Lista 5 - Cálculo

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1) a) Sim, é possível que $f(3) = -4$. O limite $\lim_{x \to 3} f(x) = 7$ indica que, quando x se aproxima de 3, o valor de $f(x)$ se aproxima de 7. O $f(3)$ pode ser qualquer valor, inclusive -4, sem afetar o limite.

b) Não, $\lim_{x \to 1} g(x)$ não existe. Para que exista, os limites laterais $\lim_{x \to 1} g(x)$ e $\lim_{x \to 1^+} g(x)$ devem ser iguais. Neste caso, os limites laterais são diferentes $(2 \neq 3)$, portanto, o limite $\lim_{x \to 1} g(x)$ não existe.

c) Se $\lim_{x \to 0} h(x) = 7$, então os limites laterais $\lim_{x \to 0^-} h(x)$ e $\lim_{x \to 0^+} h(x)$ também devem ser iguais a 7. Isso significa que, tanto quando x se aproxima de 0 pela esquerda quanto pela direita, o valor de $h(x)$ se aproxima de 7.

2) a) $\lim\limits_{x \to -1^+} f(x) = 1$

b) $\lim\limits_{x \to 0^-} f(x) = 0$

c) $\lim\limits_{x \to 0^+} f(x) = 0$

d) $\lim\limits_{x \to 1^-} f(x) = 1$

e) $\lim\limits_{x \to 1^+} f(x) = 0$

f) $\lim\limits_{x \to 2^-} f(x) = 0$

gráfico $g(x)$

g) $\lim\limits_{x \to 1^-} g(x) = 1$

h) $\lim\limits_{x \to 1^+} g(x) = 0$

i) $\lim\limits_{x \to 2^-} g(x) = 1$

j) $\lim\limits_{x \to 2^+} g(x) = 1$

k) $\lim\limits_{x \to 3^-} g(x) = 0$

l) $\lim\limits_{x \to 3^+} g(x) = 0$

3) a) $\lim_{x \to 3} (3x^2-4) = 3 \cdot 3^2 - 4 = 3 \cdot 9 - 4 = 23$

b) $\lim_{t \to 6} 3(t-5)(t-7) = 3(6-5)(6-7) = 3 \cdot 1 \cdot (-1) = -3$

c) $\lim_{z \to 4} \sqrt{z^2-7} = \sqrt{4^2-7} = \sqrt{16-7} = \sqrt{9} = 3$

d) $\lim_{x \to 2} \dfrac{2x+5}{11-x^3} = \dfrac{4+5}{11-8} = \dfrac{9}{3} = 3$

e) $\lim_{x \to 3} [\cos(x^2-5x+6)] = [\cos(9-15+6)] = [\cos(0)] \Rightarrow \cos(0) = 1$

f) $\lim_{x \to -1} 3^{(2x^3-3x+2)} = 2(-1)^3 - 3(-1) + 2 = -2+3+2 = 3$

$3^3 = 27$

4) $\lim_{x \to c} f(x) = 3$ e $\lim_{x \to c} g(x) = -2$

a) $\lim_{x \to c} f(x) g(x) = \lim_{x \to c} f(x) = 3 \cdot g(x) = -2 = -6$

b) $\lim_{x \to c} [f(x) + 3g(x)] = 3 + (3 \cdot -2) = 3 + (-6) = -3$

c) $\lim_{x \to c} \dfrac{f(x)}{f(x)-g(x)} = \dfrac{3}{3-(-2)} = \dfrac{3}{5}$

d) $\lim_{x \to c} \sqrt{|f(x) g(x)|} = \sqrt{3 \cdot (-2)} = \sqrt{6}$

e) $\lim_{x \to c} [g(x)]^3 = [-2]^3 = -8$

f) $\lim_{x \to c} \dfrac{g(x)}{f(x)-1} = \dfrac{-2}{3-1} = \dfrac{-2}{2} = -1$

5) $\lim_{x \to 5} \dfrac{3f(x)-5}{x-2} = 8 \Rightarrow \dfrac{3f(4)-5}{4-2} = 8 \Rightarrow 3f(4) = 16+5$

$3f(4) = 21 \Rightarrow f(4) = \dfrac{21}{3} = 7$

$f(4) = 7$

6) a) $\frac{x^2+x-6}{x+3} = x-2$

x^2+x-6

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$ $\Delta = b^2-4ac$
$= 1+24 = 25$

$x = \frac{-1 \pm 5}{2} = x' = -6/2 = -3$ $(x+3)(x-2)$//
$x'' = 4/2 = 2$

$\Rightarrow \frac{\cancel{(x+3)}(x-2)}{\cancel{x+3}}$ ↳ essa simplificação só é válida se $x \neq -3$!

b) Está correta, pois ao calcular o limite, consideramos o comportamento da função para os valores de x próximos a -3, logo, a expressão pode ser simplificada!

