Challenging Problem 1

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March 5, 2019

All the work below is my own.

1 Filters for Speckle Reduction

I have chosen the following four algorithms for speckle reduction. While all of these have their pros and cons, and more advanced techniques are available in literature, I felt that these four algorithms best suit my coding capabilities. I have picked the most cited papers among the ones I found suitable for implementation (756, 101, 124, 272 citations respectively).

1.1 Adaptive Weighted Median Filter (AWMF)

This algorithm is based off of Loupas et al.'s work [1]. The central idea here is to assign 'weights' to pixel intensities before running the median filtering operation. Therefore, instead of taking the median of all the pixels within a target pixel's neighborhood, this algorithm first adaptively figures out which neighboring pixels are more important, and then replicates those pixels a 'weight' number of times so as to give them a proportionate chance of coming out as the median.

The weight of the pixel (k, l) in the $2K + 1 \times 2K + 1$ neighborhood of (i, j) is computed as:

$$w_{kl} = round(w_0 - \frac{cd\sigma^2}{\mu}), \tag{1}$$

where, w_0 and c are empirically determined parameters, $d = \sqrt{(i-k)^2 + (j-l)^2}$ is the Euclidean distance between (k, l) and (i, j), σ^2 is the local variance and μ is the local mean of the neighborhood. w_{kl} is made 0 if the rounded value is negative. As can be seen from the equation, the farther a pixel is from the neighborhood center (larger d), the lower its prominence is.

The ratio $\frac{\sigma^2}{\mu}$, often referred to as the 'Speckle Index', is of importance here. Because speckle noise is multiplicative, the ratio of intensity deviation to mean is taken to be a reasonable measure of the amount of speckle noise present. This ratio is therefore a proxy for how uniform-valued the pixels in a neighborhood are. If the ratio is large, more weight is given to the central pixels, as a discontinuity in the image is expected to cut through the neighborhood window. If the ratio is small, it is understood that the neighborhood is relatively uniform in pixel intensity, and therefore distant pixels are also given considerable importance. This ratio thus helps capture the speckle texture.

Pros: The use of Speckle Index to adaptively give prominence to certain pixels over others helps preserve speckle texture. Furthermore, this algorithm is simple, computationally less complex, and runs relatively fast—making it an attractive option. AWMF is an improvement over the

standard median filter in the sense that median filters cannot capture impulsive artifacts with an area larger than half the neighborhood area. To the contrary, the use of pixel replication in AWMF removes the requirement that speckle artifacts be smaller than half the neighborhood size [2].

Cons: The algorithm is sensitive to the empirically determined parameters w_0 and c. The authors recommended using $w_0 = 99$ and $c = 20$ for a 9×9 kernel window in a $576 \times 530 \times 8-$bit image, but these values did not give satisfactory results. Instead, I used $w_0 = 10$ and $c = 0.25$, as suggested by Chen et al. [2], and it worked well for the test images in hand. Furthermore, the operator is still fixed in shape (round), and two pixels equidistant from the center receives the same weight—which might not always be ideal. The algorithm causes some blurring effects.

1.2 Directional Median Filter (DMF)

This algorithm is based off of Czerwinski et al.'s work [3]. The idea is to take medians along different directions in a pixel neighborhood, and then use the max median as a replacement of the center pixel. Mathematically, for the pixel value $I(i, j)$, it can be written as,

$$I_{out}(i, j) = max_\theta med_{\zeta,\eta}[I(i - \zeta, j - \eta)s_\theta(\zeta, \eta)], \tag{2}$$

where $s_\theta(i, j)$ corresponds to a stick of orientation θ that goes through the center (i, j). In my implementation, I have used four sticks corresponding to $\theta \in \{0^0, 45^0, 90^0, 135^0\}$. For each of the sticks, the median of the pixel intensities lying along the stick is taken, and the maximum of the four medians is taken as the final output intensity for the target pixel.

Pros: The algorithm does really well in preserving sharp discontinuities and therefore avoiding blurriness effects. It applies smoothing in only one direction at a time, and takes the median in the direction of the most significant straight line component, which helps it reduce blurriness. It is also an improvement over the AWMF algorithm since it is not fixed in shape (AWMF is round).

Cons: This filter can sometimes retain noise along the edges and make the discontinuities too sharp, resulting in a 'cracked mirror' look on the output image. Also, it is tough to have a lot of theta values to allow for a smoother rotation of the stick. Therefore, not all directional information can be captured. My implementation retains information from only four theta values.

