Lista 4 - Cálculo

Nome: Raina Nunes Peret 2024.1.08.021

1) a) $f(x) = x^9 + x^4 + 2$

função polinomial de grau 9

b) $g(x) = \sqrt[7]{x}$

função raiz

c) $h(x) = \sqrt{1 - x^2}$

função raiz

d) $r(x) = \dfrac{x^2 + 3x - 1}{x^3 + x}$ → função racional

e) $s(x) = \tan 2x$

função trigonométrica

f) $t(x) = \log_{10} x$

função logarítmica

g) $f(x) = \dfrac{x-6}{x+6}$ → função racional

h) $g(x) = x + \dfrac{x^2}{\sqrt{x-1}}$ → função algébrica

i) $h(x) = 10^x$

função exponencial

j) $r(x) = x^{10}$

função potência

k) $s(t) = 2t + \pi$

função afim

l) $t(\theta) = \cos(\theta) + \operatorname{sen}(\theta)$

função trigonométrica

2) a) $f(x) = 2x^2 - x$ $\qquad g(x) = 3x + 2$

$f \circ g \Rightarrow f(g(x))$

$(3x+2)(3x+2) = 9x^2 + 6x + 6x + 4$

$2\,g(x)^2 - g(x) = 2(3x+2)^2 - 3x + 2$

$2(9x^2 + 12x + 4) - 3x + 2$ $\qquad$ Dom = $\mathbb{R}$

$18x^2 + 24x + 8 - 3x + 2 = 18x^2 + 21x + 10$ //

$g \circ f$

$3f(x) + 2 = 3(2x^2 - x) + 2$ $\qquad$ Dom = $\mathbb{R}$

$6x^2 - 3x + 2$ //

$f \circ f$

$(2x^2 - x)(2x^2 - x) = 4x^4 - 2x^3 - 2x^3 + x^2$

$2f(x)^2 - f(x) = 2(2x^2 - x)^2 - (2x^2 - x)$

$2(4x^4 - 4x^3 + x^2) - 2x^2 + x$ $\qquad$ Dom = $\mathbb{R}$

$8x^4 - 8x^3 + \cancel{2x^2} - \cancel{2x^2} + x \Rightarrow 8x^4 - 8x^3 + x$ //

$g \circ g$

$3(gx) + 2 = 3(3x+2)+2$ $\quad$ Dom $= \mathbb{R}$

$9x + 6 + 2 \Rightarrow 9x + 8$

b) $f(x) = \sqrt{x-1}$ $\quad$ $g(x) = x^2$

$f \circ g$

$\sqrt{g(x) - 1} = \sqrt{(x)^2 - 1}$

$\rightarrow x^2 - 1 \geq 0$ $\quad$ Dom $=]-\infty, -1] \cup [+1, +\infty[$

$x^2 \geq 1$ $\quad$ $x \geq \pm\sqrt{1} \Rightarrow x \geq \pm 1$

$g \circ f$

$f(x)^2 = (\cancel{\sqrt{x-1}})^{\cancel{2}} = x - 1$ $\quad$ Dom $= [1, +\infty[$

$x - 1 \geq 0 \Rightarrow x \geq 1$

$f \circ f$

$\sqrt{f(x) - 1} = \sqrt{\sqrt{x-1} - 1}$

$\sqrt{x-1} - 1 \geq 0$

$(\cancel{\sqrt{x-1}})^{\cancel{2}} \geq 1^2$ $\quad$ Dom $= [2, +\infty[$

$x - 1 \geq 1 \Rightarrow x \geq 2$

$g \circ g$

$(x^2)^2 = x^4$ $\quad$ Dom $= \mathbb{R}$

c) $f(x) = x + \frac{1}{x}$, $\quad$ $g(x) = x^3 + 2x$

$f \circ g$

$g(x) + \frac{1}{g(x)} = x^3 + 2x + \frac{1}{x^3 + 2x}$

$x^3 + 2x \neq 0 \rightarrow x\underbrace{(x^2+2)}_{\text{nunca é zero}} = 0$ $\quad$ Dom $= \mathbb{R} - \{0\}$