$\Rightarrow \frac{(x-2)\cancel{(x+3)}}{\cancel{x+3}} = x-2$
↳ $-3-2 = -5$//

7) a) $\lim_{x \to 1} \frac{f(x)-3}{x-1} = 5$ $\Rightarrow f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ } $a=1$

$f'(1) = \lim_{x \to 1} \frac{f(x)-f(1)}{x-1} = 5$ } $f(1) = 3$ $f(1)-3 = 5(1-1) = 0$//

se $f(1)=3$;
$\lim_{x \to 1} f(x) = f(1) = 3$

b) $\lim_{x \to -2} \frac{3x^2+ax+a+3}{x^2+x-2} =$

$3x^2+ax+a+3$

x^2+x-2 $\Delta = 1+8 = 9$ { Precisamos que o numerador também seja zero em $x=-2$

$x = \frac{-1 \pm 3}{2} = x' = -2$ $(x+2)$; $x'' = 1$ $(x-1)$ } $(x+2)(x-1)$

logo, substituindo...

$3(-2)^2+a(-2)+a+3=0$

$3\cdot 4-2a+a+3=0$

$12+3-a=0$

$a=15$ ... substituindo:

$3x^2+15x+15+3 = 3x^2+15x+18$

$3(\underbrace{x^2+5x+6})$

$\Delta = b^2-4ac = 25-24=1$

$x = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{-5\pm 1}{2} = \begin{matrix} x' = -3 \\ x'' = -2 \end{matrix} \quad (x+3)(x+2)$

$3(x+3)(x+2)$

$\lim\limits_{x\to 2} \frac{3\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-1)} = \frac{3(x+3)}{x-1}$

$= \frac{3(-2+3)}{-2-1} = \frac{3\cdot 1}{-3} = -1$ //

8) a) $\lim\limits_{x\to 2} \frac{-x^3-2x^2}{2x+4} \to \frac{-x^2\cancel{(x+2)}}{2\cancel{(x+2)}} = \frac{-x^2}{2}$

substituindo: $\frac{-2^2}{2} = \frac{-4}{2} = -2$ //

b) $\lim\limits_{x\to 4} \frac{x-4}{x^2-16} = \frac{x-4}{x^2-4^2} = \frac{\cancel{x-4}}{(x+4)\cancel{(x-4)}} = \frac{1}{x+4}$

substituindo: $\frac{1}{4+4} = \frac{1}{8}$ //

c) $\lim\limits_{x\to 2} \frac{x^2-7x+10}{x-2}$

$\Delta = b^2-4ac = 49-40=9$

$x = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{7\pm 3}{2} = \begin{matrix} x' = 5 \\ x'' = 2 \end{matrix} \quad (x-5)(x-2)$

$\Rightarrow \frac{(x-5)\cancel{(x-2)}}{\cancel{x-2}} = x-5$

substi $= 2-5 = -3$ //

d) $\lim\limits_{x \to -3} \dfrac{x+3}{x^2+4x+3}$

$\Delta = b^2 - 4ac = 16 - 12 = 4$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{-4 \pm 2}{2} = \begin{matrix} x' = -3 \\ x'' = -1 \end{matrix} \quad (x+3)(x+1)$

$\Rightarrow \dfrac{x \cancel{+3}}{(\cancel{x+3})(x+1)} = \dfrac{1}{x+1} \Rightarrow \text{subst} = \dfrac{1}{-3+1} = \dfrac{1}{-2}$ //

e) $\lim\limits_{t \to -1} \dfrac{t^2+3t+2}{t^2-t-2}$

$\Delta = 9 - 8 = 1 \qquad x = \dfrac{-3 \pm 1}{2} = \begin{matrix} x' = -2 \\ x'' = -1 \end{matrix} \quad (x+2)(x+1)$

