

**Question 1**

The Pure Expectations Hypothesis (PEH) suggests that forward interest rates reflect market expectations of future spot rates, assuming investors are indifferent to bond maturities. It explains why rates of bonds with different maturities move together and why yield curves change direction with short-term rates. However, PEH cannot fully explain why yield curves are usually upward-sloping and ignores investors' risk aversion, making it incomplete in describing real market conditions.

The Liquidity Premium Theory (LPT) builds on PEH by acknowledging that investors demand compensation for the risk of holding long-term bonds, resulting in a liquidity premium. This premium increases with the length of the bond's maturity, as investors perceive higher risks over longer horizons. LPT explains the typically upward-sloping yield curve, as the liquidity premium adds to expected future spot rates. However, LPT lacks specificity regarding the size of the liquidity premium, limiting its predictive power.

The Segmented Markets Hypothesis (SMH) posits that bonds of different maturities are not substitutes, as investors have fixed preferences based on their unique liabilities and investment goals. For example, pension funds may prefer long-term bonds to match their liabilities, while banks might prefer short-term bonds for liquidity management. This segmentation explains the often upward-sloping yield curve, as supply and demand for bonds of various maturities differ. However, SMH assumes that investors strictly adhere to their preferred maturities, ignoring potential arbitrage opportunities and flexibility in investor behaviour.

The Preferred Habitat Theory (PHT) bridges LPT and SMH, suggesting that investors have preferred maturities but can shift to others if compensated for the additional risk. This compensation reflects the risk premium that investors require to move away from their preferred habitat. PHT explains why yields of different maturities move together, why yield curves are typically upward-sloping, and why they change direction with short-term rate movements. By incorporating investor preferences and the willingness to accept compensation, PHT provides a more comprehensive view of the term structure. However, it remains limited by varying investor preferences and other factors affecting interest rates, such as economic policy and macroeconomic conditions.

In summary, while these theories help explain certain patterns in the term structure of interest rates, none fully account for all empirical observations. PEH ignores risk aversion, LPT lacks precision regarding liquidity premiums, SMH restricts investor behaviour, and PHT, while more flexible, still cannot capture the full complexity of interest rate movements influenced by diverse economic factors.

## Question 2(a)

Gilt Name	Close of Business Date	ISIN	Type	Coupon	Maturity	Clean Price	Dirty Price	Yield	Mod Duration	Accrued Interest
UKT 0.25 01/25	3/9/2024	GB00BLPK7110	Conventional	0.25	31/1/2025	98.419	98.442777	4.186068	0.401361	0.023777
UKT 1.625 10/54	3/9/2024	GB00BLR0J16	Conventional	1.625	22/10/2054	53.46	54.059385	4.44182	20.384223	0.599385

Table 1: Data of UKT 0.25 01/25 and UKT 1.62 10/54.

UKT 0.25 01/25 is a short-term government bond maturing in January 2025, while UKT 1.625 10/54 is a long-term bond maturing in October 2054. The short-term bond has a coupon rate of 0.25%, whereas the long-term bond offers a higher coupon rate of 1.625%. The yield for the long-term bond is 4.44182%, higher than the 4.186068% yield of the short-term bond, indicating additional compensation for the increased interest rate risk of holding a longer-term bond.

The modified duration of UKT 0.25 01/25 is 0.401361, reflecting low sensitivity to interest rate changes, while the modified duration of UKT 1.625 10/54 is 20.384223, showing high sensitivity to interest rate fluctuations. This means the price of the long-term bond will be more affected by rate changes compared to the short-term bond.

Accrued interest is 0.023777 for the short-term bond and 0.599385 for the long-term bond, consistent with the higher coupon rate of the latter. In summary, UKT 0.25 01/25 is less risky due to its lower duration and coupon rate, while UKT 1.625 10/54 offers higher yields but carries greater interest rate risk.

