Lista 2 - cálculo

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1) a) $(1+i)-(2-3i)$

$1+i-2+3i \Rightarrow z=-1+4i \qquad \bar{z}=1-4i$

$|z|=\sqrt{a^2+b^2}=\sqrt{-1^2+4^2}=\sqrt{17}$

b) $\left(4-\frac{1}{2}i\right)-\left(9+\frac{5i}{2}\right)$

$5-\frac{1i}{2}-\frac{5i}{2}=5-\frac{6i}{2}=5-3i \qquad \bar{z}=5+3i$

$|z|=\sqrt{a^2+b^2}=\sqrt{-5^2+3^2}=\sqrt{25+9}=\sqrt{34}$

c) $(4-7i)(1+3i) \qquad i^2=-1$

$4+12i-7i-21\overset{\to(-1)}{i^2}=4+5i-21\cdot(-1)=25+5i \qquad \bar{z}=25-5i$

$|z|=\sqrt{25^2+5^2}=\sqrt{625+25}=\sqrt{650}$

d) $\frac{5-i}{3+4i}\times\frac{(3-4i)}{(3-4i)}=\frac{15-20i-3i+4\overset{\to -1}{i^2}}{\sqrt{3^2-4\underset{\to -1}{i^2}}}=\frac{15-23i-4}{\sqrt{3^2+4^2}}=\frac{11-23i}{\sqrt{9+16}}=\frac{11-23i}{\sqrt{25}}=\frac{11-23i}{5}$

$|z|=\sqrt{11^2+23^2}=\sqrt{121+529}=\sqrt{650} \qquad \bar{z}=\frac{11+23i}{5}=\frac{11}{5}+\frac{23i}{5}$

e) $\frac{3}{4-3i}\times\frac{(4+3i)}{(4+3i)}=\frac{12+9i}{\sqrt{4^2+3i^{2^2}}}=\frac{12+9i}{\sqrt{16+9}}=\frac{12+9i}{\sqrt{25}} \Rightarrow \bar{z}=\frac{12}{25}-\frac{9i}{25}$

$\hookrightarrow \frac{12+9i}{25}$

101 | 4
21 25
1

$|z|=\sqrt{12^2+9^2}=\sqrt{144+81}=\sqrt{225}=15$

f) $1+i^{101}=1+i^1=1+i$

2) a) $(3+4i)^2-2\bar{z}=z$

$(3+4i)(3+4i)-2\bar{z}=z$

$9+12i+12i+16\cdot(-1)-2\bar{z}=z$

$-7+24i-2(a-bi)=a+bi$

$-7+24i-2a+2bi=a+bi$

$(-7-2a)+(24i+2bi)=a+bi$

$\begin{cases}-7-2a=a \\ 24i+2bi=bi\end{cases} \qquad \to -3a=7 \qquad a=-\frac{7}{3}$

$\hookrightarrow 24+2b=b$

$b=-24$

$$\boxed{z=\frac{-7}{3}-2i}$$

$\frac{-16}{8} + \frac{33}{8} = \quad 33 - 16 = 17$

$\frac{+3}{1} \times \frac{17}{8} = \frac{+33}{8}$

b) $iz + 3\bar{z} = 5 - 2i$

$i(a+bi) + 3(a-bi) = 5-2i$

$ai + bi^2 + 3a - 3bi = 5 - 2i$

$ai - b + 3a - 3bi = 5 - 2i$

$(3a-b) + (a-3b)i = 5 - 2i$

$\rightarrow a = -2 + 3b$

$\begin{cases} 3a - b = 5 \\ a - 3b = -2 \end{cases} \rightarrow 3(-2+3b) - b = 5$

$-6 + 9b - b = 5 \qquad 8b = 11 \rightarrow b = \frac{11}{8}$

$\rightarrow a = -2 + 3\left(\frac{11}{8}\right)$

$a = -2 + \frac{33}{8} = \frac{17}{8}$

DÚVIDA!

d) $z^2 = 4 + 2i\sqrt{5}$ $\rightarrow$ muita DÚVIDA

$|4 + 2i\sqrt{5}| = \sqrt{4^2 + (2\sqrt{5})^2} = \sqrt{16+20} = \sqrt{36} = 6$

$|z^2| = 6 \qquad |z| = \sqrt{6}$

$\theta = \tan^{-1}\left(\frac{2\sqrt{5}}{4}\right) = \tan^{-1}\left(\frac{\sqrt{5}}{2}\right)$

$z = \sqrt{6} \cdot \left(\cos \frac{\theta_0}{2} + i \operatorname{sen} \frac{\theta_0}{2}\right)$

$z = \sqrt{6}\left(\cos\left(\frac{\theta_0}{2} + \pi\right) + i \operatorname{sen}\left(\frac{\theta_0}{2} + \pi\right)\right)$

