Lista 7 - Cálculo

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1) a) $y = 2x - 3x^2$, $(\overset{x}{2}, \overset{y}{-8})$

$$\boxed{m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$m = \lim_{h \to 0} \frac{2(x+h) - 3(x+h)^2 - (2x - 3x^2)}{h} = \frac{2x + 2h - 3(x^2 + 2xh + h^2) - 2x + 3x^2}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{2x} + \cancel{3x^2}}{h} = \frac{\cancel{h}(2 - 6x - 3h)}{\cancel{h}} = 2 - 6x - 3\cancel{h}^{\,0}$$

$$\boxed{2 - 6x}$$

$m = 2 - 6 \cdot (2) = 2 - 12 = \underline{\underline{-10}}$

$\boxed{y = mx + n}$

$-8 = -10 \cdot 2 + n$ $\boxed{y = -10x + 12}$ //

$-8 + 20 = n$

$n = 12$

b) $y = x^3 - 3x + 1$ $(\overset{x}{2}, \overset{y}{3})$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$m = \lim_{h \to 0} \frac{(x+h)^3 - 3(x+h) + 1 - (x^3 - 3x + 1)}{h} = \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{3x} - 3h + \cancel{1} - \cancel{x^3} + \cancel{3x} - \cancel{1}}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh - h^2 - 3)}{\cancel{h}} \Rightarrow 3x^2 + 3x\cancel{h}^{\,0} - \cancel{h^2}^{\,0} - 3$$

$$3x^2 - 3$$

$m = 3(2)^2 - 3 = 3 \cdot (4) - 3 = 9$

$y = mx + n$

$3 = 9 \cdot 2 + n$ $\boxed{y = 9x - 15}$

$3 = 18 + n$

$3 - 18 = n$ $n = -15$

c) $2\sqrt{x}$ $\quad \left(\overset{x}{1}, \overset{y}{2}\right)$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow m = \lim_{h \to 0} \frac{2\sqrt{(x+h)} - 2\sqrt{x}}{h} \times \frac{2\sqrt{(x+h)} + 2\sqrt{x}}{2\sqrt{(x+h)} + 2\sqrt{x}} =$$

$$= (2\sqrt{x+h})^{\cancel{2}} - (2\sqrt{x})^{\cancel{2}} = 4(x+h) - 4(x) = \frac{4\cancel{h}}{\cancel{h}\left(2\sqrt{(x+h)} + 2\sqrt{x}\right)} = \frac{4}{2\sqrt{(x+h)} + 2\sqrt{x}}$$

$$\hookrightarrow = \cancel{4x} + 4h - \cancel{4x}$$

$$= \frac{4}{2\sqrt{1+\cancel{h}} + 2\sqrt{1}} = \frac{4}{2\sqrt{1} + 2} = \frac{4}{2+2} = 1$$

$y = mx + n$

$2 = 1 + n$

$\underline{n = 1}$

$$\boxed{y = 1x + 1}$$

d) $f(x) = \frac{1}{x^2}$ $\quad \left(\overset{x}{-2}, \overset{y}{\frac{1}{4}}\right)$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{\frac{1}{x^2+2xh+h^2} - \frac{1}{x^2}}{h}$$

$$\frac{1}{h} \times \frac{-1}{\cancel{x^2}+2xh+h^2} \times \frac{1}{\cancel{x^2}} = \frac{1}{\cancel{h}} \times \frac{-1}{\cancel{h}(h+2x)} = \frac{-1}{\cancel{h}+2x} = \frac{-1}{2x}$$

$$\frac{-1}{2(-2)} = \frac{-1}{-4} = \frac{1}{4}$$

$y = mx + n$

$\frac{1}{4} = \frac{-1}{4} \cdot (-2) + n$

$n = \frac{1}{2}$

$$\boxed{y = \frac{1}{4}x + \frac{1}{2}}$$

2) a) $f(x) = 3x^2 - 4x + 1$ $\quad \rightsquigarrow (x^2 + 2xh + h^2)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 3(x+h)^2 - 4(x+h) + 1 - (3x^2 - 4x + 1)$$

