DEPARTMENT OF PETROLEUM ENGINEERING
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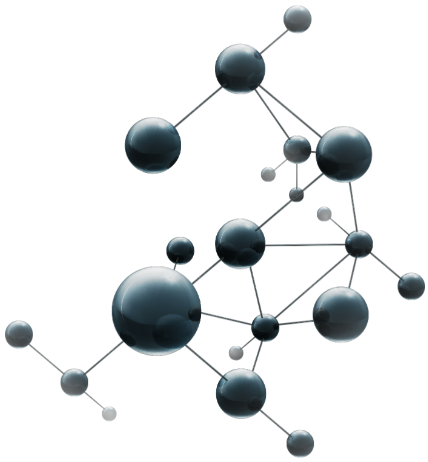
# Application of Buckley-Leverett and Fraction Flow Theory in Water Flooding & Polymer Flooding Analysis



*A Project Report*

*Submitted by*

**SHUBHAM DOBLIYAL**

*For the award of the degree*

*Of*

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# PROJECT CERTIFICATE

This is to undertake that the Project Report titled **APPLICATION OF BUCKLEY-LEVERETT AND FRACTION FLOW THEORY IN WATER FLOODING & POLYMER FLOODING ANALYSIS**, submitted by me to the Indian Institute of Technology (Indian School of Mines) Dhanbad, for the award of **Bachelor of Technology**, is a bona fide record of the research work done by me under the supervision of **Prof. Ajay Suri**. The contents of this Project Report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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# ABSTRACT

**KEYWORDS** Polymer flood; Water flood; Fractional Flow Theory; Buckley-Leverett Theory

The importance of oil in determining the world's economy and technological advancement is paramount. Therefore, it is imperative to invest sufficient time in operations or techniques targeted to improve the ultimate recovery of the field. In this project, a critical review of Polymer flooding and Water flooding is performed by employing various mathematical tools such as Multi-phase Conservation Equation, Fractional Flow theory, Buckley-Leverett Analysis in order to understand the process, which can then be utilised to further improvise the techniques available at hand . Fractional flow curves, Saturation profiles is plotted, and Recovery calculations are achieved by using various assumptions and models to perform a comparative study of the polymer flood with that of the waterflood. Finally, various factors affecting polymer flooding are discussed.

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# GLOSSARY

| | |
|---|---|
| **Adsorption** | The inherent property of any phase to attract any other different phase to their surfaces. |
| **ASP flooding** | A chemical enhanced oil recovery flooding operation in which an aqueous solution containing alkali, surfactant and polymer are simultaneously injected into the reservoir. the alkali reacts with the acidic components of oil to form an in-situ surfactant. This and the synthetic surfactant injected leads to reduction of inter-facial tension which leads to reduced residual oil saturation. While the polymer enhances the sweep efficiency of the system by improving the mobility ratio favourably. |
| **Breakthrough** | A condition of the reservoir in which the previously isolated fluid gains access to the the production well through different means and starts producing. |
| **Conformance** | A measure of the areal and vertical uniformity of displacement front as it transmits through the reservoir displacing the oil ahead of it. |
| **Dispersion** | It is the phenomenon in which the breakdown of agglomerates happens due to the shear or agitation forces. |
| **Displacement front** | The boundary or the interface between the displaced and the displacing fluid. |
| **Enhanced Oil Recovery** | A recovery process in which the original propertise of the oil are altered by injection of foreign chemical into the reservoir. It aims to improve the sweep efficiency of the reservoir and also decrease the residual oil saturation. |
| **Estimated Ultimate Recovery** | It refers to the amount of hydrocarbon fluid that can be recovered within its financial feasibility throughout the life of a well, a field, or a basin. |
| **Fingering** | A condition in which the displacing fluid bypasses sections of displaced fluid, leading to creation of a non-uniform fingered profile at the interface. |

| | |
|---|---|
| **Imbibition** | It is the process in which the wetting phase gets adsorbed into the rock surface leading to alteration in wettability of the reservoir. |
| **Improved oil recovery** | Any form of activity that leads to an increment on the oil production and ultimate recovery of the reservoir.Some examples are horizontal or multilateral drilling, infill drilling, enhanced oil recovery methods, hydraulic fracturing, etc. |
| **Infill drilling** | It refers to the process in which additional wells are drilled to improve the continuity and communication between injectors and producers in case of heterogeneous reservoirs. |
| **Interfacial tension** | It is the energy per unit area associated with interface between two immiscible liquid phases at a fixed temperature and pressure conditions. |
| **Mobility** | It is ratio of the effective or the relative permeability of a phase to that of its viscosity. The productivity of a well is directly related to the product of layer thickness and the mobility of reservoir. |
| **Mobility ratio** | It is the ratio of the mobility of the displacing fluid to that of the displaced fluid. |
| **Polyacrylamide** | A high molecular weight polymer formed after the polymerisation of acrylamide monomers. It is used to enhance the viscosity of the water, and hence, are used as mobility-control tool in various polymer flooding processes. |
| **Polymer flooding** | An enhanced oil recovery technique in which water containing some viscosifiers such as xanthan and partially hydrolyzed polyacrylamide, are injected into the reservoir to improve the sweep efficiency of the flood-front, thus, helps i reducing viscous fingering related conformance problems. |
| **Recovery factor** | It is ratio of oil recovered to that of oil initially in place. |
| **Reservoir heterogeneities** | It refers to the variations and non-uniformity in different rock properties like permeability in the reservoir caused due to geological processes, such as diagenesis, sedimentation, erosion, etc. |
| **Residual oil** | Oil that is trapped into the reservoir and cannot be recovered using primary or secondary recovery stages. |

**Secondary recovery** The recovery stage in which either water or gas is injected into the reservoir in order to maintain the reservoir pressure and to displace the hydrocarbons towards the production well.

**Sweep efficiency** It is the measure of the efficacy of an enhanced oil recovery technique related to the portion of reservoir directly in contact by the injection fluid. It is dependent on several factors such as thickness of reservoir, its heterogeneity and the presence of fractures in it, the pattern of flooding used, density and viscosity difference between the displacing and the displaced fluid, flow rate and the position of oil-water and gas-oil contacts.

**Tertiary recovery** Recovery stage in which the enhanced oil recovery methods are used to further improve the ultimate recovery. Chemical, thermal or the miscible flooding are a few of the examples of the recovery process.

**Waterflood** A secondary recovery process in which the water is injected into the reservoir to maintain the pressure of the reservoir leading to improvement in the recovery of the field.

