Lista 8 - Cálculo I

Nome: Raissa Nunes Peret 2024.1.08.021

1) a) $y = 2xe^x$, $P = (0,0)$

$(2x)' \cdot e^x + 2x \cdot (e^x)'$

$2e^x + 2x \cdot e^x$

$e^x(2+2x)$

subst = $e^0(2+0) = 2$ $m=2$

$y - y^0 = m(x - x^0)$

$y - 0 = 2(x-0)$

$y = 2x$ //

b) $y = \frac{3^x}{x}$, $P = (1,3)$

$\frac{(3^x)' \cdot x - 3^x \cdot (x)'}{x^2} = \frac{3^x \cdot \ln 3 \cdot x - 3^x}{x^2}$

$\frac{3^x(\ln 3 \cdot x - 1)}{x^2}$

Subs: $\frac{3^1(\ln 3 - 1)}{1^2} = 3(\ln 3 - 1)$

equação:

$y - y_0 = m(x - x_0)$

$y - 3 = 3(\ln 3 - 1)(x-1)$

$y = 3 + 3(\ln 3 - 1)(x-1)$ //

c) $y = \tan x$, $P = (\pi/4, 1)$

$y' = (\tan x)'$

$y' = \sec^2(x) \to \sec^2(\pi/4)$

$= (\sqrt{2})^2 = 2$

equação:

$y - y_0 = m(x - x_0)$

$y - 1 = 2(x - \frac{\pi}{4})$

$y = 1 + 2(x - \frac{\pi}{4})$

$y = 1 + 2x - \frac{2\pi}{4} \Rightarrow y = 1 + 2x - \frac{\pi}{2}$ //

d) $y = \dfrac{1}{\text{sen } x + \cos x}$, $P = (0,1)$

$$\frac{\cancel{(1)' \cdot (\text{sen } x + \cos x)} - (1) \cdot (\text{sen } x + \cos x)'}{(\text{sen } x + \cos x)^2} = \frac{-(\cos x - \text{sen } x)}{(\text{sen } x + \cos x)^2}$$

$$= \frac{-\cos x + \text{sen } x}{\text{sen}^2 x + \cos^2 x}$$

$\cos(0) = 1$ //

subst =

$$= \frac{-\cos(0) + \text{sen}(0)}{(0 + 0)} \Rightarrow \frac{-1 + 0}{0 + 1} = \frac{-1}{1} = -1$$

equação:

$y - y_0 = m(x - x_0)$

$y - 1 = -1(x - 0)$

$y = -x + 1$ //

2) $f(x) = 2x^3 - 3x^2 - 6x + 87$

derivada $f'(x) = 0$

↳ Tangentes horizontais

$f'(x) = 6x^2 - 6x - 6 \quad (\div 6)$

$f'(x) = x^2 - x - 1$ //

$x^2 - x - 1 = 0$

$\Delta = b^2 - 4ac = 1 + 4 = 5$

$$x = \frac{-b \pm \sqrt{5}}{2a} = \frac{1 \pm \sqrt{5}}{2}$$

$x' = \dfrac{1 + \sqrt{5}}{2}$ //

$x'' = \dfrac{1 - \sqrt{5}}{2}$ //

b) inclinação $3x - y = 5$

$y = 3x - 5$

$y' = 3 - 0 \quad \underline{m = 3}$ //

↳ a derivada de $\underline{y = 1 + 2e^x - 3x = 3}$

$y' = (0) + 2e^x - 3$

$y' = 2e^x - 3$

$2e^x - 3 = 3$

$2e^x = 6$

$e^x = 3$

$\underline{x = \ln 3} \rightarrow$ ponto x

agora p/ descobrir y: (substitui valor de x)