## Question 2(b)

1. Calculate the redemption yield (YTM),  $y$ :

$$P = \frac{C}{(1+y)^n} + \frac{F}{(1+y)^n}$$

$$y = \sqrt[n]{\frac{P}{(C+F)}} - 1$$

Where:

- $P$  = Clean price of the bond
- $C$  = Coupon payment
- $F$  = Face value of the bond (assumed to be 100)
- $n$  = Number of years to maturity
- $y$  = Yield to maturity

2. Calculate the Bond Price:

$$P = \frac{C}{y} \left( 1 - \frac{1}{(1+y)^n} \right) + \frac{F}{(1+y)^n}$$

3. Calculate YTM,  $y$  by trial and error from the bond valuation formula:

$$y = y_1 - \left( \frac{y_1 - y_2}{P_1 - P_2} \right) * (P_1 - P)$$

4. Calculate the Effective annual yield:

$$EAY = (1 + y)^2 - 1$$

Result:

Bond	Clean Bond Price	Yield, y	Effective Annual Yield (EAY)
UKT 0.25 01/25	98.419	2.09%	4.22%
UKT 1.625 10/54	53.46	2.24%	4.52%

Table 2: Table of result.

### **Question 2(c)**

According to Table 2, the short-term bond (UKT 0.25 01/25) has an effective annual yield (EAY) of 4.22%, which is lower than the EAY of the long-term bond (UKT 1.625 10/54) at 4.52%. This aligns with several theories of the term structure of interest rates.

The Pure Expectations Hypothesis (PEH) suggests that the yield curve reflects market expectations of future interest rates. The upward slope observed here, with the long-term bond yielding more than the short-term bond, indicates expectations of rising future rates. However, PEH alone does not account for the liquidity premium, or risks associated with longer maturities, making it incomplete in fully explaining the yield difference.

The Liquidity Premium Theory (LPT) explains that investors require additional compensation for holding longer-term bonds due to greater interest rate risk and reduced liquidity. The higher yield on the long-term bond is consistent with LPT, as it reflects the liquidity premium investors demand for bearing additional risks over a longer time horizon.

The Segmented Markets Hypothesis (SMH) suggests that bonds of different maturities are not substitutes, as investors have preferences based on their specific needs. The yield difference can be attributed to different market demands for short-term and long-term bonds. However, SMH alone does not fully explain the upward sloping yields without considering factors like risk premiums.

The Preferred Habitat Theory (PHT) suggests that investors may be willing to invest outside their preferred maturities if adequately compensated. The higher yield of 4.52% for the long-term bond compared to 4.22% for the short-term bond suggests that investors are being compensated for leaving their preferred shorter maturities, consistent with PHT.