**Wettability** The ability of solid rock surface to prefer one liquid or gas over the other.

# ABBREVIATIONS

| | |
|---|---|
| **1D** | One Dimension. |
| **3D** | Three Dimension. |
| **ASP** | Alkaline Surfactant Polymer. |
| **ATBS** | Acrylamide Tertiary Butyl Sulphonate. |
| **BL** | Buckley-Leverett. |
| **CEOR** | Chemical Enhanced Oil Recovery. |
| **EOR** | Enhanced Oil Recovery. |
| **EUR** | Estimated Ultimate Recovery. |
| **HAP** | Hydrophobically Associated Polymer. |
| **HPAM** | Partially Hydrolysed Polyacrylamide. |
| **IFT** | Inter-facial Tension. |
| **IOR** | Improved Oil Recovery. |
| **OIIP** | Oil initially in place. |
| **PV** | Pore Volume. |
| **PVT** | Pressure Volume Temperature. |
| **RF** | Recovery Factor. |
| **SAGD** | Steam Assisted Gravity Drainage. |
| **WOR** | Water-Oil Ratio. |

# NOTATION

| | |
|---|---|
| $\bar{S_w}$ | Average water saturation |
| $\Delta f_w$ | Change in fractional flow |
| $\Delta S_w$ | Change in water saturation |
| $\Delta t$ | Change in time |
| $\lambda_o$ | Mobility of oil |
| $\lambda_t$ | Total mobility |
| $\lambda_w$ | Mobility of water |
| $\pi$ | Ratio of the circumference of a circle to its diameter |
| $\rho_o$ | Density of oil, $M/L^3$, $kg/m^3$ |
| $\rho_w$ | Density of water, $M/L^3$, $kg/m^3$ |
| $\varphi$ | Porosity pf reservoir |
| $A$ | Area, $L^2$, $m^2$ |
| $a_L$ | Empirical constant in the Langmuir equation |
| $b_L$ | Empirical constant in the Langmuir equation, unit of the inverse of concentration |
| $B_w$ | Water formation volume factor, $L^3/L^3$ |
| $C_a$ | Adsorbed solute i concentration, various units |
| $C_i$ | Concentration of fluid species i, $m/L^3$, $meq/mL$, $mol/L$ |
| $D_i$ | General retardation term, fraction |

$f_w$ Water cut, $L_3/L_3$, %, fraction

$f_{w_{bank}}$ Water cut of the oil bank, $L_3/L_3$, %, fraction

$g$ Acceleration due to gravity, $L/t^2$, $m/s^2$

$IFT$ Interfacial tension, $dynes/cm$, $mN/m$

$k$ Permeability, $L^2$, $mD$

$k_{ro^\circ}$ Endpoint relative permeabiltity of oil, $L^2$, $mD$

$k_{ro}$ Relative permeabiltity of oil, $L^2$, $mD$

$k_{rw^\circ}$ Endpoint relative permeabiltity of water, $L^2$, $mD$

$k_{rw}$ Relative permeabiltity of water, $L^2$, $mD$

$L$ Length of the reservoir, $L$, $m$

$M$ Mobility ratio

$n_o$ Exponent of relative permeability curve of oil phase

$n_w$ Exponent of relative permeability curve of water phase

$N_{PD}$ Pore volume of oil produced, $L^3/L^3$

$P_c$ Capillary pressure, $psi$

$P_o$ Pressure of non-wetting or oil phase, $psi$

$P_w$ Pressure of wetting or water phase, $psi$

$q_o$ Volumetric flow of oil per unit time, $L^3/T$, $bpd$, $ft^3/d$

$q_t$ Total volumetric flow of fluid per unit time, $L^3/T$, $bpd$, $ft^3/d$

$q_w$ Volumetric flow of water per unit time, $L^3/T$, $bpd$, $ft^3/d$

$S_o$ Saturation of oil phase

$S_w$ Saturation of water phase

$S_{orw}$ Residual oil saturation to flowing water

$S_{w_{bank}}$ Saturation of water phase in the oil bank

$S_{w_{bt}}$ Breakthrough saturation of water phase

$S_{wc}$ Connate water saturation

$S_{wi}$ Initial water saturation

$S_{wr}$ Residual saturation of water phase

$t$ Time, $days$, $sec$

$t_D$ Dimensionless time

$t_{bt}$ Breakthorugh time, $days$, $sec$

$v$ Velocity of a phase, $L^2/T$, $ft^2/s$

$v_D$ Dimensionless velocity of a phase

$v_s$ Shock velocity of a phase, $L^2/T$, $ft^2/s$

$v_{C_i}$ Dimensionless velocity of constant saturation front or the specific velocity

$v_{D_s}$ Dimensionless shock velocity of a phase

$v_{w2}$ specific velocity of the solution if no chemical retention exists

$v_{w3}$ specific velocity of the injected chemical $C_i$ with retention $C_a$

$W_{ID_{bt}}$ Pore volume of water injected at breakthrough, $L^3/L^3$

$W_{ID}$ Pore volume of water injected, $L^3/L^3$

$x$ Distance, $L$, $ft$

$x_D$ Dimensionless distance

$x_{D_{bank}}$ Dimensionless distance at which the oil bank starts

$x_{D_{bt}}$ Dimensionless distance at breakthrough

# CHAPTER 1

# INTRODUCTION

In today's scenario, the significance of oil in impacting the world's economy is substantial and techniques for recovering the shall continue to be the matter of technical research and engineering innovation in the coming years. It is alarming to note that as much as 70 % of the original oil content may remain in the reservoir even after deploying primary depletion and secondary depletion processes. Therefore it is eminent to develop, improved and enhanced methods of oil recovery focused on recovering some portions of the remaining that was bypassed.

The methods applied to extract this bypassed oil is broadly termed as Improved Oil Recovery (IOR). IOR is basically any activity done in order to improvise the ultimate recovery of the field. The process of IOR involves two mechanisms, one where the production part of the petroleum extraction process is modified without directly disturbing the reservoir, for example, infill drilling, hydraulic fracturing, horizontal and multilateral drilling , etc. The other one is where a foreign substance is introduced into the formation reservoir in order to change the properties of the fluid to be produced or the reservoir rock, in a favourable manner, which in turn, leads to improved recovery of the field. These methods are generally termed as Enhanced Oil Recovery (EOR) or tertiary methods. Moreover, these classification methods differ from one another in terms of the target oil to be recovered or extracted from the field. For example, in IOR, the aim is to increase the sweep efficiency of the oil, while in tertiary recovery, the focus is on mobilising and recovering capillary or the residual oil trapped in the reservoir.

Enhanced oil recovery operations are mainly of three types: chemical flooding (micellar-polymer flooding or ASP flooding), thermal flooding processes (in-situ combustion, SAGD or steam flooding), miscible flooding or displacement process (hydrocarbon or carbon dioxide [CO2] injection). The optimal selection and application of each method depends a wide range of criterion that needs to be considered. Rerservoir properties such as net pay, reservoir pressure, reservoir temperature, porosity and permeability, depth, water & residual oil saturations, wettability and fluid properties such as viscosity, API or specific gravity and composition are few of the factors that plays vital role in determining the EOR technique to be applied.

# CHAPTER 2

# CONCEPTS AND DERIVATONS

## 2.1 MULTIPHASE CONSERVATION EQUATION

In a one dimensional system, in accordance with the law of conservation of mass and energy, the mass of a given system will always remain constant. Henceforth, the difference in the mass of a phase entering an volumetric element with that of mass of the phase exiting the volumetric element will be equal to the mass accumulated in the system.

