Homework 5

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1 Question 1

Prove the following theorem:

Theorem 1 *Let H_1 and H_2 be groups, and define*

$$H_1 \times \{e\} := \{(h_1, e) | h_1 \in H_1\} \subset H_1 \times H_2$$

and

$$\{e\} \times H_2 := \{(e, h_2) | h_2 \in H_2\} \subset H_1 \times H_2.$$

Then

a) $H_1 \times \{e\}$ and $\{e\} \times H_2$ are normal subgroups of $H_1 \times H_2$,

b) $(H_1 \times \{e\}) \cap (\{e\} \times H_2) = \{(e, e)\}$, and

c) $(H_1 \times \{e\})(\{e\} \times H_2) = H_1 \times H_2$.

Proof

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2 Question 2

Prove the following theorem:

Theorem 2 *Let G be a group, and H a subgroup of G. Then H is normal if and only if $gH = Hg$ for all $g \in G$.*