Homework 8 Ryan Coyne

1) No, because, in the Schrödinger equation, the non-potential terms do not vary over space.

2) Yes because in quantum mechanics conservation of energy can be violated over breif periods of time.

3) (a) TISE: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$

$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)$

$\frac{d\psi}{dx} = \frac{3\pi}{L}\sqrt{\frac{2}{L}} \cos\left(\frac{3\pi}{L}x\right)$

$\frac{d^2\psi}{dx^2} = -\frac{9\pi^2}{L^2}\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)$

$\frac{\hbar^2}{2m}\cdot\frac{9\pi^2}{L^2}\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right) = E\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)$ when $0 < x < L$

$E = \frac{\hbar^2 9\pi^2}{2mL^2}$

(b) TDSE: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$

$\Psi = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t}$

$\frac{\partial\Psi}{\partial x} = \frac{3\pi}{L}\sqrt{\frac{2}{L}} \cos\left(\frac{3\pi}{L}x\right)e^{-i9\omega t}$

$\frac{\partial^2\Psi}{\partial x^2} = -\frac{9\pi^2}{L^2}\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t}$

$\frac{\partial\Psi}{\partial t} = -i9\omega\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t}$

$\frac{\hbar^2}{2m}\cdot\frac{9\pi^2}{L^2}\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t} = i\hbar\cdot(-i9\omega)\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t}$

$\omega = \frac{\hbar\pi^2}{2mL^2}$

c) $\psi = \sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right) + \sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)$

$\frac{d^2\psi}{dx^2} = \frac{9\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right) + \frac{16\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)$

$\frac{\hbar^2}{2m}\left(\frac{9\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right) + \frac{16\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)\right) = E\left(\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right) + \sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)\right)$

$9\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right) + 16\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right) \neq \sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right) + \sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)$

d) $\frac{\partial^2\Psi}{\partial x^2} = -\left(\frac{9\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t} + \frac{16\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)e^{-i16\omega t}\right)$

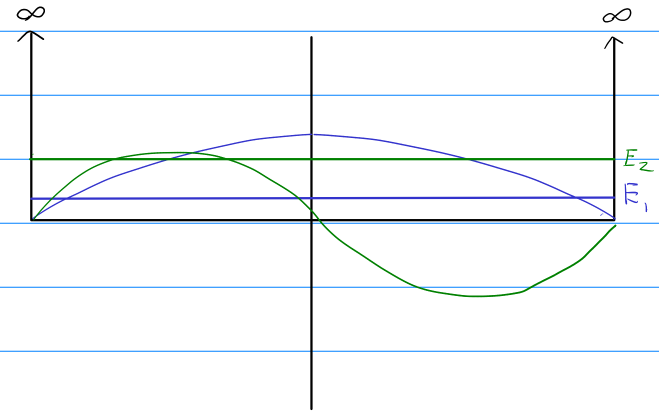
$\frac{\partial\Psi}{\partial t} = -\left(i9\omega\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t} + i16\omega\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)e^{-i16\omega t}\right)$

$\frac{\hbar^2}{2m}\left(\frac{9\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega} + \frac{16\pi^2}{L^2}\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)e^{-i16\omega t}\right)$

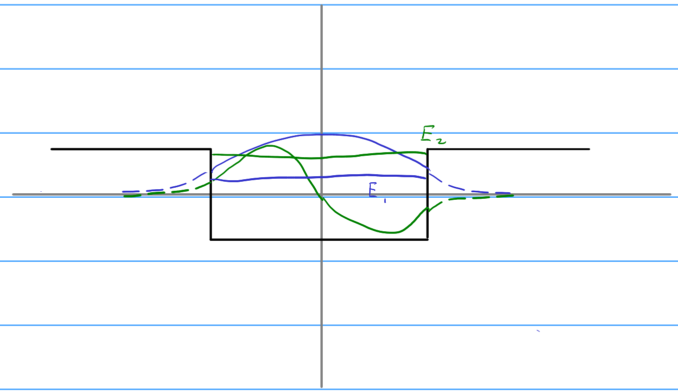
$= \hbar\left(9\omega\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{3\pi}{L}x\right)e^{-i9\omega t} + 16\omega\sqrt{\frac{\sqrt{2}}{L}} \sin\left(\frac{4\pi}{L}x\right)e^{-i16\omega t}\right)$