The mathematical equation for this is as follows:

$$\varphi A \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0$$

### 2.1.1 Derivation

For 1D flow,

At time $t$, Mass in box = $A \Delta x \varphi \; S_w|_t \, \rho_w$

At time $t + \Delta t$, Mass in box = $A \Delta \varphi \; S_w|_{t+\Delta t} \, \rho_w$

Mass accumulated =

$$\Delta x \varphi \rho_w [\, S_w|_{t+\Delta t} - S_w|_t] \tag{2.1}$$

Mass that enters in time $\Delta t = q_w|_x \, \Delta t \rho_w$

Mass that enters in time $\Delta t = q_w|_{x+\Delta x} \, \Delta t \rho_w$

Change in mass =

$$\Delta t \rho_w [\, q_w|_x - q_w|_{x+\Delta x}] \tag{2.2}$$

Equation (2.1) = Equation (2.2) :-

$$\Longrightarrow \Delta x \varphi \rho_w [S_w|_{t+\Delta t} - S_w|_t] = \Delta t \rho_w [q_w|_x - q_w|_{x+\Delta x}]$$
$$\Longrightarrow \varphi A \frac{(S_w|_{t+\Delta t} - S_w|_t)}{\Delta t} + \frac{q_w|_x - q_w|_{x+\Delta x}}{\Delta x} = 0$$

Using limits $\Delta x \longrightarrow 0$ and $\Delta t \longrightarrow 0$

$$\varphi A \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0 \tag{2.3}$$

Equation 2.3 is the final mass conversation equation for a phase in one dimension.

For 3D flow, the equation will be converted to

$$\varphi A \frac{\partial S_w}{\partial t} + \vec{\boldsymbol{\nabla}} q_w = 0 \tag{2.4}$$

## 2.2 FRACTIONAL FLOW THEORY

Fractional flow theory has been a very important tool with wide applications in understanding and validation of various reservoir simulation and numerical models. It still has been applied in understanding the mechanisms of various Chemical-Enhanced Oil Recovery (CEOR) process and for the purposes of interpreting the transport behaviour of various chemicals in porous and permeable media.

For a 1D, incompressible , two phase flow, the fractional flow equation is stated as follows :-

$$q_w = \frac{\lambda_w}{\lambda_t} q_t + k \frac{\lambda_W \lambda_o}{\lambda_t} \frac{\partial P_c}{\partial x} + k \frac{\lambda_W \lambda_o}{\lambda_t} (\rho_w - \rho_o) g_x$$

First term is the advection term which basically symbolizes the flow rate that is going with the flow. The second term is the flow rate of water due to capillary pressure effects and third one is due to gravity or buoyancy effects.

### 2.2.1 Derivation

Before we start the derivation we have to define a few concepts:-

Darcy's law (Momentum balance): $q_w = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial P_w}{\partial x} - \rho_w g_x\right)$

Mobility: $\lambda_w = \frac{k_{rw}}{\mu_w}$

Total Mobility: $\lambda_t = \lambda_w + \lambda_o$

Total flow rate: $q_t = q_w + q_o$

Total saturation: $S_o + S_w = 1$

Now, from multiphase conservation theory,

$$\varphi A \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0 \tag{2.5}$$

$$\varphi A \frac{\partial S_o}{\partial t} + \frac{\partial q_o}{\partial x} = 0 \tag{2.6}$$

Adding equations (2.5) and (2.6), we get

$$\implies \varphi A \frac{\partial (S_w + S_o)}{\partial t} + \frac{\partial q_t}{\partial x} = 0$$

$$\implies \frac{\partial q_t}{\partial x} = 0$$

This implies that $q_t$ is a function that is displacement independent i.e., it is a function of time only. In 3D flow, we can write it as

$$\vec{\nabla} q_t = 0$$

Now, coming back to 1D flow, Darcy's law is applied to the get the total flow rate as follows:

$$\implies q_t = -k\lambda_w \frac{\partial P_w}{\partial x} + k\lambda_w \rho_w g_x - k\lambda_o \frac{\partial P_o}{\partial x} + k\lambda_o \rho_o g_x$$

Since, we know that, capillary pressure is the pressure differential between non-wetting and wetting phase, i.e., $P_c = P_o - P_w$

Therefore, the previous equation now becomes,

$$\implies q_t = -k\lambda_w \frac{\partial P_w}{\partial x} + k\lambda_w \rho_w g_x - [k\lambda_o \frac{\partial P_c}{\partial x} + k\lambda_o \frac{\partial P_w}{\partial x}] + k\lambda_o \rho_o g_x$$

$$\implies q_t = -k\lambda_t \frac{\partial P_w}{\partial x} + k\lambda_w \rho_w g_x - k\lambda_o \frac{\partial P_c}{\partial x} + k\lambda_o \rho_o g_x$$

$$\implies -k\frac{\partial P_w}{\partial x}\lambda_w = \frac{\lambda_w}{\lambda_t}\left[q_t + k\lambda_o \frac{\partial P_c}{\partial x} - k\lambda_w \rho_w g_x - k\lambda_o \rho_o g_x\right]$$

In accordance with Darcy's Law, $q_w = -k\lambda_w \frac{\partial P_w}{\partial x}$

The above equation becomes,

$$\implies q_w = \frac{\lambda_w}{\lambda_t} q_t + k\frac{\lambda_w \lambda_o}{\lambda_t}\frac{\partial P_c}{\partial x} - k\frac{\lambda_w^2}{\lambda_t}\rho_w g_x - k\frac{\lambda_w \lambda_o}{\lambda_t}\rho_o g_x + k\frac{\lambda_w(\lambda_w + \lambda_o)}{\lambda_t}\rho_w g_x$$

Finally, the fractional flow equation turns out to be :-

$$q_w = \frac{\lambda_w}{\lambda_t} q_t + k\frac{\lambda_W \lambda_o}{\lambda_t}\frac{\partial P_c}{\partial x} + k\frac{\lambda_W \lambda_o}{\lambda_t}(\rho_w - \rho_o) g_x \tag{2.7}$$

### 2.2.2 Waterflooding and Spontaneous Imbibition

Waterflooding is secondary recovery process, where water from different sources is injected into formation to maintain reservoir pressure and to displace the oil that has been bypassed during primary recovery. With respect to fractional flow, the advection term dominated here while the gravity and capillary terms are neglected. Matematically,

$$q_w = \frac{\lambda_w}{\lambda_t} q_t$$

Now, fractional flow of water is defined as the ration of flow rate of water with respect to the total flow rate. Therefore,

$$f_w = \frac{q_w}{q_t} = \frac{\lambda_w}{\lambda_t} = \frac{\lambda_w}{\lambda_w + \lambda_o} = \frac{1}{1 + \frac{\lambda_o}{\lambda_w}}$$

$$\implies f_w = \frac{1}{1 + \frac{k_{ro}\mu_w}{k_{rw}\mu_o}} = \frac{1}{1 + \frac{1}{M}} \tag{2.8}$$

where M = Mobility Ratio = $\frac{\text{Mobility of displacing fluid}}{\text{Mobility of displaced fluid}} = \frac{k_{rw}\mu_o}{k_{ro}\mu_w}$

