Homework 2

Wednesday, September 13, 2023 6:46 PM

Questions

1. Almost all galaxies are moving away from us.
2. A world line relates the time and distance of an event from an observer.
3. If event A were to have caused event B, information from event A would have had to travel to the location of event B faster than the speed of light.
4. They would each see the other moving extra quickly because they are moving toward one another and so the relativistic Doppler effect would increase the perceived frequency of the light emitted from each.

Problems

1) $\lambda' = 650\,nm \quad \lambda' = 550\,nm$

$T = \frac{\lambda}{c}$

$\lambda' = \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} cT$

$\lambda'^2 = \frac{1 - v/c}{1 + v/c} c^2T^2$

$\lambda'^2 - \lambda'^2 \frac{v}{c} = c^2T^2 + vcT^2$

$v\left(\frac{\lambda'^2}{c} + cT^2\right) = -\lambda'^2 + c^2T^2$

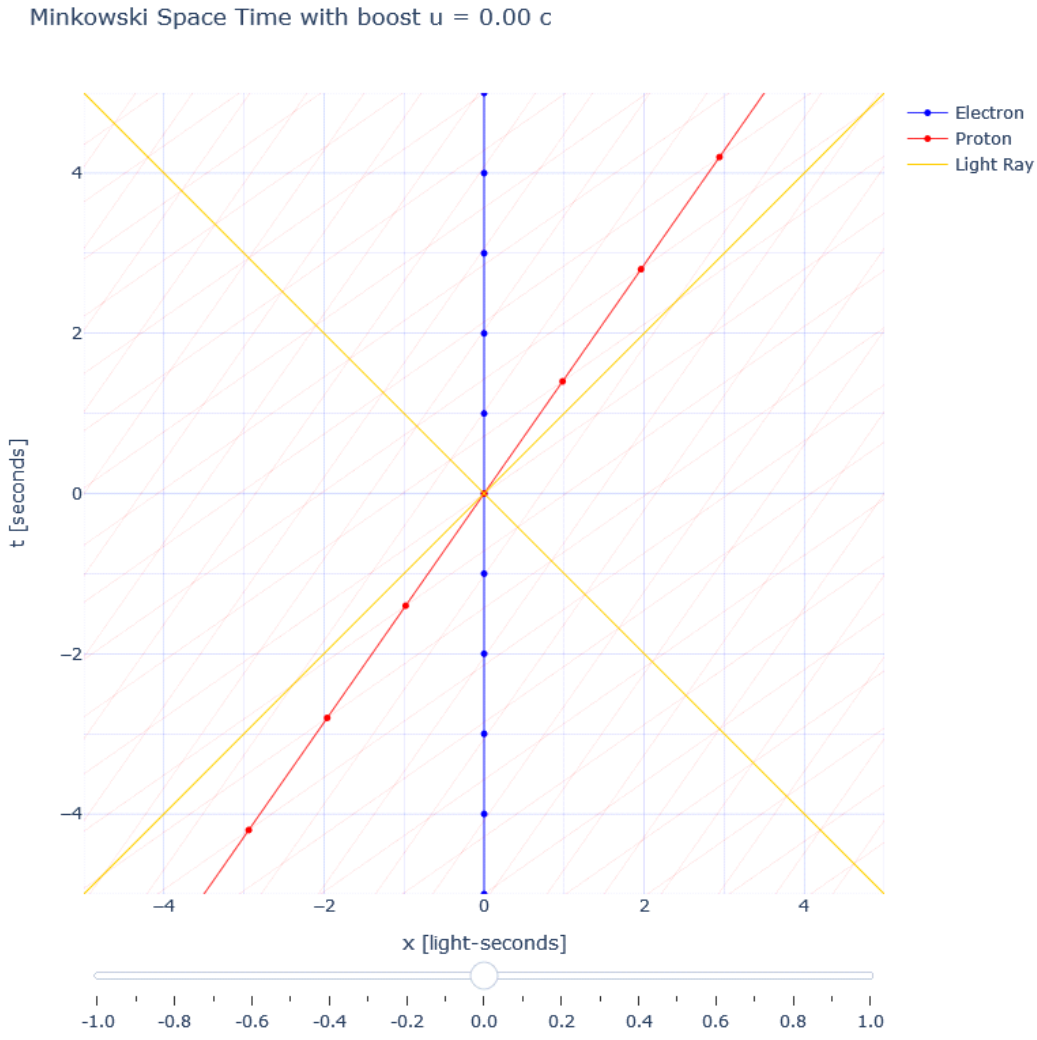
$$v = \frac{-\lambda'^2 + \lambda^2/c^2 \cdot c^2}{\lambda'^2/c + c \cdot \lambda^2/c^2}$$