$\Delta = 1 + 8 = 9 \qquad x = \dfrac{1 \pm 3}{2} = \begin{matrix} x' = 2 \\ x'' = -1 \end{matrix} \quad (x-2)(x+1)$

$\Rightarrow \dfrac{(x+2)(\cancel{x+1})}{(x-2)(\cancel{x+1})} = \dfrac{x+2}{x-2} \Rightarrow \text{subst} = \dfrac{-1+2}{-1-2} = \dfrac{1}{-3}$ //

f) $\lim\limits_{x \to 2} \dfrac{x^3-5x^2+6x}{x^2-7x+10} \rightsquigarrow x(x^2-5x+6)$

$\Delta = 25 - 24 = 1 \qquad x = \dfrac{5 \pm 1}{2} = \begin{matrix} x' = 2 \\ x'' = 3 \end{matrix} \quad (x-3)(x-2)$

$\Delta = 49 - 40 = 9 \qquad x = \dfrac{7 \pm 3}{2} = \begin{matrix} x' = 5 \\ x'' = 2 \end{matrix} \quad (x-5)(x-2)$

$\Rightarrow \dfrac{(x-3)(\cancel{x-2})}{(x-5)(\cancel{x-2})} = \dfrac{x-3}{x-5} \Rightarrow \text{subst} = \dfrac{2-3}{2-5} = \dfrac{-1}{-3} = \dfrac{1}{3}$ //

g) $\lim\limits_{u \to 1} \dfrac{u^4-1}{u^3-1} \begin{matrix} \rightsquigarrow \\ \rightsquigarrow \end{matrix} \dfrac{(u^2+1)(u^2-1)}{(u-1)(u^2+u+1)} = \dfrac{(u^2+1)(u+1)\cancel{(u-1)}}{\cancel{(u-1)}(u^2+u+1)}$

$= \dfrac{(u^2+1)(u+1)}{u^2+u+1} = \dfrac{(1+1)\cdot(1+1)}{1+1+1} = \dfrac{4}{3}$ //

h) $\lim\limits_{t \to 1} \dfrac{t^{-1}-1}{t-1} \qquad \boxed{t^{-1} = 1/t} \qquad = \dfrac{1/t - 1}{t-1}$

$$\frac{1}{t} - 1 = \frac{1-t}{t}$$

$$\Rightarrow \lim_{t \to 1} \frac{\frac{1-t}{t}}{t-1} = \frac{1-t}{t(t-1)} \qquad \to 1-t = -(t-1)$$

$$\Rightarrow \frac{-(t-1)}{t(t-1)} = \frac{-1}{t}$$

subst: $\frac{-1}{1} = -1$ //

DÚVIDA $\to$ i) $\lim_{x \to 1} \frac{x^3-1}{x^4+3x-4} \to \frac{(\cancel{x-1})(x^2+x+1)}{(\cancel{x-1})(x^3+x^2+x+4)}$

$$= \frac{x^2+x+1}{x^3+x^2+x+4} = \frac{1^2+1+1}{1^3+1^2+1+4} = \frac{3}{7} \text{ //}$$

9) a) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \frac{\sqrt{x}-\sqrt{9}}{}$ $\to (\sqrt{x})^2 (\sqrt{9})^2 = (\sqrt{x}+\sqrt{9}) \cdot (\sqrt{x}-\sqrt{9})$

$$= \frac{\cancel{\sqrt{x}-\sqrt{9}}}{(\sqrt{x}+\sqrt{9}) \cdot \cancel{(\sqrt{x}-\sqrt{9})}} = \frac{1}{\sqrt{x}+\sqrt{9}}$$

subst: $\frac{1}{\sqrt{9}+\sqrt{9}} = \frac{1}{3+3} = \frac{1}{6}$ //

b) $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} \times \frac{(\sqrt{x+3}+2)}{(\sqrt{x+3}+2)} = \frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3})^2 - (2)^2} = \frac{(x-1)(\sqrt{x+3}+2)}{x+3-4}$

$$\frac{(\cancel{x-1})(\sqrt{x+3}+2)}{\cancel{x-1}} \Rightarrow \sqrt{x+3} + 2 \text{ //}$$

subst: $\sqrt{1+3} + 2 = \sqrt{4} + 2 = 4$ //

c) $\lim_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1} \times \frac{(\sqrt{x^2+8}+3)}{(\sqrt{x^2+8}+3)} = \frac{(\sqrt{x^2+8})^2 - (3)^2}{(x+1)(\sqrt{x^2+8}+3)} = \frac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$

