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SUBJECT :- NAD

SEM :- MTech IT IVth SEM

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Assignment 3

Q1 Find Roots of polynomial equation $f(x) = x^2 - x - 1 = 0$ using

(i) BISECTION METHOD

$m = (a+b)/2$

i	a	b	m	f(m)	f(a) * f(m)
1	1.0000	2.0000	1.5000	-0.2500	0.2500
2	1.5000	2.0000	1.7500	0.3125	-0.7812
3	1.5000	1.7500	1.6250	0.0156	-0.0039
4	1.5000	1.6250	1.5625	-0.1210	0.0302
5.	1.5625	1.6250	1.5937	-0.0537	0.0065
6	1.5937	1.6250	1.6093	-0.0192	0.0010
7	1.6093	1.6250	1.6171	-0.0018	0.0000
8.	1.6171	1.6250	1.6210	0.0068	-0.0000
9	1.6171	1.6210	1.6191	0.0024	-0.0000
10	1.6171	1.6191	1.6181	0.0002	-0.0000
11	1.6171	1.6181	1.6176	-0.0008	+0.0000
12	1.6176	1.6181	1.6179	-0.0002	+0.0000
13	1.6179	1.6181	1.6180	0.0000	-0.0000

Root of given equation by Bisection Method is 1.6180

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2. REGULA - FALSI METHOD.

$$m = \frac{af(b) - bf(a)}{f(b) - f(a)}.$$

i	a	b	m	f(m)	f(a) * f(m)
1	1.0000	2.0000	1.5000	-0.2500	0.2500
2	1.5000	2.0000	1.6000	-0.0400	0.0100
3	1.6000	2.0000	1.6153	-0.0059	0.0002
4	1.6153	2.0000	1.6176	-0.0008	0.0000
5	1.6176	2.0000	1.6179	-0.0001	0.0000
6	1.6179	2.0000	1.6180	-0.0000	0.0000

Roots of given equation by Regular falsi method is 1.6180

3. SECANT METHOD.

$$m = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

i	a	b	m	f(m)	f(a) * f(m)
1	1.0000	2.0000	1.5000	-0.2500	0.2500
2	2.0000	1.5000	1.6000	-0.0400	-0.0400
3	1.5000	1.6000	1.6190	0.0022	-0.0005
4	1.6000	1.6190	1.6180	-0.0000	0.0000

Root of given equation By Secant Method is 1.6180

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4 NEWTON RAPHSON METHOD

$$m = b - \frac{f(b)}{f'(b)}$$

i	b	m	f(m)
1	2.0000	1.7500	0.3125
2	1.7500	1.6607	0.0972
3	1.6607	1.6314	0.0301
4	1.6314	1.6221	0.0093
5	1.6221	1.6193	0.0028
6	1.6193	1.6184	0.0008
7	1.6184	1.6181	0.0002
8	1.6181	1.6180	0.0000

Root of given equation by newton Raphson method is 1.6180.

5 FIXED ITTERATION METHOD.

$$m = g(b)$$

i	b	m	m - b.
1	1.0000	1.4142	0.4142
2	1.4142	1.5537	0.1395
3	1.5537	1.5980	0.0442
4	1.5980	1.6118	0.0137
5	1.6118	1.6161	0.0042
6	1.6161	1.6174	0.0013
7	1.6174	1.6178	0.0004
8	1.6178	1.6179	0.0001
9.	1.6179	1.6180	0.0000

Root of given equation by Fixed Iteration Method is 1.6180.

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Q2 Apply interpolation using data (5,150), (7,392), (11,1452), (13,2366), (17,5202) and evaluate $f(9)$

1) LAGRANGE'S INTERPOLATION

x	5	7	11	13	17
y	150	392	1452	2366	5202

Formula :- $f(x) = \frac{(x-x_1)(x-x_0)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0 +$

$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2 +$

$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$

$f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150 + \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392 +$

$\frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1452 + \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366 +$

$\frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \times 5202.$

$f(9) = \frac{-50}{3} + \frac{8 \times 392}{5 \times 3} + \frac{8 \times 1452}{3 \times 3} - \frac{23660}{3} + \frac{5202}{15 \times 3}.$