### Question 3

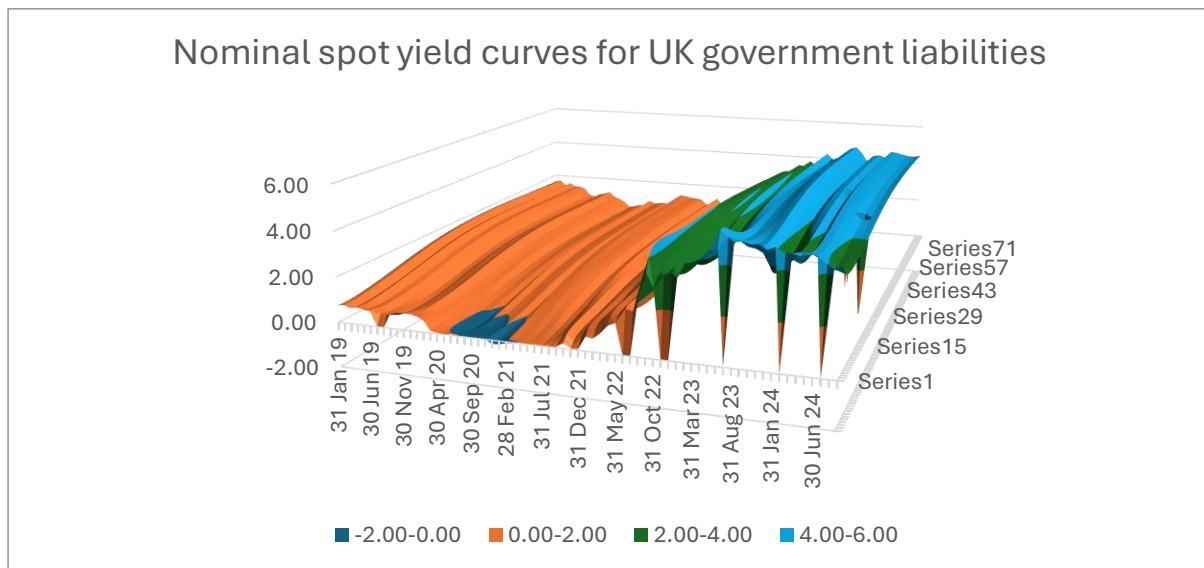


Figure 1: Plot of nominal spot yield curves from January 2019 to September 2024.

Based on Figure 1, the nominal spot yield curves for UK government liabilities from January 2019 to September 2024 reveal several key trends in interest rates over this period. Initially, from early 2019 to 2020, yields were relatively stable, ranging between 0% and 4%. However, a notable drop in yields occurred in 2020, with some periods even showing negative yields. This reflects the economic stimulus measures undertaken during the COVID-19 pandemic, where central banks significantly lowered interest rates, leading to investors accepting negative returns due to heightened risk aversion and the need for liquidity.

From mid-2021 onwards, yields began to rise sharply, moving from negative and low levels to positive territory. This was likely driven by rising inflation concerns and the shift towards tightening monetary policy by central banks to address increasing price levels. The period from 2022 to 2024 shows elevated yields, mostly above 4%, indicating the economic tightening phase as central banks worked to combat inflationary pressures.

Throughout this period, the yield curve experienced periods of flattening and even inversion, signalling market expectations of potential economic downturns. Flattening occurred as short-term and long-term yields converged, while inversion indicated higher short-term rates relative to long-term rates, often viewed as a predictor of recession.

Additionally, the chart highlights considerable volatility in short-term rates, especially during times of economic disruptions. By 2023 to 2024, yields appeared to stabilize at a higher level compared to pre-pandemic times, reflecting a broader adjustment to the macroeconomic environment and responses to challenges such as inflation and geopolitical uncertainties. Overall, the yield movements illustrate the impact of monetary policy shifts, inflation dynamics, and economic uncertainty on UK government bond yields.

#### **Question 4**

In August 2024, the Bank of England lowered interest rates for the first time after nearly three years of high rates. This decision likely had an immediate impact on the yield curve, particularly by flattening or lowering the short-term end of the curve. Generally, when central banks lower interest rates, short-term yields decline as they are directly influenced by the policy rate. Consequently, a rate cut signals lower borrowing costs, reducing returns on short-term government bonds, which might lead to a steeper yield curve if investors anticipate future economic growth.

In September 2023, the Bank of England decided not to raise interest rates further, holding the policy rate at 5.25%. This decision marked a shift from an extended period of rate increases, signaling that the tightening cycle might be nearing its end. The impact on the yield curve can be understood by considering market expectations. A flattening yield curve may have resulted, as the pause indicated to investors that further rate hikes were unlikely, leading to stabilization or declines in long-term rates.

The changes in monetary policy can be explained by different term structure theories. According to the Pure Expectations Hypothesis (PEH), the yield curve reflects expectations of future rates. The pause in rate hikes in September 2023 likely led investors to adjust their expectations downward, flattening the curve. Later, the rate cut in August 2024 reinforced expectations of declining rates, contributing to changes across different maturities.

The Liquidity Premium Theory (LPT) helps explain why long-term yields may remain higher than short-term yields, even during rate cuts. Investors demand a liquidity premium as compensation for holding longer-term bonds, which are riskier due to potential interest rate changes and economic uncertainties. Therefore, the yield curve might still slope upwards despite a rate cut, as the liquidity premium keeps long-term yields elevated.