3) a) $9z^2+16=0$

$z^2=-\frac{16}{9} \rightarrow z=\pm\sqrt{\frac{-16}{9}} = z=\pm\frac{4}{3}i$

b) $z^4-1=0$

$z^4=1 \rightarrow z=\pm\sqrt[4]{1} = z=\pm 1$

c) $2z^2-2z+1=0$

$\Delta=b^2-4ac = 4-8=-4$

$x=\frac{-b\pm\sqrt{\Delta}}{2a} = \frac{2\pm\sqrt{4}\sqrt{-1}}{4} = \frac{2\pm 2i}{4} = \quad z=\frac{1}{2}+\frac{i}{2} \quad e \quad z=\frac{1}{2}-\frac{i}{2}$

d) $z^2+z+2=0$

$\Delta=b^2-4ac = 1-8=-7$

$x=\frac{-b\pm\sqrt{\Delta}}{2a} = \frac{-1\pm\sqrt{7}\sqrt{-1}}{2} = \frac{-1\pm\sqrt{7}i}{2} = \quad z=\frac{-1}{2}+\frac{\sqrt{7}i}{2} \quad e \quad z=\frac{-1}{2}-\frac{\sqrt{7}i}{2}$

e) $z^4+3z^2+2=0$ $\quad \boxed{w=z^2}$

$w^2+3w+2=0$

$\Delta=b^2-4ac= 9-8=1$

$x=\frac{-b\pm\sqrt{\Delta}}{2a} = \frac{-3\pm 1}{2} = \begin{matrix} x'=-2 \\ x''=-1 \end{matrix}$

$z^2=-2 \Rightarrow z=\pm\sqrt{-2} = z=\pm 2i$

$z^2=-1 \Rightarrow z\pm\sqrt{-1} = z=\pm i$

f) $z^4-2z^2+4=0 \qquad w=z^2$

$w^2-2w+4=0$

$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{array} \quad 2\sqrt{3}$

$\Delta=b^2-4ac = 4-16=-12$

$x=\frac{-b\pm\sqrt{\Delta}}{2a} = \frac{2\pm\sqrt{12}\sqrt{-1}}{2} = \frac{2\pm 2\sqrt{3}i}{2} = \frac{\cancel{2}}{\cancel{2}}\pm\frac{\cancel{2}\sqrt{3}i}{\cancel{2}} \quad w=1\pm\sqrt{3}i$

$1+\sqrt{3}i \qquad \cos\theta=\frac{1}{2} \qquad z=2(\cos\frac{\pi}{3}+i\,\text{sen}\,\frac{\pi}{3})$

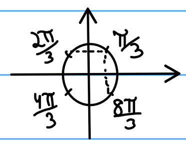
$r=\sqrt{1+3}$

$=\sqrt{4}=2 \qquad \text{sen}\,\theta=\frac{\sqrt{3}}{2}$

$$z = 2^{1/2}\left(\cos\left(\frac{\pi/3 + 2i\pi\cdot 0}{2}\right) + i\,\text{sen}\,\frac{\pi}{6}\right) \Rightarrow \sqrt{2}\left(\cos\frac{\pi}{6} + i\,\text{sen}\,\frac{\pi}{6}\right)$$

$$\Rightarrow \pm\sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$\underline{1-\sqrt{3}i}$ $\quad \cos\theta = \frac{1}{2} \quad \text{sen}\,\theta = \frac{\sqrt{3}}{2}$



$$z = \sqrt{2}\left(\cos\frac{\pi}{3} + i\,\text{sen}\,\frac{\pi}{3}\right)$$

$$r = 2^{1/2}\left(\cos\left(\frac{\frac{\pi}{3} + 2\pi\cdot 0}{2}\right) - i\,\text{sen}\left(\frac{\pi}{6}\right)\right) \Rightarrow \pm\sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

4) a) $z + \bar{z} = 2Re(z)$

$(a+bi) + (a-bi)$

$2a + bi - bi = 2a \Rightarrow 2Re(z)$

b) $z - \bar{z} = 2i\,Im(z)$

$(a+bi) - (a-bi)$

$a + bi - a + bi = 2bi \Rightarrow 2i\,Im(z)$

c) $|Re(z)| \leq |z|$

$|a| \leq |a + bi|$

$\sqrt{a^2} \leq \sqrt{a^2 + b^2}$

$a^2 \leq a^2 + b^2$

$0 \leq b^2$

d) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z_2})$

$|(a_1 + ib_1) + (a_2 + ib_2)| = z_1 + z_2$

$|(a_1 + a_2) + (b_1 + b_2)i| = z_1 + z_2 \Rightarrow \sqrt{(a_1+a_2)^2 + (b_1+b_2)^2} = |z_1 + z_2|$

$(a_1 + a_2)^2 + (b_1 + b_2)^2 = |z_1 + z_2|^2$

$(a_1 + a_2)(a_1 + a_2) + (b_1 + b_2)(b_1 + b_2) = |z_1 + z_2|^2$

$a_1^2 + 2a_1a_2 + a_2^2 + b_1^2 + 2b_1b_2 + b^2 = |z_1 + z_2|^2$

$(a_1^2 + b_1^2) + (a_2^2 + b^2) + 2(a_1a_2 + b_1b_2) = |z_1 + z_2|^2$

$|z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z_2}) = |z_1 + z_2|^2$

e) $|z_1 + z_2| \leq |z_1| + |z_2|$

$|(a_1 + b_1 i) + (a_2 + b_2 i)| \leq \sqrt{(a_1)^2 + (b_2)^2} + \sqrt{(a_2)^2 + (b_2)^2}$