$= \frac{1}{3}(2430)$

$= 810$

$y(9) = 810$

2) DIVIDED DIFFERENCE INTERPOLATION.

x	5	7	11	13	17
y	150	392	1452	2366	5202

divided difference Table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42.		
		709			
17	5202				

divided difference formula

$$f(x) = f(x_0) + (x-x_0)\Delta f(x) + (x-x_0)(x-x_1)\Delta^2 f(x) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x)$$

$$f(9) = f(5) + (9-5)\Delta f(5) + (9-5)(9-7)\Delta^2 f(5) + (9-5)(9-7)(9-11)\Delta^3 f(5)$$

$$f(9) = 150 + 4 \times 121 + 8 \times 24 + (-16) \times 1$$

$$f(9) = 150 + 484 + 192 - 16$$

$$f(9) = \underline{\underline{810}}$$

3) FORWARD AND BACKWARD INTERPOLATION

Here both are not applicable to find $f(9)$ as they uses value that are equidistant ~~at~~ ~~as~~ and the value here ~~is~~ are not equidistant.

Q3. Apply Numerical differentiation technique on function from (5, 150), (7, 392), (11, 1452), (13, 2366), (17, 5202).

1) METHOD BASED ON INTERPOLATION

- divided difference

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

Newtons divided difference interpolation formula. $f(x) = f(x_0) + (x-x_0)\Delta f(x) + (x-x_0)(x-x_1)\Delta^2 f(x) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x)$

$y = 150 + (x-5) \times 121 + (x-5)(x-7)\,24 + (x-5)(x-7)(x-11) \times 1$

$y = 150 + (x-5) \times 121 + (x^2 - 12x + 35) \times 24 + (x^3 - 23x^2 + 16x - 385) \times 1$

$y = x^3 + x^2$

Now differentiating with respect to x

$y' = 3x^2 + 2x$

again

$y'' = 6x + 2$

So first order derivative of function is $3x^2 + 2x$ and second order derivative is $6x + 2$.

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2) • By Lagrange's Interpolation

$$f(x) = \frac{(x-7)(x-11)(x-13)(x-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150 + \frac{(x-5)(x-11)(x-13)(x-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392$$

$$+ \frac{(x-5)(x-7)(x-13)(x-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1452 + \frac{(x-5)(x-7)(x-11)(x-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366 +$$

$$\frac{(x-5)(x-7)(x-11)(x-13)}{(17-5)(17-7)(17-11)(17-13)} \times 5202$$

$f(x) = (x^4 - 48x^3 + 838x^2 - 6288x + 17017) \times 0.1302 + (x^4 - 46x^3 + 756x^2 - 518x + 12155) \times (-0.8167) + (x^4 - 42x^3 + 616x^2 - 3702x + 7735) \times (5.0417) + (x^4 - 40x^3 + 55x^2 - 3224x + 6545) \times (-0.1615) + (x^4 - 36x^3 + 466x^2 - 2556x + 5005) \times (1.8062)$

On solving further

$f(x) = x^3 + x^2$.

Now differentiating with respect to x.

$y' = 3x^2 + 2x$.

Again

$y'' = 6x + 2$.

$\therefore$ First order derevative is $3x^2 + 2x$ and second order derevative is $6x + 2$.

ii) METHOD BASED ON FINITE DIFFERENCE

The method based on finite difference i.e. forward and backward interpolation methods are not applicable because the interval are not equidistance and finite diffence uses values that are equidistance Hence not applicable.

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Q4. Derive formulas for algorithm for Numerical Quadrature and obtain trapezoidal, simpson's 1/3, 3/8 and weedle's rule from it.

