CALIFORNIA STATE UNIVERSITY **LONG BEACH**
COLLEGE OF ENGINEERING
COMPUTER ENGINEERING COMPUTER SCIENCE DEPARTMENT

Digital Signal Processing

Project-3

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The labs and projects materials are taken from Dr. Thomas Johnson

Part 1: The Z-Transform

The Z-transform is defined as

$$X(z) = \sum_{n=-\infty}^{n=\infty} x(n)z^{-n} \quad where\ z = |z|e^{j\omega} = re^{j\omega}$$

The area of the complex plane bounded by non-negative values of r=R- and r=R+ wherein the Z-transform exists (R-<|z|< R+) is call the Region of Convergence (ROC). The ROC may be the entire complex plane, or it may be the null space if R+ < R- and the Z-transform fails to exist. The inverse transform is a complex line integral of no importance to us. If the unit circle (r=|z|=1) is within the ROC, the evaluation of the Z-transform at frequencies around the unit circle yields the DTFT, a special case of the Z-transform. The use of the Z-transform is facilitated by use of table-look-up methods which match the often used sequences x[n] with their previously calculated Z-transform counterparts. Various properties of the transform allow for the quick determination of combinations of operations done on sequences to the resultant Z-transform for which the resultant sequence can be looked up in a table. Verification of that the correct Z-transform has been found for a sequence can often be done with the help of Matlab.

1. Determine the resulting vector x of coefficients and vector n of corresponding powers of z of the following polynomial operations using Matlab. See Example 4.5 on p.108 of text. Use function ***conv_m*** (see p. 44 of text) to produce the correct support vector along with the corresponding output vector.
   Let $X(z) = (z^3 + 1 - z^{-1} + 3z^{-2} - 2z^{-3})$ and $Y(z) = (z^2 - 2z + 4 + 3z^{-1} - 2z^{-2} + z^{-3})$.

   (a) $X_1(z) = X(z)Y(z)$
   (b) $X_2(z) = (1 + z^{-1} + z^{-2})^3$
   (c) $X_3(z) = (X(z) - Y(z))^2$
   (d) $X_4(z) = X^2(z) - Y^2(z)$

2. Determine the z–transform of each sequence using the definition (Eq. 4.1 of text). Use ***fprintf*** to output the region of convergence for the sequence and verify the z–transform expression using Matlab if indicated.
   To verify with Matlab, notice that the Z-transform has a related constant coefficient difference equation associated with it. The difference equation's output using an impulsive input (use function ***y=filter(b,a,δ(n))*** should give the same results as the associated sequencex(n) shown below. See pp. 110-111 of text for an example.

   (a) $x(n) = (3/5)^n\, u(n)$ and verify using Matlab.
   (b) $x(n) = -2^n\, u(-n-1)$ and verify using Matlab.
   (c) $x(n) = (0.8)^n\, u(n-2)$ and verify using Matlab.
   (d) $x(n) = (n+1)\,(3^n)\, u(n)$ and verify using Matlab.

Part 2: The Z- Transform Properties

Signal Type	Sequence x(n)	Transform	ROC
Unit Impulse	$\delta(n)$	1	$\forall z$
Unit Step	$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
Unit Step Folded	$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
Exponential	$a^n\,u(n)$	$\frac{1}{1-a\,z^{-1}}$	$\|z\| > \|a\|$
Exponential Folded	$b^n\,u(-n-1)$	$\frac{1}{1-b\,z^{-1}}$	$\|z\| < \|b\|$
Cosine	$[a^n \cos(\omega_o n)]\,u(n)$	$\frac{1-(a\cos(\omega_0))\,z^{-1}}{1-(2a\cos(\omega_0))\,z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
Sine	$[a^n \sin(\omega_o n)]\,u(n)$	$\frac{1-(a\sin(\omega_0))\,z^{-1}}{1-(2a\cos(\omega_0))\,z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
Exponential x n	$na^n\,u(n)$	$\frac{az^{-1}}{(1-a\,z^{-1})^2}$	$\|z\| > \|a\|$
Exponential x n Folded	$-nb^n\,u(-n-1)$	$\frac{bz^{-1}}{(1-bz^{-1})^2}$	$\|z\| < \|b\|$

1. Determine the z–transform of these sequences using the z–transform Table 4.1 (p. 110 of text) and the z–transform properties. Express X(z) as a rational function of z^{-1}. Verify your results using Matlab. Indicate the region of convergence in each case and provide a pole–zero plot of the transform.
   (a) $x_1(n) = 2\delta(n-2) + 3u(n-3)$
   (b) $x_2(n) = n^2\,(2/3)^{n-2}\,u(n-1)$

2. The z–transform of $x(n)=(-0.5)^n\,u(n)$ is $X(z) = 1/(1+0.5z^{-1})$ for $|z| > 0.5$. Determine the z–transform of each sequence and indicate the region of convergence. Verify your results with Matlab. See pp. 107-8 of text for various properties of the z-transform.
   (a) $x_1(n) = (0.5)^n\,x(n-2)$
   (b) $x_2(n) = x(n+2)*x(n-2)$ where * is convolution.