$$\Rightarrow \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(\sqrt{x^2+8}+3)} = \frac{x-1}{\sqrt{x^2+8}+3} \text{ //}$$

subst: $\frac{-1-1}{\sqrt{(-1)^2+8}+3} = \frac{-2}{3+3} = \frac{-2}{6} = -\frac{1}{3}$ //

d) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x} \times \frac{(\sqrt{x+3}+\sqrt{3})}{(\sqrt{x+3}+\sqrt{3})} = \frac{(\sqrt{x+3})^{\cancel{2}}-(\sqrt{3})^{\cancel{2}}}{x(\sqrt{x+3}+\sqrt{3})}$

$\frac{\cancel{x}}{\cancel{x}(\sqrt{x+3}+\sqrt{3})} = \frac{1}{\sqrt{x+3}+\sqrt{3}} \Rightarrow$ subst: $\frac{1}{\sqrt{0+3}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow \frac{\sqrt{3}}{6}$ //

e) $\lim_{x\to -3} \frac{2-\sqrt{x^2-5}}{x+3} \times \frac{(2+\sqrt{x^2-5})}{(2+\sqrt{x^2-5})} = \frac{(2)^2-(\sqrt{x^2-5})^{\cancel{2}}}{(x+3)(2+\sqrt{x^2-5})} = \frac{4-x^2+5}{(x+3)(2+\sqrt{x^2-5})}$

$\Rightarrow x^2+9 \Rightarrow \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(2+\sqrt{x^2-5})} = \frac{x-3}{2+\sqrt{x^2-5}}$ // $\Rightarrow$ subst. $= \frac{-3-3}{2+\sqrt{(-3)^2-5}} = \frac{-6}{4} \Rightarrow \frac{-3}{2}$ //

f) $\lim_{x\to 4} \frac{4-x}{5-\sqrt{x^2-9}} \times \frac{(5+\sqrt{x^2-9})}{(5+\sqrt{x^2-9})} = \frac{(4-x)(5+\sqrt{x^2-9})}{(5)^2-(\sqrt{x^2-9})^{\cancel{2}}} = \frac{(4-x)(5+\sqrt{x^2-9})}{25-x^2-9}$

$\frac{(4-x)(5+\sqrt{x^2-9})}{x^2-16} = \frac{\cancel{(4-x)}(5+\sqrt{x^2-9})}{(x+4)\cancel{(x-4)}} = \frac{5+\sqrt{x^2-9}}{4+x}$

$\Rightarrow$ subst. $\frac{5+\sqrt{4^2-9}}{4+4} = \frac{5+5}{8} = \frac{10}{8} = \frac{5}{4}$ //

10) a) $\lim_{x\to 0^-} \frac{x}{|x|}$ $\quad |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\frac{x}{-x} = -1$ //

b) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right) \to \frac{1}{x} - \frac{1}{x} = 0$ //

c) $\lim_{x \to -3} |x+3| \rightarrow \begin{cases} x+3, & x \geq -3 \\ -|x+3|, & x < -3 \end{cases}$

$\lim_{x \to -3^-} -(x+3) = -x-3 \Rightarrow -(-3)-3 = 0$

$\lim_{x \to -3^+} x+3 \Rightarrow -3+3 = 0$

d) $\lim_{x \to 2} (x+3) \dfrac{|x-2|}{x-2} \rightarrow \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases}$

$\lim_{x \to 2^+} (x+3) \cdot \left(\dfrac{x-2}{x-2}\right) = x+3$
$\hookrightarrow 2+3 = 5$

$\lim_{x \to 2^-} (x+3) \cdot \left(\dfrac{-x+2}{x-2}\right) = (x+3) \cdot (-1)$
$\hookrightarrow -x+3$
$\hookrightarrow -2+3 = 1$

não existe, pois os limites laterais são diferentes!

e) $\lim_{x \to 3/2} \dfrac{2x^2 - 3x}{|2x-3|} \rightarrow \begin{cases} 2x-3, & \text{se } x \geq 3/2 \\ -(2x-3), & \text{se } x < 3/2 \end{cases}$ $\quad \frac{3}{2} = \frac{9}{4}$