1.3 Aggressive Region Growing Filter (ARGF)

This algorithm is based off of Chen et al.'s work [2]. It focuses on the window size itself, and changes the size adaptively to preserve speckle texture while at the same time removing noise. In order to do so, the algorithm initializes with a standard window size of 11×11. For each pixel's neighborhood, it considers how homogeneous the pixel intensities are, and then undergoes consecutive phases of contraction and extension of the window size. In the contraction phase, the window around the pixel (i, j) is shrunk as long as the following homogeneity condition is met:

$$h_{ij} > h_0 + \sigma_0 \implies \frac{\sigma_{ij}^2}{\mu_{ij}} > \frac{a||w||}{b + ||w||} + c + ke^{-d||w||} \tag{3}$$

where, the left hand side corresponds to the speckle index around the $(i, j)^{\text{th}}$ pixel as discussed earlier, and the right hand is a function of the window size $||w||$. The parameters a, b, c, d and k are estimated empirically. A minimum window size, S_{min}, is also enforced in the contraction phase.

In the growth phase, the window size is grown as long as the following complimentary condition is met:

$$h_{ij} \leq h_0 + \sigma_0 \implies \frac{\sigma_{ij}^2}{\mu_{ij}} \leq \frac{a||w||}{b + ||w||} + c + ke^{-d||w||}. \tag{4}$$

A maximum window size, S_{max}, is enforced; and it is also ensured that h_{ij} changes by less than σ_0 in each iteration. Lastly, if the final window size is larger than the initial window size, the mean of the neighborhood pixels is taken as the target value; otherwise, the median is taken.

Pros: This method reduces noise better than AWMF, as shown by the authors. It is also able to deal with a variable window size, which helps it retain speckle features reasonably.

Cons: The algorithm runs contraction and growth phases for each center pixel, which makes it slower than the standard median filter. Also, there are a lot of parameters that need to be empirically determined for satisfactory results. In my implementation, I have used the parameter values recommended in the paper, which worked fine—but that does not guarantee an across-the-board solution. There remain considerable blurriness in the images.

1.4 Geometric Filter (GF)

This algorithm is based off of Crimmins et al.'s work [4], and uses a convex hull approach. The author argues that a speckle has a 'wormy' appearance. If we consider (and actually code the algorithm in such a way) that the image intensities form a surface in the 3D space with the z-axis heights proportional to the intensities, then the speckles appear as narrow winding walls and valleys in that surface. The geometric filter attempts to tear down the narrow walls and fill up the narrow valleys. However, it reduces narrow walls and valleys faster than it reduces wider towers. Thus, the algorithm does not hurt the higher/wider towers much (textures we care about) while removing smaller noisy elements. In order to achieve this, the major steps are:

1. Scan the image line by line in different directions. In my implementation, I have scanned the image line by line in N-S, E-W, SE-NW and SW-NE directions consecutively.

2. For each line of pixels, construct a vertical 'pixel grid' of the dimension $(256\times$ length(pixel line) $)$. Assign 1 to the bottom i number of pixels in each column, where i is the intensity $(0 - 255)$ of the corresponding pixel in the line. Assign 0 elsewhere.

3. Run the 'Complimentary 8-hull Algorithm' on the pixel grid. This algorithm first runs the '8-hull Algorithm' on the grid, swaps all the 0's with 1's, runs the 8-hull algorithm on the resulting grid, and swaps 0's and 1's again. The 8-hull algorithm replaces a 0 with a 1 if its 3×3 neighborhood satisfies one of 8 pre-specified patterns.

4. Map the resulting pixel grid back to the line of pixels, projecting the column heights as pixel intensities.

5. Repeating the above for all of the lines of pixels in various directions completes one iteration of the algorithm.

Pros: The algorithm attempts to solve the problem in a very intuitive and interesting manner, which was why I'd initially chosen it. Besides from that, unfortunately, I later found no benefit of using it.

Cons: The algorithm is *very* slow. The reason is imaginable, it is computationally very complex with all the grid projections and back. Furthermore, the 8-hull algorithm by definition works

with a 3×3 window, making the window size very rigidly small. This in turn makes the algorithm impractical for larger noise artifacts. For the test images in this assignment, the algorithm could not accomplish any meaningful noise reduction, despite taking a lot more time than the other ones.

```
In [1]: from IP import *
        import IP
        import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib import colors
        import skimage
        from skimage.util import random_noise
        from skimage.filters import median
        %matplotlib inline
        from scipy.io import loadmat
```