$$\hookrightarrow = \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{4x} - 4h + \cancel{1} - \cancel{3x^2} + \cancel{4x} - \cancel{1}}{h}$$

$$\frac{\cancel{h}(6x+3h-4)}{\cancel{h}} = 6x+\cancel{3h}^{0}-4$$

$$6x-4$$

Dom da função e da derivada $: \mathbb{R}$

b) $f(x) = \dfrac{2x+1}{x+3}$

$$\lim_{x\to 0} \frac{f(x+h)-f(x)}{h} = \frac{\frac{2(x+h)+1}{(x+h)+3} - \left(\frac{2x+1}{x+3}\right)}{h}$$

$$= \frac{\frac{\cancel{2x}+2h+\cancel{1}}{\cancel{x}+h\cancel{+3}} - \frac{\cancel{2x}-1}{\cancel{x+3}}}{h}$$

$$\frac{2h}{h} \times \frac{1}{h} = \frac{2h}{h^2} = \frac{\cancel{h}(2)}{\cancel{h}(h)} = \frac{2}{h}$$

Dom $\mathbb{R}-\{0\}$

c) $f(x) = \sqrt{1-2x}$

$$\lim_{x\to 0} \frac{f(x+h)-f(x)}{h} = \frac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} \times \frac{\sqrt{1-2(x+h)}+\sqrt{1-2x}}{\sqrt{1-2(x+h)}+\sqrt{1-2x}} = \left(\left(\sqrt{1-2(x+h)}\right)^{\cancel{2}} - \left(\sqrt{1-2x}\right)^{\cancel{2}}\right)$$

$$\hookrightarrow \cancel{1}-\cancel{2x}-2h-\cancel{1}+\cancel{2x}$$

$$\frac{-2h}{h\left(\sqrt{1-2(x+h)}+\sqrt{1-2x}\right)} = \frac{\cancel{h}(-2)}{\cancel{h}\left(\sqrt{1-2(x+h)}+\sqrt{1-2x}\right)} = \frac{-2}{\sqrt{1-2(x+h)}+\sqrt{1-2x}}$$

$\hookrightarrow \sqrt{1-2(x+h)}>0 \quad \hookrightarrow \sqrt{1-2x}>0$

$1-2(x+h)>0 \qquad 1-2x>0$

$1-2x-\cancel{2h}^{0}>0 \qquad x<\frac{1}{2}$

$x>\frac{-1}{-2}$

$\hookrightarrow x<\frac{1}{2}$

Domínio função $\sqrt{1-2x}\geq 0$

$x \geq \frac{-1}{-2}$ $\qquad$ Dom $=]-\infty, \frac{1}{2}]$

$x \leq \frac{1}{2}$

Domínio derivada $=]-\infty, \frac{1}{2}[$

3)a) $S(t) = t^2 - 6t - 5$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow (x+h)^2 - 6(x+h) - 5 - (x^2 - 6x - 5)$$

$$\hookrightarrow \cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h - \cancel{5} - \cancel{x^2} + \cancel{6x} + \cancel{5}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}} = 2x + \cancel{h}^{\to 0} - 6$$

$$2x - 6$$

$t = 2s$

$2(2) - 6 = 4 - 6 = -2 \;\to\; -2m/s$ DÚVIDA!

b) $s(t) = t^{-1} - t$

$t^{-1} = t^{-2} \quad -t = -1$

$t^{-2} - 1$

$\hookrightarrow (2)^{-2} - 1 = \frac{1}{4} - 1 = -\frac{5}{4} = -1{,}25 m/s$

c) $s(t) = \dfrac{t+1}{t-1}$

$$= \frac{(t + \Delta t + 1)}{t + \Delta t - 1} - \frac{t+1}{t-1}$$

$$= \frac{(t+\Delta t+1)(t-1) - (t+1)(t+\Delta t-1)}{(t+\Delta t-1)(t-1)}$$

$= t^2 - t + t\Delta t - \Delta t + t - 1$

$= t^2 + t\Delta t - \Delta t - 1$

$= t^2 + t\Delta t - \cancel{t} + \cancel{t} + \Delta t - 1$

$= t^2 + t\Delta t + \Delta t - 1$

$(\cancel{t^2} + \cancel{t\Delta t} - \Delta t - \cancel{1}) - (\cancel{t^2} + \cancel{t\Delta t} + \Delta t - \cancel{1})$