$\sqrt{(a_1 + b_1)^2 + (b_1 + b_2)^2} \leq \sqrt{(a_1)^2 + (b_1)^2} + \sqrt{(a_2)^2 + (b_2)^2}$

Se $z_2 : z_1 + z_2$ e $z_1 = 1$

$|z_1 + z_2|^2 = |z_1|^2 + 2Re(z_1\bar{z_2}) + |z_2|^2$

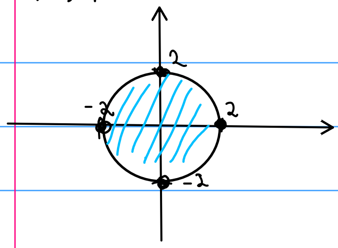
$|z_1 + z_2|^2 \leq |z_1|^2 + 2|z_1||z_2| + |z_2|^2 \Rightarrow (|z_1| + |z_2|)^2$

$|z_1 + z_2|^2 \leq (|z_1| + |z_2|)^2$

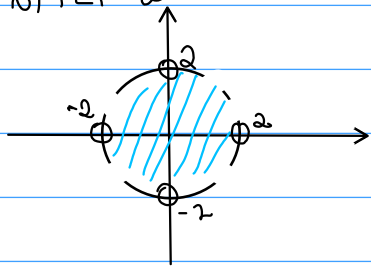
$|z_1 + z_2| \leq |z_1| + |z_2|$

*2 não incluso!

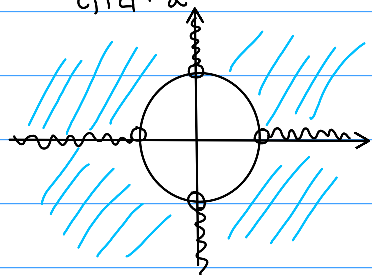
5) a) $|z| = 2$



b) $|z| < 2$



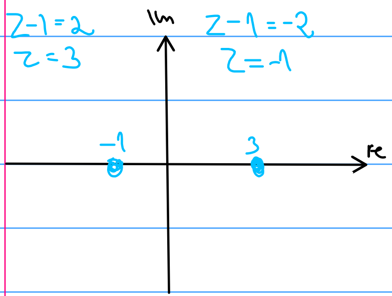
c) $|z| > 2$



d) $|z-1| = 2$

$Z-1=2$
$z=3$

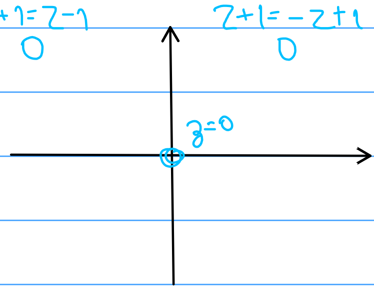
$Z-1=-2$
$Z=-1$



e) $|z+1| = |z-1|$

$Z+1=Z-1$
0

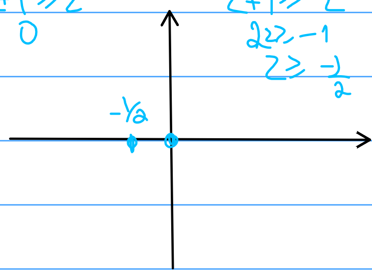
$Z+1=-Z+1$
0



f) $|z+1| \geq |z|$

$Z+1 \geq Z$
0

$Z+1 \geq -Z$
$2Z \geq -1$
$Z \geq -\frac{1}{2}$



6) $\frac{1}{Z} = \frac{1}{r}(\cos\theta - i \operatorname{sen}\theta)$

$Z = r(\cos\theta + i \operatorname{sen}\theta) \rightarrow \bar{Z} = r(\cos\theta - i \operatorname{sen}\theta)$

$\frac{1}{Z} = \frac{1}{r(\cos\theta + i \operatorname{sen}\theta)}$ $\xrightarrow{\text{multip. pelo conj.}}$ $\frac{1}{r(\cos\theta + i \operatorname{sen}\theta)} \times \frac{\cos\theta - i \operatorname{sen}\theta}{\cos\theta - i \operatorname{sen}\theta}$

$\Rightarrow (\cos\theta + i \operatorname{sen}\theta)(\cos\theta - i \operatorname{sen}\theta) = \cos^2\theta + i \operatorname{sen}^2\theta = \boxed{1}$

$\frac{1}{Z} = \frac{\cos\theta - i \operatorname{sen}\theta}{r}$

$\hookrightarrow \frac{1}{Z} = \frac{1}{r}(\cos\theta - i \operatorname{sen}\theta)$

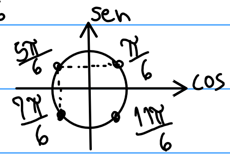
zw, z/w e $1/z$

$z \cdot w$ = multip. módulos e soma argumentos!
z/w = divide módulos e subtrai argumentos!
$1/z$ = invert. o módulo e trocamos o sinal do argumento!

