
1.45	9.9		
1.50	9.5		
1.55	9.2		
1.60	8.9		
1.65	8.6		
1.70	8.4		

Experiment Data (3)

Accelerating Voltage:

$$V \pm \delta V = 400 \text{ V}$$

Coil Current $I \pm \delta I$	Beam Loop Diameter $d \pm \delta d$	Magnetic Field $B \pm \delta B$	Charge-to-Mass Ratio $e/m_e \pm \delta (e/m_e)$
1.55	10.6		
1.60	10.1		
1.65	9.8		
1.70	9.5		
1.75	9.3		
1.80	9.0		
1.85	8.7		
1.9	8.5		
1.95	8.4		
2.0	8.1		

Experiment Data (4)

Accelerating Voltage:
 $V \pm \delta V = 500\text{V}$

Coil Current $I \pm \delta I$	Beam Loop Diameter $d \pm \delta d$	Magnetic Field $B \pm \delta B$	Charge-to-Mass Ratio $e/m_e \pm \delta (e/m_e)$
1.85	9.3		
1.90	9.1		
1.95	9.0		
2.00	8.9		
2.05	8.8		
2.10	8.6		
2.15	8.5		
2.20	8.3		
2.25	8.1		

Problem 2. Derivation of Magnetic Field Strength

Derive the expression for the magnetic field due to the Helmholtz coils:

- (a.) Use the Biot-Savart Law to derive the general expression for the magnetic field at a point $P = (x, 0, 0)$ along the axis of symmetry of a circular current-carrying coil with N turns which lies in the y - z -plane and whose center is at the origin $\mathcal{O} = (0, 0, 0)$. Your result should be

$$\vec{B} = \frac{\mu_0 N I R^2}{2(x^2 + R^2)^{3/2}} \hat{x}$$

where $\mu_0 = 1.256\,637\,062\,12(19) \times 10^{-6} \text{ H/m} \approx 4\pi \times 10^{-7} \text{ H/m}$ is the magnetic constant (a.k.a., vacuum permeability), I is the current flowing through the wire, R is the radius of the coil, x is the distance from the center of the coil along its axis of symmetry to the point at which the field is evaluated, and \hat{x} is the unit vector in the x -direction. Hint: Start by deriving the magnetic field expression due to a single current-carrying loop and modify this expression to get the expression for a solenoid of N turns.

- (b.) Now consider a pair of identical coils connected in series. Think of the coils in a coordinate system such that $x = 0$ and $x = R$ represent the centers of the two coils. For the case where the current through both coils is flowing in the same direction, the general expression for the total magnetic field along the axis of symmetry can be derived using the Biot-Savart law and the superposition principle. Show that this magnetic field is given by,

$$B = \frac{\mu_0 N I R^2}{2} \left(\frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{(R^2 + (R - x)^2)^{3/2}} \right).$$

- (c.) Assume that the current through each coil is flowing in the same direction. Starting from the result of part (b.), show that the magnetic field at $x = R/2$ is given by

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R}.$$

Problem 3. Finding δB

Assuming measured values (with uncertainties) of coil current ($I \pm \delta I$) and coil radius ($R \pm \delta R$), use the uncertainty propagation rules to derive an expression for the uncertainty δB in the magnetic field given by

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R}.$$

Problem 4. Deriving the Mathematical Expression for e/m

In the **Principle of Operation** section of the Charge-to-Mass Ratio Lab Manual, the derivation of the relation for e/m_e is outlined. Work through this derivation outline and fill in the mathematical details (using Newton's laws, Conservation of Energy, doing all the algebra, etc.) to give a full, step-by-step derivation of the relation

$$\frac{e}{m_e} = \frac{2V}{r^2 B^2}$$

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Problem 5. Finding $\delta(e/m)$

Assuming measured values (with uncertainties) of accelerating voltage ($V \pm \delta V$), beam loop radius ($r \pm \delta r$), and magnetic field ($B \pm \delta B$) derive an expression for the uncertainty in e/m_e using the relation

$$\frac{e}{m_e} = \frac{2V}{r^2 B^2}$$

with the uncertainty propagation rules.

Problem 6. Estimating e/m From The Data

Appropriately use, and combine as necessary, the data collected in Problem 1, along with the expressions for $B \pm \delta B$ and $e/m_e \pm \delta(e/m_e)$ that you derived in Problems 2, 3, 4, & 5, to determine an estimated value, with uncertainty, for $e/m_e \pm \delta(e/m_e)$. Show and explain your work.

Problem 7. Determining Measurement Accuracy

According to the National Institute of Standards and Technology (NIST), the accepted value of the electron charge-to-mass ratio is

$$e/m_e = (-1.758\,820\,010\,76 \pm 0.000\,000\,000\,53) \times 10^{11} \text{ C/kg}$$

(See: <https://physics.nist.gov/cgi-bin/cuu/Value?esme>).

(a.) Does the value of e/m_e which you found in Problem 6 agree with the accepted value reported by NIST? What is the percent difference between your value for e/m_e and the accepted value? Show your work!

(b.) What does this tell you about your measurements?

Design Considerations

Problem 8. Improving the Experiment

Use the work you did in the previous problems to determine the dominant source(s) of uncertainty in your measurements. Answer the following:

- Rank your measurement uncertainties from largest to smallest. What was the dominant source of uncertainty?
- Given this information, what could you change about the experimental apparatus to improve your measurements?
- How would you need to modify the apparatus if you replaced the Helmholtz coils which are currently on the device with new coils which have a diameter of 1 *ft* and 300 *turns*?