$y = 1 + 2e^{\ln 3} - 3\ln 3$

$y = 1 + 2(3) - 3\ln 3$

$y = 7 - 3\ln 3$

$R = \left(\underbrace{\ln 3}_{x}, \underbrace{7 - 3\ln 3}_{y} \right)$

3) $y = x + x^2 \qquad P = (1, 2)$

$y' = 1 + 2x$

subst $=$ $y' = 1 + 2(1) = 3$

$m_t \cdot m_n = -1$

$m_t = 3$

$3 \cdot m_n = -1 \quad m_n = -\frac{1}{3}$

equação:

$y - y_0 = m(x - x_1)$

$y - 2 = -\frac{1}{3}(x - 1)$

$y = -\frac{x}{3} + \frac{1}{3} + 2$

$y = -\frac{1}{3}x + \frac{7}{3}$ //

4) $f(x) \approx L(x) = f(a) + f'(a)(x-a)$

$x = -1$

$f(a) = f(-1) = \sqrt[3]{-1} = -1$

$f'(a) = f'(x) = (\sqrt[3]{x})$

$x^{1/3} = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

subst:

$f'(-1) = \frac{1}{3\sqrt[3]{(-1)^2}} = \frac{1}{3 \cdot 1} = \frac{1}{3}$ //

subst:

$L(x) = -1 + \frac{1}{3}(x-(-1))$

$= -1 + \frac{1}{3}(x+1)$

$= -1 + \frac{1}{3}x + \frac{1}{3}$ $\qquad \frac{1}{3} - \frac{3}{3} = \frac{-2}{3}$

$= \frac{-2}{3} + \frac{1}{3}x$ //

para $x = \sqrt[3]{-0{,}96} = -0{,}96$

$L(-0{,}96) = \frac{-2}{3} + \frac{1}{3}(-0{,}96)$

$L(-0{,}96) = \frac{-2}{3} - \frac{0{,}96}{3} = \frac{-2{,}96}{3} \approx -0{,}987$ //

para $x = \sqrt[3]{-1{,}29} = -1{,}29$

$L(-1{,}29) = \frac{-2}{3} + \frac{1}{3}(-1{,}29)$

$= \frac{-2}{3} - \frac{1{,}29}{3} = \frac{-3{,}29}{3} \approx -1{,}097$ //

5) a) $y = (\underbrace{x^2+4x+6}_{u})^5 = (u^5)' = 5u^4 \cdot u'$

$y' = 5(x^2+4x+6)^4 \cdot (2x+4)$ //

b) $y = \cos(\underbrace{\tan x}_{u}) = (\cos u)' = -\operatorname{sen} u \cdot u'$

$-\operatorname{sen}(\tan x) \cdot \sec^2 x$

c) $y = e^{\underbrace{\sqrt{x}}_{u}} = (e^u)' = e^u \cdot u'$

$u' = x^{1/2} = \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2\sqrt{x}}$

$= e^u \cdot \frac{1}{2\sqrt{x}} = \frac{e^u}{2\sqrt{x}}$

d) $y = \tan \underbrace{3x}_{u} = (\tan u)' = \sec^2 u \cdot u'$

$\sec^2(3x) \cdot 3$

e) $y = \sqrt[3]{\underbrace{1+x^3}_{u}} = (u^{1/3})' = \frac{1}{3}u^{-2/3} = \frac{1}{3\sqrt[3]{u^2}}$

$= \frac{1}{3\sqrt[3]{u^2}} \cdot (1+x^3)' = \frac{1}{3\sqrt[3]{x^2}} \cdot 3$

$= \frac{3}{3\sqrt[3]{x^2}}$

f) $y = \operatorname{sen}(\underbrace{e^x}_{u}) = (\operatorname{sen} u)' = \cos u \cdot u'$

$\cos e^x \cdot e^x$

$= \cos 2e^x$

6) a) $f(x) = (\underbrace{x^3 + 4x}_{u})^7 = (u^7)' = 7u^6 \cdot u'$

$f'(x) = 7(x^3+4x)^6 \cdot (3x^2+4)$ //

b) $g(x) = \sqrt{x^2 - 7x}$

$= (\underbrace{x^2-7x}_{u})^{1/2} = (u^{1/2})' = \frac{1}{2} u^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$