7) a) $z = 2\sqrt{3} - 2i$, $w = 8i$

$|z| = \rho = \sqrt{(2\sqrt{3})^2 - (2)^2} = \sqrt{4 \cdot 3 + 4} = \sqrt{16} = 4$

$\left.\begin{array}{l} \operatorname{sen}\theta = \dfrac{b}{\rho} = \dfrac{-2}{4} = \dfrac{-1}{2} \\ \\ \cos\theta = \dfrac{a}{\rho} = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2} \end{array}\right\} 30° \quad \pi/6$



$\boxed{z = 4\left(\cos \frac{5\pi}{6} + i \operatorname{sen} \frac{5\pi}{6}\right)}$

$z = 4\left(\cos \frac{\pi}{6} - i \operatorname{sen} \frac{\pi}{6}\right)$

$z = 4 \cdot \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = \frac{4\sqrt{3}}{2} - \frac{4}{2} = 2\sqrt{3} - 2i$

$|w| = \sqrt{0 + 8^2} = \sqrt{64} = 8$

$\left.\begin{array}{l} \operatorname{sen}\theta = \dfrac{b}{\rho} = \dfrac{8}{8} = 1 \end{array}\right\} \dfrac{\pi}{2}$

$\boxed{w = 8\left(\cos \frac{\pi}{2} + i \operatorname{sen} \frac{\pi}{2}\right)}$

$\cos\theta = \dfrac{a}{\rho} = \dfrac{0}{8} = 0$

$z \cdot w = 8 \cdot 4\left(\cos\left(\frac{\pi}{2} + \frac{5\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{2} + \frac{5\pi}{6}\right)\right)$

$\frac{3\pi}{6} + \frac{5\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$

$zw = 32\left(\cos \frac{4\pi}{3} + i \operatorname{sen} \frac{4\pi}{3}\right)$

$\frac{5\pi}{6} - \frac{3\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

$z/w = \frac{4}{8}\left(\cos\left(\frac{5\pi}{6} - \frac{\pi}{2}\right) + i \operatorname{sen}\left(\frac{5\pi}{6} - \frac{\pi}{2}\right)\right)$

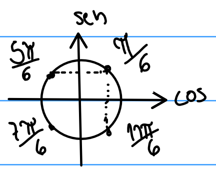
$z/w = \frac{1}{2}\left(\cos \frac{\pi}{3} + i \operatorname{sen} \frac{\pi}{3}\right)$

$\frac{1}{z} = \frac{1}{4}\left(\cos \frac{-5\pi}{6} + i \operatorname{sen} \frac{-5\pi}{6}\right) \Rightarrow \frac{1}{4}\left(\cos \frac{5\pi}{6} - i \operatorname{sen} \frac{5\pi}{6}\right)$

b) $z = \sqrt{3} + i$, $w = 1 + \sqrt{3}i$

$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$\operatorname{sen}\theta = \dfrac{b}{\rho} = \dfrac{1}{2}$



$\boxed{z = 2\left(\cos \frac{\pi}{6} + i \operatorname{sen} \frac{\pi}{6}\right)}$

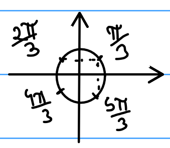
$z = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \frac{\cancel{2}\sqrt{3}}{\cancel{2}} + 1 = \sqrt{3} + 1i$

$\cos\theta = \dfrac{a}{\rho} = \dfrac{\sqrt{3}}{2}$

$|w| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\cos\theta = \dfrac{a}{\rho} = \dfrac{1}{2}$

$\boxed{z = 2\left(\cos \frac{\pi}{3} + i \operatorname{sen} \frac{\pi}{3}\right)}$



$z = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 1 + \sqrt{3}i$

$\operatorname{sen}\theta = \dfrac{b}{\rho} = \dfrac{\sqrt{3}}{2}$

$$\frac{\pi}{6}+\frac{2\pi}{6}=\frac{3\pi}{6}=\frac{\pi}{2}$$

$$z\cdot w = 2\cdot 2\left(\cos\left(\tfrac{\pi}{6}+\tfrac{\pi}{3}\right)+i\,sen\left(\tfrac{\pi}{6}+\tfrac{\pi}{3}\right)\right)$$

$$zw = 4\left(\cos \tfrac{\pi}{2}+i\,sen\,\tfrac{\pi}{2}\right)$$

$$\frac{z}{w} = \frac{\cancel{2}}{\cancel{2}}\left(\cos\left(\tfrac{\pi}{6}-\tfrac{\pi}{3}\right)+i\,sen\left(\tfrac{\pi}{6}-\tfrac{\pi}{3}\right)\right) \qquad \frac{\pi}{6}-\frac{2\pi}{6}=-\frac{\pi}{6}$$

$$\frac{z}{w} = 1\left(\cos -\tfrac{\pi}{6}+i\,sen -\tfrac{\pi}{6}\right) \to 1\left(\cos\tfrac{\pi}{6}-i\,sen\,\tfrac{\pi}{6}\right)$$

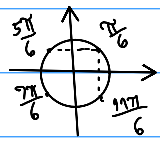
$$\frac{1}{z} = \frac{1}{2}\left(\cos -\tfrac{\pi}{6}+i\,sen -\tfrac{\pi}{6}\right) \to \frac{1}{2}\left(\cos\tfrac{\pi}{6}+i\,sen\,-\tfrac{\pi}{6}\right)$$

c) $z = 5(\sqrt{3}+i)$, $w = -3-3i$

$\hookrightarrow 5\sqrt{3}+5i$

$$|z| = \sqrt{(5\sqrt{3})^2+5^2} = \sqrt{25\cdot 3+25} = \sqrt{100} = 10$$

$$\cos\theta = \frac{a}{\rho} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$