$g \circ f$

$f(x)^3 + 2f(x)$

$\left(x+\frac{1}{x}\right)^3 + 2\left(x+\frac{1}{x}\right) =$

$x^3 - 3x^2\left(\frac{1}{x}\right) + 3x\left(\frac{1}{x^2}\right) - \frac{1}{x^3} = x^3 + 3x + 3\frac{1}{x} + \frac{1}{x^3}$

$x^3 + 3x + 3\frac{1}{x} + \frac{1}{x^3} + 2x + 2\frac{1}{x} = x^3 + 5x + 5\frac{1}{x} + \frac{1}{x^3}$

$Dom = \mathbb{R} - \{0\}$

$f \circ f$

$f(x) + \frac{1}{f(x)} = \left(x+\frac{1}{x}\right) + \frac{1}{\left(x+\frac{1}{x}\right)}$ $\qquad x + \frac{1}{x} \neq 0$

$Dom = \mathbb{R} - \{0\}$

$x + \frac{1}{x} = 0 \Rightarrow x = \frac{-1}{x}$

$g \circ g$

$(g(x))^3 + 2g(x)$

$(x^3+2x)^3 + 2(x^3+2x)$

$x^9 - 3(x^3)^2 2x + 3(x^3)(2x)^2 - 2x^3$

$= x^9 + 6x^7 + 12x^5 + 8x^3$

$= x^9 + 6x^7 + 12x^5 + 8x^3 + 2x^3 + 4x$

$= x^9 + 6x^7 + 12x^5 + 10x^3 + 4x$ $\qquad Dom = \mathbb{R}$

d) $f(x) = \cos x \qquad g(x) = 1 - \sqrt{x}$

$f \circ g$

$\cos g(x) = \cos(1-\sqrt{x}) \qquad Dom = [0, +\infty[$

$g \circ f$

$1 - \sqrt{f(x)} = 1 - \sqrt{\cos x}$

$\cos(x) \geq 0 \qquad Dom = [0, +\infty[$

$f \circ f$

$\cos(f x) = \cos(\cos(x))$

$Dom = \mathbb{R}$

$g \circ g$

$1 - \sqrt{g(x)} = 1 - \sqrt{1 - \sqrt{x}}$

$1 - \sqrt{x} \geq 0$ $\quad$ $Dom = [0, 1]$

$-\sqrt{x} \geq -1 \to x \leq 1$

3) $f \circ g \circ h$

a) $f(x) = 3x - 2$, $g(x) = \text{sen}\, x$, $h(x) = x^2$

$3(g(x)) - 2 = 3\,\text{sen}(h(x)) - 2$

$3\,\text{sen}(x^2) - 2$

b) $f(x) = |x - 4|$, $g(x) = 2^x$, $h(x) = \sqrt{x}$

$|g(x) - 4| = |2^x - 4|$

$= |2^{\sqrt{x}} - 4|$

c) $f(x) = \sqrt{x - 3}$, $g(x) = x^2$, $h(x) = x^3 + 2$

$\sqrt{g(x) - 3} = \sqrt{(h(x))^3 - 3}$

$= \sqrt{(x^3 + 2)^2 - 3}$

d) $f(x) = \tan x$, $g(x) = \dfrac{x}{x-1}$, $h(x) = \sqrt[3]{x}$

$\tan\left(\dfrac{h(x)}{h(x) - 1}\right) \Rightarrow \tan \dfrac{\sqrt[3]{x}}{\sqrt[3]{x} - 1}$

4) $f(-x) = \dfrac{1 + (-x)}{1 - (-x)} = \dfrac{1 - x}{1 + x}$

$\dfrac{1}{f(x)} = \dfrac{1}{\frac{1+x}{1-x}} = \dfrac{1-x}{1+x}$ $\quad$ portanto; $f(-x) = \dfrac{1}{f(x)}$

$f \circ f$

$$f\left(\frac{1+x}{1-x}\right) \Rightarrow \frac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}}$$

num $= 1 + \frac{1+x}{1-x} = \frac{(1-x)+(1+x)}{1-x} = \frac{2}{1-x}$

den $= 1 - \frac{1+x}{1-x} = \frac{(1-x)-(1+x)}{1-x} = \frac{-2x}{1-x}$

$$f(f(x)) = \frac{\cancel{\frac{2}{1-x}}}{\cancel{\frac{-2x}{1-x}}} = \frac{2}{-2x} = \frac{-1}{x}$$