1) GENERAL QUADRATURE FORMULA

$$I = \int_a^b f(x)$$

Expressing in term of n, Putting $x = a + hu$

at $x = a$, $u = 0$ & $x = a + nh$, $u = n$

$dx = h\,du$

$$\int_a^{a+nh} f(x)\,dx = \int_0^n f(a+hu)\,h\,du$$

$$\int_a^{a+nh} f(x)\,dx = h\int_0^n f(a+hu)\,du$$

Expanding using Newtons forward difference interpolation formula

$$= h\int_0^n \left[f(a) + \frac{u}{\lfloor 1}\Delta f(a) + \frac{u(u-1)}{\lfloor 2}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 f(a) + \cdots\right]du$$

$$= h\left[f(a)\cdot u + \Delta f(a)\frac{u^2}{2} + \frac{\Delta^2 f(a)}{\lfloor 2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right) + \left(\frac{u^4}{4} - u^3 + u^2\right)\frac{\Delta^3 f(a)}{\lfloor 3}\right]$$

$$\Rightarrow \int_a^{a+nh} f(x)\,dx = h\left[n f(a) + \frac{n^2}{2}\Delta f(a) + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 f(a)}{\lfloor 2} + \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^3 f(a)}{\lfloor 3} \cdots\right] \quad \text{--- (1)}$$

This is general quadrature formula.

- Derivation of Trapezoidal from general quadrature formula.
  Put $n = 1$ in eq (1) and ignore higher power terms.

$$\int_a^{a+h} f(x)\,dx = h\left[f(a) + \frac{1}{2}\Delta f(a)\right]$$

$$\int_a^{a+h} f(x)\,dx = h\left[f(a) + \frac{1}{2}\left[f(a+2h) - 2f(a+h) + f(a)\right]\right]$$

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$$\int_a^{a+h} f(x)\,dx = h\left[f(a) + \frac{f(a+h) - f(a)}{2}\right]$$

$$\int_a^{a+h} f(x)\,dx = \frac{h}{2}\left[f(a) + f(a+h)\right]$$

- Derivation of Simpson's 1/3 rule from general quadrature formula.

Putting $n=2$ in eq (1) and ignoring higher form terms

$$\int_a^{a+2h} f(x)\,dx = h\left[2f(a) + \frac{2^2}{2}\Delta f(a) + \left(\frac{2^3}{3} - \frac{2^2}{2}\right)\frac{\Delta^2 f(a)}{2}\right]$$

$$= h\left[2f(a) + 2\Delta f(a) + \left(\frac{8}{3} - 2\right)\frac{\Delta^2 f(a)}{2}\right]$$

$$= \frac{h}{3}\left[6f(a+h) + \Delta^2 f(a)\right]$$

$$= \frac{h}{3}\left[6f(a+h) + f(a+2h) - 2f(a+h) + f(a)\right]$$

$$\int_a^{a+2h} f(x)\,dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$$

- Derivation of Simpson's 3/8 rule from general quadrature formula.

Putting $n=3$ in equation 1 and ignoring higher terms

$$\int_a^{a+3h} f(x)\,dx = h\left[3f(a) + \frac{9}{2}\{f(a+h) - f(a)\} + \left(\frac{27}{3} - \frac{9}{2}\right)\{f(a+2h) - 2f(a+h) + f(a)\} + \left(\frac{81}{4} - 27 + 9\right)\{f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)\}\right.$$

$$\int_a^{a+3h} f(x)\,dx = \frac{3h}{8}\left(f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)\right)$$

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- Derivation of Weddle's Rule from ~~most~~ general quadrature formula.

Putting $n = 6$ in eq ① and ignoring higher terms of Δ^6

$$\int_a^{a+6h} f(x)\,dx = n\Big(6f(x) + \frac{6^2}{2}\big(f(a+h) - f(a)\big) + \Big(\frac{6^3}{3} - \frac{6^2}{2}\Big)\big(f(a+2h) - 2f(a+h) + f(a)\big) + \Big(\frac{6^4}{4} - 6^3 + 6^2\Big)\big(f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)\big) + \Big(\frac{6^5}{5} - \frac{3}{4}(6^4) + \frac{8}{3}(6^3) - 3(6^2)\Big)\big(f(a+4h) - 4f(a+3h) + 6f(a+2h) - 4f(a+h) + f(a)\big) + \cdots + \big(f(a+6h) - 6f(a+5h) + 15f(a+4h) - 20f(a+3h) + 15f(a+2h) - 6f(a+h) + f(a)\big)$$

$$\Rightarrow \int_a^{a+6h} f(x)\,dx = \frac{3h}{10}\Big[f(a) + 5f(a+h) + f(a+2h) + 6f(a+3h) + f(a+4h) + 5f(a+5h) + f(a+6h)\Big]$$

2) NEWTON COTES FORMULA

$$I = \int_a^b f(x)\,dx = \int_a^b y\,dx$$

Put $x = a + hu$.

at $x = a$ $\quad u = 0$.

