理论力学第 6 次作业

2.17

$$L = T(q_i,\dot{q}_i) - V(q_i)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = 0$$

其中

$$\frac{\partial T}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i}\sum_j f_j(q_j)\dot{q}_j^2 = 2f_i(q_i)\dot{q}_i$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) = \frac{d}{dt}(2f_i(q_i)\dot{q}_i) = 2\frac{df_i}{dq_i}\dot{q}_i^2 + 2f_i(q_i)\ddot{q}_i$$

$$\frac{\partial T}{\partial q_i} = \frac{\partial}{\partial q_i}\left(\sum_j f_j(q_j)\dot{q}_j^2\right) = \frac{df_i(q_i)}{dq_i}\dot{q}_i^2$$

$$\frac{\partial V}{\partial q_i} = \frac{\partial}{\partial q_i}\sum_j V_j(q_j) = \frac{\partial V_i}{\partial q_i} = \frac{dV_i(q_i)}{dq_i}$$

$$\therefore 2f_i(q_i)\ddot{q}_i + \frac{df_i(q_i)}{dq_i}\dot{q}_i^2 + \frac{dV_i(q_i)}{dq_i} = 0$$

记

$$s_i(q_i) = \dot{q}_i$$

$$\ddot{q}_i = \dot{s}_i$$

$$2f_i(q_i)\dot{s}_i + \frac{df_i(q_i)}{dq_i}s_i^2 + \frac{dV_i(q_i)}{dq_i} = 0$$

$$\frac{d}{dq_i}\left(f_i(q_i)s_i^2\right) + \frac{dV_i(q_i)}{dq_i} = 0$$

即

$$\frac{d}{dq_i}\left(f_i(q_i)s_i^2+V_i(q_i)\right)=0$$

所以

$$f_i(q_i)s_i^2+V_i(q_i)=C$$

即

$$f_i(q_i)\dot{q}_i^2+V_i(q_i)=C$$

$$\dot{q}_i=\sqrt{\frac{C-V_i(q_i)}{f_i(q_i)}}$$

$$\int\sqrt{\frac{f_i(q_i)}{C-V_i(q_i)}}dq_i=t+D$$