Homework 6

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Question 1

Consider the group $G = D_8$. You are given that $H = \{e, r^4, s, sr^4\}$ is a subgroup of D_8.

a) Either show that H is a normal subgroup of D_8, or show that it is not a normal subgroup.
Proof For H to be normal, it is necessary that $gH = Hg$ for all $g \in G$. Now, consider $g = r$:

$$\begin{aligned} rH &= \{r, r^5, rs, rsr^4\} \\ &= \{r, r^5, sr^7, sr^{11}\} \\ &= \{r, r^5, sr^7, sr^3\} \\ Hr &= \{r, r^5, sr, sr^5\}. \end{aligned}$$

Thus, H is not normal because $rH \neq Hr$. ■

b) List all the left cosets of H.
$\{e, r^4, s, sr^4\}$
$\{r, r^5, sr, sr^5\}$
$\{r^2, r^6, sr^2, sr^6\}$
$\{r^3, r^7, sr^3, sr^7\}$

Question 2

Give the composition table for the group $\mathrm{Aut}(\mathbb{Z}_{24})$.

$\circ$	f_1	f_5	f_7	f_{11}	f_{13}	f_{17}	f_{19}	f_{23}
f_1	f_1	f_5	f_7	f_{11}	f_{13}	f_{17}	f_{19}	f_{23}
f_5	f_5	f_1	f_{11}	f_7	f_{17}	f_{13}	f_{23}	f_{19}
f_7	f_7	f_{11}	f_1	f_5	f_{19}	f_{23}	f_{13}	f_{17}
f_{11}	f_{11}	f_7	f_5	f_1	f_{23}	f_{19}	f_{17}	f_{13}
f_{13}	f_{13}	f_{17}	f_{19}	f_{23}	f_1	f_5	f_7	f_{11}
f_{17}	f_{17}	f_{13}	f_{23}	f_{19}	f_5	f_1	f_{11}	f_7
f_{19}	f_{19}	f_{23}	f_{13}	f_{17}	f_7	f_{11}	f_1	f_5
f_{23}	f_{23}	f_{19}	f_{17}	f_{13}	f_{11}	f_7	f_5	f_1