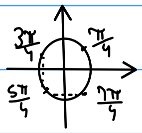
$$\boxed{z = 10\left(\cos\tfrac{\pi}{6}+i\,sen\,\tfrac{\pi}{6}\right)}$$

$$10\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right) = 5\sqrt{3}+5i$$

$$sen\,\theta = \frac{b}{\rho} = \frac{5}{10} = \frac{1}{2}$$

$$|w| = \sqrt{(-3)^2+(-3)^2} = \sqrt{18} = 3\sqrt{2}$$

$$\cos\theta = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$



$$\boxed{w = 3\sqrt{2}\left(\cos\tfrac{5\pi}{4}+i\,sen\,\tfrac{5\pi}{4}\right)}$$

$$w = 3\sqrt{2}\cdot\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right) = \frac{6}{2}+\frac{6}{2} = 3+3i$$

$$sen\,\theta = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\begin{array}{c|l} 6,4 & 2 \\ 3\ \ 2 & 2 \\ & 3 \end{array} \quad 4 \quad 12$$

$$z\cdot w = 10\cdot 3\sqrt{2}\left(\cos\left(\tfrac{\pi}{6}+\tfrac{5\pi}{4}\right)+i\,sen\left(\tfrac{\pi}{6}+\tfrac{5\pi}{4}\right)\right)$$

$$zw = 30\sqrt{2}\left(\cos\tfrac{17\pi}{12}+i\,sen\,\tfrac{17\pi}{12}\right) \qquad \frac{2\pi}{12}+\frac{15\pi}{12}=\frac{17\pi}{12}$$

$$\frac{z}{w} = \frac{10}{3\sqrt{2}}\left(\cos\left(\tfrac{\pi}{6}-\tfrac{5\pi}{4}\right)+i\,sen\left(\tfrac{\pi}{6}-\tfrac{5\pi}{4}\right)\right)$$

$$\frac{z}{w} = \frac{10}{3\sqrt{2}}\left(\cos -\tfrac{13\pi}{12}+i\,sen -\tfrac{13\pi}{12}\right)$$

$$\frac{1}{z} = \frac{1}{10}\left(\cos -\tfrac{\pi}{6}+i\,sen -\tfrac{\pi}{6}\right) \Rightarrow \frac{1}{10}\left(\cos\tfrac{\pi}{6}+i\,sen -\tfrac{\pi}{6}\right)$$

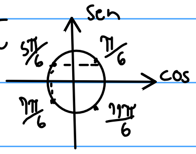
d) $z = 4\sqrt{3}-4i$, $w = -1+i$

$$|z| = \sqrt{(4\sqrt{3})^2+(-4)^2} = \sqrt{16\cdot 3+16} = \sqrt{48+16} = \sqrt{64} = 8$$

$$|w| = \sqrt{(-1)^2+1^2} = \sqrt{2}$$

z

$\cos\theta = \frac{a}{\rho} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$



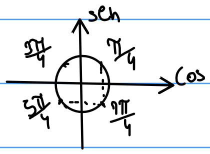
$\boxed{z = 8\left(\cos \frac{5\pi}{6} + i\,\text{sen}\,\frac{5\pi}{6}\right)}$

$8\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 4\sqrt{3} + 4i$

$\text{sen}\,\theta = \frac{b}{\rho} = \frac{-4}{8} = \frac{-1}{2}$

w

$\cos\theta = \frac{a}{\rho} = \frac{-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$



$\boxed{w = \sqrt{2}\left(\cos \frac{7\pi}{4} + i\,\text{sen}\,\frac{7\pi}{4}\right)}$

$\text{sen}\,\theta = \frac{b}{\rho} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

6, 4 | 2
3, 2 | 2
3, 1 | 3

$z \cdot w = 8\sqrt{2}\left(\cos\left(\frac{5\pi}{6} + \frac{7\pi}{4}\right) + i\,\text{sen}\left(\frac{5\pi}{6} + \frac{7\pi}{4}\right)\right)$

$\frac{10\pi}{12} + \frac{21\pi}{12} = \frac{31\pi}{12}$

$zw = 8\sqrt{2}\left(\cos \frac{31\pi}{12} + i\,\text{sen}\,\frac{31\pi}{12}\right)$

$\frac{10\pi}{12} - \frac{21\pi}{12} = \frac{-11\pi}{12}$

$\frac{z}{w} = \frac{8}{\sqrt{2}}\left(\cos\left(\frac{5\pi}{6} - \frac{7\pi}{4}\right) + i\,\text{sen}\left(\frac{5\pi}{6} - \frac{7\pi}{4}\right)\right)$

$\frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

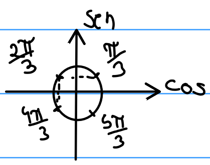
$\frac{z}{w} = 4\sqrt{2}\left(\cos \frac{-11\pi}{12} + i\,\text{sen}\,\frac{-11\pi}{12}\right) \rightarrow 4\sqrt{2}\left(\cos \frac{11\pi}{12} + i\,\text{sen}\,\frac{11\pi}{12}\right)$

