Lista 9 - Cálculo I

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1) a) $f(x) = x \ln x - x$

1) Derivada de $x \ln x$

$f'(x) = x \ln x = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

2) Derivada de $-x$

$f'(x) = -x = -1$

portanto;

$f'(x) = \ln x + 1 - 1 = \ln x$ //

$f'(x) = \ln x$ //

b) $f(x) = \text{sen}(\ln x)$

derivada de $\text{sen}(x)$ é $\cos(u) \cdot u'$

$\ln x' = \frac{1}{x}$

$f'(x) = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$ /

$f'(x) = \frac{\cos(\ln x)}{x}$ //

c) $y = \ln \sqrt{x}$

$\ln x^{1/2} = \frac{1}{2} \ln x$

$y' = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$ // $y' = \frac{1}{2x}$ //

d) $y = \sqrt[3]{\ln x}$

$(\ln x)^{1/3}$ $* \; u^n = n u^{n-1}$ $u = \ln x$ $u' = \frac{1}{x}$ //

$y' = \frac{1}{3} (\ln x)^{-2/3} \cdot \frac{1}{x} = y' = \frac{1}{3x(\ln x)^{2/3}}$

e) $y = \ln(\operatorname{sen}^2 x)$

$= 2\ln(\operatorname{sen} x$

$y' = 2 \cdot \frac{1}{\operatorname{sen} x} \cdot \cos x = \quad y' = 2\cot x$ //

f) $f(t) = \ln(t\sqrt{t^2 - 1})$

$= \ln t + \frac{1}{2}\ln(t^2 - 1)$

$f'(t) = \frac{1}{t} + \frac{1}{2} \cdot \frac{1}{t^2 - 1} \cdot 2t$

$f'(t) = \frac{1}{t} + \frac{t}{t^2 - 1}$

g) $f(u) = \frac{u}{1 + \ln u}$

$f'(u) = \frac{(1 + \ln u) \cdot 1 - u \cdot \frac{1}{u}}{(1 + \ln u)^2} = \frac{1 + \ln u - 1}{(1 + \ln u)^2}$

$f(u) = \frac{\ln u}{(1 + \ln u)^2}$

h) $y = 2x \log_{10}\sqrt{x}$

$= \log_{10}\sqrt{x} = \frac{1}{2}\log_{10} x$

$y = x \log 10 x$

$y' = 2 \cdot \left[1 \cdot \log 10 x + x \cdot \frac{1}{x\ln 10}\right]$

$y' = 2 \cdot \left(\log_{10} x + \frac{1}{\ln 10}\right)$

i) $y = \log_5 (xe^x)$

$= \log_5 x + \log_5 e^x = \log_5 x + x\log_5 e$

$y' = \frac{1}{x\ln 5} + \ln 5 \cdot \frac{1}{\ln 5}$

$y' = \frac{1}{x\ln 5} + \frac{\ln e}{\ln 5} = \frac{1}{x\ln 5} + \frac{1}{\ln 5}$ //

2) a) $y = (2x+1)^5 (x^4-3)^6 \rightarrow \underbrace{\ln((2x+1)^5) + \ln((x^4-3)^6)}_{5\ln(2x+1) + 6\ln(x^4-3)}$

$$\left(\frac{1}{y} \cdot y'\right) = 5 \cdot \frac{1}{2x+1} \cdot (2x+1)' + 6 \cdot \frac{1}{x^4-3} \cdot (x^4-3)'$$

$(2x+1)' = 2 \qquad 5 \cdot \frac{1}{2x+1} \cdot 2 = \frac{10}{2x+1}$

$(x^4-3)' = 4x^3 \qquad 6 \cdot \frac{1}{x^4-3} \cdot 4x^3 = \frac{24x}{x^4-3}$

$$y' = (2x+1)^5 (x^4-3)^6 \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3}\right) //$$

b) $\dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

$$\ln y = \frac{3}{4} \ln x - \frac{1}{2} \ln(x^2+1) - 5\ln(3x+2)$$

$$\frac{y'}{y} = \frac{3}{4} \cdot \frac{1}{x} - \frac{1}{2} \cdot \frac{2x}{x^2+1} - 5 \cdot \frac{3}{3x+2}$$