The Preferred Habitat Theory (PHT) suggests that investors have preferences for certain maturities but may be willing to deviate if sufficiently compensated. Following the pause in rate hikes and the rate cut, investors may have required extra compensation to hold longer-term bonds, particularly in an environment with falling short-term rates. This additional compensation maintains an upward-sloping yield curve, even when the central bank is lowering rates.

## Question 5

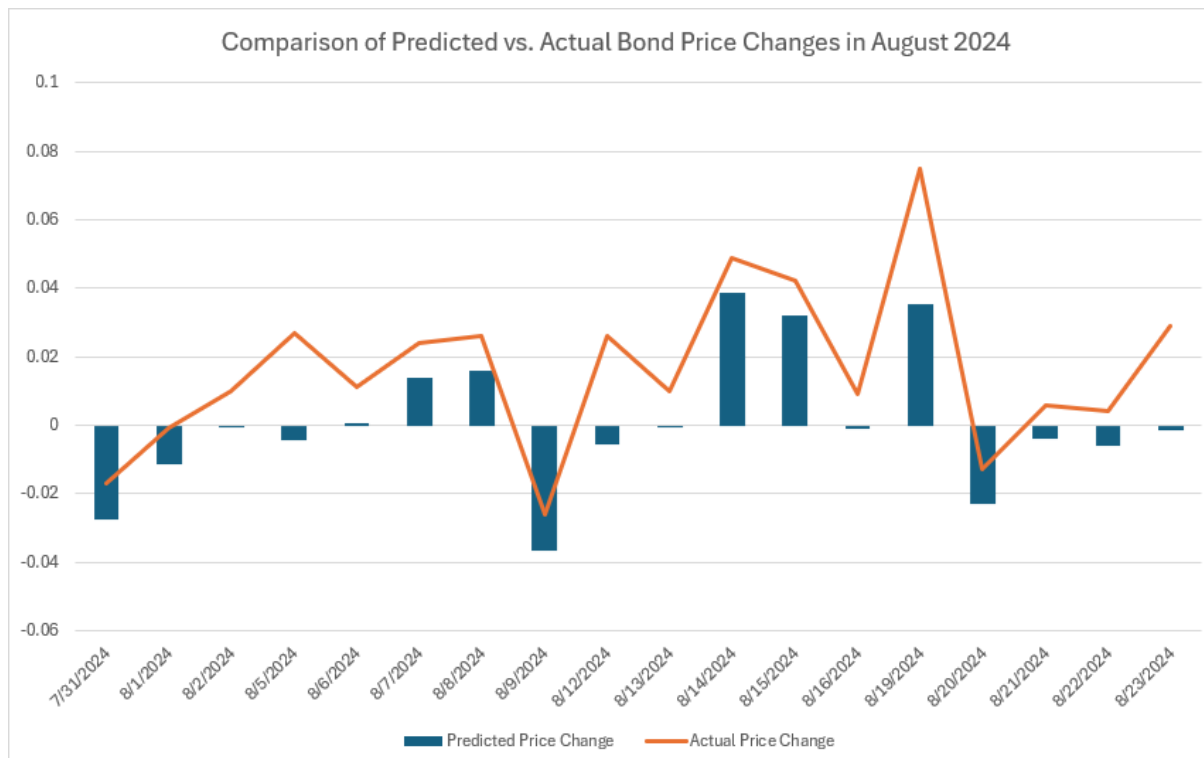


Figure 2: Comparison of Predicted and Actual Bond Price Changes for (UKT 0.25 01/25).

In Figure 2, the predicted changes are mostly small with some minor upward and downward fluctuations in response to the given yield changes while, the actual price changes display more variability compared to predicted changes, suggesting market reactions to additional factors.

For most days, the actual price changes are relatively close to the predicted price changes in terms of directions and magnitude. This indicates that the modified duration model provides a good approximation for the bond's price sensitivity to interest rate changes.

There are a few noteworthy exceptions, though. For instance, the expected price change on August 22, 2024, is 0.035, whereas the actual price change is 0.075. This implies that a larger-than-expected movement might have been caused by additional factors like market sentiment, liquidity circumstances, or macroeconomic news.