$= -2\Delta t$

$$v(t) = \lim_{\Delta t \to 0} \frac{-2}{(t + \cancel{\Delta t} - 1)(t-1)}$$

$$v(t) = \frac{-2}{(t-1)^2}$$

4)

a) $P=(0,0)$

à esquerda (0^-)

$$\lim_{x\to 0^-} \frac{(x+h)^2-x^2}{h} = \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x+\cancel{h}^{\to 0}$$

$$= 2x$$

à direita (0^+)

$$\lim_{x\to 0^+} \frac{(x+h)-x}{h} = \frac{\cancel{x}+h-\cancel{x}}{h} = \frac{h}{h} = 1$$

subst $= 2\cdot 0 = 0$ e $1 = 1$

logo a função não é diferenciável

b) $P=(1,2)$

à esquerda (1^-)

$y=2 \to$ derivada $=0$

à direita (1^+)

$2x = 2\cancel{x}^{0} = 2$

logo, não é diferenciável

$0 \neq 2$

5) $f(x) = |3x - 6|$

a) $3x - 6$

$$\lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \frac{3(x+h) - 6 - 3x + 6}{h} = \frac{\cancel{3x} + 3h \cancel{-6} \cancel{-3x} \cancel{+6}}{h}$$

$$\frac{\cancel{h}(3)}{\cancel{h}} = 3$$

$$-(3x-6) = -3x + 6$$

$$\lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = \frac{-3(x+h) + 6 + 3x - 6}{h} = \frac{\cancel{-3x} - 3h \cancel{+6} \cancel{+3x} \cancel{-6}}{h}$$

$$\frac{h(-3)}{h} = -3$$

$f(x)$ não é diferenciável pois os limites laterais são diferentes!

b) $f(x) = \begin{cases} (3x-6), & \text{se } x \geq 2 \\ -(3x-6), & \text{se } x < 2 \end{cases}$

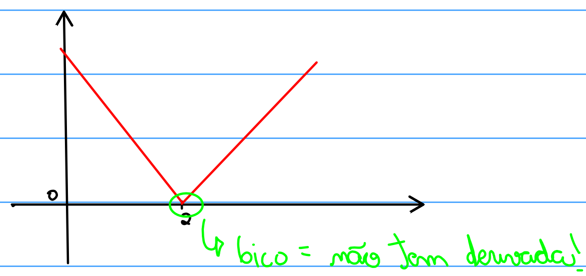
para $3x - 6 = 3$

para $-(3x-6) = -3$, logo :

$$f'(x) = \begin{cases} 3, & \text{se } x \geq 2 \\ -3, & \text{se } x < 2 \end{cases}$$

Como $f'(x)$ não é diferenciável em $x = 2$, então $f(x)$ não está definida nesse ponto

c)



6) a) $f(x) = \begin{cases} \dfrac{2\,\text{sen}\,x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

$$\boxed{\text{I} \quad \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = f(0)}$$

$\boxed{<0}$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{2\,\text{sen}\,x}{x} = 2$$

$\boxed{>0}$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{2\,\text{sen}\,x}{x} = 2$$

$$f(0) = 2$$

é contínua em $x = 0$

II derivadas laterais

à direita de 0

$$\lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^+} \frac{\frac{2\,\text{sen}(x+h)}{x} - \cancelto{2}{\frac{2\,\text{sen}\,x}{x}}}{h} = \frac{\frac{2\,\text{sen}(x+\cancel{h})}{x} - 2}{\cancel{h}}$$

$$\frac{2\left(\frac{\text{sen}\,x}{x} - 1\right)}{1} \times \frac{1}{x}$$

b) $f(x) = \begin{cases} 2x-1, & x \geq 0 \\ x^2+2x+7, & x < 0 \end{cases}$

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$2x-1 = -1$ $\quad$ $x^2+2x+7 = 7$ $\quad$ $f(0) = 2x-1 = -1$

a função não é contínua!