Spontaneous imbitition, on the other hand, is microscopic displacement of oil from the capillary tubes of the reservoir by water. Here, the advection term is equal to zero as the darcy flow rate is assumed to be zero and the gravity term is neglected, while the capillary term dominates the fractional flow equation. Spontaneous imbitition is a non-linear function of saturation and saturation gradient, and hence, is very difficult to calculate. Mathematically,

$$q_w = k\frac{\lambda_W\lambda_o}{\lambda_t}\frac{\partial P_c}{\partial x} = k\frac{\lambda_W\lambda_o}{\lambda_t}\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x} \tag{2.9}$$

## 2.3 BUCKLEY-LEVERETT THEORY

Buckley- Leverett Theory is widely used for the evaluating the movement of a fluid displacing front for an immiscible displacement process in a porous media. The theory is based on the fractional flow theory and made use of the following assumptions to estimate the rate of injected fluid bank movement :-

- Linear and horizontal 1D flow
- Water is used as the injected fluid in the oil reservoir
- Both oil and water are incompressible in nature
- Both oil and water are immiscible with one another

- Capillary and gravity pressure effects are neglected

Mathematically, using Buckley Leverett theory, we calculate the velocity of the constant saturation front by applying the multiphase conservation and fractional flow theory.

### 2.3.1 Front Velocity Calculations

From multiphase conservation equation (2.3), we have:

$$\varphi A \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0$$

Fractional flow concept, we know that:

$$q_w = f_w * q_t$$

Putting this in the equation (2.3), we get:

$$\varphi A \frac{\partial S_w}{\partial t} + q_t \frac{\partial f_w}{\partial x} = 0$$

$$\varphi A \frac{\partial S_w}{\partial t} + q_t \frac{df_w}{dS_w} \frac{\partial S_w}{\partial x} = 0 \tag{2.10}$$

Now we define the initial and boundary conditions. At $t = 0,\ x > 0,\ S_w = S_{wi}$ and at $x = 0,\ t > 0,\ S_w = 1 - S_{or}$. We also define two terms,

$$x_D = \frac{x}{L} \implies 0 \leq x_D \leq 1$$

$$t_D = \text{Pore volume injected} = \frac{\int_0^t q_t(t)\,dt}{\phi AL} = \frac{q_t t}{\varphi AL}$$

Therefore, we get,

$$\frac{\partial S_w}{\partial x} = \frac{\partial S_w}{\partial x_D}\frac{\partial x_D}{\partial x} = \frac{\partial S_w}{\partial x_D}\frac{1}{L} \tag{2.11}$$

$$\frac{\partial S_w}{\partial t} = \frac{\partial S_w}{\partial t_D}\frac{\partial t_D}{\partial t} = \frac{\partial S_w}{\partial t_D}\frac{qt}{\varphi AL} \tag{2.12}$$

Using equations (2.10), (2.11) and (2.12), we get,

$$\frac{\partial S_w}{\partial t_D} + \frac{df_w}{dS_w}\frac{\partial S_w}{\partial x_D} = 0 \tag{2.13}$$

As we know, velocity is,

$$v = \frac{x}{t} = \frac{q_t}{A\varphi}\frac{x_D}{t_D}$$

and the dimensionless velocity is,

$$v_D = \frac{x_D}{t_D}$$

Therefore, we get,

$$\frac{\partial S_w}{\partial x_D} = \frac{\partial S_w}{\partial v_D}\left.\frac{\partial v_D}{\partial x_D}\right|_{t_D} = \frac{1}{t_D}\frac{\partial S_w}{\partial v_D} \tag{2.14}$$

$$\frac{\partial S_w}{\partial t_D} = \frac{\partial S_w}{\partial v_D}\left.\frac{\partial v_D}{\partial t_D}\right|_{x_D} = -\frac{x_D}{t_D^2}\frac{\partial S_w}{\partial v_D} = -\frac{v_D}{t_D}\frac{\partial S_w}{\partial v_D} \tag{2.15}$$

Now, combining the equations (2.13), (2.14) and (2.15), we get,

$$\frac{dS_w}{dv_D}\left[-\frac{v_D}{t_D} + \frac{df_w}{dS_w}\frac{1}{t_D}\right] = 0 \tag{2.16}$$

As we can see equation (2.16) has two solutions:

1. $\frac{dS_w}{dv_D} = 0$, this is a trivial solution whose answer is $S_w = constant$.
2. $\left[-\frac{v_D}{t_D} + \frac{df_w}{dS_w}\frac{1}{t_D}\right] = 0$. This is called a rarefraction and answer in this case is $v_D = \frac{df_w}{dS_w}$.

### 2.3.2 Shocks in multiphase flow in porous media

A Shock is a discontinuity in saturation that propagates throughout the reservoir rock, displacing the fluid ahead of it. Here, we do the mass balance of an elemental volume to find the velocity of the shock front.

**Derivation**

Mass in = $q_t \Delta t f_w^L \rho_w$

Mass out = $q_t \Delta t f_w^R \rho_w$

Change in mass = $(S_w^L - S_w^R)\varphi A v_s \Delta t \rho_w$

Now, as (Mass in - Mass out) = Change in mass, therefore,

$$\implies v_s = \frac{q_t(f_w^L - f_w^R)}{\varphi A(S_w^L - S_w^R)} = \frac{q_t}{A\varphi}\frac{\Delta f_W}{\Delta S_w} \tag{2.17}$$

$$\implies v_{Ds} = \frac{\Delta f_W}{\Delta S_w} \tag{2.18}$$

### 2.3.3 Recovery Calculations

One of the major applications of the Buckley-Leverett theory is to estimate the recovery of a reservoir before and after a CEOR process. Using the Buckley-Leverett theory, the pore volumes of oil produced with respect to the pore volume of water (or any other fluid) can be calculated and plotted before and after the breakthrough of the flood-front.

Now we define,

$N_{PD}$ = Pore volumes of oil produced

$t_D \ or \ W_{ID}$ = Pore volumes of water injected

$N_{PD}^{Max}$ = Maximum recovery = $(1 - S_{or}) - S_{wi}$

**Case I: Before Breakthrough**

In this case, since the flood-front hasn't reached the production well yet, therefore, only oil is getting produced. Therefore, the amount of pore volume of oil produced is equal to the amount of pore volume of water injected, i.e., $N_{PD} = t_D = \dfrac{x_D}{v_D} = \dfrac{1}{\frac{df_w}{dS_w}}$.

