4.2-1

$P_1 = 1\times 8 - 1\times 2 = 6$

$P_2 = 1\times 2 + 3\times 2 = 8$

$P_3 = 7\times 6 + 5\times 6 = 72$

$P_4 = 5\times 4 - 5\times 6 = -10$

$P_5 = 1\times 6 + 1\times 2 + 5\times 6 + 5\times 2 = 48$

$P_6 = 3\times 4 + 3\times 2 - 5\times 4 - 5\times 2 = -12$

$P_7 = 1\times 6 + 1\times 8 - 7\times 6 - 7\times 8 = -84$

$\Rightarrow C_{11} = P_5 + P_4 - P_2 + P_6 = 18$

$C_{12} = P_1 + P_2 = 6 + 8 = 14$

$C_{21} = P_3 + P_4 = 62$

$C_{22} = P_5 + P_1 - P_3 - P_7 = 66$

$\therefore$ 结果为 $\begin{bmatrix} 18 & 14 \\ 62 & 66 \end{bmatrix}$

4.3-1

证明 $T(n) = T(n-1) + n$ 的解为 $O(n^2)$

$$
\begin{aligned}
T(n) &= T(n-1) + n = T(n-2) + (n-1) + n \\
&= T(n-3) + (n-2) + (n-1) + n \\
&= \cdots = T(1) + 2 + 3 + \cdots + n \\
&= \frac{(n+2)(n-1)}{2} + T(1) \\
&= \frac{n^2+n-2}{2} + T(1) \\
&= O(n^2) + C = O(n^2)
\end{aligned}
$$

4.4-1

$$2\frac{\left(\frac{3}{2}\right)^{n}-1}{\frac{3}{2}-1}n \qquad \frac{1}{2}$$

$$
\begin{array}{c}
n \\
| \\
3 \cdot \frac{n}{2} \\
| \\
3^2 \cdot \frac{n}{2^2} \\
\vdots \\
3^{\log n} \cdot \Theta(1)
\end{array}
$$

$$\Rightarrow \text{Total} = n\left[\left(\frac{3}{2}\right)^0 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^2 + \cdots\right]$$

$$= n$$

$$\Rightarrow \text{Total} = n + \frac{3}{2}n + (\frac{3}{2})^2 n + \dots + (\frac{3}{2})^{\log n - 1} n + \Theta(n^{\log 3})$$

$$= n \sum_{i=0}^{\log n - 1} (\frac{3}{2})^i + \Theta(n^{\log 3})$$

$$= 2[(\frac{3}{2})^{\log n} - 1] n + \Theta(n^{\log 3})$$

$$= 2(n^{\log \frac{3}{2}} - 1) n + \Theta(n^{\log 3})$$

$$= 2(n^{\log 3 - \log 2} - 1) n + \Theta(n^{\log 3})$$

$$= 2(n^{\log 3} - n) + \Theta(n^{\log 3})$$

$$= O(n^{\log 3})$$

猜测：$T(n) = cn^{\log 3} - dn$

假设，对于所有的 $k < n$，$T(k) = ck^{\log 3} - dk$

通过归纳法证明：$T(n) \le cn^{\log 3} - dn$

证：$T(n) = 3T(\lfloor n/2 \rfloor) + n$

$$\le 3 \cdot [c(\frac{n}{2})^{\log 3} - d(\frac{n}{2})] + n$$

$$= \frac{3}{2^{\log 3}} \cdot cn^{\log 3} + n - \frac{3}{2}dn$$

$$= cn^{\log 3} + (1 - \frac{3}{2}d)n$$

$\leq cn^{\log 3} + dn$，当 $d \geq 2$ 时

4.5-1

a. $T(n) = 2T(\frac{n}{4}) + 1$

因为 $a=2$，$b=4$，$n^{\log_b a} = n^{\log_4 2} = n^{\frac{1}{2}}$

$f(n) = 1 = O(n^{\log_b a - \varepsilon}) = O(n^{\frac{1}{2} - \varepsilon})$

其中 $\varepsilon = \frac{1}{2}$，符合主定理的情况(1)

$T(n) = \Theta(n^{\frac{1}{2}})$

b.

$T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

因为 $a=2$，$b=4$，$n^{\log_b a} = n^{\log_4 2} = n^{\frac{1}{2}}$

$f(n) = \sqrt{n} = O(n^{\log_b a}) = n^{\frac{1}{2}}$

符合主定理情况(2)

$T(n) = \Theta(n^{\frac{1}{2}} \log n)$

c.

$T(n) = 2T(\frac{n}{4}) + n$

因为 $a=2$，$b=4$，$n^{\log_b a} = n^{\log_4 2} = n^{\frac{1}{2}}$

$f(n) = n = O(n^{\log_b a + \varepsilon}) = O(n^{\frac{1}{2} + \varepsilon})$

其中 $\varepsilon = \frac{1}{2}$.

对于足够大的 n,

$2 \cdot \frac{n}{4} = \frac{n}{2} \leq cn$, 其中 $c \geq \frac{1}{2}$

符合主定理情(3), 则 $T(n) = \Theta(n)$

d.

$T(n) = 2T(\frac{n}{4}) + n^2$

因为 $a = 2$, $b = 4$, $n^{\log_b a} = n^{\log_4 2} = n^{\frac{1}{2}}$

$f(n) = n^2 = \Omega(n^{\log_b a + \varepsilon}) = \Omega(n^{\frac{1}{2} + \varepsilon})$

其中 $\varepsilon = \frac{3}{2}$

对于足够大的 n,

$2 \cdot (\frac{n}{4})^2 = \frac{1}{8} n^2 \leq cn^2$, 其中 $c \geq \frac{1}{8}$

符合主定理的情况(3)

所 $T(n) = \Theta(n^2)$