$\frac{1}{z} = \frac{1}{8}\left(\cos \frac{-5\pi}{6} + i\,\text{sen}\,\frac{-5\pi}{6}\right) \rightarrow \frac{1}{8}\left(\cos \frac{5\pi}{6} + i\,\text{sen}\,\frac{-5\pi}{6}\right)$

8) a) $(1 - \sqrt{3}i)^6$

$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$

$\cos\theta = \frac{1}{2}$



$z = 2\left(\cos \frac{2\pi}{3} + i\,\text{sen}\,\frac{2\pi}{3}\right)$

$\text{sen}\,\theta = \frac{-\sqrt{3}}{2}$

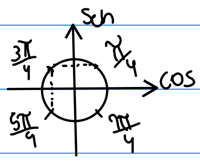
$\frac{12\pi}{3} = 4\pi$

$z^6 = 2^6\left(\cos 6 \cdot \frac{2\pi}{3} + i\,\text{sen}\; 6 \cdot \frac{2\pi}{3}\right)$

$z = 64\left(\cos 4\pi + i\,\text{sen}\, 4\pi\right)$

b) $(1 - i)^8$

$|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$



$z = 2\left(\cos \frac{3\pi}{4} + i\,\text{sen}\,\frac{3\pi}{4}\right)$

$\cos\theta = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\text{sen}\,\theta = \frac{-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

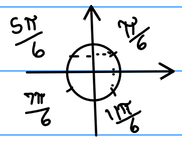
$z^8 = 2^8\left(\cos 8\cdot\frac{3\pi}{4} + i\,\text{sen}\, 8\cdot\frac{3\pi}{4}\right)$ $\qquad \frac{8\cdot 3\pi}{4} = \frac{24\pi}{4} = 6\pi$

$z^8 = 256\left(\cos 6\pi + i\,\text{sen}\,6\pi\right)$

c) $(2\sqrt{3}+2i)^7$

$|z| = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{4\cdot 3+4} = \sqrt{16} = 4$

$\cos\theta = \frac{a}{\rho} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$



$z = 4\left(\cos\frac{\pi}{6} + i\,\text{sen}\,\frac{\pi}{6}\right)$

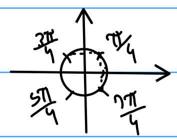
$\text{sen}\,\theta = \frac{b}{\rho} = \frac{2}{4} = \frac{1}{2}$

$z^7 = 4^7\left(\cos\frac{\pi}{6} + i\,\text{sen}\,\frac{\pi}{6}\right)$

d) $(1+i)^{40}$

$|z| = \sqrt{1^2+1^2} = \sqrt{2}$

$\cos\theta = \frac{a}{\rho} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right)$

$\text{sen}\,\theta = \frac{b}{\rho} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

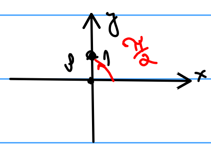
$z^{40} = \sqrt{2}^{40}\left(\cos 40\cdot\frac{\pi}{4} + i\,\text{sen}\,40\cdot\frac{\pi}{4}\right)$ $\qquad \frac{40\pi}{4} = 10\pi$

$(2^{1/2})^{40} \to 2^{20}$

$z^{40} = 1048576\left(\cos 10\pi + i\,\text{sen}\,10\pi\right)$

9) a) raízes quadradas de i

$\sqrt{i}$, $n=2$ e $z=1$



$z = 1\left(\cos\frac{\pi}{2} + i\,\text{sen}\,\frac{\pi}{2}\right)$

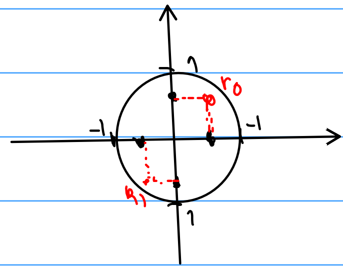
* módulo das raízes

$\sqrt[2]{1} = 1 \qquad \rho = 1$

$\rho = 1 \qquad \theta = \frac{\pi}{2}$

$\frac{\pi}{2} + \frac{4\pi}{2} = \frac{5\pi}{2}$ $\qquad$ $\frac{5\pi}{2} \times \frac{1}{2} = \frac{5\pi}{4}$

$K=0$, $\theta_0 = \dfrac{\pi/2 + \cancel{2} \cdot \overset{0}{0} \cdot \pi}{2} = \dfrac{\pi/2}{2}$ $\qquad$ $\frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$



$V_0 = 1\left(\cos \frac{\pi}{4} + i \,\text{sen}\, \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

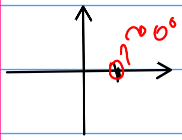
K_1, $\theta_0 = \dfrac{\pi/2 + 2 \cdot 1 \cdot \pi}{2} = \dfrac{5\pi/2}{2} = \dfrac{5\pi}{4}$

$V_1 = 1\left(\cos \frac{5\pi}{4} + i \,\text{sen}\, \frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$

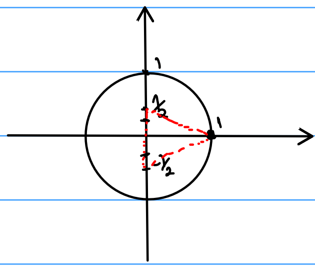
b) raízes cúbicas de 1

$\sqrt[3]{1}$ $\quad n=3$ $\quad z=1$

$\rho = 1$



$K_0 = 0$ $\qquad \theta_0 = \dfrac{0 + 2 \cdot \pi \cdot 0}{3} = 0 \rightarrow 1$