c) $\begin{cases} x^{2/3}, & x \geq 0 \\ x^{1/3}, & x < 0 \end{cases}$

$f_+' : x^{2/3} = \frac{2}{3}x^{\frac{2}{3}-\frac{3}{3}} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}} = \frac{2}{3\sqrt[3]{0}} = +\infty$

$f_-' : x^{1/3} = \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}} = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} = +\infty$

derivadas laterais não são finitas, não há $f'(0)$

d) $f(x) \begin{cases} 2x+\tan x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$

$\lim_{x \to 0^+} 2x + \tan x = 2(0) + \tan(0) = 0$

$\lim_{x \to 0^-} x^2 = 0^2 = 0$

$f_-' = x^2 = 2x^{2-1} = 2x$

$f_+' = (2x+\tan x)$

$2+\sec^2(x)$

$2+1=3$

A derivada não existe, pois os limites laterais divergem!

7) a) $h(x) = 5x - \cancel{x}^{0}$

$5x^0 =$ $f'(x) = 5$ //

b) $f(x) = -4x^{10}$

$f'(x) = -40x^9$

c) $f(x) = x^3 + 6x - \cancel{4}^{0}$

$f'(x) = 3x^2 + 6$

d) $g(x) = 5x^8 - 2x^5 + \cancel{6}^{0}$

$40x^7 - 10x^4$

e) $y = x^{-2/5}$

$y'(x) = \frac{-2}{5} x^{-7/5}$

f) $V(r) = \frac{4}{3} \pi r^3$

$V'(r) = \frac{12}{3} \pi r^2$

g) $y(t) = 7t^{-9}$

$y'(t) = -63t^{-10}$

h) $R(x) = \frac{\sqrt{10}}{x^7} = \left(\frac{\sqrt{10}}{1}\right) \times x^{-7}$

$= \left(\frac{-7\sqrt{10}}{1}\right) \times x^{-8} = \frac{-7\sqrt{10}}{x^8}$

i) $y = \sqrt[3]{x}$ $\qquad \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

$= (x^3)^{1/2} = x^{3/2}$

$= \frac{3}{2} x^{1/2}$ // $\Rightarrow y' = \frac{3}{2}\sqrt{x}$

j) $g(x) = x^2 + \frac{1}{x^2} \rightarrow 2x^1 + 1 \times X^{-2}$

$g(x) = 2x + -2x^{-3}$

$\frac{4}{3} - \frac{3}{3} = \frac{1}{3}$ $\frac{2}{3} - \frac{3}{3} = \frac{-1}{3}$

k) $y = x^{4/3} - x^{2/3}$

$y' = \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$

l) $v = x\sqrt{x} + \frac{1}{x^2\sqrt{x}} \rightarrow \frac{x^{-2} \cdot x^{-1/2}}{x^{-5/2}}$

$= x^1 \cdot x^{1/2}$

$x^{3/2} + X^{-5/2}$

$v' = \frac{3}{2}x^{1/2} - \frac{5}{2}X^{-7/2}$ //

$\rightarrow v' = \frac{3}{2}\sqrt{x} - \frac{5}{2}\frac{1}{x^{7/2}}$

m) $g(x) = (\overbrace{1+\sqrt{x}}^{f})(\overbrace{x - x^3}^{g}) \rightarrow f' \cdot g + f \cdot g'$

$(\cancel{1}^{0} + \sqrt{x})' \cdot (x - x^3) + (1+\sqrt{x}) \cdot (x - x^3)'$

$x^{1/2}$ $\cancel{1} \rightarrow 3x^2$

$\rightarrow \frac{1}{2}x^{-1/2} \cdot (x - x^3) + (1+\sqrt{x})(1 - 3x^2)$

$\rightarrow \frac{1}{2\sqrt{x}} \cdot (x - x^3) + (1+\sqrt{x})(1-3x^2)$

$\frac{x - x^3}{2\sqrt{x}} + (1+\sqrt{x})(1-3x^2)$

8) a) $(2f + g)'(3)$

$2f'(3) \cdot g'(3)$

$2 \cdot (-6) + 7 = -12 + 7 = -5$

b) $(f g)'(3) \rightarrow f' \cdot g + f \cdot g'$

$\underbrace{-6 \cdot 2}_{} + -4 \cdot 7$

$-12 + -28 = -40$ //

c) $(f/g)'(3) = \dfrac{f' \cdot g - f \cdot g'}{g^2}$

$-6 \cdot 2 - (-4 \cdot 7)$

$\dfrac{-12 + 28}{4} = \dfrac{16}{4} = 4$

$\rightarrow (-6-7) = (-13)$

d) $\left(\dfrac{f}{f-g}\right)'(3) = \dfrac{f' \cdot (f-g) - f \cdot (f-g)'}{(f-g)^2}$