$\frac{1}{2\sqrt{u}} \cdot (x^2-7x)' = \frac{1}{2\sqrt{u}} \cdot (2x-7) = \frac{2x-7}{2\sqrt{x^2-7x}}$ //

c) $s(t) = \sqrt[3]{1 + \tan t}$

$= (1+\tan t)^{1/3} = [(\underbrace{1+\tan t}_{u})^{1/3}]' = \frac{1}{3} u^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{u^2}}$

$\frac{1}{3\sqrt[3]{u^2}} \cdot (1+\tan t)' = \frac{1}{3\sqrt[3]{u^2}} \cdot \sec^2 = \frac{\sec^2}{3\sqrt[3]{(1+\tan t)^2}}$

d) $f(y) = \left(\underbrace{\frac{y-6}{y+7}}_{u}\right)^3 = (u^3)' = 3u^2 \cdot u'$

$\hookrightarrow \left(\frac{y-6}{y+7}\right)' = \frac{(y-6)' \cdot (y+7) - (y-6)(y+7)'}{(y+7)^2} = \frac{1 \cdot \cancel{y}+7) - (\cancel{y}-6) \cdot 1}{(y+7)^2} = \frac{13}{(y+7)^2}$

$3 \cdot \left(\frac{y-6}{y+7}\right)^2 \cdot \frac{13}{(y+7)^2} = \frac{39 \cdot (y-6)^2}{(y+7)^3}$ //

e) $\operatorname{senh} x = \frac{(e^x - e^{-x})}{2} \to \frac{1}{2} u \cdot u'$

$\hookrightarrow (e^x - (-e^{-x}) = e^x + e^{-x}$

$\frac{1}{2}(e^x + e^{-x})$

f) $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$\frac{1}{2}(u)\cdot(u)' = \frac{1}{2}(e^{x} - e^{-x})$ //
$= \operatorname{senh} x$

g) $y = \dfrac{e^{3x}}{1+e^{x}}$ $\curvearrowright 3e^{3x}$

$y' = \dfrac{(3e^{3x})(1+e^{x}) - (e^{3x})(e^{x})}{(1+e^{x})^{2}} = 3e^{3x} + 3e^{4x} - e^{4x} = \dfrac{3e^{3x} + 2e^{4x}}{(1+e^{x})^{2}}$ //

h) $f(x) = x\underbrace{e^{-x^{2}}}_{u}$ $(x(u))' \cdot u'$

$1(u)' \cdot u' \rightarrow -2x$

$e^{-x^{2}} \cdot -2x$ //

i) $y = \dfrac{\operatorname{sen}^{2} x}{\cos x} = \operatorname{sen}^{2} x \cdot (\cos x)^{-1} = (\operatorname{sen}^{2} x \cdot (u)^{-1})' = 2\operatorname{sen} x \cdot (u)^{-2} \cdot u'$

$y' = \dfrac{(\operatorname{sen}^{2} x)' \cdot (\cos x) - (\operatorname{sen}^{2} x) \cdot (\cos x)'}{((\cos x))^{2}} = \dfrac{\overset{2\operatorname{sen}}{\cancel{\operatorname{sen}}}(x) \cdot (\cos x) - (\operatorname{sen}^{2} x) \cdot -\operatorname{sen}(x)}{(\cos x)^{2}}$

$y' = \dfrac{2\operatorname{sen}(x)\cos^{2}(x) + \operatorname{sen}^{3}(x)}{\cos^{2} x}$

$= \dfrac{\operatorname{sen}(x)(2\cos^{2} x + \operatorname{sen}^{2} x)}{\cos^{2} x}$

j) $y = \operatorname{sen}(\underbrace{\operatorname{sen}(\operatorname{sen} x)}_{u})$ $= (\operatorname{sen}(u))' = \cos(u) \cdot u'$

