Lista 6 - Cálculo 1

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a) $\lim_{x \to -2} f(x) = +\infty$

O limite $f(x)$ tende a $+\infty$ à medida que x se aproxima de -2.

$\lim_{x \to -2^-} f(x) = +\infty$, $\lim_{x \to -2^+} f(x) = +\infty$

Portanto, para que o limite bilateral ($\lim_{x \to -2}$) seja $+\infty$, ambos os limites laterais precisam tender a $+\infty$.

b) Não, os limites laterais não podem ter valores diferentes. A função apresenta descontinuidade infinita em $x=1$, onde os limites laterais se divergem.

Limite bilateral = $\nexists$

c) Não, pois a função diverge para $-\infty$, indicando uma assíntota vertical em $x=4$.

d) Sim, isso indica que, para os valores de x muito grandes ou muito pequenos, os valores da função se aproximam dessas retas, mas sem necessariamente alcançá-las.

2)

a) $\lim_{x \to 0^+} \frac{1}{2x} = +\infty$

b) $\lim_{x \to 0^-} \frac{3}{5x^1} = -\infty$

c) $\lim_{x \to 2^-} \frac{3}{x-2} = -\infty$

d) $\lim_{x \to 3^+} \frac{1}{x-3} = \frac{1}{+\infty - 3} = +\infty$

e) $\lim_{x \to -5^-} \frac{3x}{2x+10} = \frac{3 \cdot -\infty}{2 \cdot -\infty + 10} = \frac{-\infty}{-\infty} = +\infty$

f) $\lim_{x \to 7} \frac{4}{(x-7)^2} = +\infty$

$\lim_{x \to 7^+} \frac{4}{(x-7)^2} = +\infty$ $\lim_{x \to 7^-} \frac{4}{(x-7)^2} = +\infty$

DÚVIDA g) $\lim_{x \to -2^+} \frac{2x}{x^3+8} = \frac{2 \cdot +\infty}{-\infty + 8} = \frac{+\infty}{-\infty} = -\infty$

h) $\lim_{x \to 0^-} \frac{2}{3x^{1/3}} = -\infty$

i) $\lim_{x \to \pi/2^-} \tan x = +\infty$

3) a) $y = \frac{X^2+4}{X-3}$ $\rightarrow$ $X-3=0$

$X=3$

$\lim_{x \to 3^+} \frac{\overbrace{X^2+4}^{13}}{X-3} = +\infty$

$\lim_{x \to 3^-} \frac{\overbrace{X^2+4}}{X-3} = -\infty$

b) $y = \frac{X^2-X-6}{X^2+2X-8} = \frac{(x+3)(x-2)}{(x+4)(x-2)}$

$\Delta = b^2 - 4ac = 1 + 24 = 25$ $(x+3)(x-2)$

$X = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 5}{2} =$ $x' = -3$

$x'' = 2$

$\Delta = b^2 - 4ac = 4 + 32 = 36$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm 6}{2} = \begin{matrix} x' = -4 \\ x'' = 2 \end{matrix}$ $\quad (x+4)(x-2)$

$y = \frac{(x+3)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}} = \frac{x+3}{x+4}$

$x + 4 = 0$

$\underline{x = -4}$ //

$\lim_{x \to -4^+} \frac{x+3}{x+4} = +\infty$

$\lim_{x \to -4^-} \frac{x+3}{x+4} = -\infty$

c) $y = \ln(x-1)$

↳ função logarítmica está definida apenas quando o argumento é positivo.

$x - 1 > 0$

$\underline{x > 1}$

$\lim_{x \to 1^+} \ln(x-1) = -\infty$

d) $y = \sec(x)$

$x = \frac{\pi}{2} + k\pi \, , \; k \in \mathbb{Z}$

$\lim_{x \to (\frac{\pi}{2} + k\pi)^+} \sec(x) = +\infty$

$\lim_{x \to (\frac{\pi}{2} + k\pi)^-} \sec(x) = -\infty$

4) a) $f(x) = \dfrac{x - \sqrt{1 + x^2}}{3x - 1}$ → $3x - 1 = 0$

$x = \dfrac{1}{3}$

AV

$f(x) = \dfrac{x - \sqrt{1 + x^{\cancel{2}}}}{3x - 1} = \dfrac{\cancel{x} - 1 - \cancel{x}}{3x - 1} = \dfrac{-1}{3x - 1}$

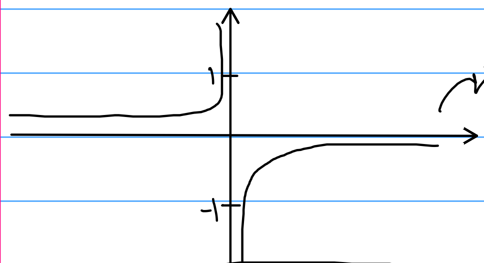
$\lim\limits_{x \to \frac{1}{3}^+} \dfrac{\sqrt{x^{\cancel{2}}}}{3x} = \dfrac{\cancel{x}}{3\cancel{x}} = \dfrac{1}{3}$ // → assíntota vertical!