$$= c\frac{-\lambda'^2 + \lambda^2}{\lambda'^2 + \lambda^2}$$

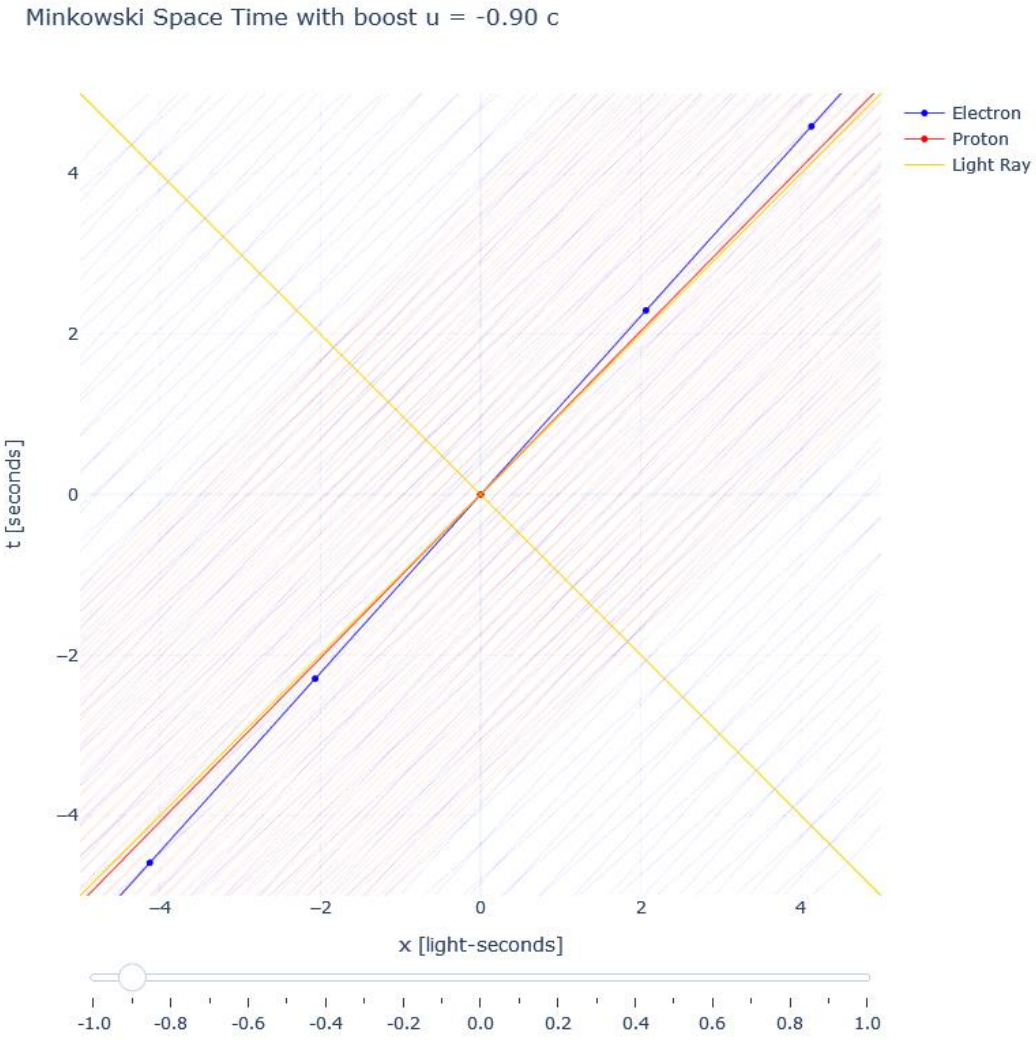
$$= 3\times10^8 \text{ m/s} \frac{-(550\text{ nm})^2 + (650\text{ nm})^2}{(550\text{ nm})^2 + (650\text{ nm})^2}$$

$$= 4.96 \times 10^7 \text{ m/s}$$

2) (a)



(b)



(c) $v_p = \frac{v_p' + v_e}{1 + \frac{v_p v_e}{c^2}}$

$= \frac{0.7c + 0.9c}{1 + 0.7 \cdot 0.9}$

$= 0.982c$

3) $t_\mu = 26\,ns \qquad d = 10.0\,m$

$t = \gamma t_\mu$

$d = vt$

$= \gamma v t_\mu$

$= \frac{v t_\mu}{\sqrt{1 - v^2/c^2}}$

$d^2 v^2 = \ldots^2 + ^2$

$= \frac{}{\sqrt{1 - v^2/c^2}}$

$d^2 - \frac{d^2 v^2}{c^2} = v^2 t_\mu^2$

$d^2 = v^2 (t_\mu^2 + \frac{d^2}{c^2})$

$\frac{d}{v^2 \sqrt{t_\mu^2 + a^2/c^2}}$

$= 2.37 \times 10^8$

4) $v = 0.95c$, $t = 4.4y$, $d = 4.2ly$

(a) $t' = \frac{t}{\gamma}$

$= 4.4y \sqrt{1 - 0.95^2}$

$= 1.4y$

(b) $d' = vt'$

$= 0.95c \cdot 1.4y$

$= 1.3l$

Challengers

1) $v = 0.95c$ $a = 1g$

$t = \frac{v}{a}$

$= \frac{0.95 \cdot 3 \times 10^8 m/s}{9.8 m/s^2}$

$= 2.91 \times 10^7 s$

$= 331 days$

2) $a' = \frac{dv'}{dt'} = \frac{1}{\gamma^3} a = a(1 - \frac{u^2}{c})^{3/2}$

$\int_{v_i}^{v_f}$ $\int_{t_i}^{t_f}$

$$\int_0^{v_f} \frac{1}{\left(1-\frac{v'^2}{c^2}\right)^{3/2}}\, dv' = \int_0^{t_f} a\, dt'$$