$$\frac{y'}{y} = \frac{3}{4x} - \frac{x}{x^2+1} - \frac{15}{3x+2}$$

$$y' = y\left(\frac{3}{4x} - \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$$

$$y' = \frac{x^{3/4}(\sqrt{x^2+1})}{(3x+2)^5} \cdot \left(\frac{3}{4x} - \frac{x}{x^2+1} - \frac{15}{3x+2}\right) //$$

c) $y = x^{1/x^2}$

$\frac{1}{x^2} \ln x$

$$\frac{d}{dx}\left(\frac{\ln x}{x^2}\right)$$

$$\frac{1/x \, x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x\ln x}{x^3} = \frac{1-2\ln x}{x^3}$$

$$y' = y \cdot \frac{1 - 2\ln x}{x^3}$$

$$y' = x^{1/x^2} \cdot \frac{1 - 2\ln x}{x^3}$$

d) $y = x^{\operatorname{sen} x}$

$= \operatorname{sen} x \ln x$

$$\frac{y'}{y} = \cos x \ln x + \operatorname{sen} x \cdot \frac{1}{x}$$

$$y' = y \left(\cos x \ln + \frac{\operatorname{sen} x}{x} \right)$$

$$y' = x^{\operatorname{sen} x} \left(\cos x \ln x + \frac{\operatorname{sen} x}{x} \right)$$

e) $y = (\operatorname{sen} x)^{\ln x}$

$\ln y = \ln x \cdot \ln(\operatorname{sen} x)$

$$\frac{y'}{y} = [\ln x \cdot \ln(\operatorname{sen} x)]$$

$= (\ln x)' \cdot \ln(\operatorname{sen} x) + (\ln(\operatorname{sen} x))' \cdot \ln x$

$= \frac{1}{x} + \cot x$

$= \frac{1}{x} \ln(\operatorname{sen} x) + \cot x \ln x$

$$y' = y \cdot \left(\frac{1}{x} \ln(\operatorname{sen} x) + \cot x \ln x \right)$$

$$y' = (\operatorname{sen} x)^{\ln x} \cdot \left(\frac{\ln(\operatorname{sen} x)}{x} + \cot x \ln x \right)$$

f) $y = (\ln x)^{\cos x}$

$\ln y = \cos x \cdot \ln(\ln x)$

$$\frac{y'}{y} = [\cos x \cdot \ln(\ln x)]$$

$= (\cos x)' \cdot \ln(\ln x) + (\ln(\ln x))' \cdot \cos x$

$*(\cos x)' = -\operatorname{sen} x$

$$(\ln(\ln x))' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

$$y' = -\operatorname{sen} x \ln(\ln x) + \frac{\cos x}{x \ln x}$$

$$y' = y \cdot \left(-\operatorname{sen} x \ln(\ln x) + \frac{\cos x}{x \ln x}\right)$$

$$y' = (\ln x)^{\cos x} \cdot \left(-\operatorname{sen} \ln(\ln x) + \frac{\cos x}{x \ln x}\right)$$

3) a) $y = 7^x \operatorname{sen} x$

1º $y' = (7^x)' \operatorname{sen} x + 7^x (\operatorname{sen} x)'$

$$y' = 7^x \ln 7 \operatorname{sen} x + 7^x \cos x$$

2º $y'' = \underbrace{(7^x \ln 7 \operatorname{sen} x)'}_{7^x \ln^2 7 \operatorname{sen} x + 7^x \ln 7 \cos x} + \underbrace{(7^x \cos x)'}_{7^x \ln 7 \cos x - 7^x \operatorname{sen} x}$

$$y'' = 7^x (\ln^2 7 \operatorname{sen} x + 2 \ln 7 \cos x - \operatorname{sen} x)$$