5) a) $f(3)=7$

$f^{-1}(7)=3$

b) $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$ $\qquad f^{-1}=(3)$

$3 + x^2 + \tan\left(\frac{\pi x}{2}\right) = 3$

$x^2 + \tan\left(\frac{\pi x}{2}\right) = 0$ $\qquad = x^2 = 0$

$\tan\left(\frac{\pi \cdot 0}{2}\right) = \tan(0) = 0$

$f^{-1}(3) = 0$

c) $f^{-1}(4)$, $f(x) = 3 + x + e^x$

$3 + x + e^x = 4$

$x + e^x = 1$

$0 + e^0 = 1$ $\qquad x = 0$

$f^{-1}(4) \approx 0$

d) $f(x) = 2x + \ln x$, $f^{-1}(2)$

$2x + \ln x = 2$

$\ln x = 2 - 2x$

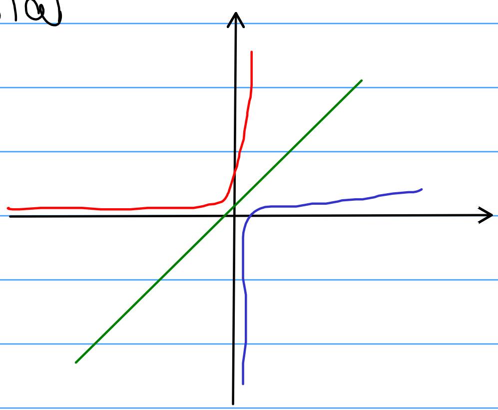
supondo: $x = 1$

$\ln(1) = 0$ e $2 - 2(1) = 0$

$f(1) = 2$

$f^{-1}(2) = 1$ //

6) a)

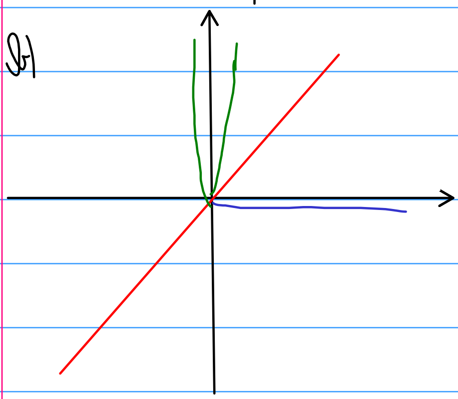


$y = e^x$ => função exponencial, cresce a medida que x aumenta

$y = x$ => função linear com inclinação 1 que passa pela origem

$y = \ln x$ => função logarítmica que cresce lentamente à medida que x aumenta

b)

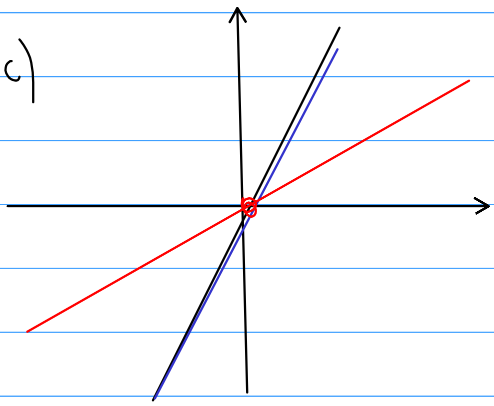


$y = x^2$ => função quadrática, cresce à medida que x aumenta

$y = \sqrt{x}$ => função raiz quadrada, cresce lentamente de acordo com x

$y = x$ => função linear com inclinação 1 que passa pela origem

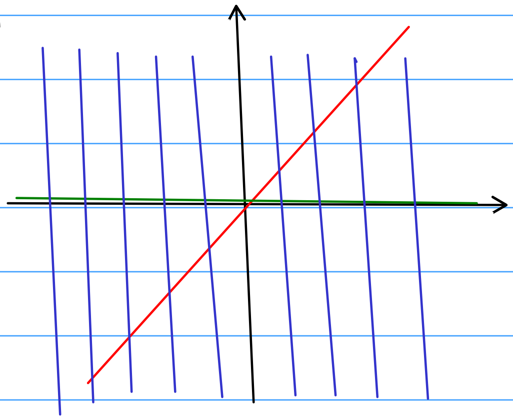
c)