$\dfrac{(-6 \cdot -6) - (-4 \cdot -13)}{36} = \dfrac{36 - 52}{36} = \dfrac{-16}{36} = -\dfrac{4}{9}$ //

9) a) $\dfrac{d}{dx}(\tan x) = \sec^2 x$

$\tan(x) = \dfrac{\operatorname{sen} x}{\cos x}$

$= \dfrac{\cancel{(\operatorname{sen} x)'}^{\cos x} \cdot (\cos x) - (\operatorname{sen} x) \cdot \cancel{(\cos x)'}^{-\operatorname{sen} x}}{\cos^2 x}$

$= \dfrac{\cos^2 x + \operatorname{sen}^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} \Rightarrow \left(\dfrac{1}{\cos x}\right)^2 = \sec^2 x$

b) $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

$\sec(x) = \dfrac{1}{\cos(x)} \qquad = \dfrac{\cancel{(1)'}^{0} \cdot (\cos x) - (1) \cdot \cancel{(\cos x)'}^{-\operatorname{sen} x}}{\cos^2 x}$

$= \dfrac{+\operatorname{sen} x}{\cos^2 x} \Rightarrow \dfrac{1}{\cos x} \cdot \dfrac{\operatorname{sen} x}{\cos x}$

$\sec(x) \quad \tan(x) \qquad = \sec x \cdot \tan x$ //

c) $\dfrac{d}{dx}(\cot x) = -\operatorname{cossec}^2 x$

$\cot(x) = \dfrac{\cos(x)}{\operatorname{sen}(x)} \quad = \dfrac{\cancel{(\cos x)'}^{-\operatorname{sen}} \cdot (\operatorname{sen} x) - (\cos x) \cdot \cancel{(\operatorname{sen} x)'}^{\cos}}{\operatorname{sen}^2 x}$

$= \dfrac{-\operatorname{sen}^2 x - \cos^2 x}{\operatorname{sen}^2 x} = \dfrac{-(\operatorname{sen}^2 x + \cos^2 x)}{\operatorname{sen}^2 x} = \dfrac{-1}{\operatorname{sen}^2 x} \Rightarrow -\left(\dfrac{1}{\operatorname{sen} x}\right)^2 = -\operatorname{cossec}^2 x$ //

d) $\frac{d}{dx}(\text{cossec}\, x) = -\text{cossec}\, x \cot x$

$$\text{csc}(x) = \frac{1}{\text{sen}(x)} = \frac{\cancel{(1)'}^{\to 0} \cdot (\text{sen}\, x) - (1) \cdot \cancel{(\text{sen}\, x)'}^{\to \cos(x)}}{\text{sen}^2 x}$$

$$= \frac{-\cos(x)}{\text{sen}^2 x} = \frac{-1}{\text{sen}\, x} \cdot \frac{\cos x}{\text{sen}\, x}$$

$$\hookrightarrow -\text{csc}\, x \quad \hookrightarrow \cot x \quad = -\text{cossec}\, x \cot x$$

10) a) $f(x) = \overset{f}{x}\, \overset{g}{\text{sen}\, x}$ $\boxed{\text{regra do produto} = (f \cdot g)' = f' \cdot g + f \cdot g'}$

$$f'(x) = \cancel{(x)'}^{\to 1} \cdot (\text{sen}\, x) + (x) \cdot \cancel{(\text{sen}\, x)'}^{\to \cos(x)}$$

$$f'(x) = \text{sen}\, x + x \cos(x)$$

b) $y = \cos x - 2 \tan x$

$$y' = (\cos x)' - 2(\tan x)'$$

$$y' = -\text{sen}\, x - 2 \sec^2 x$$

c) $g(t) = t^3 \cos t \rightsquigarrow$ 2 prod.