b) $y = x(2x+1)^4$

$y' = \underbrace{(x)'}_{1}(2x+1)^4 + x\underbrace{((2x+1)^4)'}_{8(2x+1)^3}$

$$y' = (2x+1)^4 + 8x(2x+1)^3$$

$$y'' = \underbrace{8(2x+1)^3}\ \underbrace{8(2x+1)^3 + 48x(2x+1)^2}$$

$$y'' = 8(2x+1)^3 + 8(2x+1)^3 + 48(2x+1)^2$$

$$y'' = 16(2x+1)^3 + 48x(2x+1)^2 \ //$$

c) $y = \cos(\ln x)$

$y' = -\operatorname{sen}(\ln x) \cdot (\ln x)' \to \frac{1}{x}$

$$y' = -\frac{\operatorname{sen}(\ln x)}{x}$$

$$y'' = -\left(\frac{\operatorname{sen}(\ln x)}{x}\right)'$$

$$\cos \ln(x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$$

$$y'' = -\frac{[\operatorname{sen}(\ln x)]' \cdot x - \operatorname{sen}(\ln x) \cdot \overset{1}{(x)'}}{x^2}$$

$$y'' = -\frac{\frac{\cos(\ln x)}{x} \cdot x - \operatorname{sen}(\ln x) \cdot 1}{x^2}$$

$$y'' = -\frac{\cos(\ln x) - \operatorname{sen}(\ln x)}{x^2}$$

4) a)

$f_0(x) = x^{1/3}$

$$f_0'(x) = \frac{1}{3} x^{-2/3}, \quad f''(x) = -\frac{2}{9} x^{-5/3}, \quad f_0'''(x) = \frac{10}{27} x^{-8/3}$$

em $a = 1$

$f_0(1) = 1^{1/3} = 1$

$f_0'(1) = \frac{1}{3}$

$f_0''(1) = -\frac{2}{9}$

$f_0'''(1) = \frac{10}{27}$

→polinomio de taylor

$$p3(x) = f_0(1) + f_0'(1)(x-1) + \frac{f_0''(1)}{2!}(x-1)^2 + \frac{f_0'''(1)}{3!}(x-1)^3$$

$$p3(x) = 1 + \frac{1}{3}(x-1) - \frac{2/9}{2}(x-1)^2 + \frac{10/27}{6}(x-1)^3$$

$$p3(x) = 1 + \frac{1}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{5}{81}(x-1)^3$$

b) $f'(x) = e^x$, $f''(x) = e^x$, $f'''(x) = e^x$

em $a = 0$

$f(0) = e^0 = 1$

$f'(0) = 1$

$f''(0) = 1$

$f'''(0) = 1$

$$p_3(x) = f(0) + f'(0) \cdot x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3$$

$$p_3(x) = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 \text{ //}$$

c)

$f'(x) = \cos x$, $f''(x) = -\text{sen}\, x$, $f'''(x) = -\cos x$

em $a = 0$

$f(0) = \text{sen}(0) = 0$

$f'(0) = \cos(0) = 1$

$f''(0) = -\text{sen}(0) = 0$

$f'''(0) = -\cos(0) = -1$

$$p_3(x) = f(0) + f'(0) x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3$$

$$p_3(x) = 0 + x + 0 + \frac{-1}{6} x^3$$

$$p_3(x) = x - \frac{1}{6} x^3 \text{ //}$$

5) a) $x \to 1$

$x^4 - 1 = (-1)^4 - 1 = 1 - 1 = 0$

$x + 1 = -1 + 1 = 0$

$\Big\} \frac{0}{0}$

$(x^4 - 1)' = 4x^3$ $\quad (x+1)' = 1$

$$\lim_{x \to -1} \frac{x^4 - 1}{x + 1} = \lim_{x \to -1} \frac{4x^3}{1}$$

$x = -1$

$$\frac{4(-1)^3}{1} = \frac{-4}{1} = -4 \,//$$

b) $x \to 1$

$$\left.\begin{array}{l} x^9 - 1 = 1^9 - 1 = 0 \\ x^5 - 1 = 1^5 - 1 = 0 \end{array}\right\} \frac{0}{0}$$