2 Preprocessing

To make the runtimes feasible for testing purposes, I have chosen to resize the raw images to a dimension of 128x128 pixels.

```
In [2]: figA1 = loadmat('FigA1.mat')
        figB1 = loadmat('FigB1.mat')
        data1 = loadmat('data1.mat')
        data2 = loadmat('data2.mat')
        data3 = loadmat('data3.mat')
```

```
In [4]: # resizing
        figA1_resized = imresize(figA1['image'],(128,128),anti_aliasing=True)
        figB1_resized = imresize(figB1['image'],(128,128),anti_aliasing=True)
        data1_resized = imresize(data1['image'],(128,128),anti_aliasing=True)
        data2_resized = imresize(data2['image'],(128,128),anti_aliasing=True)
        data3_resized = imresize(data3['image'],(128,128),anti_aliasing=True)

        # converting them to 0-255 uint8 images
        figA1_resized= figA1_resized-figA1_resized.min()
        figA1_resized= (figA1_resized/figA1_resized.max()*255).astype("int")
        figB1_resized= figB1_resized-figB1_resized.min()
        figB1_resized= (figB1_resized/figB1_resized.max()*255).astype("int")
        data1_resized= data1_resized-data1_resized.min()
        data1_resized= (data1_resized/data1_resized.max()*255).astype("int")
        data2_resized= data2_resized-data2_resized.min()
        data2_resized= (data2_resized/data2_resized.max()*255).astype("int")
        data3_resized= data3_resized-data3_resized.min()
        data3_resized= (data3_resized/data3_resized.max()*255).astype("int")
```

3 Implementation of Adaptive Weighted Median Filter (AWMF)

In [5]:
```
# this function creates a distance matrix where each entry holds the
# corresponding pixel location's Euclidean distance from the center pixel
def generate_d(w_len):
    dist_mat = np.zeros((w_len,w_len))
    padding_len = int(np.floor(w_len/2.))
    for i in range(w_len):
        for j in range(w_len):
            dist_mat[i,j]=\
            np.linalg.norm(np.array([i,j])\
                           -np.array([padding_len,padding_len]))
    return dist_mat
```

In [6]:
```
# calculate the weight coefficients according to Loupas et al. [1]
def generate_w(w0,c,d,sig2,miu):
    temp_w = w0-(c*d*sig2/miu)
    temp_w[temp_w<0]=0
    return temp_w.round()
```

In [7]:
```
# replicates each pixel under the window 'weight' number of times
# to update its prominence in median calculation
def weighted_values(values,weights):
    weighted_list = []
    for i in range(values.shape[0]):
        for j in range(values.shape[1]):
            weighted_list.extend([values[i,j]]*weights[i,j])
    return weighted_list
```

In [8]:
```
def AWMF(I,w_len,w0,c,plot_steps=False):
    if(plot_steps):
        IP.plt.figure(figsize=(12,4))
        IP.plt.subplot(1,2,1);IP.plt.title('Input Image')
        IP.imshow(I,cmap='gray')
        print("Shape of input image:"+str(I.shape))

    if(w_len % 2 == 0):
        print("Please insert an odd kernel window length.")
        return

    #compute number of layers to pad
    padding_len = int(np.floor(w_len/2.))

    #initialize padded image with all zeros
    padded_I = np.zeros((I.shape[0]+2*padding_len,I.shape[1]+2*padding_len))

    # assign main image to the middle of the padded_image
    padded_I[padding_len:-padding_len,padding_len:-padding_len]=I
```

```
#nearest neighbor assignments to the horizontal and vertical padded segments
padded_I[0:padding_len,padding_len:-padding_len] = I[0,:]
padded_I[-padding_len:,padding_len:-padding_len] = I[-1,:]
padded_I[padding_len:-padding_len,0:padding_len] = I[:,0:1]
padded_I[padding_len:-padding_len,-padding_len:] = I[:,-1:]

#taking care of the four square corners of the padded layers
padded_I[0:padding_len,0:padding_len] = \
np.repeat(I[0,0], padding_len*padding_len).reshape(padding_len,padding_len)

padded_I[-padding_len:,0:padding_len] = \
np.repeat(I[-1,0], padding_len*padding_len).reshape(padding_len,padding_len)

padded_I[0:padding_len:,-padding_len:] = \
np.repeat(I[0,-1], padding_len*padding_len).reshape(padding_len,padding_len)

padded_I[-padding_len:,-padding_len:] = \
np.repeat(I[-1,-1], padding_len*padding_len).reshape(padding_len,padding_len)

#initialize filtered image with all zeros
filtered_I = np.zeros(I.shape)

# compute distance matrix
distance_mat = generate_d(w_len=w_len)

#scan padded image with a kernel window of w_len*w_len dimension
for id_r,row in enumerate(range(padding_len,padded_I.shape[0]-padding_len)):
    for id_c,col in enumerate(range(padding_len,padded_I.shape[1]-padding_len)):
        #grab values within kernel window and take median
        values = padded_I[row-padding_len:row+padding_len+1,\
                          col-padding_len:col+padding_len+1]

        # adaptively generate each pixel value's weight
        weights = generate_w(w0,c,distance_mat,\
                             sig2=np.var(values),miu=np.mean(values))

        # replicate pixels according to weights
        weighted_values_list = weighted_values(values.astype("int"),\
                                               weights.astype("int"))

        #assign median to filtered image
        filtered_I[id_r,id_c] = np.median(weighted_values_list)

if(plot_steps):
    IP.plt.subplot(1,2,2);
    IP.imshow(filtered_I,cmap='gray');IP.plt.title('AWMF Filtered Image')
    print("Shape of filtered image:"+str(filtered_I.shape))
return filtered_I
```