$\lim_{x \to 3/2^+} \dfrac{2x^2-3x}{2x-3} \Rightarrow \dfrac{4(3/2)-3}{2} = \dfrac{6-3}{2} = \dfrac{3}{2}$

$\lim_{x \to 3/2^-} \dfrac{2x^2-3x}{-(2x-3)} = \dfrac{4x-3}{2} = \dfrac{-3}{2}$

$\nexists$, limites diferentes!

f) $\lim_{x \to 0} \dfrac{|\sin x|}{\sin x} \longrightarrow \begin{cases} x > 0, & = \sin x \\ x < 0, & = -\sin x \end{cases}$

$\lim_{x \to 0^+} \dfrac{\sin x}{\sin x} = 1$ $\qquad$ $\lim_{x \to 0^-} \dfrac{-\sin x}{\sin x} = -1$

limite não existe, pois $1 \neq -1$

g) $\lim_{x \to 0} \dfrac{\cos x - 1}{|\cos x - 1|} =$ cos possui o mesmo comportamento!

$\dfrac{\cos x - 1}{-(\cos x - 1)} = -1$ $\qquad$ $\dfrac{\cos x - 1}{(\cos x - 1)} = -1$

11)

a) $f(x) = \begin{cases} 7x-2, & se\ x \geq 2 \\ x^2-2x+1, & se\ x<2 \end{cases}$

$\lim_{x \to 2^+} f(x) = 7 \cdot 2 - 2 = 12$

$\lim_{x \to 2^-} f(x) = 2^2 - 2 \cdot 2 + 1$

$4-5 = -1$

$\lim_{x \to 2} f(x) = \nexists$

b) $f(x) = \begin{cases} x+1, & se\ x<2 \\ \sqrt{x^3+1}, & se\ x \geq 2 \end{cases}$

$\lim_{x \to 2^+} f(x) = \sqrt{2^3+1}$

$\sqrt{9} = 3$

$\lim_{x \to 2^-} f(x) = 2+1 = 3$

$\lim_{x \to 2} f(x) = 3 //$

c) $f(x) = \begin{cases} x+1, & se\ x<0 \\ 2, & se\ =0 \\ \sqrt{x+5}, & se\ x>0 \end{cases}$

$\lim_{x \to 0^+} f(x) = \sqrt{5}$

$\lim_{x \to 0^-} f(x) = 0+1 = 1$

$\lim_{x \to 0} f(x) = 2, \nexists$

12) $f(x) = \begin{cases} x^2-4x, & se\ x \leq -2 \\ 4-K, & se\ x>-2 \end{cases}$

$\lim_{x \to -2} \to (-2)^2 - 4 \cdot (-2)$

$4+8 = 12$

$4-K = 12$

$-K = 12-4$

$-K = 8 \ (\times -1)$

$\boxed{K = -8}$

13)
$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = L_0\sqrt{1-1} = L_0 \cdot 0 = 0$$

O resultado indica que, conforme a velocidade de um objeto se aproxima da velocidade da luz, seu comprimento tende a zero.
O limite à esquerda nos permite entender o comportamento do comprimento do objeto à medida que sua velocidade se aproxima da velocidade da luz, mas sem realmente alcançá-la.

14) Limites tendendo ao infinito podemos utilizar o maior do polinômio maior (dominante)

a) $\lim_{x\to\infty} \frac{4x+7}{x^2-2} = \lim_{x\to\infty} \frac{4x}{x^2} = \frac{4}{x} = 0$ ↳ $\frac{c}{\pm\infty} = 0$ → teorema

b) $\lim_{t\to\infty} \frac{7+t^3+4t}{2t^3-t^2+3} = \lim_{x\to\infty} \frac{7\cancel{t^3}}{2\cancel{t^3}} = \frac{7}{2}$

c) $\lim_{r\to\infty} \frac{r^4-r^2+1}{r^5+r^3-r} = \lim_{r\to\infty} \frac{r^4}{r^5} = \frac{1}{r} = 0$

d) $\lim_{x\to-\infty} \frac{(1-x)(2+x)}{(1+2x)(2-3x)} = \frac{2+2x-2x-x^2}{2-3x+4x-6x^2}$