$K_1 = 1$ $\qquad \theta_1 = \dfrac{0 + 2 \cdot \pi \cdot 1}{3} = \dfrac{2\pi}{3} \Rightarrow \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$K_2 = 2$ $\qquad \theta_2 = \dfrac{0 + 2 \cdot \pi \cdot 2}{3} = \dfrac{4\pi}{3} \Rightarrow -\frac{1}{2} - \frac{\sqrt{3} i}{2}$

c) raízes quartas de $1+i$

$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\qquad n=4$

$\cos = \frac{a}{\rho} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Big\} \frac{\pi}{4}$ $\qquad z = \sqrt{2}\left(\cos \frac{\pi}{4} + i \,\text{sen}\, \frac{\pi}{4}\right)$

$\frac{\pi}{4} \times \frac{1}{4} = \frac{\pi}{16}$

$K_0 = 0$ $\qquad \theta_0 = \dfrac{\pi/4 + 2 \cdot \pi \cdot 0}{4} = \dfrac{\pi/4}{4} = \dfrac{\pi}{16}$

$\frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$ $\qquad \frac{9\pi}{4} \times \frac{1}{4} = \frac{9\pi}{16}$

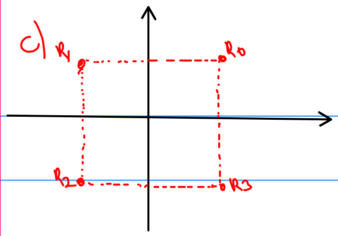
$K_1 = 1$ $\qquad \theta_1 = \dfrac{\pi/4 + 2\pi \cdot 1}{4} = \dfrac{9\pi/4}{4} = \dfrac{9\pi}{16}$

$\frac{\pi}{4} + \frac{16\pi}{4} = \frac{17\pi}{4} \times \frac{1}{4} = \frac{17\pi}{16}$

$K_2 = 2$ $\qquad \theta_2 = \dfrac{\pi/4 + 2\pi \cdot 2}{4} = \dfrac{17\pi/4}{4} = \dfrac{17\pi}{16}$

$\frac{\pi}{4} + \frac{24\pi}{4}$

$K_3 = 3$ $\qquad \theta_3 = \dfrac{\pi/4 + 2\pi \cdot 3}{4} = \dfrac{25\pi/4}{4} = \dfrac{25\pi}{16}$

c) 

d) as raízes cúbicas de $-8i$

$n=3, \quad z=-8i$ $\qquad \operatorname{sen}\theta = \frac{b}{\rho} = \frac{-8}{8} = -1 \quad \Big| \; \frac{3\pi}{2}$

$|z| = \sqrt{0^2+(-8i)^2} = \sqrt{64} = 8$

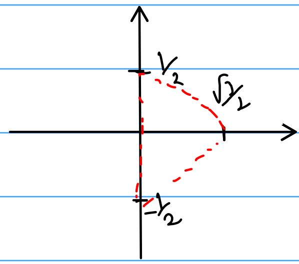
$z = 8\left(\cos \frac{3\pi}{2} + i \operatorname{sen} \frac{3\pi}{2}\right)$ $\qquad \frac{3\pi}{2} \cdot \frac{1}{3} = \frac{3\pi}{6} = \frac{\pi}{2}$ $\qquad \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2} \times \frac{1}{3} = \frac{7\pi}{6}$

$\frac{3\pi}{2} + \frac{8\pi}{2} = \frac{11\pi}{2} \times \frac{1}{3} = \frac{11\pi}{6}$

$K_0 = 0 \qquad \theta_0 = \frac{\frac{3\pi}{2} + 2\cdot\pi\cdot 0}{3} = \frac{\frac{3\pi}{2}}{3} = \frac{\pi}{2}$

$K_1 = 1 \qquad \theta_1 = \frac{\frac{3\pi}{2} + 2\cdot\pi\cdot 1}{3} = \frac{\frac{7\pi}{2}}{3} = \frac{7\pi}{6} \rightarrow \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$K_2 = 2 \qquad \theta_2 = \frac{\frac{3\pi}{2} + 2\pi\cdot 2}{3} = \frac{\frac{11\pi}{2}}{3} = \frac{11\pi}{6} \rightarrow -\frac{\sqrt{3}}{2} + \frac{1}{2}i$



e) As raízes quintas de -32

$n=5, \quad z=-32$ $\qquad \cos = \frac{a}{\rho} = \frac{-32}{32} = -1$

$|z| = \sqrt{(-32)^2} = 32$ $\qquad \pi$

$z = 32(\cos \pi + i \operatorname{sen} \pi)$

$K_0 = 0 \qquad \theta_0 = \frac{\pi + 2\cdot\pi\cdot 0}{5} = \frac{\pi}{5}$

$K_1 = 1 \qquad \theta_1 = \frac{\pi + 2\pi\cdot 1}{5} = \frac{3\pi}{5}$

$K=2 \qquad \theta_2 = \frac{\pi + 2\cdot\pi\cdot 2}{5} = \frac{5\pi}{5} = \pi = 180°$

$K=3 \qquad \theta_3 = \frac{\pi + 2\cdot\pi\cdot 3}{5} = \frac{7\pi}{5}$

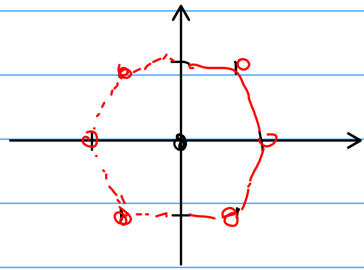
$K=4 \qquad \theta_4 = \frac{\pi + 2\cdot\pi\cdot 4}{5} = \frac{9\pi}{5}$

g) as raízes sextas de 64.