$\omega = \frac{\hbar\pi^2}{2m}$

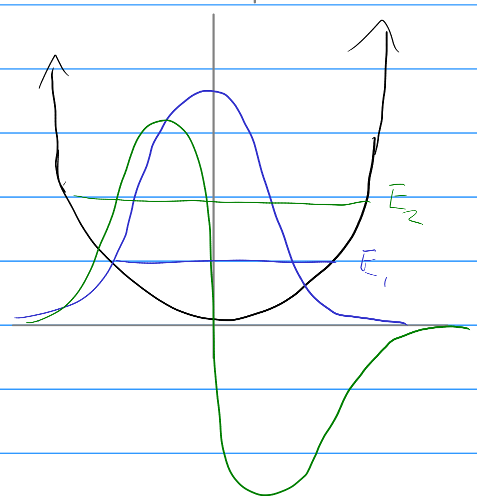
4)



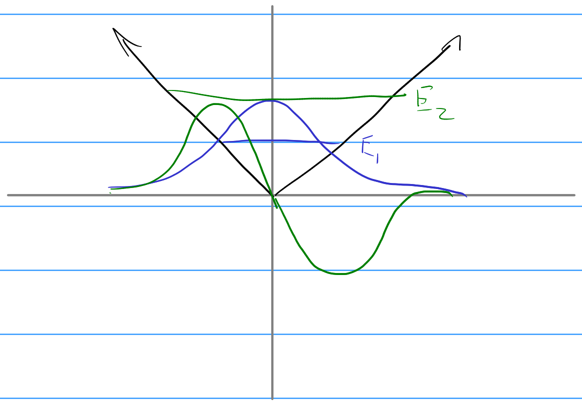
5)



6)



7)



8)

9) (a) $\lambda = \frac{h}{p}$ $\qquad k = 50\,nm^{-1} = \frac{p}{\hbar}$

$\lambda = \frac{1}{2\pi k}$

$= \frac{1}{2\pi 50}\ nm$

$= 3.2\ pm$

(b) $p = \frac{h}{3.2\,pm}$

$= \frac{4.135\times10^{-15}\,eV\cdot s}{3.2\times10^{-12}\,m}\cdot\frac{c}{c}$

$= 3.87\times10^{5}\ eV/c$

(c) $K = \frac{p^2}{2m}$

$= \frac{(3.87\times10^{5}\,eV/c)^2}{2\cdot 5.11\times10^{5}\,eV/c^2}$

$= 1.47\times10^{5}\ eV$

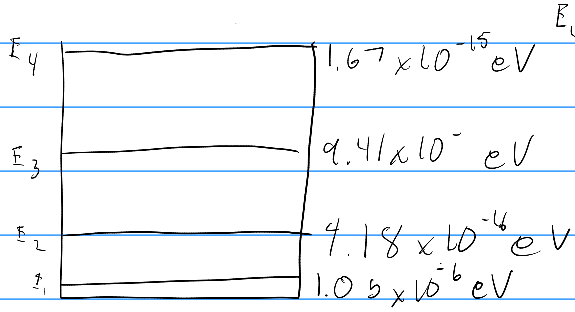
(d) $\Psi(x,t) = Ae^{i(50\times10^{-9}\,m^{-1}x\ -\ 2482\,s^{-1}t)}$

10) (a) $E_n = \frac{n^2 h^2}{8 m L^2}$

$E_1 = \frac{h^2}{8\,0.511\,MeV/c^2\,(0.2\,nm)^2}$; $E_2 = 4.18\times10^{-6}\ eV$