$\lim\limits_{x \to \frac{1}{3}^-} \dfrac{\sqrt{x^{\cancel{2}}}}{3x} = \dfrac{\cancel{x}}{3\cancel{x}} = \dfrac{1}{3}$ //

AH

$\lim\limits_{x \to +\infty} \dfrac{x - \sqrt{1 + x^{\cancel{2}}}}{3x - 1} = \dfrac{\cancel{x} - 1 - \cancel{x}}{3x - 1} = \dfrac{-1}{3x - 1} = \dfrac{-1}{+\infty} = 0$ // → $\dfrac{c}{+\infty}$

$\lim\limits_{x \to -\infty} \dfrac{x - \sqrt{1 + x^2}}{3x - 1} = \dfrac{-1}{3x - 1} = \dfrac{-1}{-\infty} \to 0$ //



→ aproxima de 0 mas nunca alcança!

b) $g(x) = \dfrac{\sqrt{x^2 + 4}}{x}$ → $x = 0$

AV

$\lim\limits_{x \to 0^+} = \dfrac{\sqrt{x^{\cancel{2}} + 4}}{x} = \dfrac{\cancel{x} + 2}{\cancel{x}} = 2$

$\lim\limits_{x \to 0^-} = \dfrac{\sqrt{x^{\cancel{2}} + 4}}{x} = \dfrac{\cancel{x} + 2}{\cancel{x}} = 2$

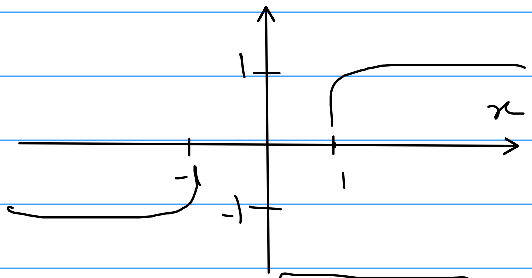
} não há assíntotas verticais!

AH

$$\lim_{x\to+\infty} = \frac{\sqrt{\cancel{x^2}}}{x} = 1$$

"metáfora"

$$\lim_{x\to-\infty} = \frac{\sqrt{\cancel{x^2}}}{x} = \frac{\infty}{-\infty} = -1$$

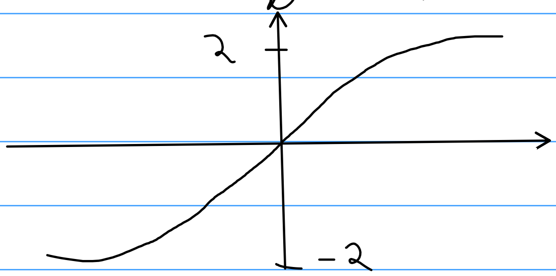


c) $h(x) = \dfrac{\sqrt{4x^2+1}}{x+2} \rightarrow x = -2$

AH

$$\lim_{x\to\infty^+} \frac{\sqrt{4\cancel{x^2}}}{x} = \frac{2\cancel{x}}{\cancel{x}} = 2$$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2}}{x} = \frac{2\cancel{x}}{\cancel{x}} = -2$$



5) a) $\displaystyle\lim_{\theta\to0} \frac{\operatorname{sen}\sqrt{2}\,\theta}{\theta} = \lim_{\theta\to0} \sqrt{2} \cdot \lim_{\theta\to0} \frac{\operatorname{sen}\theta}{\theta} = \sqrt{2}\cdot 1 = \sqrt{2}$ //

(limite $\to \sqrt{2}$, $\to 1$)

b) $\displaystyle\lim_{t\to0} \frac{\operatorname{sen} 8t}{\operatorname{sen} 9t} = \frac{\frac{\operatorname{sen}(8t)\cdot 8t}{8t}}{\frac{\operatorname{sen}(9t)\cdot 9t}{9t}}$

$$\Rightarrow \lim_{t\to0} \frac{8\cancel{t}}{9\cancel{t}} \cdot \lim_{t\to0} \frac{\frac{\operatorname{sen}(8t)}{8t}}{\frac{\operatorname{sen}(9t)}{9t}} = \frac{8}{9} \cdot \frac{1}{1} = \frac{8}{9}$$ //

c) $\lim_{\theta \to \frac{\pi}{2}} \frac{\sin(\cos\theta)}{\cos\theta}$

$= \frac{\sin(\cos\theta)}{\frac{\cos\theta}{\sin\theta}} = \frac{\sin(\cos\theta)\cdot\sin\theta}{\cos\theta}$

$\cos\theta \to 0 \quad \sin\theta \to 1$

$\lim_{\theta \to \pi/2} \frac{\sin(\cos\theta)\cdot 1}{0} \Rightarrow \infty$

d) $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\cos\theta + 1}{\cos\theta + 1} = \frac{(\cos\theta - 1)(\cos\theta + 1)}{\sin\theta(\cos\theta + 1)} \rightsquigarrow (\cos\theta)^2 - (1)^2$

$= \lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\sin\theta(\cos\theta + 1)} = \frac{-\sin^2\theta}{\sin\theta(\cos\theta + 1)}$

