Problem Set 10

Ryan Coyne

November 17, 2023

Result 1: Find a formula for

$$1 + 4 + 7 + \cdots (3n - 2)$$

for positive integers then verify your formula by mathematical induction.

PROOF: ■

Result 2: Prove the following inequality for every positive integer n:

$$2! \cdot 4! \cdot 6! \cdots (2n)! \geq ((n+1)!)^n.$$

PROOF: We proceed by induction. Since $(2 \cdot 1)! = (2!)^1$, the statement is true when $n = 1$. Assume that

$$2! \cdot 4! \cdot 6! \cdots (2k)! \geq ((k+1)!)^k$$

for some integer, k. We show,

$$2! \cdot 4! \cdot 6! \cdots (2k)! \cdot (2(k+1)) \geq ((k+2)!)^{k+1}.$$

Now observe that,

$$2! \cdot 4! \cdot 6! \cdots (2k)! \cdot (2(k+1))! \geq ((k+1)!)^k \cdot (2(k+1))!.$$

We divide by $2! \cdot 4! \cdot 6! \cdots (2k)!$,

$$\begin{aligned} (2(k+1))! &\geq ((k+1)!)^k \frac{(2(k+1))!}{2! \cdot 4! \cdot 6! \cdots (2k)!} \\ &\geq ((k+1)!)^k (2k+2) \end{aligned}$$

■

Result 3: Prove that for every real number $x > -1$ and every positive integer n.

$$(1+x)^n \geq 1 + nx.$$

PROOF: ■