The differences can also be observed on 8/1/2024 and 8/16/2024, where the actual price changes are greater than the predicted values. Such deviations could be due to changes in investor risk perception or market expectations about future rate changes.

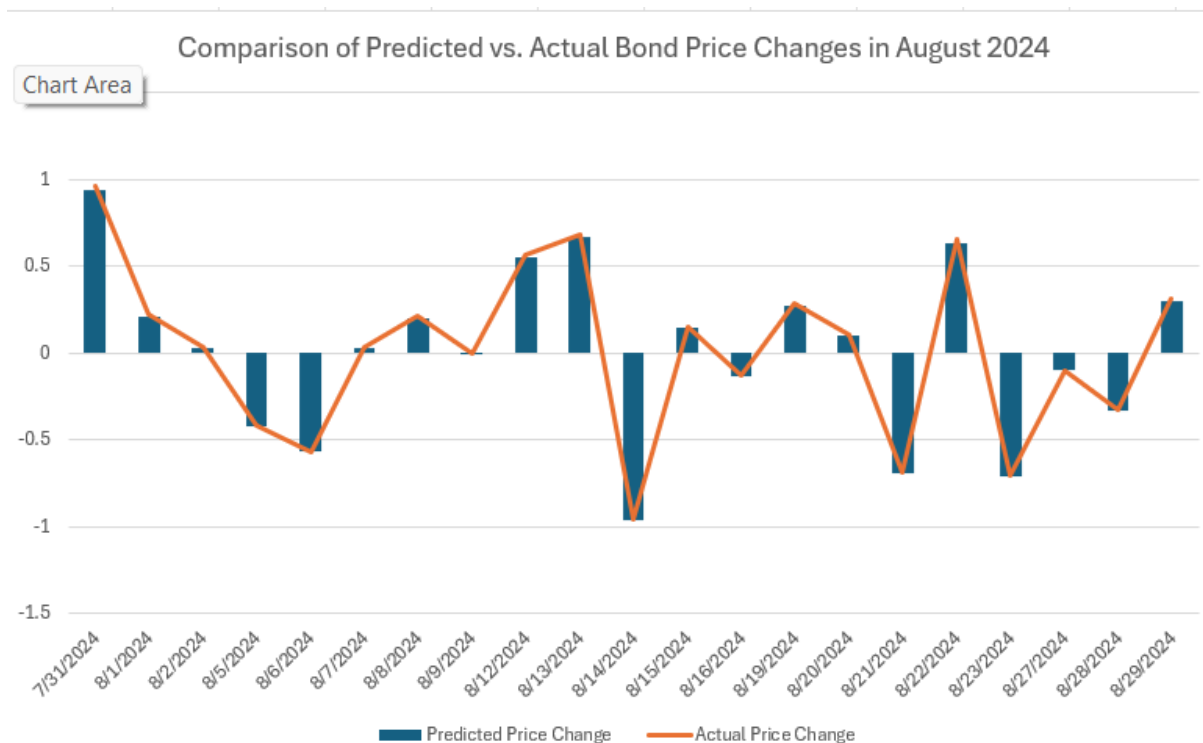


Figure 3: Comparison of Predicted and Actual Bond Price Changes for (UKT 1.625 10/54).

In Figure 3, the predicted price changes are based on a linear relationship between yield changes and bond price movements, which is characteristic of using modified duration. The actual price changes display more volatility compared to the predicted changes, with greater fluctuations on certain dates.

The price adjustments for the bond generally show consistency in direction with the theoretical predictions made using modified duration, suggesting that the model is effective in capturing the immediate response of bond prices to interest rate changes.

However, the magnitude of price changes differs considerably between the predicted and actual values on certain dates. This indicates that the modified duration model, while useful, does not fully account for the nonlinearity in bond pricing (convexity) and market-specific factors like investor sentiment and liquidity conditions.

To better match the actual market behaviour, incorporating convexity into the analysis would provide a more accurate estimate of price changes, especially for larger yield shifts. Additionally, understanding broader market conditions and investor expectations can help explain the deviations observed in actual price changes compared to theoretical predictions.