4 Implementation of Directional Median Filter (DMF)

```
In [9]: def generate_sticks(w_len):
            padding_len = int(np.floor(w_len/2.))

            # horizontal stick
            a = np.zeros((w_len,w_len), int)
            a[padding_len,:]=1

            # diagonal stick
            d = np.zeros((w_len,w_len), int)
            np.fill_diagonal(d,1)

            # the other diagonal stick
            b=np.flip(d,axis=1)

            # vertical stick
            c = np.zeros((w_len,w_len), int)
            c[:,padding_len]=1

            return a,b,c,d

In [10]: def DMF(I,w_len,plot_steps=False):
             if(plot_steps):
                 IP.plt.figure(figsize=(12,4))
                 IP.plt.subplot(1,2,1);IP.plt.title('Input Image')
                 IP.imshow(I,cmap='gray')
                 print("Shape of input image:"+str(I.shape))

             if(w_len % 2 == 0):
                 print("Please insert an odd kernel window length.")
                 return

             #compute number of layers to pad
             padding_len = int(np.floor(w_len/2.))

             #initialize padded image with all zeros
             padded_I = np.zeros((I.shape[0]+2*padding_len,I.shape[1]+2*padding_len))

             # assign main image to the middle of the padded_image
             padded_I[padding_len:-padding_len,padding_len:-padding_len]=I

             #nearest neighbor assignments to the horizontal and vertical padded segments
             padded_I[0:padding_len,padding_len:-padding_len] = I[0,:]
             padded_I[-padding_len:,padding_len:-padding_len] = I[-1,:]
             padded_I[padding_len:-padding_len,0:padding_len] = I[:,0:1]
             padded_I[padding_len:-padding_len,-padding_len:] = I[:,-1:]
```

```
    #taking care of the four square corners of the padded layers
    padded_I[0:padding_len,0:padding_len] = \
    np.repeat(I[0,0], padding_len*padding_len).reshape(padding_len,padding_len)

    padded_I[-padding_len:,0:padding_len] = \
    np.repeat(I[-1,0], padding_len*padding_len).reshape(padding_len,padding_len)

    padded_I[0:padding_len:,-padding_len:] = \
    np.repeat(I[0,-1], padding_len*padding_len).reshape(padding_len,padding_len)

    padded_I[-padding_len:,-padding_len:] = \
    np.repeat(I[-1,-1], padding_len*padding_len).reshape(padding_len,padding_len)

    # initialize filtered image with all zeros
    filtered_I = np.zeros(I.shape)

    # get the four sticks
    a,b,c,d = generate_sticks(w_len)

    #scan padded image with a kernel window of w_len*w_len dimension
    for id_r,row in enumerate(range(padding_len,padded_I.shape[0]-padding_len)):
        for id_c,col in enumerate(range(padding_len,padded_I.shape[1]-padding_len)):
            #grab values within kernel window and take median
            values = padded_I[row-padding_len:row+padding_len+1,\
                              col-padding_len:col+padding_len+1].astype("int")

            # consider the values lying along the sticks
            stick_a = values[values*a>0]
            stick_b = values[values*b>0]
            stick_c = values[values*c>0]
            stick_d = values[values*d>0]

            # compute median along each stick orientation
            median_list = [np.median(stick_a),\
                           np.median(stick_b),np.median(stick_c),\
                           np.median(stick_d)]

            #assign max median to filtered image
            filtered_I[id_r,id_c] = max(median_list)

    if(plot_steps):
        IP.plt.subplot(1,2,2);
        IP.imshow(filtered_I,cmap='gray')
        IP.plt.title('Directional Median Filtered Image')
        print("Shape of filtered image:"+str(filtered_I.shape))
    return filtered_I
```