$\Rightarrow \frac{-\cancel{x^2}}{-6\cancel{x^2}} = \frac{1}{6}$

e) $\lim_{x\to-\infty} \left(\frac{1-x^3}{x^2+7x}\right)^5 = \frac{1-x^8}{x^7+7x^6} \Rightarrow \frac{-x^8}{x^7} = -x = -(-\infty) = +\infty$

f) $\lim_{x\to-\infty} \sqrt{\frac{2x^2+x+1}{3x^2+4}} \Rightarrow \frac{\sqrt{2x^{\cancel{2}}}}{\sqrt{3x^{\cancel{2}}}} = \frac{\sqrt{2}\,\cancel{x}}{\sqrt{3}\,\cancel{x}} = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{\cancel{2}} = \frac{2}{3}$

g) $\lim_{x\to\infty} \frac{\sqrt{1+4x^2}}{4+x} \Rightarrow \frac{\sqrt{4x^{\cancel{2}}}}{x} = \frac{2\cancel{x}}{\cancel{x}} = 2$

h) $\lim_{x\to-\infty} \frac{\sqrt{x^2+4x}}{4x+1} \Rightarrow \frac{\sqrt{x^{\cancel{2}}}}{4x} = \frac{\cancel{x}}{4\cancel{x}} = \frac{1}{4}$

i) $\lim\limits_{x \to -\infty} \dfrac{7x+2}{\sqrt{5x^2-3}} = \dfrac{7x}{\sqrt{5x^{\cancel{2}}}} = \dfrac{7\cancel{x}}{\sqrt{5}\,\cancel{x}} = \dfrac{7}{\sqrt{5}}$ //

pegando pelo maior

$\dfrac{x^2}{x} = x = \infty$

15) a) $\lim\limits_{x \to \infty} (x - \sqrt{x})$

$$\lim_{x \to \infty} \frac{(x-\sqrt{x})(x+\sqrt{x})}{(x+\sqrt{x})} = \frac{(x)^{2} - (\cancel{\sqrt{x}})^{\cancel{2}}}{(x+\sqrt{x})} = \frac{x^2 - x}{x + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{x(x-1)}{x+\sqrt{x}} = \frac{x-1}{1+\frac{\sqrt{x}}{x}} = \frac{x-1}{1+\frac{1}{\sqrt{x}}}$$

↳ $\dfrac{x-1}{1} = \infty$ //

b) $$\frac{(\sqrt{x^2+3}+x)(\sqrt{x^2+3}-x)}{(\sqrt{x^2+3}+x)} = \frac{(\cancel{\sqrt{x^2+3}})^{\cancel{2}} - \cancel{(x)^2}}{(\sqrt{x^2+3}+x)} = \frac{3}{\sqrt{\cancel{x^2}+3}+x}$$

⇝ $\dfrac{3}{x} = 0$ //, pois $\dfrac{c}{\pm\infty} = 0$

c) $$\lim_{x \to \infty} \frac{(\sqrt{x^2+3x+1}-x)(\sqrt{x^2+3x+1}+x)}{(\sqrt{x^2+3x+1}-x)} = \frac{(\cancel{\sqrt{x^2+3x+1}})^{\cancel{2}} - (x)^2}{(\sqrt{x^2+3x+1}-x)} = \frac{\cancel{x^2}+3x+1-\cancel{x^2}}{(\sqrt{x^2+3x+1}-x)}$$

$$\lim_{x \to \infty} \frac{3+\frac{1}{x}}{\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+1} = \frac{3+0}{\sqrt{1+0}+1} = \frac{3}{2}$$ //

d) $$\lim_{x \to \infty} \frac{(\sqrt{9x^2+x}-3x)(\sqrt{9x^2+x}+3x)}{(\sqrt{9x^2+x}-3x)} = \frac{(\cancel{\sqrt{9x^2+x}})^{\cancel{2}} - (3x)^2}{(\sqrt{9x^2+x}-3x)} = \frac{\cancel{9x^2}+x-\cancel{9x^2}}{(\sqrt{9x^2+x}-3x)}$$

$\Rightarrow \dfrac{x}{\sqrt{9x^2}} = \dfrac{\cancel{x}}{\sqrt{9}\,\cancel{x}} = \dfrac{1}{3}$ //

e) $$\lim_{x \to -\infty} \frac{(x+\sqrt{x^2+2x})(x-\sqrt{x^2+2x})}{(x+\sqrt{x^2+2x})} = \frac{(x)^2 - (\cancel{\sqrt{x^2+2x}})^{\cancel{2}}}{(x+\sqrt{x^2+2x})} = \frac{2x}{(x+\sqrt{x^2+2x})}$$