$n = 6,$

$|z| = \sqrt{64^2} = 64$

$\cos = \frac{64}{64} = 1 \rightsquigarrow \theta$



$K_0 = 0 \quad \frac{0 + \pi \cdot 2 \cdot 0}{6} = 0$

$K_1 = 1 \quad \frac{0 + \pi \cdot 2 \cdot 1}{6} = \frac{1}{6} \Rightarrow \frac{\sqrt{3}}{2} + \frac{1}{2} i$

$K_2 = 2 \quad \frac{0 + \pi \cdot 2 \cdot 2}{6} = \frac{4\pi}{6} \to \frac{2\pi}{3} \to \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$K_3 = 3 \quad \frac{0 + \pi \cdot 2 \cdot 3}{6} = \pi$

$K = 4 \quad \frac{0 + \pi \cdot 2 \cdot 4}{6} = \frac{8\pi}{6} = \frac{4\pi}{3} \Rightarrow \frac{1}{2} + \frac{\sqrt{3}}{2} i$

10) $\operatorname{sen} x \approx x - \frac{x^6}{6} \qquad \cos x \approx 1 - \frac{x^2}{2} \qquad e^x \approx 1 + x + \frac{x^2}{2} + x^3$

$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \dots \qquad \cos + i \operatorname{sen} x \approx \left(1 - \frac{x^2}{2}\right) + i\left(x - \frac{x^3}{6}\right)$

$= 1 + ix - \frac{x^2}{2} - \frac{ix^3}{6}$

$= \left(1 - \frac{x^2}{2}\right) + i\left(x - \frac{x^3}{6}\right)$

↳ resultados idênticos até a ordem x^3; $e^{ix} \approx \cos x + i \operatorname{sen} x$

11) a) $e^{-i\pi/2}$

$= \cos \pi/2 - i \operatorname{sen} \pi/2$

$= 0 - i$

b) $e^{2\pi i}$ $= \cos 2\pi + i \operatorname{sen} 2\pi$

$= 1 + 0i \quad = 1$

c) $e^{i\pi/3}$

$= \cos \pi/3 + i \operatorname{sen} \pi/3$

$\frac{1}{2} + \frac{\sqrt{3}i}{2}$

d) $e^{-i\pi}$

$= \cos(-\pi) + i \operatorname{sen}(-\pi)$

$= \cos \pi - i \operatorname{sen} \pi$

$= -1 - 0i$

e) $e^{2+i\pi} = e^2 \cdot e^{i\pi}$

$= e^2 (\cos \pi + i \operatorname{sen} \pi)$

$e^2 = -1 + 0i$

$= -e^2$

f) $e^{\pi+1} = e^{\pi} \cdot e^{1i}$

$= e^{\pi} (\cos 1 + i \operatorname{sen} 1)$

$= e^{\pi} \cos(1) + i\, e^{\pi} \operatorname{sen}(1)$

12) $\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$\cos x = \frac{(\cos x + i \operatorname{sen}) + (\cos(-x) + i \operatorname{sen}(-x))}{2} \Rightarrow \frac{\cos(x) + \cos(x) - \cancel{\operatorname{sen}(x)} + \cancel{\operatorname{sen}(x)}}{2}$

$\frac{\cancel{2}\cos(x)}{\cancel{2}} \Rightarrow \cos(x)$

$\operatorname{sen} x = \frac{e^{ix} - e^{-ix}}{2i}$

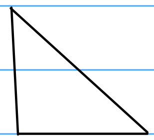
$\operatorname{sen} x = \frac{(\cos x + i \operatorname{sen} x) - (\cos(-x) + i \operatorname{sen}(-x))}{2i} \Rightarrow \frac{(\cos x + i \operatorname{sen} x) - (\cos(+x) + i \operatorname{sen}(-x))}{2i}$

$= \frac{(\cancel{\cos x}) - (\cancel{\cos x}) + i \operatorname{sen} x + i \operatorname{sen} x}{2i} = \frac{\cancel{2i} \operatorname{sen} x}{\cancel{2i}} = \operatorname{sen} x$ //

Provando = $sen^2 x + cos^2 x = 1$

$x = cos(\theta)$ $y = sen(\theta)$

↳ $x^2 + y^2 = 1$ → pitágoras



$$sen^2 + cos^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a + b^2}{c^2} \Rightarrow \frac{c^2}{c^2} = 1$$

↳ $sen^2 + cos^2 = 1$

↳ $a^2 + b^2 = c^2$