

# Project I

Vivek Rai, 12BT30025

a)

Let us make the following substitution  $z = \dot{y}/\mu$

After simplification, we obtain the following two ordinary differential equations:

$$\dot{z} = \mu(1 - y^2)z - y\mu^{-1} \quad \text{and} \quad \dot{y} = z\mu$$

c)

Several profile runs reveal that ODE15s runs almost 10x-15x faster than ODE45.

d)

The Van Der Pol oscillator is a non-conservative oscillator. The phase-plane and the time-axis plots from the oscillation reveal the non-linear damping in the oscillator. After running the simulation for several values of  $\mu$ , we can observe that the system quickly goes into an oscillatory behavior compared to the situation when the value of  $\mu$  is high. It is the simplest and best understood limit cycle (self-sustaining) that exhibits relaxation behavior, that is, alternating fast and slow time scales. This can be observed from the phase plane plots of the equation, where the curve moves with a slow time constant along the null clines but with a very fast time scale otherwise. This effect is further pronounced by increasing the value of  $\mu$  which is the non-linearity parameter here.

It has been found that study of ion-channels and generation of action potential, when studied, reveals a model characteristic of this Van Der Pol oscillator. That is, a quick rise in the potential across the membrane in form of a spike and then gradual decay. This pattern is typical yet important feature of a nerve impulse.