# 理论力学第 12 次作业

## 5.2

$$
\begin{aligned}
I &= m_i(\boldsymbol{r}_i \times \boldsymbol{n}) \cdot (\boldsymbol{r}_i \times \boldsymbol{n}) \\
&= m_i \boldsymbol{r}_i \cdot (\boldsymbol{n} \times (\boldsymbol{r}_i \times \boldsymbol{n})) \\
&= m_i \boldsymbol{r}_i \cdot (\boldsymbol{r}_i - (\boldsymbol{r}_i \cdot \boldsymbol{n})) \\
&= m_i\left[r_i^2 - (\boldsymbol{r}_i \cdot \boldsymbol{n})\right]
\end{aligned}
$$

## 5.3

$$\frac{dT}{dt} = \boldsymbol{\omega} \cdot \boldsymbol{N}$$

$$T = \frac{1}{2}\boldsymbol{\omega} \cdot \boldsymbol{I} \cdot \boldsymbol{\omega} = \frac{1}{2}\omega_\alpha I_{\alpha\beta}\omega_\beta$$

$$\frac{dT}{dt} = \omega_\alpha I_{\alpha\beta}\dot{\omega}_\beta = \omega_\alpha \dot{L}_\alpha$$

又因为

$$\frac{d\boldsymbol{L}}{dt} + \boldsymbol{\omega} \times \boldsymbol{L} = \boldsymbol{N}$$

所以

$$\frac{dT}{dt} = \boldsymbol{\omega} \cdot (\boldsymbol{N} - \boldsymbol{\omega} \times \boldsymbol{L}) = \boldsymbol{\omega} \cdot \boldsymbol{N} - \boldsymbol{L} \cdot (\boldsymbol{\omega} \times \boldsymbol{\omega}) = \boldsymbol{\omega} \cdot \boldsymbol{N}$$

## 5.4

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

其中

$$q_3 = \psi,\ Q_j = \boldsymbol{F}_i \frac{\partial \boldsymbol{r}_i}{\partial q_j}$$

欧拉角为

$$
\begin{aligned}
\omega_1 &= \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \\
\omega_2 &= \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \\
\omega_3 &= \dot{\phi}\cos\theta + \dot{\psi}
\end{aligned}
$$

$$T = \frac{1}{2} I_i \omega_i^2, \ \frac{\partial \boldsymbol{r}_i}{\partial \psi} = \hat{z}' \times \boldsymbol{r}_i$$

$$I_3 \frac{d}{dt}\left(\omega_3 \frac{\partial \omega_3}{\partial \dot{\psi}}\right) - I_1 \omega_1 \frac{\partial \omega_1}{\partial \dot{\psi}} - I_2 \omega_2 \frac{\partial \omega_2}{\partial \dot{\psi}} = \boldsymbol{F}_i \cdot \left(\hat{z}' \times \boldsymbol{r}_i\right)$$

$$\hat{z}' \cdot (\boldsymbol{r}_i \times \boldsymbol{F}_i) = \hat{z}' \cdot \boldsymbol{N} = \boldsymbol{N}_3$$

$$\therefore I_3 \, \dot{\omega}_3 - \omega_1 \omega_2 (I_1 - I_2) = N_3$$