$x = a + nh$ $\quad u = n$

$dx = du$.

$$y = L_0(x) * y_0 + L_1(x) * y_1 + \cdots + L_i(x) * y_i$$

$$y = \sum_{i=0}^{n} L_i(x) * y_i$$

$$I = \sum_{i=0}^{n} y_i \int_a^b L_i(x)\,dx$$

This is Newton cotes Formula.

$$I = \sum_{i=0}^{n} y_i \int_a^n L_i(a+hu)\, h\,du.$$

$$I = h \sum_{i=0}^{n} y_i \int_a^n L_i(a+hu)\,du.$$

Multiplying and dividing by n

$$I = nh \sum_{i=0}^{n} y_i \frac{1}{n} \int_0^n L_i(u)\,du.$$

$$I = nh \sum_{i=0}^{n} y_i \, {}^nC_i.$$

- Derivation of Trapezoidal rule from Newton cotes formula.

Put $n=1$

$$\cancel{I = h \sum_0^1 {}^1C_0 y_0 + {}^1C_1 y_1}$$

$$I = h \sum_0^1 {}^1C_i \, y_i \qquad \text{where } {}^1C_i = \int_0^1 L_i(u)\,du.$$

$$I = h\left[{}^1C_0 y_0 + {}^1C_1 y_1\right]$$

$$I = h\left[\int_0^1 L_0(u)\,du * y_0 + \int_0^1 L_1(u)\,du * y_1\right]$$

$$I = h\left[\int_0^1 \frac{u-1}{0-1} * du * y_0 + \int_0^1 \frac{u-0}{1-0}\,du \cdot y_1\right]$$

$$I = h\left[\int_0^1 (1-u)\,du * y_0 + \int_0^1 u\,du * y_1\right]$$

$$I = h\left[\left(u - \frac{u^2}{2}\right)_0^1 * y_0 + \left(\frac{u^2}{2}\right)_0^1 * y_1\right]$$

$$I = h\left[\left(1 - \frac{1}{2}\right) y_0 + \left(\frac{1}{2}\right) y_1\right]$$

$$I = h\left[\frac{1}{2} y_0 + \frac{1}{2} y_1\right]$$

$$I_1 = \frac{h}{2}\left[y_0 + y_1\right]$$

$$I_2 = \frac{h}{2}\left[y_1 + y_2\right]$$

$$I_3 = \frac{h}{2}[y_2 + y_3].$$

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$$I_n = \frac{h}{2}[y_{n-1} + y_n]$$

$$I = I_1 + I_2 + I_3 + \cdots + I_n.$$

$$I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1})\right]$$

This is trapezoidal formula.

Q5. Apply pivoting and solve system of linear equations
$x + 2y + 3z = 14$, $3x + 2y + z = 10$, $2x + 3y + 2z = 14$.

a) direct Method → Gauss Elimination Method
First we have to convert system of equation into upper triangular matrix.
To do this, we will rearrange equation such that the ~~first~~ pivot value is greatest among all the value for that variable column. So our equations become like

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 10 \\ 1 & 2 & 3 & 14 \\ 2 & 3 & 2 & 14 \end{array}\right]$$

Row 1 interchanged by Row 2.

$$R_2 \leftarrow R_2 - \frac{1}{3}R_1$$

$$R_3 \leftarrow R_3 - \frac{2}{3}R_1$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 10 \\ 0 & 4/3 & 8/3 & 32/3 \\ 0 & 5/3 & 4/3 & 22/3 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{2}{4/3}R_2$$

$$R_3 \leftarrow R_3 - \frac{5/3}{4/3}R_2$$

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$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 10 \\ 0 & 4/3 & 8/3 & 32/3 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

Now modified equations are

$3x + 2y + z = 10$ — (I)

$\frac{4}{3}y + \frac{8}{3}z = \frac{32}{3}$ — (II)

$-2z = -6$. — (III)

from eq (III) $z = \frac{-6}{-2}$, $z = 3$.