$= \lim_{\theta \to 0} \frac{-\sin\theta}{\cos\theta + 1} = \frac{-0}{1+1} = 0 //$

e) $\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta} =$

$\lim_{\theta \to 0} \cancel{\sin\theta} \xrightarrow{} 0 \quad \lim_{\theta \to 0} \cancel{\frac{\sin\theta}{\theta}} \xrightarrow{} 1 = 0 \cdot 1 = 0 //$

f) $\lim_{x \to 0} \frac{\tan x}{4x} = \frac{\frac{\sin x}{\cos x}}{4x} \quad \frac{\sin x}{\cos x} \cdot \frac{1}{4x}$

$= \cancel{\frac{\sin x}{x}} \to 1 \cdot \frac{1}{4} \cdot \cancel{\frac{1}{\cos x}} \to 1 = 1 \cdot \frac{1}{4} \cdot 1 = \frac{1}{4} //$

g) $\lim_{x \to 0} \frac{x + x\cos x}{\sin x \cos x}$

$\frac{x(1+\cos x) \div x}{\sin x \cos x \div x} \Rightarrow \lim_{x \to 0} \frac{1 + \cos x}{\sin x \cos x}$

$$= \left(\lim_{x\to 0} \frac{1+\cos x}{\cos x}\right) \cdot \left(\lim_{x\to 0} \frac{1}{\sin x}\right)$$

limite indeterminado!

h) $\displaystyle\lim_{x\to \frac{\pi}{4}} \frac{\operatorname{sen} x - \cos x}{\cos 2x}$

$\cos 2x = \cos^2 x - \sin^2 x$

cálculo numerador: $\sin \frac{\pi}{4} - \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

cálculo denominador: $\cos 2\left(\frac{\pi}{4}\right) = \cos \frac{\pi}{2} = 0$

$\frac{0}{0}$

$$= \lim_{x\to \frac{\pi}{4}} \frac{\cos x + \operatorname{sen} x}{-2\operatorname{sen} 2x}$$

$\cos \frac{\pi}{4} + \operatorname{sen} \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

$-2 \operatorname{sen}\left(\frac{\pi}{2}\right) = -2 \cdot 1 = -2$

$\frac{\sqrt{2}}{-2} = -\frac{\sqrt{2}}{2}$

i) $\displaystyle\lim_{x\to 1} \frac{\operatorname{sen}(x-1)}{x^2+x-2}$

$\Delta = b^2 - 4ac = 1+8 = 9$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 3}{2} = \begin{matrix} x' = -2 \\ x'' = 1 \end{matrix} \quad (x+2)(x-1)$

$\Rightarrow u = x - 1$

$$\lim_{u\to 0} \cancel{\frac{\operatorname{sen}(u)}{u}}^{\to 1} \cdot \frac{u}{u(u+3)}$$

$$1 \cdot \frac{1}{0+3} = \frac{1}{3}$$

6) a) $\lim_{x\to\infty}\left(1+\frac{7}{5x}\right)^{x}$

$=\left(\left(\left(1+\frac{1}{\frac{5x}{7}}\right)^{\frac{5x}{7}}\right)\right)^{\frac{7}{5}}$ $\quad \frac{5x}{7}\to\infty$

$\lim_{x\to\infty}\left(1+\frac{7}{5x}\right)=e^{\frac{7}{5}}$

b) $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}$

$=\left(1-\frac{1}{x+1}\right)^{x}$ $\quad 1-\frac{1}{x}\approx e^{-\frac{1}{x}}$

$=\lim_{x\to\infty}\left(1-\frac{1}{x+1}\right)^{x}=e^{-1}=\frac{1}{e}$

c) $\lim_{x\to\infty}\left(\frac{7x+3}{7x+4}\right)^{x+1}$

$\frac{7x+3}{7x+4}=1-\frac{1}{7x+4}$

$=\lim_{x\to\infty}\left(1-\frac{1}{7x+4}\right)^{x+1}$ $\quad = e^{-1/7}$

d) $\lim_{x\to 0}\frac{5^{x+3}-125}{x}$

$5^{x+3}=5^{x}\cdot 5^{3}=5^{x}\cdot 125$

$\frac{5^{x+3}-125}{x}=\frac{125(5^{x}-1)}{x}$

$a=5$

$\lim_{x\to 0}\frac{125(5^{x}-1)}{x}=125\cdot\ln 5$

e) $\lim\limits_{x \to 0} \dfrac{7^{x+2} - 49}{14x}$

$7^{x+2} = 7^x \cdot 7^2 = 7^x \cdot 49$

$\dfrac{7^{x+2} - 49}{14x} = \dfrac{49(7^x - 1)}{14x}$

$\lim\limits_{x \to 0} \dfrac{49(7^x - 1)}{14x} = \dfrac{49}{14} \cdot \ln 7 = \dfrac{7}{2} \ln 7$

f) $\lim\limits_{x \to 3} \dfrac{15^{x-3} - 1}{x - 3}$

$a = 15$ e $x \to 3$

$\lim\limits_{x \to 3} \dfrac{15^{x-3} - 1}{x - 3} = \ln 15$