5 Implementation of Aggressive Region Growing Filter (ARGF)

```
In [11]: def ARGF(I,w_len,a=2.,b=19.5,c=0.3,d=0.004,k=0.5,S_min=49,plot_steps=False):
             if(plot_steps):
                 IP.plt.figure(figsize=(18,4))
                 IP.plt.subplot(1,3,1);IP.plt.title('Input Image')
                 IP.imshow(I,cmap='gray')
                 print("Shape of input image:"+str(I.shape))

             if(w_len % 2 == 0):
                 print("Please insert an odd kernel window length.")
                 return

             #compute number of layers to pad
             padding_len = int(np.floor(w_len/2.))

             #initialize padded image with all zeros
             padded_I = np.zeros((I.shape[0]+2*padding_len,I.shape[1]+2*padding_len))

             # assign main image to the middle of the padded_image
             padded_I[padding_len:-padding_len,padding_len:-padding_len]=I

             #nearest neighbor assignments to the horizontal and vertical padded segments
             padded_I[0:padding_len,padding_len:-padding_len] = I[0,:]
             padded_I[-padding_len:,padding_len:-padding_len] = I[-1,:]
             padded_I[padding_len:-padding_len,0:padding_len] = I[:,0:1]
             padded_I[padding_len:-padding_len,-padding_len:] = I[:,-1:]

             #taking care of the four square corners of the padded layers
             padded_I[0:padding_len,0:padding_len] = \
             np.repeat(I[0,0], padding_len*padding_len).reshape(padding_len,padding_len)

             padded_I[-padding_len:,0:padding_len] = \
             np.repeat(I[-1,0], padding_len*padding_len).reshape(padding_len,padding_len)

             padded_I[0:padding_len:,-padding_len:] = \
             np.repeat(I[0,-1], padding_len*padding_len).reshape(padding_len,padding_len)

             padded_I[-padding_len:,-padding_len:] = \
             np.repeat(I[-1,-1], padding_len*padding_len).reshape(padding_len,padding_len)

             if(plot_steps):
                 IP.plt.subplot(1,3,2);
                 IP.imshow(padded_I,cmap='gray');IP.plt.title('Padded Image')
                 print("Shape of padded image:"+str(padded_I.shape))

             #initialize filtered image with all zeros
             filtered_I = np.zeros(I.shape)
```

```
#scan padded image with a kernel window of w_len*w_len dimension
for id_r,row in enumerate(range(padding_len,padded_I.shape[0]-padding_len)):
    for id_c,col in enumerate(range(padding_len,padded_I.shape[1]-padding_len)):
        # grab values within kernel window and take median
        values = padded_I[row-padding_len:row+padding_len+1,\
                          col-padding_len:col+padding_len+1]

        # region contraction phase
        hij = np.var(values)/np.mean(values)
        h0wd = (a*values.size)/(b+values.size)
        sig0wd = c+k*np.exp(-1*d*(values.size))
        temp_len = padding_len

        # keep contracting until condition is violated
        while(hij>(h0wd+sig0wd) and values.size>S_min):
            temp_len = temp_len-1
            values = padded_I[row-temp_len:row+temp_len+1,\
                              col-temp_len:col+temp_len+1]
            hij = np.var(values)/np.mean(values)
            h0wd = (a*values.size)/(b+values.size)
            sig0wd = c+k*np.exp(-1*d*(values.size))

        # region growth phase
        hij = np.var(values)/np.mean(values)
        h0wd = (a*values.size)/(b+values.size)
        sig0wd = c+k*np.exp(-1*d*(values.size))
        hij_old = hij
        temp_len2 = temp_len

        # keep growing until condition is violated
        while(hij<=(h0wd+sig0wd) and values.size<w_len**2 \
              and np.abs(hij-hij_old)<sig0wd):
            hij_old = hij
            temp_len2 = temp_len2+1
            values = padded_I[row-temp_len2:row+temp_len2+1,\
                              col-temp_len2:col+temp_len2+1]
            hij = np.var(values)/np.mean(values)
            h0wd = (a*values.size)/(b+values.size)
            sig0wd = c+k*np.exp(-1*d*(values.size))

        #assign mean or median to filtered image
        if(values.size > w_len**2):
            filtered_I[id_r,id_c] = np.mean(values)
        else:
            filtered_I[id_r,id_c] = np.median(values)

if(plot_steps):
```

```
        IP.plt.subplot(1,3,3);
        IP.imshow(filtered_I,cmap='gray');IP.plt.title('Filtered Image')
        print("Shape of filtered image:"+str(filtered_I.shape))
    return filtered_I
```

