PROJECT PROPOSAL

CS 254

The Minimal Enclosing Circle Problem

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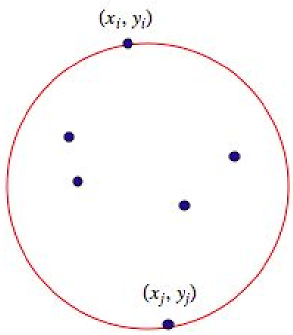
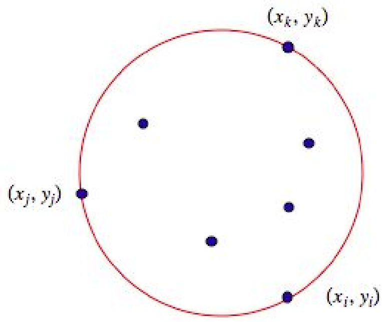
1. Introduction

The problem is defined as follows: Given a set of S of 2D points, find the circle C with the smallest radius such that all the points in S are contained in either C or its circumference.



A smallest enclosing circle defined by 2 points.



A smallest enclosing circle defined by 3 points.

This is one of the most fundamental problems in computational geometry with many applications as well. Moreover, the simplicity of the problem is so appealing that it inspires a reader to instantaneously ponder over it in search for its efficient algorithm.

This problem was first introduced in 1857 - long before the discipline of algorithm was formalized. But the researchers started working on it seriously only in 1960s. There is a simple $O(n^4)$ algorithm for this problem which will take days for any moderate size input even on the ultrafast computers present today. For many decades, there was no linear time algorithm for this problem. The first linear time algorithm was designed in 1983. However, it is too complex and uses many sophisticated algorithms and tools, e.g. linear programming in finite dimensions.

As a simple problem like this deserves a simple and elegant algorithm, one such elegant algorithm was discovered in 1991.

A) There are algorithm that make use of the convex hull, and has the running time ranging from $O(nh)$ to $O(h^3n)$ where h is the number of vertices in the convex hull.

B) There also exist several algorithms with running time ranging from $O(n^4)$ to $O(n \log n)$.

C) The best known algorithms are the Megiddo's Algorithm (O(n) running time) and Welzl's Algorithm (expected O(n) running time).

Further we will have a closer look at the Welzl's Randomized Algorithm for solving the problem.

2. Applications

A) The minimal enclosing circle is useful when we want to plan the location of a shared facility. Consider, for example, a hospital servicing a number of communities. If we think of the communities as points in the plane, finding their minimal enclosing circle gives a good place to put the hospital: the circle's center. Placing the hospital at the point defined by the circle's center minimizes the distance between the hospital and the farthest community from it.

B) In the military, the minimal enclosing circle is known as the Bomb Problem. If the points in the set are viewed as targets on a map, the center of the minimal circle surrounding them is a good place to drop a bomb to destroy the targets, and the circle's radius can be used to calculate how much explosive is required.

C) If we want to rotate a set of points into any arbitrary alignment, e.g. rotate the shape to make the line connecting some two chosen points vertical, then the minimal enclosing circle of the points is the amount of space in the plane that must be empty to permit this rotation.

D) The minimal enclosing circle provides a rough approximation to its points. In other words, any application involving a test of proximity between sets of points and a new point could benefit from having computed the minimal enclosing circles of the point sets to quickly determine whether the new point is close to a given set.

3. Goals/Objectives

1. Analysis of various Smallest Enclosing Circle Algorithms.

2. Study and Implementation of the Welzl's Algorithm.

3. Optimization of some Smallest Enclosing Circle Algorithms.

4. Advantages of Randomised Algorithms.

5. Application of Jung's Theorem in 2-Dimension.

4. References

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E)Jung, H.W.E. "Über die kleinste Kugel, die eine räumliche Figur einschliesst." *J. reine angew. Math.* 123, 241-257, 1901.

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