$(\operatorname{sen}(\operatorname{sen} x))' = \cos(\operatorname{sen} x)$

$\cos(\operatorname{sen}(\operatorname{sen} x)) \cdot (\cos(\operatorname{sen} x))$ //

k) $y = \sqrt{\underbrace{x + \sqrt{x}}}$

$(x+\sqrt{x})^{1/2} = (\underbrace{x+(x)^{1/2}}_{u})^{1/2} = (u^{1/2})' = \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$

$\frac{1}{2\sqrt{u}} \cdot \underbrace{[(x+(x)^{1/2}]]'}$

$1 + \frac{1}{2} x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x+(x)^{1/2}}} \cdot 1 + \frac{1}{2\sqrt{x}}$ //

l) $y = \underbrace{e}_{u}^{e\,\text{sen}\,x} = ((u)^{e\,\text{sen}\,x})' = u^{e\,\text{sen}\,x} \cdot u'$

$= e^{e\,\text{sen}\,x} \cdot e^{\text{sen}\,x} \cdot \cos(x)$

7) a) $x^3 + x^2y + 4y^2 = 6$

$\underbrace{x^2y} + x^3 + 4y^2 = 6$

$(2x \cdot y + x^2 \cdot 1 \cdot y') + x^3 + 4y^2 = 6$

$2x \cdot y + x^2 \cdot y' + x^3 + 3x^2 + 8y \cdot y' = 6'$

$y'(2x^3 + 8y) = -3x^2 - x^3$

$y' = \frac{-3x^2 - x^3}{2x^3 + 8y}$ //

b) $\sqrt{xy} = 1 + x^2y$

$(xy)^{1/2} = 1 + x^2y$

$\frac{d}{dx}(xy)^{1/2} = \frac{d}{dx} 1 + \frac{d}{dx} x^2y$

$\frac{1}{2\sqrt{x}} \cdot y^{1/2} + x^{1/2} \cdot \frac{1}{2\sqrt{y}} \cdot y' = 2x \cdot y + x^2 \cdot 1 \cdot y'$

$$y'\left(x^{1/2}\cdot\frac{1}{2\sqrt{y}}-x^2\right)=2xy-\frac{1}{2\sqrt{x}}\cdot y^{1/2}$$

$$y'=\frac{2xy-\frac{1}{2\sqrt{x}}\cdot y^{1/x}}{x^{1/2}\cdot\frac{1}{2\sqrt{y}}-x^2}$$

c) $\operatorname{sen} x+\cos y=\operatorname{sen} x\cos y$

$$\frac{d}{dx}\operatorname{sen} x+\frac{d}{dx}\cos y=\frac{d}{dx}\operatorname{sen} x\cdot\cos y$$

$$\cos x+-\operatorname{sen} y\cdot y'=\cos x\cdot\cos y+\operatorname{sen} x\cdot(-\operatorname{sen} y)\cdot y'$$

$$y'(-\operatorname{sen} y+\operatorname{sen} x\cdot\operatorname{sen} y)=\underline{\cos x\cdot\cos y}-\cos x$$

$$y'=\frac{\cos x\cdot\cos y-\cos x}{-\operatorname{sen} y+\operatorname{sen} x\cdot\operatorname{sen} y}$$

d) $y=\operatorname{arc}\underbrace{\tan\sqrt{x}}$

$y=\operatorname{arc}\tan y$

$$\frac{d}{dx}y=\frac{d}{dx}\operatorname{arc}\tan\sqrt{x}$$

$$y'=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}$$

$$y'=\frac{1}{2\sqrt{x}(1+x)}$$

e) $y=\underbrace{\operatorname{arcsen}}_{\frac{1}{\sqrt{1-u^2}}}(2x+1)$

$$y'=\frac{1}{\sqrt{1-(2x+1)^2}}\cdot\frac{d}{dx}[2x+1]$$

$$y'=\frac{2}{\sqrt{1-(2x+1)^2}}$$

f) $y = x \arccos(x)$

$$y' = \frac{d}{dx}[x] \cdot \arccos(x) + x \cdot \frac{d}{dx}[\arccos(x)]$$

$$y' = \arccos x + x \cdot \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$y = \arccos(x) - \frac{x}{\sqrt{1-x^2}}$$

