理论力学第 1 次作业

1.1

$$\boldsymbol{F}\cdot\boldsymbol{v}=\boldsymbol{v}\cdot\frac{d\boldsymbol{p}}{dt}=\frac{m\boldsymbol{v}d\boldsymbol{v}}{dt}=\frac{d\left(\frac{1}{2}mv^2\right)}{dt}=\frac{dT}{dt}$$

1.2

$$\boldsymbol{R}=\frac{\sum_i m_i\boldsymbol{r}_i}{\sum_i m_i}=\frac{\sum_i m_i\boldsymbol{r}_i}{M}$$

$$M^2\boldsymbol{R}^2=\left(\sum_i m_i\boldsymbol{r}_i\right)^2=\sum_i m_i^2r_i^2+\sum_{i\neq j}m_im_j\boldsymbol{r}_i\boldsymbol{r}_j$$

$$M\sum_i m_i\boldsymbol{r}_i^2-\frac{1}{2}\sum_{i\neq j}m_im_jr_{ij}^2=\sum_i m_i^2r_i^2+\sum_{i\neq j}m_im_jr_i^2-\frac{1}{2}\sum_{i\neq j}m_im_j\left(r_i^2-2\boldsymbol{r}_i\boldsymbol{r}_j+r_j^2\right)$$

$$=\sum_i m_i^2r_i^2+\sum_{i\neq j}m_im_jr_i^2-\frac{1}{2}\sum_{i\neq j}m_im_j\left(r_i^2+r_j^2\right)+\sum_{i\neq j}m_im_j\boldsymbol{r}_i\boldsymbol{r}_j$$

$$=\sum_i m_i^2r_i^2+\sum_{i\neq j}m_im_j\boldsymbol{r}_i\boldsymbol{r}_j$$

$$\therefore M\sum_i m_i\boldsymbol{r}_i^2-\frac{1}{2}\sum_{i\neq j}m_im_jr_{ij}^2=M^2\boldsymbol{R}^2$$

1.3

双粒子系统有

$$M\frac{d^2\boldsymbol{R}}{dt^2}=\sum_i\boldsymbol{F}_i^{(e)}$$

总动量为

$$\boldsymbol{P}=M\frac{d\boldsymbol{R}}{dt}$$

所以

$$\dot{\boldsymbol{P}} = \frac{d}{dt} M \frac{dR}{dt} = M \frac{d^2 \boldsymbol{R}}{dt^2}$$

所以有

$$\dot{\boldsymbol{P}} = \sum_i \boldsymbol{F}_i^{(e)}$$

即

$$\dot{\boldsymbol{p}}_1 + \dot{\boldsymbol{p}}_2 = \boldsymbol{F}_1^{(e)} + \boldsymbol{F}_2^{(e)}$$

又因为

$$\dot{\boldsymbol{p}}_1 = \boldsymbol{F}_1^{(e)} + \boldsymbol{F}_{21}$$

$$\dot{\boldsymbol{p}}_2 = \boldsymbol{F}_2^{(e)} + \boldsymbol{F}_{12}$$

所以

$$\boldsymbol{F}_{21} + \boldsymbol{F}_{12} = 0$$

即

$$\boldsymbol{F}_{21} = - \boldsymbol{F}_{12}$$

满足相互作用力的弱形式。

再从动量矩定理出发

$$\dot{\boldsymbol{L}} = \sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i^{(e)} = \boldsymbol{N}^{(e)}$$

$$\dot{\boldsymbol{L}} = \sum_i \boldsymbol{r}_i \times \dot{\boldsymbol{p}} = \sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i^{(e)} + \sum_{i,j} \boldsymbol{r}_i \times \boldsymbol{F}_{ji}^{(e)}$$

其中

$$\sum_{i,j} \boldsymbol{r}_i \times \boldsymbol{F}_{ji}^{(e)} = \sum_{i,j} \boldsymbol{r}_i \times \boldsymbol{F}_{ji} + \sum_{i,j} \boldsymbol{r}_j \times \boldsymbol{F}_{ij}$$

因为只有两个粒子，上式即

$$\boldsymbol{r}_1 \times \boldsymbol{F}_{21} + \boldsymbol{r}_2 \times \boldsymbol{F}_{12} = \boldsymbol{r}_1 \times \boldsymbol{F}_{21} - \boldsymbol{r}_2 \times \boldsymbol{F}_{21}$$
$$= (\boldsymbol{r}_1 - \boldsymbol{r}_2) \times \boldsymbol{F}_{21} = \boldsymbol{r}_{12} \times \boldsymbol{F}_{21}$$

所以

$$\sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i^{(e)} + \boldsymbol{r}_{12} \times \boldsymbol{F}_{21} = \dot{\boldsymbol{L}} = \boldsymbol{N}^{(e)}$$

可得

$$\boldsymbol{r}_{12} \times \boldsymbol{F}_{21} = 0$$

所以两粒子之间的相互作用力与两粒子共线，满足相互作用力的弱形式。