$(x^9 - 1)' = 9x^8$

$(x^5 - 1)' = 5x^4$

$$\lim_{x \to 1} \frac{x^9 - 1}{x^5 - 1} = \lim_{x \to 1} \frac{9x^8}{5x^4}$$

$$\lim_{x \to 1} \frac{9x^8}{5x^4} = \lim_{x \to 1} \frac{9x^4}{5} = \frac{9(1)^4}{5} = \frac{9}{5} \,//$$

c) $\lim_{x \to 0} \frac{e^x - 1}{\operatorname{sen} x}$

$x \to 0$

$$\left.e^x - 1 = e^0 - 1 = 0 \qquad \operatorname{sen} x = \operatorname{sen}(0) = 0\right\} \frac{0}{0}$$

$(e^x - 1)' = e^x$

$(\operatorname{sen} x)' = \cos x$

$$\lim_{x \to 0} \frac{e^x - 1}{\operatorname{sen} x} = \frac{e^x}{\cos x}$$

$x = 0$

$$\frac{e^0}{\cos(0)} = \frac{1}{1} = 1 \,//$$

d) $\lim\limits_{x \to 0} \dfrac{\operatorname{sen} x}{x^3}$

$x \to 0 \quad , \; x^3 \to 0 \quad \Big\} \; 0/0$

$(\operatorname{sen} x)' = \cos x$

$(x^3)' = 3x^2$

$(\cos x)' = -\operatorname{sen} x \qquad (3x^2) = 6x$

$\lim\limits_{x \to 0} \dfrac{\cos x}{3x^3} = \dfrac{-\operatorname{sen} x}{6x}$

$(-\operatorname{sen} x)' = -\cos x \qquad (6x) = 6$

$\lim\limits_{x \to 0} \dfrac{-\operatorname{sen} x}{6x} = \dfrac{-\cos x}{6} = \dfrac{-1}{6}$ //

e) $\lim\limits_{x \to \infty} \dfrac{\ln x}{x}$

$\dfrac{\infty}{\infty} \qquad (\ln x)' = 1/x$

$(x)' = 1$

$\lim\limits_{x \to \infty} \dfrac{\ln x}{x} = \dfrac{1/x}{1} = \dfrac{1}{x} = 0$

f) $\lim\limits_{x \to 0} 3x \csc x \qquad * \; \csc x = \dfrac{1}{\operatorname{sen} x}$

$= \dfrac{3x}{\operatorname{sen} x} \quad \Big\} \; 0/0$

$(3x)' = 3 \qquad (\operatorname{sen} x)' = \cos x$

$\lim\limits_{x \to 0} \dfrac{3x}{\operatorname{sen} x} = \dfrac{3}{\cos x} = \dfrac{3}{1} = 3$ //

g) $\lim\limits_{x \to 0^+} \sqrt{x} \ln x$

$\sqrt{x} \to 0 \qquad \ln x \to -\infty \quad \Big\} \; \dfrac{-\infty}{\infty}$

$(\ln x)' = 1/x$

$(1/\sqrt{x})' = -\dfrac{1}{2x^{3/2}}$

$$\lim_{x\to 0^+} \frac{\ln x}{1/\sqrt{x}} = \frac{1/x}{-\frac{1}{2x^{3/2}}}$$

$$\lim_{x\to 0^+} 2\sqrt{x} = 0$$

h) $\displaystyle\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

$x \to 0 \quad e^x - 1 - x \to 0 \ \} \ 0/0$

$(e^x - 1 - x)' = e^x - 1$

$(x^2)' = 2x$

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \frac{e^x - 1}{2x}$$

$(e^x - 1)' = e^x$

$(2x)' = 2$

$\Big\} \displaystyle\lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$ //

i) $\displaystyle\lim_{x\to 0} x \cot x \qquad * \cot x = \frac{\cos x}{\operatorname{sen} x}$

$= \dfrac{x \cos x}{\operatorname{sen} x} \ \} \ 0/0$

$(x \cos x)' = \cos x - x \operatorname{sen} x$

$(\operatorname{sen} x)' = \cos x$

$$\lim_{x\to 0} \frac{\cos x - x \operatorname{sen} x}{\cos x}$$

$x = 0$

$$\frac{\cos(0) - 0 \cdot \operatorname{sen}(0)}{\cos(0)} = \frac{1-0}{1} = 1$$ /

j) $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

$x^2 \to 0$ y $0/0$

$(1-\cos x)' = \operatorname{sen} x \qquad (x^2)' = 2x$

$\lim_{x \to 0} \frac{\operatorname{sen} x}{2x} = \frac{\cos x}{2} = \frac{1}{2}$ //