**Case II: After Breakthrough**

After the breakthrough, the water saturation and fractional flow of water in the production well will slowly increase. But the total reservoir length remains same. Therefore, we take the average of the total water saturation present before the producing well ($\bar{S_w}$). Now, the pore volume of oil produced equals,

$$N_{PD} = \bar{S_w} - S_{wi} \tag{2.19}$$

where,

$$\bar{S_w} = \int_0^1 S_w \, dx_D = [x_D S_w]_0^1 - \int x_D \frac{dS_w}{dx_D} \, dx_D = [x_D S_w]_0^1 - \int_1^{f_w^1} t_D \, df_w$$

$$\bar{S_w} = S_w^1 + t_D \left[1 - f_w^1\right] \tag{2.20}$$

Putting the value of $\bar{S_w}$ from equation (2.20) into equation (2.19), we get the amount of pore volume of oil produced to be,

$$N_{PD} = \left(S_w^1 + t_D \left[1 - f_w^1\right]\right) - S_{wi} \tag{2.21}$$

# CHAPTER 3

# FRACTIONAL FLOW, SATURATION AND RECOVERY PROFILES

## 3.1 WATER FLOOD

Waterflood is a secondary enhanced oil recovery process used to maintain the reservoir pressure, thus providing energy to the reservoir and improving its recovery by displacing the previously bypassed or the residual oil. Reservoir heterogeneities in reservoirs in the form of channels and fractures leads to decrease in the efficiency of the waterflood and can cause early breakthrough of water in the production wells.

In this project, the relative permeability curve, fractional flow curve, saturation profile and the recovery plot were plotted in MS-Excel using the concepts mentioned in the previous chapter.

### 3.1.1 Assumptions

In order to plot the above mentioned curves some assumption were made. Those assumptions are as follows:

- Two phase flow
- 1D flow
- No capillary pressure or gravity
- Incompressible fluids
- No source/sinks

### 3.1.2 Initial Condition:

The initial/interstitial water saturation is equal to the residual water saturation.

$$S_{wi} = S_{wr} = S_{wc}$$

### 3.1.3 Boundary Conditions:

Constant injection and production rates at $x = 0$ and $x = L$ respectively.

### 3.1.4 Profiles Construction Mechanism

Initially , different parameters were assumed to be given, according to which different plots were made. By varying these parameters one could see how the curve would change in response to it. Given below is the table of the parameters that were assumed for further calculations.

Table 3.1: Assumed Parameters for Waterflood

| **Parameters** | **Values** |
|---|---|
| $S_{wc}$ | 0.1 |
| $S_{orw}$ | 0.2 |
| $k_{rw^\circ}$ | 0.6 |
| $k_{ro^\circ}$ | 0.92 |
| $n_w$ | 2.5 |
| $n_o$ | 6 |
| $\mu_w$ | 0.5 $cp$ |
| $\mu_o$ | 17 $cp$ |
| $A$ | 1000000 $ft^2$ |
| $L$ | 100 $ft$ |
| $\varphi$ | 0.2 |
| $q_t$ | 20000 $ft^3/d$ |
| $t$ | 100 $days$ |

Now, values of water saturation was taken from connate water saturation i.e., 0.1 to 0.6 with the assumption that each day there is an increment in water saturation by 0.01. For each saturation, relative permeability to water and oil were calculated by using Corey's Model. The formulae for the Corey's Model is as follows:

$$k_{rw} = k_{rw^\circ} \left( \frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right)^{n_w} \tag{3.1}$$

$$k_{ro} = k_{ro^\circ} \left( \frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right)^{n_o} \tag{3.2}$$

Now,the relative permeability curve ($k_{rw}$ or $k_{ro}$ vs $S_w$ curve) was plotted in the MS-Excel was shown in the figure 3.1.

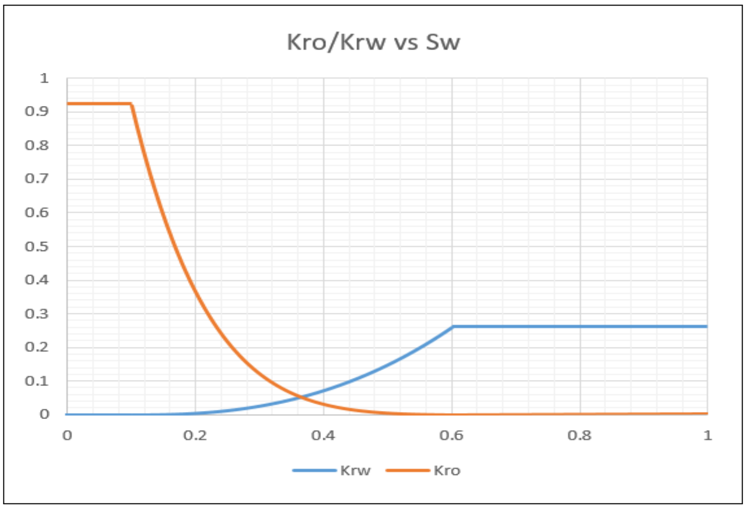


Figure 3.1: Relative Permeability Curves for Waterflood

After this, the mobility ratio followed by the fractional flow ($f_w$) was calculated using the equation (2.8). With the fractional flow curve ($f_w$ vs $S_w$ curve) was plotted in the MS-Excel as shown in figure 3.2.

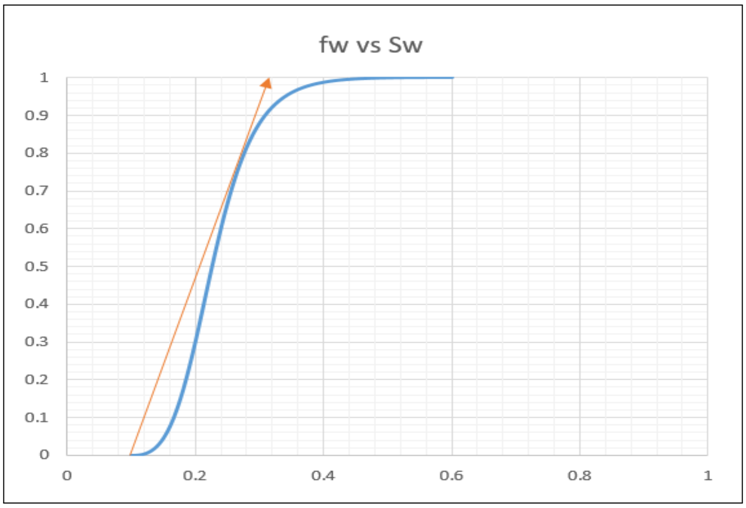


Figure 3.2: Fractional Flow Curve for Waterflood

Now, the values of $\frac{\partial f_w}{\partial S_w}$ and $\frac{\Delta f_w}{\Delta S_w}$ were calculated. Breakthrough point is the point at which the value of the difference between the $\frac{\Delta f_w}{\Delta S_w}$ and $\frac{\partial f_w}{\partial S_w}$ is minimum (equal to zero). This can also be shown from the fractional flow curve. In fig 3.2, the point at which the tangent to the curve from the connate water saturation intersects the curve is called the breakthrough point. From this the breakthrough saturation was known either using the fractional flow curve or by using 'vlookup' in MS-Excel for the minimum difference. In this case, the breakthrough saturation $S_{wbt} = 0.271$.