Puting in eq (II)

$\frac{4}{3}y + \frac{8}{3} \times 3 = \frac{32}{3}$.

$\frac{4}{3}y = \frac{8}{3}$

$y = 2$.

Puting y & z in eq (I)

$3x + 2 \times 2 + 3 = 10$

$3x = 10 - 7$

$3x = 3$

$x = 1$.

So final answer is $x = 1$, $y = 2$, $z = 3$.

b) Iterative Methods. →

(i) ~~Gauss~~ Jacobi Method.

Here in given system of equation the diagonal are not dominent, hence we have manage it. So equations are rearranged

$3x + 2y + z = 10$.

$2x + 3y + 2z = 14$.

$x + y + 3z = 14$

$x = \frac{1}{3}(10 - 2y - z)$; $y = \frac{1}{3}(14 - 2x - 2z)$; $z = \frac{1}{3}(14 - x - 2y)$

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Initial values of x, y & z are 0.

→ 1st iteration

$x_1 = \frac{1}{3}(10 - 2\times0 - 0) = 3.33$

$y_1 = \frac{1}{3}(14 - 2\times0 - 2\times0) = \frac{14}{3} = 4.667$

$z_1 = \frac{1}{3}(14 - 0 - 0) = 4.667$

→ 2nd iteration

$x_2 = \frac{1}{3}(10 - 2(4.667) - 4.667) = -1.333$

$y_2 = \frac{1}{3}(14 - 2(3.33) - 2(4.667)) = -0.667$

$z_2 = \frac{1}{3}[14 - (3.33) - 2(4.667) = 0.444$

→ 3rd iteration

$x_3 = \frac{1}{3}(10 - 2(-0.667) - (0.447)) = 3.63$

$y_3 = \frac{1}{3}[14 - 2(-1.333) - 2(0.444)] = 5.259$

$z_3 = \frac{1}{3}[14 - (-1.333) - 2(\cancel{5.25} - 0.667)] = 5.556$

→ 4th iteration

$x_4 = \frac{1}{3}[10 - 2(5.259) - (5.556)] = -2.025$

$y_4 = \frac{1}{3}[14 - 2(3.63) - 2(5.556)] = -1.457$

$z_4 = \frac{1}{3}[14 - (3.63) - 2(5.259)] = -0.049$

→ 5th iteration

$x_5 = \frac{1}{3}[10 - 2(-1.457) - (-0.049)] = 4.321$

$y_5 = \frac{1}{3}[14 - 2(-2.025) - 2(-0.049)] = 6.049$

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$z_5 = \frac{1}{3}[14 - (-2.025) - 2(-~~01~~.457)] = 6.313$

→ 6^{th} iteration.

$x_6 = \frac{1}{3}[10 - 2(6.049) - (6.313)] = -2.804$

$y_6 = \frac{1}{3}[14 - 2(4.321) - 2(6.313)] = -2.422$

$z_6 = \frac{1}{3}[14 - (4.321) - 2(6.049)] = -0.807$

→ 7^{th} iteration

$x_7 = \frac{1}{3}[10 - 2(-2.422) - (-0.807)] = 5.217$

$y_7 = \frac{1}{3}[14 - 2(-2.804) - 2(-0.807)] = 7.074$

$z_7 = \frac{1}{3}[14 - (-2.804) - 2(-2.422)] = 7.216$

→ 8^{th} iteration

$x_8 = \frac{1}{3}[10 - 2(7.074) - (7.216)] = -3.788$

$y_8 = \frac{1}{3}[14 - 2(5.217) - 2(7.216)] = -3.622$

$z_8 = \frac{1}{3}[14 - (5.217) - 2(7.074)] = -1.788$

Since we have perform 8 iterations but still we not getting close to the intersection point. It means we can say that through Jacobi we can't find the solution of equation as they are divergent.