$y = x$ => função linear com inclinação 1 que passa pela origem

$y = \arcsen x$ => inversa da função seno restrita

$y = \operatorname{sen} x$ => crescente no intervalo d)



$y = x \Rightarrow$ função linear

$y = \arctan x \Rightarrow$ inversa função tangente restrita

$y = \tan x \Rightarrow$ função crescente no intervalo

7) a) $f(x) = x^2 + 1 \quad , \quad x \geq 0$

$x = y^2 + 1$

$-y^2 = 1 - x \ (\times -1) \Rightarrow y^2 = x - 1$

$y = \sqrt{x-1}$

b) $f(x) = x^3 - 1$

$x = y^3 - 1$

$-y^3 = 1 - x \ (\times -1) \Rightarrow y^3 = x - 1$

$y = \sqrt[3]{x-1}$

c) $f(x) = x^2 - 2x + 1 \quad , \quad x \geq 1$

$f(x) = (x-1)^2$

$x = (y-1)^2$

$\sqrt{x} = y - 1$

$y = \sqrt{x} + 1$

d) $f(x) = x^{2/3}$

$x = y^{2/3}$

$y = x^{3/2}$

8) a) $f(x) = 1 + \sqrt{2+5x}$

$f(x) - 1 = \sqrt{2+5x}$

$(f(x)-1)^2 = (\sqrt{2+5x})^2$

$(f(x)-1)^2 = 2+5x$

$x = \dfrac{(f(x)-1)^2 - 2}{5}$ $\Rightarrow f^{-1}(y) = \dfrac{(f(y)-1)^2 - 2}{5}$

$2+5x \geq 0 \iff 5x \geq -2 \Rightarrow x \geq -\dfrac{2}{5}$ $\quad Im(f^{-1}) = [-2/5, +\infty[$

$1 + \sqrt{2+5x} \geq 1$ $\quad Dom = [1, +\infty[$

b) $f(x) = \dfrac{4x-1}{2x+3}$

$y(2x+3) = 4x-1$

$2xy + 3y = 4x-1 \Rightarrow 2xy - 4x = -3y - 1$

$x(2y-4) = -3y-1$

$x = \dfrac{-3y-1}{2y-4}$

$y = \dfrac{-3x-1}{2x-4}$

$2x+3 \neq 0 \Rightarrow 2x = -3 \Rightarrow x = \dfrac{-3}{2}$ $\quad Im = \mathbb{R} - \{-3/2\}$

$Dom =]-\infty, -2[\cup]2, \infty[$

c) $f(x) = e^{2x-1}$

$y = e^{2x-1}$ $\quad Im =]0, \infty[$

$\ln(y) = 2x-1$

$2x = \ln(y) + 1$ $\quad Dom =]0, \infty[$

$x = \dfrac{\ln(y)+1}{2}$

d) $y = 2^{10x}$

$\log_2(y) = 10x$

$x = \frac{\log_2(y)}{10}$

Dom $= [0, \infty[$

$f^{-1}(y) = \frac{\log_2(y)}{10}$

Im $= \mathbb{R}$

e) $y = \ln(x+3)$

$e^{x} = x+3$

$x = e^{y} - 3$

$f^{-1}(y) = e^{y} - 3$

Dom $= \mathbb{R}$

$x+3>0 \quad x>-3$

Im $=]-3, \infty[$

f) $f(x) = \frac{1+3x}{5-2x}$

$y = \frac{1+3x}{5-2x}$

$y(5-2x) = 1+3x$

$5y - 2xy = 1+3x$

$-2xy - 3x = 1-5y$

$x(-2y-3) = 1-5y$

$x = \frac{1-5y}{-2y-3} \rightarrow (\cdot -1) \rightarrow f^{-1}(x) = \frac{5y-1}{2y+3}$