6 Implementation of Geometric Filter (GF)

In [12]:
```
# this function checks if a 0 should be converted into a 1 according
# to the 8-hull template patterns

def check_hull8_pattern(block,mode='all'):
    pattern1 = np.array([[1,0,0],[1,0,0],[1,1,0]])
    pattern2 = np.array([[0,0,0],[1,0,0],[1,1,1]])
    pattern3 = np.array([[0,0,0],[0,0,1],[1,1,1]])
    pattern4 = np.array([[0,0,1],[0,0,1],[0,1,1]])
    patterns_a = [pattern1,pattern2,pattern3,pattern4]

    if(mode=='a'):
        for pattern in patterns_a:
            block_temp = block*pattern
            if(np.array_equal(block_temp, pattern)):
                # found match!
                return True

    pattern5 = np.array([[1,1,0],[1,0,0],[1,0,0]])
    pattern6 = np.array([[1,1,1],[1,0,0],[0,0,0]])
    pattern7 = np.array([[1,1,1],[0,0,1],[0,0,0]])
    pattern8 = np.array([[0,1,1],[0,0,1],[0,0,1]])
    patterns_b = [pattern5,pattern6,pattern7,pattern8]

    if(mode=='b'):
        for pattern in patterns_b:
            block_temp = block*pattern
            if(np.array_equal(block_temp, pattern)):
                # found match!
                return True

    patterns_all = patterns_a+patterns_b

    if(mode=='all'):
        for pattern in patterns_all:
            block_temp = block*pattern
            if(np.array_equal(block_temp, pattern)):
                # found match!
                return True

    # no match found
```

```
            return False
```

In [13]:
```
# implements 8-hull algorithm

def hull8(I,mode='all'):
    output = np.zeros((I.shape))
    output[0:1,:] = I[0,:]
    output[-1:,:] = I[-1,:]
    output[:,0:1] = I[:,0:1]
    output[:,-1:] = I[:,-1:]
    for row in range(1,I.shape[0]-1):
        for col in range(1,I.shape[1]-1):
            # if center pixel is already 1, we don't care
            if(I[row,col] == 0):
                neighborhood_block = I[row-1:row+2,col-1:col+2]
                if (check_hull8_pattern(neighborhood_block,mode)):
                    output[row,col] = 1
            else:
                output[row,col] = 1
    return output
```

In [14]:
```
# converts 0's to 1's and vice versa.
# Wonder if there was a better way though.

def complement_mat(i):
    return 1*(i<0.5)
```

In [15]:
```
# implements the complimentary hull algorithm
def complimentary_hull(I):
    I1 = hull8(I,'a')
    I1 = complement_mat(I1)
    I1 = hull8(I1,'b')
    I1 = complement_mat(I1)
    return I1
```

In [16]:
```
# from a line of pixels, generates a vertical pixel grid
def pixelrow2vertgrid(pixel_row):
    vertical_grid = np.zeros((256,len(pixel_row)),int)
    for idx,intensity in enumerate(pixel_row):
        vertical_grid[255-intensity:,idx] = 1
    return vertical_grid
```

In [17]:
```
# from a vertical pixel grid, goes back to line of pixels
def vertgrid2pixelrow(vertical_grid):
    return np.sum(vertical_grid,axis=0)-1
```