$= 1.05\times10^{-16}\ eV$ $\qquad E_3 = 9.41\times10^{-6}\ eV$

$E_4 = 1.67\times10^{-15}\ eV$



(b) $\Delta E = 1.568 \times 10^{-15}\ eV$

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{4.136\times10^{-15}\ eV\cdot s\cdot c}{1.568\times10^{-15}\ eV}$$

$$= 7.91\ Mm$$

(c) $P_n = 2\int_0^{0.04nm} |\psi|^2\, dx$

$$= 20nm^{-1}\int_{0nm}^{0.04nm} \sin^2\left(\frac{n\pi}{0.2nm}x\right) dx$$

$$= 20nm^{-1}\int_{0nm}^{0.04nm} \frac{1}{2}\left(1-\cos\left(\frac{2n\pi}{0.2nm}x\right)\right)dx$$

$$= 10\,nm^{-1}\left(x - \frac{0.2nm}{2n\pi}\sin\left(\frac{2n\pi}{0.2nm}x\right)\right)\Big|_0^{0.04nm}$$

$P_1 = 0.049$

$P_2 = 0.15$

$P_3 = 0.23$

$P_4 = 0.24$

11) $\Psi(x,t) = \frac{1}{\sqrt{L}}\sin\left(\frac{n\pi}{L}x\right)e^{-i\frac{E_n}{\hbar}t} + \frac{1}{\sqrt{L}}\sin\left(\frac{m\pi}{L}x\right)e^{-i\frac{E_m}{\hbar}t}$

$$\frac{\partial^2\Psi}{\partial x^2} = -\frac{n^2\pi^2}{L^{5/2}}\sin\left(\frac{n\pi}{L}x\right)e^{-iE_nt/\hbar} - \frac{m^2\pi}{L^{5/2}}\sin\left(\frac{m\pi}{L}x\right)e^{-iE_mt/\hbar}$$

$$\frac{\partial\Psi}{\partial t} = -i\frac{E_n}{\hbar}\frac{1}{\sqrt{L}}\sin\left(\frac{n\pi}{L}x\right)e^{-iE_nt/\hbar} - i\frac{E_m}{\hbar}\frac{1}{\sqrt{L}}\sin\left(\frac{m\pi}{L}x\right)e^{-iE_mt/\hbar}$$

$$\frac{\hbar}{2m}\left(\frac{n^2\pi^2}{L^{5/2}}\sin\left(\frac{n\pi}{L}x\right)e^{-iE_nt/\hbar} - \frac{m^2\pi^2}{L^{5/2}}\sin\left(\frac{m\pi}{L}x\right)e^{-iE_mt/\hbar}\right)$$

$$= \frac{E_n}{\hbar}\frac{1}{\sqrt{L}}\sin\frac{n\pi}{L}x\ e^{-iE_nt/\hbar} + \frac{E_m}{\hbar}\frac{1}{\sqrt{L}}\sin\frac{m\pi}{L}x)e^{-iE_mt/\hbar}$$

$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$, $E_m = \frac{m^2\hbar^2\pi^2}{2mL^2}$

12) $\psi(x) = \frac{1}{\sqrt{L}}\sin\left(\frac{n\pi}{L}x\right) + \frac{1}{\sqrt{L}}\sin\left(\frac{m\pi}{L}x\right)$

$$\frac{d^2\psi}{dx^2} = -\left(\frac{n^2\pi^2}{L^{5/2}}\sin\left(\frac{n\pi}{L}x\right) + \frac{m^2\pi^2}{L^{5/2}}\sin\left(\frac{m\pi}{L}x\right)\right)$$

$$\frac{\hbar^2}{2m}\left(\frac{n^2\pi}{L^{5/2}}\sin\left(\frac{n\pi}{L}x\right) + \frac{m^2\pi^2}{L^{5/2}}\sin\left(\frac{m\pi}{L}x\right)\right) = E_n\frac{1}{\sqrt{L}}\sin\left(\frac{n\pi}{L}x\right) + E_m\frac{1}{\sqrt{L}}\sin\left(\frac{m\pi}{L}x\right)$$

$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$, $E_m = \frac{m^2\hbar^2\pi^2}{2mL^2}$