Dom $= \mathbb{R}$

$5-2x=0 \Rightarrow x = \frac{5}{2}$ Im $=]-\infty, 5/2[\cup]5/2, +\infty[$

g) $y = \dfrac{1+e^x}{1-e^x}$

$y(1-e^x) = 1+e^x$

$y - ye^x = 1+e^x$

$ye^x + e^x = y-1$

$e^x(y+1) = y-1$

$e^x = \dfrac{y-1}{y+1} \Rightarrow x = \ln\left(\dfrac{y-1}{y+1}\right)$

$1-e^x \neq 0 \quad e^x \neq 1, \; x \neq 0$

$Dom =]-\infty, -1[\cup]1, \infty[$

$Im = \mathbb{R}$

h) $y = \dfrac{e^x}{1+2e^x}$

$x = \dfrac{e^y}{1+2e^y} \quad = x(1+2e^y) = e^y$

$x + 2xe^y = e^y$

$2xe^y - e^y = -x$

$e^y(2x-1) = -x$

$e^y = \dfrac{-x}{2x-1} \qquad y = \ln\left(\dfrac{-x}{2x-1}\right)$

$* \; 2x-1=0 \Rightarrow x = \dfrac{1}{2}$

$Dom = [0, 1/2] \qquad Im = \mathbb{R}$

9) a) $e^x = 16$

$x = \ln(16)$

$x \approx \ln(16)$

≈ 2.7726

b) $e^x = \ln(2)$

$x = \ln(\ln(2))$

$x \approx \ln(0.6931) \approx -0.3665$

c) $e^{2x+3} - 7 = 0$

$e^{2x+3} = 7$

$2x + 3 = \ln(7)$

$2x = \ln(7) - 3$

$x = \frac{\ln(7) - 3}{2}$

$x \approx \frac{1.9459 - 3}{2} \approx -0.5270$

d) $\ln x = -1$

$x = e^{-1}$

$x \approx 0.3679$

e) $\ln(2x-1) = 3$

$2x - 1 = e^3$

$2x = e^3 + 1$

$x = \frac{e^3 + 1}{2}$

$x \approx \frac{20.0855 + 1}{2} = 10.5428$

f) $\ln x + \ln(x-1) = 0$

$\ln(x(x-1)) = 0$

$x(x-1) = 1$

$x^2 - x - 1 = 0$ $\quad \Delta = (-1)^2 - 4(1)(-1)$

$x = \frac{-(-1) \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{1+4}}{2}$

$x = \frac{1 \pm \sqrt{5}}{2} \approx 1,618$

↳ Descarta o $-$

g) $\ln(\ln x) = 1$

$\ln x = e^1$

$x = e^1 = e^e$

$x \approx e^{2.718} \approx 15.1543$

h) $2^{x-5} = 3$

$x - 5 = \log_2(3)$

$x = \log_2(3) + 5$

$x \approx 1.585 + 5 \approx 6.585$

i) $1 + \arctan x = \sqrt{3}$

$\arctan x = \sqrt{3} - 1$

$x = \tan(\sqrt{3} - 1)$ $\quad x \approx \tan(0.732) \approx 0.893$

10) $\cos(\text{arc sen}(x)) = \sqrt{1-x^2}$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$x^2 + \cos^2(\theta) = 1$

$\cos^2(\theta) = 1 - x^2$

$\cos(\theta) = \sqrt{1-x^2}$

$-1 \leq x \leq 1$

$\text{sen}(2 \text{ arc sen } x)$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\rightarrow \theta = \text{arc sen}(x) \Rightarrow \sin(\theta) = x \quad , \quad \cos(\theta) = \sqrt{1-x^2}$

$\sin(2 \text{ arc sen}(x)) = 2 \text{ sen}(\text{arc sen}(x)) \cos(\text{arc sen}(x))$

$\sin(2 \text{ arc sen}(x)) = 2x\sqrt{1-x^2}$

$\cos(2 \text{ arc cos } x)$

$\cos(2\theta) = 2\cos^2(\theta) - 1$

$\cos(2 \text{ arc cos}(x)) = 2x^2 - 1$

$\tan(\text{arc sen } x)$

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$

$= \dfrac{x}{\sqrt{1-x^2}}$ //