Optimizing Grocery Store Location

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Abstract

In this paper we will be considering optimization of place and how it may impact retail. More specifically we will be examining the effects on grocery stores. As grocery stores are a necessity for healthy living the question then becomes a question of which grocery store to put in a given place rather than whether to even construct one in that place. Building off of Lucrezio Figurelli model in "Store Choice in Spatially Differentiated Markets", we will attempt to first quantify the placement, type, and distance of a grocery store as a unit of money. This will provide us a common metric on which to measure utility of the store while also allowing us to optimize in terms of profitability and affordability of a grocery through consequence of these units. We consider this unit as it is probably the primary thing all households have in common and they were most in accordance with how we wished to define "optimal" for this project. Elaborating on Figurelli's model we will find however that for our target city, Boston, the distance, average income of customers, and demographic (all relating heavily to our variables) appear too clustered to notice much variance in what we will consider the optimal position. However this paper works to expose an interesting, somewhat un-initiative, aspect of cities and how they impact retail.

Background

We noticed that where Wentworth is located in Boston, there are not many decent grocery stores around. To get quality food, we would need to travel quite far, which for us, diminishes the want to go to that store. It occurred to us that maybe there is a reason that these stores are located where they are, and even for these reasons specifically. We then had the idea that if we could start over and place the grocery stores exactly where we wanted to, how would we accomplish that? This inspiration and curiosity is what made us interested in doing this project. We wanted to make a mathematical model that would be able to place specific grocery stores in locations that would be both profitable for the stores and beneficial to the customers.

Objectives

Our goal for this project was to create a model using quantifiable variables in order to locate grocery stores that would be able to benefit the profit of the businesses as well as the convience for the consumers. The steps we took to complete this goal were:

- Choose and create variables that would be most influential and important for the model
- Put choosen variables in mathematical model
- Code model into AMPL using actual raw data
- Use code to find best zip code to locate new store

Our Model

$u_{hsbt} = v_{hsbt} + \alpha_h \cdot e(b, p_{hst}) + \gamma_h(d_{hs}) + X_{hs} \cdot \beta + \xi_{hs} + \epsilon_{hst}$
Let t and b be constant
Then v_{hsbt} is constant
Now considering $\alpha_h \cdot e(b, p_{hst})$:
let I_h = income of household h

$$\Delta \frac{p_{hs}}{I_h} \propto \Delta e$$

$$\frac{\Delta e}{\Delta p_{hs}} \propto \frac{1}{I_h}$$

$$\frac{\delta e}{\delta p_{hs}} = \frac{c}{I_h}$$

$$\alpha_h \cdot e = \frac{\alpha_h \cdot c \cdot p_{hs}}{I_h}$$

We will assume $\alpha_h \cdot c = -1$

$$\alpha_h \cdot e = \frac{-p_{hst}}{I_h}$$

Now considering $X_{hs} \cdot \beta$ we will use the following:

$$X_{hs} \cdot \beta = \beta \cdot \frac{|I_h - I_s|}{|I_h|}$$

Finally let us intuitively define $\gamma_h(d_{hs})$:

$$\gamma_h(d_{hs}) = K \cdot e^{d_{hs}}$$

- Utility for price should be some ratio of the price of the bundle at a store and the individuals income.
- The store attribute, as defined by cost alone, will be defined by the price demographic.
- We considered utility of distance to a store to be exponential from intuition.

Optimal Location Model

$$u_{hsbt} = v_{hsbt} + \alpha_h \cdot e(b, p_{hst}) + \gamma_h(d_{hs}) + X_{hs} \cdot \beta + \xi_{hs} + \epsilon_{hst}$$

where
u_{hsbt} = total utility
v_{hsbt} = utility of bundle over time
$\alpha \cdot e(b, p_{hst})$ = utility of expenditure or price
$X_{hs} \cdot \beta$ = observed store component in terms of financial demographic
ξ_{hs} = error on the store demographic calculation
ϵ_{hst} = error given household h and store s
h = household by zip-code
s = store by zip-code
t = times
b = bundle

Results

$$u_{hs} = C + \frac{-p_{hs}}{I_h} + K \cdot e^{d_{hs}} + \beta \cdot \frac{|I_h - I_s|}{|I_h|}$$

where C is a constant.
Now to optimize we will maximize the following in terms of s.

$$\sum_{\forall h} + \frac{-p_{hs}}{I_h} + K \cdot e^{d_{hs}} + \beta \cdot \frac{|I_h - I_s|}{|I_h|}$$

Conclusion

When we run our model, we often come up with the same zip code. It is difficult to truly test our model because Boston is a dense city where there are all different types of stores in each zip code, so no matter where the model told us to place the store, it would not be incorrect. The most ideal location for the stores would be in the center of the city because it has the most access, has the most variety, and has the highest median incomes.

Conclusion Cont.

Although our goals were altered throughout our project, we still had a successful end product. At the end of our project, we were able to:

- Find the most valuable variables, such as income and distance, that determined someone's choice to go to a specific grocery store
- Create a model, using these variables, that gives the utility of the location of a certain store
- Code the model in AMPL
- Use raw data to put into our code for real world results
- Find the utility of specific store in each location
- Determine potential location of new grocery store by zip code

Future Work

We would like to go back more to our original idea of completely starting the city from scratch and place all new stores in new locations. Also, we would like to make the locations a bit more specific. Instead of saying just which zip code the store should be placed in, we would say which corner, street, or block it would be most beneficial at. Even though our model now is a good start, we need to add more to it to make it even more reliable and accurate. We could either add in more variables that would not have as much weight as the other ones and we could also add in an error value.

References

[1] Lucrezio Figurelli, *Store Choice in Spatially Differentiated Markets* (4 January, 2013), available at `https://www2.bc.edu/lucrezio-figurelli/Site/Home_files/jmp_figurelli.pdf`.

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