The pore volume of water injected was calculated. After the breakthrough the value of $W_{ID}$ was found.

$$t_D \text{ or } W_{ID} = \frac{1}{\left.\frac{df_w}{dS_w}\right|_{S_w}} \tag{3.3}$$

Before breakthrough, the value of $W_{ID}$ was normalised using the amount of pore volume of water injected at the breakthrough $W_{ID_{bt}} = \frac{1}{\left.\frac{df_w}{dS_w}\right|_{S_{bt}}}$.

$$t_D \text{ or } W_{ID} = W_{ID_{bt}} * \frac{t}{t_{bt}} \tag{3.4}$$

From the Buckley-Leverett theory, it is known that:

$$x_D = t_D * v_D = \frac{q_t t}{A\varphi L} \left.\frac{df_w}{dS_w}\right|_{S_w} \tag{3.5}$$

The above equation was applied to find and plot the saturation profile at time = 100 days (fig 3.3).

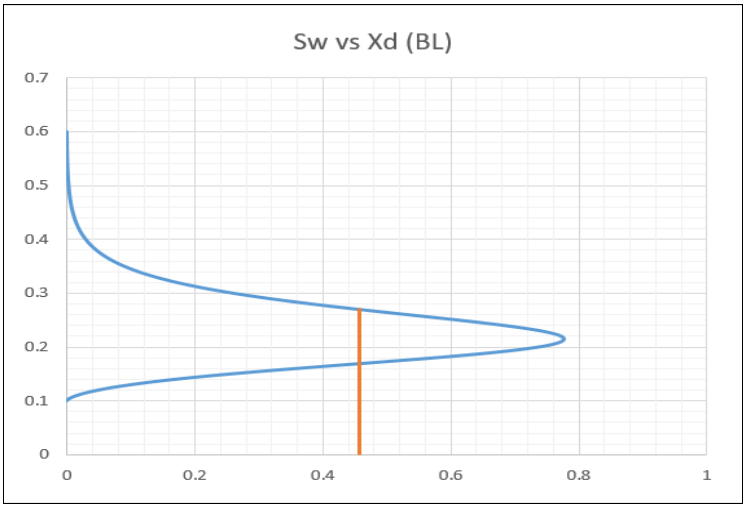


Figure 3.3: Saturation Profile of Waterflood using Buckley-Leverett Theory

In the curve (fig 3.3), there is two values of saturation at same position, which is physically impossible. There, we concluded that there must be shock front present. Hence, using the Weldge theory a new saturation profile was plotted (fig 3.4), in which after the breakthrough saturation, the saturation, the saturation was taken to be constant and equal to connate water saturation. The time of the breakthrough and the dimensionless distance $x_D$ of the breakthrough was identified by using 'vlookup' function in MS-Excel. Some of the parameters that were calculated and identified are mentioned in the table 3.2.

Table 3.2: Calculated Parameters in Waterflood

| **Parameters** | **Values** |
|---|---|
| $S_{w_{bt}}$ | 0.271 |
| $Time_{bt}$ | 171 days |
| $x_{D_{bt}}$ | 0.452 |
| $W_{ID_{bt}}$ | 0.221 |

Using the calculated parameters such as the dimensionless distance at which the breakthrough occurs, the saturation profile using the Weldge theory was plotted at time = 100 days.

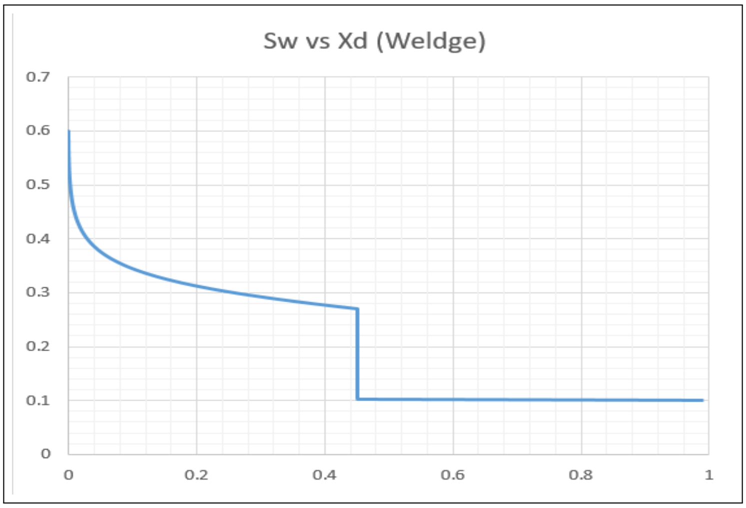


Figure 3.4: Saturation Profile of Waterflood using Weldge Theory

Finally pore volumes of oil produced was calculated by following the steps as mentioned in the section 2.3.3. The recovery curve for waterflood was plotted as shown in the figure (3.5)

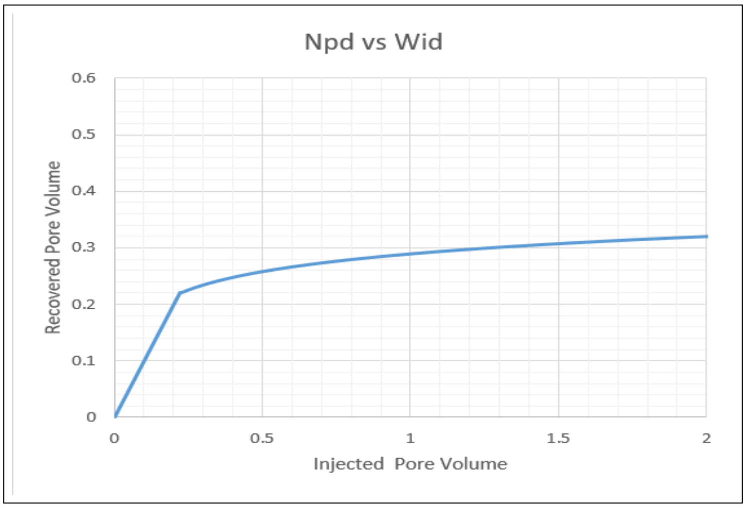


Figure 3.5: Recovery plot for Waterflood

### 3.2 POLYMER FLOOD

Polymer flooding is an tertiary recovery process or a Chemical Enhanced Oil Recovery (CEOR) process which involves introduction of polymer into the injected water stream. This increases the viscosity of the oil-recovery drive fluid, which in turn leads to improvement in the conformance or the sweep efficiency of the displacement front, thus, improving the total oil recovery or the EUR of the field. The increment in viscosity of the oil-recovery driving fluid leads to improvement in the sweep efficiency of by two different mechanisms. First it can be applied to improve the mobility ratio during the enhanced oil recovery process. This viscosity-improving mechanism leads to the reduction in viscous-fingering of the driving fluid in the displaced fluid. Second, enhancement in viscosity of the displacement fluid improve sweep efficiency, which in turn improves the conformance of the front when heterogeneous reservoir rocks with variable permeabilities.

During the polymer flooding process, a water soluble polymer whose molecular weight is high is injected in low concentrations into the water to increase its viscosity. Some of the polymers used in the industry are xanthan, HPAM, ATBS, HAP, etc. Many factors needs to be taken into consideration while injecting polymer into the water such as thermal degradation , chemical degradation and mechanical degradation of the polymer and also the retardation effect of the polymer due to its adsorption tendencies on the surface of the reservoir rocks.