8) a) $\operatorname{sen}(x+y) = 2x - 2y \qquad (\pi, \pi)$

$$\frac{d}{dx} \operatorname{sen}(x+y) = \frac{d}{dx} 2x - \frac{d}{dx} 2y$$

Regra da cadeia!

$$\cos\underbrace{(x+y)}_{1+y'} = 2 - 2y'$$

$$\cos(x+y) \cdot (1+y') = 2 - 2y'$$

$$y'(\cos(x+y) - 2) = 2 - \cos(x+y)$$

$$y' = \frac{2-\cos(x+y)}{-\cos(x+y)-2}$$

subst:

$$y' = \frac{2-\cos(\pi+\pi)}{-\cos(\pi+\pi)-2} = \frac{2-\cos(2\pi)}{-\cos(2\pi)-2}$$

$* \boxed{\cos(2\pi) = 1}$

$$= \frac{2-1}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$$ //

equação Tangente (π, π)

$$y - \pi = \frac{1}{3}(x - \pi) \qquad -\frac{1}{3} + \frac{3}{3} = \frac{2}{3}$$

$$y = \frac{1}{3}x - \frac{1}{3}\pi + \pi$$

$$y = \frac{1}{3}x + \frac{2}{3}\pi$$

b) $x^2 + xy + y^2 = 3 \qquad (1,1)$

$$\frac{d}{dx} x^2 + \frac{d}{dx}(xy) + \frac{d}{dx} y^2 = \frac{d}{dx} 3$$

$$2x + 1 \cdot y + x \cdot 1y' + 2y \cdot y' = 0$$

$$y'(x + 2y) = -2x - y$$

$$y' = \frac{-2x-y}{x+2y}$$

subst:

$$y'(1,1) = \frac{-2\cdot(1)-(1)}{1+2(1)} = \frac{-2-1}{1+2} = \frac{-3}{3} = -1$$ //

equação da reta Tangente: $(1,1)$

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x-1)$

$y - 1 = -x + 1$

$y = -x + 2$ //

c) $x^2 + 2xy - y^2 + x = 2 \qquad (1,2)$

$$\frac{d}{dx}x^2 + \frac{d}{dx}2xy - \frac{d}{dx}y^2 + \frac{d}{dx}x = \frac{d}{dx}2$$

$2x + 2\cdot(1\cdot y + x\cdot 1\cdot y') - 2y\cdot y' + 1 = 0$

$2x + 2y + 2xy' - 2y\cdot y' + 1 = 0$

$y'(2x - 2y) = -2x - 2y - 1$

$$y' = \frac{-2x-2y-1}{2x-2y}$$ //

subst: $(1,2)$

$$y'(1,2) = \frac{-2-4-1}{2-4} = \frac{-7}{-2} = \frac{7}{2}$$ //

equação da reta Tangente: $(1,2)$

$y - y_0 = m(x - x_0)$

$y - 2 = \frac{7}{2}(x-1)$ $\qquad \frac{-7}{2} - \frac{4}{2} = \frac{11}{2}$

$y = \frac{7}{2}x - \frac{7}{2} - 2$

$y = \frac{7}{2}x - \frac{11}{2}$

9) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{d}{dx}\frac{x^2}{a^2} + \frac{d}{dx}\frac{y^2}{b^2} = \frac{d}{dx}(1)$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{a^2} \cdot \frac{\cancel{2}y}{b^2} \Rightarrow \frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

equação na reta Tangente:

$$y - y_0 = m(x - x_0)$$

$$y - y_0 = \frac{-b^2 x}{a^2 y}(x - x_0)$$

$$y_0(y - y_0) = \frac{-b^2 x_0}{a^2} x + \frac{b^2 x_0}{a^2} x_0$$

$$= \frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$