

Name: _____

Date: _____

1. Show $y = c_1 e^{\frac{x}{2}} + c_2 e^{-x} - x - 2$ is a two-parameter solution to the second-order differential equation $2y'' + y' - y = x + 1$

$$y' = \frac{c_1 e^{\frac{x}{2}}}{2} - c_2 e^{-x} - 1$$

$$y'' = \frac{c_1 e^{\frac{x}{2}}}{4} + c_2 e^{-x}$$

$$\frac{2c_1 e^{\frac{x}{2}}}{4} + 2c_2 e^{-x} + \frac{c_1 e^{\frac{x}{2}}}{2} - c_2 e^{-x} - 1 - c_1 e^{\frac{x}{2}} - c_2 e^{-x} + x + 2 = x + 1$$

$$x + 1 = x + 1$$

$y = c_1 e^{\frac{x}{2}} + c_2 e^{-x} - x - 2$ is a two parameter solution to the given differential equation

2. Determine the particular solution given $y(0) = 1$ and $y'(0) = 0$

$$1 = c_1 e^0 + c_2 e^0 - 0 - 2$$

$$1 = c_1 + c_2 - 2$$

$$3 = c_1 + c_2$$

$$0 = \frac{c_1 e^0}{2} - c_2 e^0 - 1$$

$$0 = \frac{c_1}{2} - c_2 - 1$$

$$1 = \frac{c_1}{2} - c_2$$

$$2 = c_1 - c_2$$

$$3 = c_1 + c_2$$

$$2 = c_1 - c_2$$

$$5 = 2c_1$$

$$c_1 = \frac{5}{2}$$

$$c_2 = \frac{1}{2}$$

$$y = \frac{5}{2} e^{\frac{x}{2}} + \frac{1}{2} e^{-x} - x - 2$$