In [25]:
```
def GF(I):
    #E-W direction
    print('GF step: Beginning E-W direction')
```

```
temp=np.zeros(I.shape,int)
for row in range(I.shape[0]):
    # generate vertical pixel grid
    vertical_grid=pixelrow2vertgrid(list(I[row,:].\
                                         astype("int")))

    # run complimentary hull algorithm on the grid
    comp_output = complimentary_hull(vertical_grid)

    # convert updated grid back to a line of pixels
    updated_row = vertgrid2pixelrow(comp_output)

    # save the row
    temp[row,:] = updated_row

#N-S direction
print('GF step: Beginning N-S direction')
temp2=np.zeros(I.shape,int)
for col in range(I.shape[1]):
    # same as before
    vertical_grid=pixelrow2vertgrid(list(temp[:,col].\
                                         astype("int")))
    comp_output = complimentary_hull(vertical_grid)
    updated_row = vertgrid2pixelrow(comp_output)
    temp2[:,col] = updated_row

# SE-NW direction
print('GF step: Beginning SE-NW direction')
diags1 = [temp2.diagonal(i) for i in range(temp2.shape[1]-1,\
                                           -temp2.shape[0],-1)]

temp3=np.zeros(I.shape,int)
for idx, diag in enumerate(diags1):
    # same as before
    vertical_grid=pixelrow2vertgrid(list(np.round(diag).\
                                         astype("int")))
    comp_output = complimentary_hull(vertical_grid)
    updated_row = vertgrid2pixelrow(comp_output)
    if(idx<I.shape[0]):
        i = I.shape[0]-1-idx
        np.fill_diagonal(temp3[:,i:], updated_row)
    else:
        i = idx + 1-I.shape[0]
        np.fill_diagonal(temp3[i:], updated_row)

# SW-NE direction
print('GF step: Beginning SW-NE direction')
diags2 = [temp3[::-1,:].diagonal(i) for i in \
```

```
                    range(-temp3.shape[0]+1,temp3.shape[1])]

        temp4=np.zeros(I.shape,int)
        for idx, diag in enumerate(diags2):
            # same as before
            vertical_grid=pixelrow2vertgrid(list(np.round(diag).\
                                            astype("int")))
            comp_output = complimentary_hull(vertical_grid)
            updated_row = vertgrid2pixelrow(comp_output)
            row_opposite = updated_row[::-1]
            temp4=np.fliplr(temp4)
            if(idx<I.shape[0]):
                i = I.shape[0]-1-idx
                np.fill_diagonal(temp4[:,i:], row_opposite)
            else:
                i = idx + 1-I.shape[0]
                np.fill_diagonal(temp4[i:], row_opposite)
            temp4=np.fliplr(temp4)

        return temp4
```

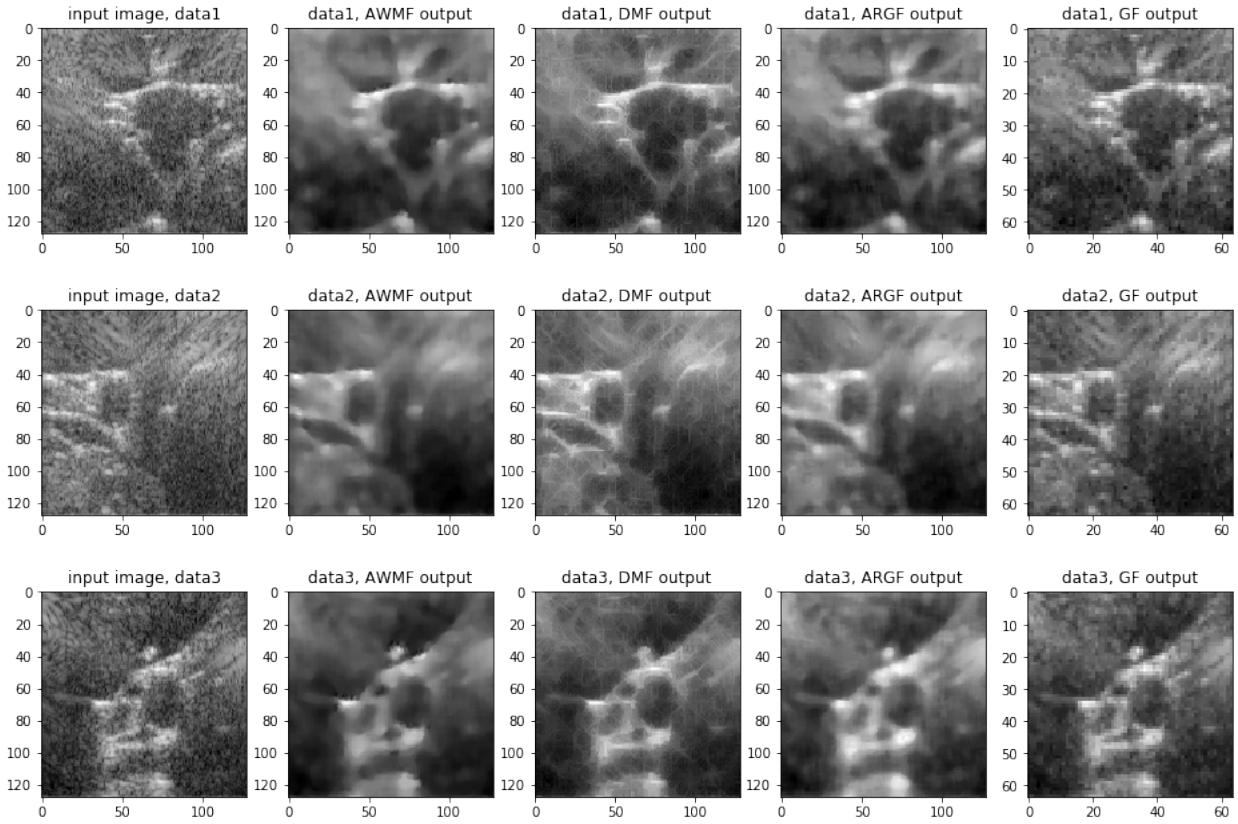
7 Output Comparison

```
In [26]: image_list = [data1_resized, data2_resized, data3_resized]
         output_dict = {1:[], 2:[], 3:[]}

         for idx, im in enumerate(image_list):
             print("Applying AWMF to data"+str(idx+1))
             data_AWMF = AWMF(im,w_len=9,w0=10,c=0.25,plot_steps=False)
             output_dict[idx+1].append(data_AWMF)

             print("Applying DMF to data"+str(idx+1))
             data_directional = DMF(im,w_len=9,plot_steps=False)
             output_dict[idx+1].append(data_directional)

             print("Applying ARGF to data"+str(idx+1))
             data_ARGF = ARGF(im,w_len=11,a=2.,b=19.5,c=0.3,\
                              d=0.004,k=0.5,S_min=49,plot_steps=False)
             output_dict[idx+1].append(data_ARGF)

             print("Applying GF to data"+str(idx+1))
             im_resized = imresize(im,(64,64),anti_aliasing=True)
             im_resized = im_resized-im_resized.min()
             im_resized = (im_resized/im_resized.max()*255).astype("int");
             data_geom = GF(im_resized)
             output_dict[idx+1].append(data_geom)
```

```
Applying AWMF to data1
```

```
Applying DMF to data1
Applying ARGF to data1
Applying GF to data1
GF step: Beginning E-W direction
GF step: Beginning N-S direction
GF step: Beginning SE-NW direction
GF step: Beginning SW-NE direction
Applying AWMF to data2
Applying DMF to data2
Applying ARGF to data2
Applying GF to data2
GF step: Beginning E-W direction
GF step: Beginning N-S direction
GF step: Beginning SE-NW direction
GF step: Beginning SW-NE direction
Applying AWMF to data3
Applying DMF to data3
Applying ARGF to data3
Applying GF to data3
GF step: Beginning E-W direction
GF step: Beginning N-S direction
GF step: Beginning SE-NW direction
GF step: Beginning SW-NE direction
```