K) $\lim_{x \to 0} \frac{\operatorname{sen} x}{e^x}$

$\frac{\operatorname{sen}(0)}{e^0} = \frac{0}{1} = 0$ y definida

l) $\lim_{x \to -\infty} \frac{x^2}{e^x}$

$\frac{\infty}{0}$ $\qquad \lim_{x \to \infty} \frac{x^2}{e^x} = 0$

//

6) a)

$\ln y = \frac{1}{x^2} \ln(\cos x)$

$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln(\cos x)}{x^2}$

$\ln(\cos x) \to \ln(1) = 0$

$x^2 \to 0$ $\qquad$ y $0/0$

$(\ln(\cos x))' = -\tan x$

$(x^2)' = 2x$

$\lim_{x \to 0^+} \frac{-\tan x}{2x} = \frac{-\tan(0)}{2(0)} = 0$

$\ln y = 0 \Rightarrow y = e^0 = 1$

$\lim_{x \to 0^+} (\cos x)^{1/x^2} = 1$

b) $\lim_{x \to 0} (1-2x)^{1/x}$

$\ln y = \frac{1}{x} \ln(1-2x)$

$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\ln(1-2x)}{x}$

$x \to 0 \; \} \; 0/0$

$(\ln(1-2x))' = \frac{-2}{1-2x}$ $\qquad (x)' = 1$

$\lim_{x \to 0} \frac{\ln(1-2x)}{x} = \frac{-2}{1-2x}$

$x \to 0$

$\lim_{x \to 0} (1-2x)^{1/x} = e^{-2}$

c) $\lim_{x \to \infty} x^{1/x}$ $\qquad \ln y = \frac{\ln x}{x}$

$\} \; \infty/\infty$ $\qquad (\ln x)' = 1/x \quad (x)' = 1$

$\lim_{x \to \infty} \frac{1/x}{1} = \frac{1}{x} = 0$

$y = e^0 = 1$

$\lim_{x \to \infty} x^{1/x} = 1$

7) $x \to 0$

$\left.\begin{array}{l} \operatorname{sen} x \to 0 \\ x \to 0 \end{array}\right\} 0/0$

$\left.\begin{array}{l} (\operatorname{sen} x)' = \cos x \\ (x)' = 1 \end{array}\right\} \lim_{x \to 0} \frac{\operatorname{sen} x}{x} = \cos x$

$x \to 0$

$\cos(0) = 1$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$

$x \to 0$

$\cos x - 1 \to 0$ $\Big\} \, 0/0$

$x \to 0$

$(\cos x - 1)' = -\operatorname{sen} x \qquad (x)' = 1$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = -\operatorname{sen} x$$

$-\operatorname{sen}(0) = 0$ //

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \;(a > 0$$

$x \to 0$

$a^x - 1 \to 0$ $\Big\} \, 0/0$

$x \to 0$

$(a^x - 1)' = a^x \ln a$

$(x)' = 1$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = a^x \ln a$$

$x \to 0$

$a^0 \ln a = 1 \cdot \ln a = \ln a$ //

$$\underbrace{\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e}$$

$$= \lim_{x \to 0} x \ln\left(1 + \frac{1}{x}\right)$$

$x \to \infty$

$$\ln\left(1+\frac{1}{x}\right) \to \ln(1) = 0$$

$x \to \infty$

$$\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right) = \frac{\ln(1+1/x)}{1/x}$$

$$\left(\ln\left(1+1/x\right)\right)' = \frac{-1/x^2}{1+1/x} = \frac{-1}{x^2-x}$$

$$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$\lim_{x\to\infty} \frac{\ln(1+1/x)}{1/x} = \frac{-1/(x^2+x)}{-1/x^2} = \frac{x^2}{x^2+x}$$

$$= \frac{x^2}{x^2+x} = \frac{1}{1+1/x} = 1$$

Logo, $\ln y = 1$, e $y = e^1 = e$.