(ii) Gauss Seidal Method

$x = \frac{1}{3}(10 - 2y - z)$; $y = \frac{1}{3}(14 - 2x - 2z)$; $z = \frac{1}{3}(14 - x - 2y)$

Initial value $x_0 = 0$, $y_0 = 0$, $z_0 = 0$

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→ 1^{st} iteration

$x_1 = \frac{1}{3}(10 - 2\times 0 - 0) = 3.333$

$y_1 = \frac{1}{3}(14 - 2\times(3.333) - 0) = 2.444$

$z_1 = \frac{1}{3}(14 - 3.333 - 2(2.444)) = 1.926$

→ 2^{nd} iteration

$x_2 = \frac{1}{3}[10 - 2(2.444) - (1.926)] = 1.062$

$y_2 = \frac{1}{3}[14 - 2(1.062) - 2(1.926)] = 2.675$

$z_2 = \frac{1}{3}[14 - (1.062) - 2(2.675)] = 2.529$

→ 3^{rd} iteration

$x_3 = \frac{1}{3}(10 - 2(2.675) - 2.529) = 0.707$

$y_3 = \frac{1}{3}[14 - 2(0.707) - 2(2.529)] = 2.509$

$z_3 = \frac{1}{3}[14 - 0.707 - 2(2.509)] = 2.758$

→ 4^{th} iteration

$x_4 = \frac{1}{3}[10 - 2(2.509) - 2.758] = 0.741$

$y_4 = \frac{1}{3}[14 - 2(0.741) - 2(2.758)] = 2.334$

$z_4 = \frac{1}{3}[14 - (0.741) - 2(2.334)] = 2.864$

→ 5^{th} iteration

$x_5 = \frac{1}{3}[10 - 2(2.334) - 2.864] = 0.823$

$y_5 = \frac{1}{3}[14 - 2(0.823) - 2(2.864)] = 2.209$

$z_5 = \frac{1}{3}[14 - (0.823) - 2(2.209)] = 2.92$

→ 6^{th} iteration

$x_6 = \frac{1}{3}[10 - 2(2.209) - 2.92] = 0.888$

$y_6 = \frac{1}{3}[14 - 2(0.888) - 2(2.92)] = 2.128$

$z_6 = \frac{1}{3}[14 - 0.888 - 2(2.218)] = 2.952$

→ 7^{th} iteration

$x_7 = \frac{1}{3}[10 - 2(2.218) - 2.952] = 0.93$

$y_7 = \frac{1}{3}[14 - 2(0.93) - 2(2.952)] = 2.078$

$z_7 = \frac{1}{3}[14 - (0.93) - 2(2.078)] = 2.971$

→ 8^{th} iteration

$x_8 = \frac{1}{3}[10 - 2(2.078) - (2.971)] = 0.957$

$y_8 = \frac{1}{3}[14 - 2(0.957) - 2(2.971)] = 2.048$

$z_8 = \frac{1}{3}[14 - 0.957 - 2(2.048)] = 2.982$

→ 9^{th} iteration

$x_9 = \frac{1}{3}[10 - 2(2.48) - 2.982] = 0.984$

$y_9 = \frac{1}{3}[14 - 2(0.974) - 2(2.982)] = 2.029$

$z_9 = \frac{1}{3}[14 - 0.974 - 2(2.029)] = 2.989$

→ 10^{th} iteration

$x_{10} = \frac{1}{3}[10 - 2(2.029) - 2.989] = 0.986$

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$y_{10} = \frac{1}{3}[14 - 2(0.984) - 2(2.989)] = 2.018.$

$z_{10} = \frac{1}{3}[14 - 0.984 - 2(2.018)] = 2.99$

Now the two digits value after decimal matches with the previous iteration, we'll stop the process.
Solution by Gauss Seidal Method

$x = 0.986 \cong 1.$

$y = 2.018 \cong 2.$

$z = 2.99 \cong 3$

Q7. Solve Ordinary Differential Equation $dy/dx = x + y$; $y(0) = 1$ using.