$$g'(t) = (t^3)' \cdot (\cos t) + (t^3) \cdot \cancel{(\cos t)'}^{\to -\text{sen}\, x}$$

$$g'(t) = 3t^2 \cdot \cos t + t^3 \cdot -\text{sen}\, x$$

$$g'(t) = 3t^2 \cos t - t^3 \text{sen}\, t$$

d) $g(t) = 4 \sec t + \tan t$

$$g'(t) = (4 \sec t)' + (\tan t)'$$

$$g'(t) = 4(\sec(t) \tan(t)) + (\sec^2(t))$$

e) $y = \frac{\tan x}{x} \begin{matrix} \rightsquigarrow f \\ \rightsquigarrow g \end{matrix}$

$$= \frac{\cancel{(\tan x)'}^{\to \sec^2(x)} \cdot (x) - (\tan x) \cdot \cancel{(x)'}^{\to 1}}{x^2} = \frac{\sec^2(x) \cdot (x) - \tan(x)}{x^2}$$

//

f) $y = \dfrac{x}{\operatorname{sen} x + \cos x} \Rightarrow \dfrac{f}{g}$

$$= \frac{\cancel{(x)'}^{\,1} \cdot (\operatorname{sen} x + \cos x) - (x) \cdot (\operatorname{sen} x + \cos x)'}{(\operatorname{sen} x + \cos x)^2} =$$

$$= \frac{\operatorname{sen} x + \cos x - x \cdot \cos x - \operatorname{sen} x}{(\operatorname{sen} x + \cos x)^2}$$

$$= \frac{(1+x)\operatorname{sen}(x) + (1-x)\cos(x)}{(\operatorname{sen} x + \cos x)^2}$$

g) $y = \dfrac{\tan x - 1}{\sec x}$

$$y' = \frac{(\tan x - \overset{0}{\cancel{1}})' \cdot (\sec x) - (\tan x - 1) \cdot \cancel{(\sec x)'} \to \cos x}{\sec^2 x}$$

$$= \frac{\sec^2 x \cdot \sec x - \tan x - \sec x}{\sec^2 x} = \sec^2 x - \tan^2 x + \tan x \,//$$

h) $y = \dfrac{\operatorname{sen} x}{x^2}$

$$= \frac{\cancel{(\operatorname{sen} x)'}^{\,\cos x} \cdot (x)^2 - (\operatorname{sen} x) \cdot (x^2)'^{\,\to 2x}}{(x^2)^2} = \frac{\cos x \cdot x^2 - \operatorname{sen} x \cdot 2x}{x^4}$$

$$= \frac{x^2 \cos x - 2x \operatorname{sen} x}{x^4} \Rightarrow \frac{\cancel{x}(x \cos x - 2 \operatorname{sen} x)}{\cancel{x^4}}$$

$$= \frac{x \cos(x) - 2 \operatorname{sen}(x)}{x^3}$$

i) $y = \tan \theta \cdot (\operatorname{sen} \theta + \cos \theta)$

$$y' = (\tan \theta)' \cdot (\operatorname{sen} \theta + \cos \theta) + (\tan \theta) \cdot (\overbrace{\operatorname{sen} \theta}^{\cos \theta} + \overbrace{\cos \theta}^{-\operatorname{sen} \theta})'$$

$$y' = \sec^2(x) \cdot (\operatorname{sen} \theta + \cos \theta) + \tan \theta \cdot (\cos \theta - \operatorname{sen} \theta) \,//$$

j) $y = \operatorname{cossec} x \cdot \cot x$

$$y' = (\operatorname{cossec} x)' \cdot (\cot x) + (\operatorname{cossec} x) \cdot (\cot x)'$$

$$y' = -\operatorname{cossec} x \cot^2 x + \operatorname{cossec} x \cdot (-\operatorname{cossec}^2 x)$$