To construct the saturation and fractional flow profiles of polymer flooding, we have to take the polymer and its retardation effects into considerations.

### 3.2.1 Assumptions

Similar to that in water flooding, some assumptions were also made in order to study and analyse the polymer flooding process and to plot the different profiles. Those assumptions are as follows:

- Two phase flow
- 1D flow
- No capillary pressure or gravity
- Incompressible fluids
- Isothermal
- Dispersion is neglected

### 3.2.2 Derivation of velocity of the saturation front in Polymer Flooding

In polymer flooding, the solution injected is made of two components, polymer as the solute and water as the solvent. Therefore, we have to apply multiphase conservation equation to both the components in order to derive the final equation.

**Mass Balance of Solute Component:**

$$\frac{\partial S_w C_i}{\partial t} = -\frac{q}{A\phi}\frac{\partial f_w C_i}{\partial x} - \frac{\partial C_a}{\partial t} \tag{3.6}$$

where,

$C_i$ = Solute 'i' concentration (mass/PV water)

$$C_a = \text{Adsorbed concentration (mass/PV)}$$

Also according to Langmuir theorem:

$$C_a = \frac{a_L C_i}{1 + b_L C_i} \tag{3.7}$$

**Mass Balance of Water Component:**

$$\frac{\partial S_w}{\partial t} = -\frac{q}{A\phi}\frac{\partial f_w}{\partial x} \tag{3.8}$$

Using the two equations of mass balance, we get:

$$S_w \frac{\partial C_i}{\partial t} = -\frac{q f_w}{A\phi}\frac{\partial C_i}{\partial x} - \frac{\partial C_a}{\partial t} \tag{3.9}$$

To make the above equation dimensionless, we have to define

$$x_D = \frac{X}{L}$$

$$t_D = \frac{qt}{A\phi L} = \frac{\text{Pore volume injected}}{\text{Pore volume of core}}$$

Therefore, equation (12) becomes

$$S_w \frac{\partial C_i}{\partial t_D} = -f_w \frac{\partial C_i}{\partial x_D} - \frac{\partial C_a}{\partial t_D} \tag{3.10}$$

Now, we know that $C_a = f(C_i)$. So,

$$\frac{\partial C_a}{\partial t_D} = -\frac{\partial C_a}{\partial C_i}\frac{\partial C_i}{\partial t_D} = -D_i \frac{\partial C_i}{\partial t_D} \tag{3.11}$$

where,

$$D_i = \frac{\partial C_a}{\partial C_i}$$

Putting this in equation (13), we get:

$$(S_w + D_i)\frac{\partial C_i}{\partial t_D} = -f_w \frac{\partial C_i}{\partial x_D} \tag{3.12}$$

For a velocity front of constant solute concentration ($dC_i = 0$). Hence,

$$dC_i = \frac{\partial C_i}{\partial x_D} dx_D + \frac{\partial C_i}{\partial t_D} dt_D \tag{3.13}$$

Hence, we get the velocity equation as follows:

$$v_{C_i} = \left|\frac{\partial x_D}{\partial t_D}\right|_{C_i} = -\frac{\left|\frac{\partial C_i}{\partial t_D}\right|_{C_i}}{\left|\frac{\partial C_i}{\partial x_D}\right|_{C_i}} = \frac{f_w}{S_w + D_i} \tag{3.14}$$

where,

$v$ = interstitial injection velocity (dimensionless) normalised by $\frac{q}{A\phi}$ = specific velocity

Corresponding to the front of the component $C_i$, we assume the water saturation is $S_{w3}$. According to the Buckley–Leverett theory, the specific velocity of $S_{w3}$ is

$$v_{S_{w3}} = \left|\frac{\partial x_D}{\partial t_D}\right|_{S_{w3}} = \left|\frac{\partial f_w}{\partial S_w}\right|_{S_{w3}} \tag{3.15}$$

Because $S_{w3}$ is the water saturation at the chemical front of $C_i$, their specific velocities must be the same, resulting in

$$\left|\frac{\partial f_w}{\partial S_w}\right|_{S_{w3}} = \left|\frac{f_w}{S_w + D_i}\right|_{S_{w3},C_i} \tag{3.16}$$

### 3.2.3 Retardation Effect in Two Phase Flow

If in a reservoir having connate water saturation, a chemical or polymer solution is injected, chemical retention will occur, leading to the formation of a denuded water zone after the injected front called the oil bank. This chemical or polymer shock at say, $x_{w3}$ causes the change in saturation at the shock from $S_{w3}$ to $S_{w2}$. This denuded water further helps in displacing the oil and connate water ahead of it. The second shock or the breakthrough shock is formed between the denuded water zone and the intial connate water and oil zone, at distance say $x_{w2}$.

The material or the mass balance of the retained chemical is used to determine the velocity of this shock front: the chemical injected into the reservoir with concentration $C_i$, should have the same amount present in the denuded water zone as well as the zone containing chemical that has been retained behind the chemical front. Mathematically,

$$C_i A\phi(x_{w3} - x_{w2})S_{w2} = C_a A\phi x_{w3} \tag{3.17}$$

$$x_{w2} = x_{w3}\left(1 + \frac{\frac{C_a}{C_i}}{S_{w2}}\right) = x_{w3}\left(1 + \frac{D_i}{S_{w2}}\right) \tag{3.18}$$

The equation is modified to get the relation between specific velocities.

$$v_{w3} = \frac{v_{w2}}{\left(1 + \frac{D_i}{S_{w2}}\right)} \tag{3.19}$$

where,

$v_{w2}$ = specific velocity of the solution if no chemical retention exists

$v_{w3}$ = specific velocity of the injected chemical $C_i$ with retention $C_a$

According to equation (3.15),

$$v_{w3} = \frac{f_{w3}}{S_{w3} + D_i} = \frac{f_{w3} - f_{w2}}{S_{w3} - S_{w2}} = \frac{f_{w2}}{S_{w2} + D_i} \tag{3.20}$$

Substituting this Eq. (3.19) yields

$$v_{w2} = \left(\frac{f_{w2}}{S_{w2} + D_i}\right)\left(1 + \frac{D_i}{S_{w2}}\right) = \frac{f_{w2}}{S_{w2}} \tag{3.21}$$

### 3.2.4 Profiles Construction Mechanism

In order to construct the saturation profiles and fractional flow curves, it is necessary to understand the mechanism of polymer flooding. In polymer flooding, two shocks are formed: first is the oil bank front, behind which the connate water present in the reservoir displaces the oil, and second is chemical front or the polymer front, behind which the polymer solution or the chemical pushes forward the oil ahead of it towards the production well. The region behind the chemical shock or front is a region with waves of constant polymer or chemical concentration spreading in all directions. Ahead of the chemical front is a clear oil bank which is formed due to stripping of polymer from the aqueous phase. The viscosity of the aqueous phase is increased due to continuous addition of polymer into the solution, thus, making a stable oil bank.