(i) EULAR METHOD
Assuming $h = 0.25$ and we have to find value at $x = 1$
We know formula $y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$

$y_1 = y_0 + h[x_0 + y_0] = 1 + 0.25(0+1) = 1.25.$

$y_2 = y_1 + h[x_1 + y_1] = 1.25 + 0.25(1.5) = 1.625.$

$y_3 = y_2 + h[x_2 + y_2] = 1.625 + 0.25(2.125) = 2.1562.$

$y_4 = y_3 + h[x_3 + y_3] = 2.1562 + 0.25(2.906) = 2.882.$

hence $y(1) = 2.882$

x	0	0.25	0.5	0.75	1
y	1	1.25	1.625	2.1562	2.882

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(ii) RANGE KUTTA METHOD

$$\frac{dy}{dx} = x + y$$

$y(0) = 1$

assuming $h = 0.25$, and we have to find $y(1)$

$K_1 = hf(x_0, y_0) = 0.25 \times 1 = 0.25$.

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}) = 0.25(1.25) = 0.3125$.

$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = 0.25(1.2812) = 0.3203$.

$K_4 = hf(x_0 + h, y_0 + k_3) = 0.25(1.5703) = 0.3925$

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} = \frac{0.25 + 2 \times 0.3125 + 2 \times 0.3203 + 0.3925}{6}$$

$k = 0.3180$

Now $y_1 = y_0 + k = 1 + 0.3180 = 1.318$.

$y(0.25) = 1.318$

Again repeating the same above process.

$k_1 = hf(x_1, y_1) = 0.25(1.568) = 0.392$

$K_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}) = 0.25(1.889) = 0.4723$.

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}) = 0.25 \times (1.929) = 0.482$

$k_4 = hf(x_1 + h, y_1 + k_3) = 0.25 \times (2.3003) = 0.5751$

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} = \frac{0.392 + 2 \times 0.4723 + 2 \times 0.482 + 0.5751}{6}$$

$k = 0.4794$.

$y_2 = y_1 + k = 1.318 + 0.4794 = 1.7974$

$y(0.5) = 1.7974$

Again repeating same process

$k_1 = hf(x_2, y_2) = 0.25(2.2974) = 0.5743$

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$k_2 = hf(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}) = 0.25 \times (2.7096) = 0.6774$

$k_3 = hf(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}) = 0.25 \times (2.7611) = 0.6903$

$k_4 = hf(x_2 + h, y_2 + k_3) = 0.25 \times (3.237) = 0.809$

$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} = \frac{0.5743 + 2\times 0.6774 + 2\times 0.6903 + 0.809}{6}$

$y_3 = y_2 + k = 2.4839$

$y(0.75) = 2.4839$

Again repeating the same process

$k_1 = hf(x_3, y_3) = 0.25 \times (3.233) = 0.808$

$k_2 = hf(x_3 + \frac{h}{2}, y_3 + \frac{k_1}{2}) = 0.25 \times (3.763) = 0.9408$

$k_3 = hf(x_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}) = 0.25 \times (3.829) = 0.957$

$k_4 = hf(x_3 + h, y_3 + k_3) = 0.25 \times (4.4412) = 1.1103$

$y_4 = y_3 + k = 2.4839 + \frac{1}{6}[0.808 + 2\times 0.9408 + 2\times 0.957 + 1.1103]$

$y_4 = 3.436$

$y(1) = 3.436$

Hence the value at $x = 1$ is 3.436.

(iii) PICARD'S METHOD

$\frac{dy}{dx} = x + y \qquad y(0) = 1$

$dy = (x+y)\,dx$

Integrating both sides

$\int_{y_0}^{y} dy = \int_{x_0}^{x} (x+y)\,dx$

$y - y_0 = \int_{x_0}^{x} (x+y)\,dx$

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$$y^1 = y_0 + \int_{x_0}^{x} (x + y_0)\, dx.$$