$\Rightarrow \dfrac{2x}{\sqrt{x^{\cancel{2}}}} = \dfrac{2\cancel{x}}{\cancel{x}} = 2$ //

f) $\lim_{x \to \infty} \frac{(\sqrt{x^2+1} - \sqrt{x^2-1})(\sqrt{x^2+1} - \sqrt{x^2-1})}{(\sqrt{x^2+1} - \sqrt{x^2-1})} = \frac{(\cancel{\sqrt{x^2+1}})^{\cancel{2}} - (\cancel{\sqrt{x^2-1}})^{\cancel{2}}}{(\sqrt{x^2+1} - \sqrt{x^2-1})}$

$= \frac{0}{\sqrt{x^{\cancel{2}}}} = \frac{0}{x} = 0$ //

g) $\lim_{x \to \infty} \frac{(\sqrt{x^4+x^2} + \sqrt{x^2+5x} - x^2 - x)(\sqrt{x^4+x^2} + \sqrt{x^2+5x} + x^2 + x)}{(\sqrt{x^4+x^2} + \sqrt{x^2+5x} - x^2 - x)}$

$\Rightarrow \frac{(\sqrt{x^4+x^2})^{\cancel{2}} - (\sqrt{x^2+5x})^{\cancel{2}} - (x^2)^2 - (x)^2}{(\sqrt{x^4+x^2} + \sqrt{x^2+5x} - x^2 - x)} = \frac{\cancel{x^4} + \cancel{x^2} - \cancel{x^2} + 5x - \cancel{x^4} - x^2}{(\sqrt{x^4+x^2} + \sqrt{x^2+5x} - x^2 - x)}$

$\Rightarrow \frac{-x^2}{\sqrt{x^4}} = \frac{x^2}{x^2} = 0$

16) a) $\lim_{x \to 1} f(x)$ $\quad 3x \leq f(x) \leq x^3 + 2$, p/ $0 \leq x \leq 2$

$\lim_{x \to 1} 3x = 3(1) = 3$

$\lim_{x \to 1} (x^3 + 2) = (1)^3 + 2 = 3$

Logo $\lim_{x \to 1} f(x) = 3$

b) $\lim_{x \to 0} x^4 \cos\left(\frac{4}{x}\right)$ $\Rightarrow$ $-1 \leq \cos\left(\frac{4}{x}\right) \leq 1$

$\lim_{x \to 0} -x^4 = 0$

$\lim_{x \to 0} x^4 = 0$

$\Bigg\}$ $\lim_{x \to 0} x^4 \cos\left(\frac{4}{x}\right) = 0$

c) $\lim_{x \to \infty} f(x)$, se $\frac{4x-1}{x} < f(x) < \frac{4x^2+3x}{x^2}$ p/ $x > 5$

limite inf: $\frac{4x-1}{x} = \frac{4\cancel{x}}{\cancel{x}} - \frac{1}{x} = 4 - \frac{1}{x}$

$$\lim_{x\to\infty} \left(4 - \frac{1}{x}\right) = 4$$

limite sup: $\frac{4x^2+3x}{x^2} = \frac{4\cancel{x^2}}{\cancel{x^2}} + \frac{3x}{x^2} = 4 + \frac{3}{x}$

$$\lim_{x\to\infty} \left(4 - \frac{1}{x}\right) = 4$$

logo, $\lim_{x\to\infty} f(x) = 4$

d) $\lim_{x\to\infty} \frac{\sin^2 x}{x^2} \to \boxed{0 \leq \sin^2 x \leq 1}$

$0 \leq \frac{\sin^2 x}{x^2} \leq \frac{1}{x^2}$

$\lim_{x\to\infty} \frac{1}{x^2} = 0$ $\qquad$ $\lim_{x\to\infty} \frac{\sin^2 x}{x^2} = 0$ //

e) $\lim_{x\to 0^+} \sqrt{x}\, e^{\sin(\pi/x)}$

$e^{-1} \leq e^{\sin(\pi/x)} \leq e$

$e^{-1}\sqrt{x} \leq \sqrt{x}\, e^{\sin(\pi/x)} \leq e\sqrt{x}$

tende a 0

$$\lim_{x\to 0^+} \sqrt{x}\, e^{\sin(\pi/x)} = 0$$ //