Mechanics Hw 1

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1. $\vec{F}_\alpha = \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \vec{F}_\alpha^{ext} = \dot{\vec{P}}_\alpha$

$\dot{\vec{P}} = \sum_\alpha \dot{\vec{P}}_\alpha$

$= \sum_\alpha \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \sum_\alpha \vec{F}_\alpha^{ext}$

$N=3$: $\dot{\vec{P}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{31} + \vec{F}_{32} + \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}$

$= \vec{F}_{12} + \vec{F}_{13} - \vec{F}_{12} + \vec{F}_{23} - F_{13} - F_{23} + \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}$

$= \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}$

2. $\vec{a} \cdot \vec{b} = |a||b| \cos(\alpha - \beta)$

$\vec{a} \cdot \vec{b} = (|a| \cos\alpha)(|b| \cos\beta) + (|a| \sin\alpha)(|b| \sin\beta)$

$|a||b| \cos(\alpha - \beta) = |a||b| (\cos\alpha \cos\beta + \sin\alpha \sin\beta)$

$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$\vec{a} \times \vec{b} = |a||b| \sin(\beta - \alpha) \hat{k}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & 0 \\ b_x & b_y & 0 \end{vmatrix} = \hat{k}(a_x b_y - a_y b_x)$

$= |a||b| (\cos\alpha \sin\beta - \sin\alpha \cos\beta)$

$\sin(\beta - \alpha) = \cos\alpha \sin\beta - \sin\alpha \cos\beta$

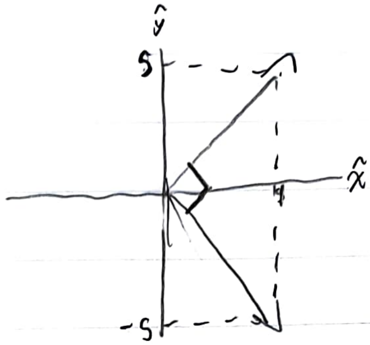
$\sin(\beta - \alpha) = -\sin(\alpha - \beta)$

$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

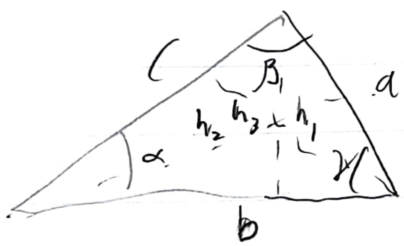
3. $\vec{b} \cdot \vec{c} = 0$

$= 1 - s^2$

$s = \pm 1$



4.



(a) $A = \frac{1}{2} b h_1 = \frac{1}{2} a h_2 = \frac{1}{2} c h_3$

$h_1 = a \sin\gamma = c \sin\alpha$

$h_2 = c \sin\beta$

$A = \frac{1}{2} ab \sin\gamma = \frac{1}{2} bc \sin\alpha = \frac{1}{2} ac \sin\beta$

$|a \times b| = ab \sin\gamma$

$|b \times c| = bc \sin\alpha$

$|a \times c| = ac \sin\beta$

$A = \frac{1}{2} |a \times b| = \frac{1}{2} |b \times c| = \frac{1}{2} |a \times c|$

(b) $|a \times b| = |b \times c| = |a \times c|$

$ab \sin\gamma = ac \sin\beta$

$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$ab \sin\gamma = bc \sin\alpha$

$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

5.(a) $\hat{x} = \hat{r}\cos\phi \quad \hat{y} = \hat{r}\sin\phi$

$$\hat{r} = \hat{r}(\cos^2\phi + \sin^2\theta)$$
$$= \underbrace{\hat{r}\cos\phi}_{\hat{x}}\cos\phi + \underbrace{\hat{r}\sin\phi}_{\hat{y}}\sin\phi$$
$$= \hat{x}\cos\phi + \hat{y}\sin\phi$$
$$\hat{\phi}\cdot\hat{r} = 0$$
$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

(b) $\dot{\hat{r}} = (-\hat{x}\sin\phi + \hat{y}\cos\phi)\dot{\phi}$

$$= \dot{\phi}\hat{\phi}$$
$$\dot{\hat{\phi}} = (-\hat{x}\cos\phi - \hat{y}\sin\phi)\dot{\phi}$$
$$= -\dot{\phi}\hat{r}$$

6. $\hat{x} = \hat{\rho}\cos\phi \quad \hat{y} = \hat{\rho}\sin\phi \quad \hat{z} = \hat{z}$

$$\hat{\rho} = \hat{\rho}\cos^2\phi + \hat{\rho}\sin^2\phi$$
$$= \hat{x}\cos\phi + \hat{y}\sin\phi$$
$$\hat{z} = \hat{z}$$
$$\hat{\phi} = \hat{z}\times\hat{\rho}$$
$$= -\hat{x}\sin\phi + \hat{y}\cos\phi$$
$$\dot{\hat{\rho}} = \dot{\phi}(\hat{x}\sin\phi + \hat{y}\cos\phi)$$
$$= \dot{\phi}\hat{\phi}$$
$$\dot{\hat{\phi}} = -\dot{\phi}\hat{x}\cos\phi - \dot{\phi}\hat{y}\sin\phi$$
$$= -\dot{\phi}\hat{r}$$
$$\dot{\hat{z}} = 0$$

7. $\frac{df}{dt} = f$

$\int \frac{df}{f} = \int dt$

$\ln f = t + c$

$f = ce^{t}$

8. $f_s = \mu_s m_A g = m_B g$

$m_B = \mu_s m_A$

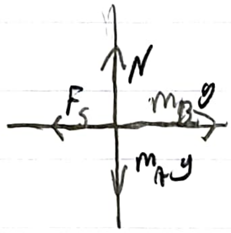
9. $F_{net} = m_B g - f_k$

$= m_B g - \mu_K m_A g$

m/s²

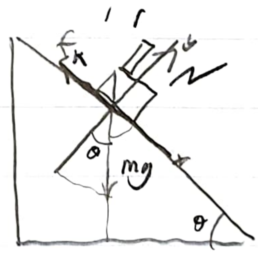
$a = \frac{F_{net}}{m_A + m_B}$

$= \frac{m_B g - \mu_K m_A g}{m_A + m_B}$ m/s²



10. (a)

$a = g\sin\theta - \mu_K g\cos\theta$



(b) $f_s = \mu_s g m_A \cos\theta$

$f_s \geq m_A g \sin\theta - m_B g \sin\theta$

$\mu_s \geq \frac{m_A \sin\theta - m_B \sin\theta}{m_A \cos\theta}$

(c) $N_B = (m_A + m_B) g \sin\theta$

$N_A = m_A g \sin\theta$