3. The deconv function is useful in dividing two causal sequences. Write a Matlab function to divide two non–causal sequences (similar to the ***conv_m*** function). The format of the function should be:

```
function [p,np,r,nr] = deconv_m (b,nb,a,na)
% Modified deconvolution routine for noncausal sequences
% function [p,np,r,nr] = deconv_m(b,nb,a,na)
%
%       p=polynomial part with support np1 <= n <= np2
%       np=[np1, np2]
%       r=remainder part with support nr1 <= n <= nr2
%       nr=[nr1, nr2]
%       b=numerator polynomial with support nb1 <= n <= nb2
%       nb=[nb1, nb2]
%       a=denominator polynomial with support na1 <= n <= na2
%       na=[na1, na2]
%
```

Check your function on the following operation

$$\frac{z^2+z+1+z^{-1}+z^{-2}+z^{-3}}{z+2+z^{-1}} = (z-1+2z^{-1}-2z^{-2}) + \frac{3z^{-2}+3z^{-3}}{z+2+z^{-1}}$$

Part 3: The System Function H(z)

The system function H(z) is a rational fraction in the complex variable z whose numerator zeros give the zero-locations of the response and denominator zeros give the location of the poles of the response. H(z) is determined from the constant coefficient difference equation by taking the z-transform of both sides of the equation and solving for Y(z)/X(z)=H(z). The system response is the z-transform of the impulse response of the system. It can be inverted to find h(n) by the method of partial fraction expansion. H(z) is written as a sum of terms of the form $r_k/(1-p_k z^{-1})$ where r_k is called the k-th residue and p_k is the k-th pole. These terms have simple table-lookup associated time domain functions. Matlab has a function ***[r,p,c] = residuez(b,a)*** that will do partial fraction expansion if the numerator coefficient vector b and denominator coefficient vector a can be found from the expression for H(z). Also the function ***zplane(b,a)*** plots the poles and zeros of H(z) on the complex plane with the unit circle as a reference.

1. Determine the following inverse z–transforms using the partial fraction expansion method.
   (a) $X_a(z) = (1 - z^{-1} - 4z^{-2} + 4z^{-3})/(1 - 2.75z^{-1} + 1.625z^{-2} - 0.25z^{-3})$ as a right-sided sequence.
   (b) $X_b(z) = z/(z^3 + 2z^2 + 1.25z + 0.25)$ where |z|>1.

2. For the linear and time–invariant systems described by the following impulse responses, determine the
   (i) system function representation H(z)
   (ii) difference equation representation y(n)
   (iii) pole–zero plot of H(z)
   (iv) output y(n) if the input is $x(n) = (1/4)^n u(n)$.

   (a) $h(n) = 5\,(0.25^n)\ u(n)$

   (b) $h(n) = [1-\sin(\omega_0 n)]\,u(n)$ where $\omega_0=\pi/2$

3. Solve the difference equation for y(n) using the one-sided z-transform approach and then generate y_{20}, the first 20 samples of y(n) by implementing the equation below. Find the error = max(abs(y(1:20) – y_{20})) of the first twenty samples of each output.

   $y(n) = 0.81\,y(n-2) + x(n) - x(n-1)$ for $n \geq 0$;
   Initial conditions: $y(-1)=2$; $y(-2) = 2$
   Input: $x(n) = (0.7)^n\,u(n+1)$

4. Determine the zero–input response y_{zi} and zero–state response y_{zs} of the system

   $y(n)=0.9801y(n-2)+x(n)+2x(n-1)+x(n-2)$ for $n \geq 0$
   Initial conditions: $y(-1)=0$; $y(-2)=1$; $x(-1)= x(-2)=0$.

   to the input $x(n)=5(-1)^n u(n)$. Generate y_{500}, the first 500 samples of the equation above, and the first 500 samples of the total response $y=y_{zs} + y_{zi}$ and then find the maximum absolute error = max(abs($y-y_{500}$)).