$$y_1 = 1 + \int_{0}^{x} (x+1)\, dx.$$

$$y_1 = 1 + x + \frac{x^2}{2}.$$

Now,

$$y_2 = y_0 + \int_{x_0}^{x} (x + y_1)\, dx.$$

$$y_2 = 1 + \int_{0}^{x} x + 1 + x + \frac{x^2}{2}\, dx.$$

$$y_2 = 1 + \int_{0}^{x} 1 + 2x + \frac{x^2}{2}\, dx.$$

$$y_2 = 1 + x + \frac{x^2}{\cancel{2}} + \frac{x^3}{6}$$

Now,

$$y_3 = y_0 + \int_{x_0}^{x} (x + y_2)$$

$$y_3 = 1 + \int_{0}^{x} x + 1 + x + x^2 + \frac{x^3}{6}\, dx.$$

$$y_3 = 1 + \int_{0}^{x} 1 + 2x + x^2 + \frac{x^3}{6}\, dx.$$

$$y_3 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}.$$

Now

$$y_4 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120} + \ldots$$

Lets find $y(1)$

$$1 + 1 + 1 + \frac{1}{3} + \frac{1}{24}.$$

$$= 3.3756$$

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(iv) TAYLOR SERIES METHOD

$\frac{dy}{dx} = x + y$, $y(0) = 1$.

$h = 0.25$, $y(1) = 2$.

$y' = x + y$ $\quad y_0' = x_0 + y_0 = 1$

$y'' = 1 + y'$ on substituting $y_0'' = 1 + y_0' = 2$

$y''' = y''$ $\quad y_0''' = y'' = 2$

$y^{iv} = y'''$ $\quad y_0^{iv} = y''' = 2$.

Putting these values in taylor series

$$y_1 = y_0 + hy_0' + \frac{h^2}{\underline{|2}} y_0'' + \frac{h^3}{\underline{|3}} y_0''' + \frac{h^4}{\underline{|4}} y_0^{iv} + \cdots$$

$$y_1 = 1 + 0.25 + \frac{(0.25)^2}{2} \times 2 + \frac{(0.25)^3}{6} \times 2 + \frac{(0.25)^4}{24} \times 2.$$

$y_1 = 1.318$.

$y(0.25) = 1.318$

again repeating the same process with x_1, y_1 in place of x_0, y_0

$y_1' = x_1 + y_1 = 1.568$.

$y_1'' = 1 + y_1' = 2.568$.

$y_1''' = y_1'' = 2.568$.

$y_1^{iv} = y_1''' = 2.568$.

Putting in taylors series

$$y_2 = 1.318 + 0.25 \times 1.568 + \frac{(0.25)^2}{2}(2.568) + \frac{(0.25)^3}{6}(2.568) + \frac{(0.25)^4}{24} \times 2.568.$$

$y_2 = 1.7974$

$\Rightarrow y(0.5) = 1.7974$

again repeating same process with x_2, y_2 in place of x_1, y_1

$y_2' = x_2 + y_2 = 2.2974$.

$y_2'' = 1 + y_2' = 3.2974$.

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$$y_2''' = y_2'' = 3.2974$$
$$y_2^{iv} = y_3'' = 3.2974$$

$$y_3 = y_2 + hy_2' + \frac{h^2}{\lfloor 2} y_2'' + \frac{h^3}{\lfloor 3} y_3''' + \frac{h^4}{\lfloor 4} y_4^{iv} + \cdots$$

$$y_3 = 1.7974 + 0.25 \times (2.2974) + \frac{0.25^2}{\lfloor 2} \times 3.2974 + \frac{0.25^3}{6} \times 3.2974 + \frac{0.25^4}{24}(3.2974)$$

$$y_3 = 2.4839$$
$$y_3(0.75) = 2.4839 \cong 2.484$$

again repeating same.

$$y_3' = x_3 + y_3 = 3.234$$
$$y_3'' = 1 + y_3' = 4.234$$
$$y_3''' = y_3'' = 4.234$$
$$y_3^{iv} = y_3''' = 4.234$$

$$y_4 = 2.484 + 0.25(3.234) + \frac{0.25^2}{2}(4.234) + \frac{0.25^3}{6}(4.234) + \frac{0.25^4}{24}(4.234)$$

$$y_4 = 3.436$$
$$y(1) = 3.436$$