In [30]:
```
# plotting outputs
IP.plt.figure(figsize=(16,11))

for idx, im in enumerate(image_list):
    IP.plt.subplot(3,5,5*idx+1)
    IP.imshow(im,cmap='gray')
    IP.plt.title('input image, data'+str(idx+1))

    IP.plt.subplot(3,5,5*idx+2)
    IP.imshow(output_dict[idx+1][0],cmap='gray')
    IP.plt.title('data'+str(idx+1)+', AWMF output')

    IP.plt.subplot(3,5,5*idx+3)
    IP.imshow(output_dict[idx+1][1],cmap='gray')
    IP.plt.title('data'+str(idx+1)+', DMF output')

    IP.plt.subplot(3,5,5*idx+4)
    IP.imshow(output_dict[idx+1][2],cmap='gray')
    IP.plt.title('data'+str(idx+1)+', ARGF output')

    IP.plt.subplot(3,5,5*idx+5)
    IP.imshow(output_dict[idx+1][3],cmap='gray')
    IP.plt.title('data'+str(idx+1)+', GF output')
```



As can be seen from the results above, the AWMF does a reasonable job of removing noises and bringing out the underlying structures vividly. However, the images are blurred. The same goes for ARGF–the blurriness is quite visible despite the decent noise reduction. In my qualitative judgment, the DMF outputs are the best. They preserve speckle textures better than all the other algorithms I have implemented. The noise is visibly reduced to a very good degree. Unfortunately, the geometric filter did a very poor job. Although it achieved some smoothing by flattening out the narrower valleys and towers, it made no difference of value. As explained before, one of the reasons behind this is the 3×3 pattern template used by the 8-hull algorithm, which does not work well when noise towers/valleys are large in size.

References

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[3] Richard N Czerwinski, Douglas L Jones, and William D O'Brien. Ultrasound speckle reduction by directional median filtering. In *Proceedings., International Conference on Image Processing*, volume 1, pages 358–361. IEEE, 1995.

[4] Thomas R Crimmins. Geometric filter for speckle reduction. *Applied optics*, 24(10):1438–1443, 1985.