Homework 7

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Question 1

Let $G = \mathbb{Z}_4 \times \mathbb{Z}_4$, and let

$$H = \{(0,0), (2,1), (2,3), (0,2)\} \subset G.$$

You may assume H is a normal subgroup. List the elements of G/H, and determine the composition table for G/H.

$$G/H = \{\{(0,0), (2,1), (2,3), (0,2)\},$$
$$\{(0,1), (2,2), (2,0), (0,3)\},$$
$$\{(1,0), (3,1), (3,3), (1,2)\},$$
$$\{(1,1), (3,2), (3,0), (1,3)\}\}$$

$\circ$	H	$(0,1)H$	$(1,0)H$	$(1,1)H$
H	H	$(0,1)H$	$(1,0)H$	$(1,1)H$
$(0,1)H$	$(0,1)H$	H	$(1,1)H$	$(1,0)H$
$(1,0)H$	$(1,0)H$	$(1,1)H$	$(0,1)H$	H
$(1,1)H$	$(1,1)H$	$(1,0)H$	H	$(0,1)H$

Question 2

You are given that there are 8 automorphisms of $G = \mathbb{Z}_2 \times \mathbb{Z}_4$. Find and list all 8 of them.

For all $x \in \mathbb{Z}_2$ and $y \in \mathbb{Z}_4$:
$f_1(x,y) = (x, y)$
$f_2(x,y) = (x, y^2)$
$f_3(x,y) = (x, y^3)$
$f_4(x,y) = (x, y^4)$
$f_5(x,y) = (x^2, y)$
$f_6(x,y) = (x^2, y^2)$
$f_7(x,y) = (x^2, y^3)$
$f_8(x,y) = (x^2, y^4)$