$$y' = -\operatorname{cossec} x \cot^2 x - \operatorname{cossec}^3 x$$

K) $y = x \operatorname{sen} x \quad \cos x$

$$y' = (x)'(\operatorname{sen} x \cos x) + (x)\cdot(\underbrace{\operatorname{sen} x}_{\cos x}\ \underbrace{\cos x}_{-\operatorname{sen} x})'$$

$$y' = \operatorname{sen} x \cos x + x \cdot \cos x - \operatorname{sen} x$$

$$y' = x \cdot \cos^2 x + \operatorname{sen} x \cdot \cos x$$

11)

a) $f(x) = (x^3 + 2x)e^x \rightarrow$ regra do produto

$$f'(x) = (x^3+2x)' \cdot (e^x) + (x^3+2x)\cdot(e^x)'$$

$$= (3x^2+2)\cdot e^x + (x^3+2x)\cdot e^x$$

$$= e^x(x^3+3x^2+2x+2)$$

$$\frac{1}{2} - \frac{2}{2} = -\frac{1}{2}$$

b) $g(x) = (e^x + 3x^2)\sqrt{x}$

$$g'(x) = (e^x + 3x^2)' \cdot (\sqrt{x}) + (e^x + 3x^2)\cdot(\sqrt{x})'$$

$$= e^x + 6x \cdot \sqrt{x} + (e^x + 3x^2)\cdot \frac{1}{2} x^{-1/2}$$

$$= (e^x + 6x)\cdot\sqrt{x} + (e^x+3x^2)\cdot\frac{1}{2\sqrt{x}}$$

c) $f(z) = (1 - e^x)(z + e^x)$

$$f'(z) = (\cancel{1} - e^z)' \cdot (z + e^z) + (1 - e^z)(\cancel{z} + e^z)'$$

$$(-e^z)(z+e^z) + \underbrace{(1-e^z)\cdot(1+e^z)}$$

$$-e^z z - e^{2x} + 1 - e^{2z}$$

$$-e^z z + 1 - 2e^{2z}$$

d) $y = \dfrac{e^x}{x^2}$

$$y' = \frac{(e^x)'(x^2) - (e^x)(x^2)'}{(x^2)^2} = \frac{e^x(x^2) - (e^x)(2x)}{x^4}$$

$$= \frac{e^x \cdot x^2 - e^x \cdot 2x}{x^4}$$

//

e) $y = \dfrac{e^x}{1+x}$

$\overset{0}{\cancel{x}} + 1$

$$y' = \frac{(e^x)'(1+x) - (e^x)(1+x)'}{(1+x)^2} = \frac{e^x \cdot (1+x) - e^x}{(1+x)^2} = \frac{\cancel{e^x} + xe^x - \cancel{e^x}}{(1+x)^2} = \frac{xe^x}{(1+x)^2}$$

f) $f(x) = \dfrac{1 - xe^x}{x + e^x}$

$$\frac{(1-xe^x)'(x+e^x) - (1-xe^x)(x+e^x)'}{}$$

$$= \frac{-x^2e^x \cdot (x+e^x) - (1-xe^x)\, 1+e^x}{(x+e^x)^2}$$

$$= \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x+e^x)^2}$$

$$f'(x) = \frac{-x^2e^x - 2e^{2x} - 1}{(x+e^x)^2}$$

g) $y = \dfrac{x + e^x}{x^3 + x - 2\,\text{sen}\,x}$

$$\frac{(x+e^x)'(x^3+x-2\,\text{sen}\,x) - (x+e^x)(x^3+x-2\,\text{sen}\,x)'}{(x^3+x-2\,\text{sen}\,x)^2}$$

$$= \frac{x^3 + x - 2\,\text{sen}\,x + e^3x^3 + e^xx - 2e^x\,\text{sen}\,x - (3x^3 + x - 2x\cos x + 3x^2e^x + e^x + e^x - 2e^x\cos x)}{(x^3 + x - 2\,\text{sen}\,x)^2}$$

$$y' = \frac{-2x^3 - 2\,\text{sen}\,x + e^xx^3 - 3x^2e^x - 2e^x\,\text{sen}\,x + 2x\cos x - 2e^x\cos x}{(x^3 + x - 2\,\text{sen}\,x)^2}$$