Now, the values of the required parameters were assumed as follows:

Table 3.3: Assumed Parameters for Polymer flood

| **Parameters** | **Values** |
|---|---|
| $S_{wc}$ | 0.1 |
| $S_{orw}$ | 0.2 |
| $k_{rw^\circ}$ | 0.6 |
| $k_{ro^\circ}$ | 0.92 |
| $n_w$ | 2.5 |
| $n_o$ | 6 |
| $\mu_w$ | 0.5 $cp$ |
| $\mu_o$ | 17 $cp$ |
| $A$ | 1000000 $ft^2$ |
| $L$ | 100 $ft$ |
| $\varphi$ | 0.2 |
| $q_t$ | 20000 $ft^3/d$ |
| $t$ | 100 $days$ |
| $D_i$ | 1.02 |

The values of water saturation was taken from connate water saturation to any value (In this case, it is 0.6) with the increment in water saturation by 0.01 in a daily basis. The relative permeability curve for polymer flooding was plotted in MS-Excel (Fig 3.6) the same way as in case of waterflood i.e., by using Corey's Model equations (3.1) and (3.2).

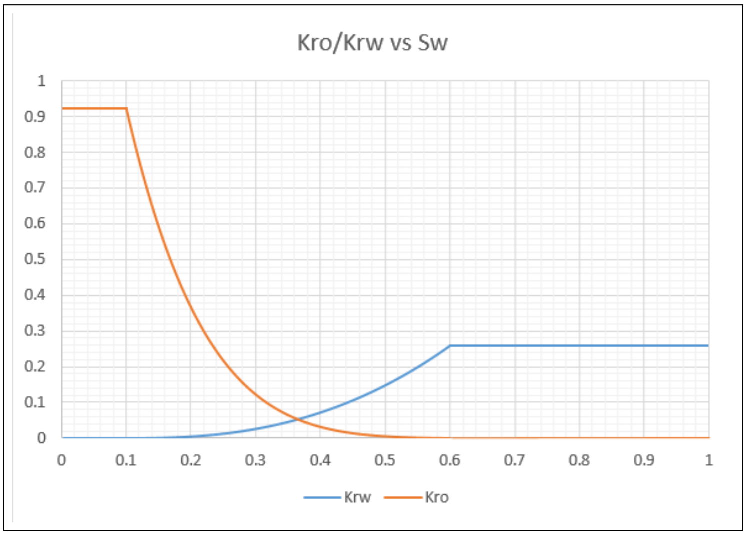


Figure 3.6: Relative Permeability Curves for Polymer flood

Now the fractional flow curves for both Polymer-Oil and Water-Oil solutions were plotted (Fig. 3.7) using the equation (2.8) mentioned in the Fractional flow theory section.

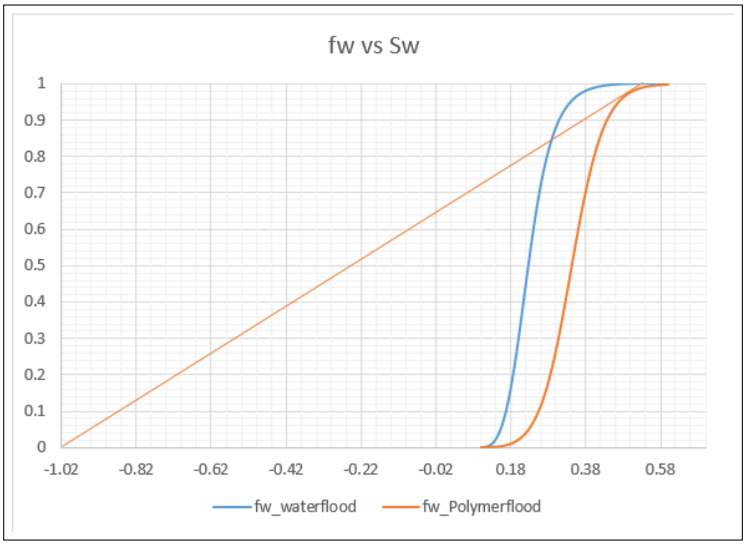


Figure 3.7: Fractional Flow Curves in case of Polymer flood

The intersection of the tangent drawn from the $D_i$ to the Polymer-Oil solution curve gives the oil bank saturation. While the breakthrough saturation was using the intersection of the same tangent from $D_i$ to the Water-Oil solution curve.

The pore volumes of aqueous polymer solution being injected ($W_{ID}$ or $t_D$ and the dimensionless distance ($x_D$) were calculated in the same manner as that in case of waterflood using the equations (3.3), (3.4) and (3.5). Moreover, their values at breakthrough and bank saturation were found using the 'vlookup' function in MS-Excel. Below is the table containing the parameters that were looked up for further purposes:

Table 3.4: Calculated Parameters in Polymer flood

| **Parameters** | **Values** |
|---|---|
| $S_{w_{bt}}$ | 0.495 |
| $Time_{bt}$ | 395 days |
| $x_{D_{bt}}$ | 0.412 |
| $W_{ID_{bt}}$ | 1.551 |
| $vel_{bt}$ | 0.645 |
| $S_{w_{bank}}$ | 0.287 |
| $f_{w_{bank}}$ | 0.841 |
| $x_{D_{bank}}$ | 0.065 |

Utilizing the above mentioned calculated parameters, the saturation profiles for the BUckley-Leverett theory (Fig. 3.8) and for Weldge theory (Fig 3.9) were plotted. In the figure 3.9 it is two fronts formed in the polymer flooding mechanism are clearly distinct and visible.

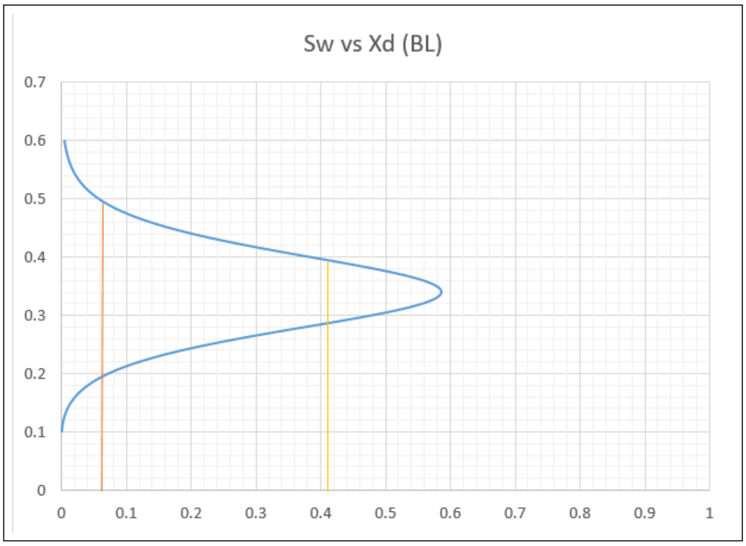


Figure 3.8: Saturation Profile of Polymer flood using Buckley-Leverett Theory


Figure 3.9: Saturation Profile of Polymer flood using Weldge Theory

At last the recovery calculations were performed for the polymer flooding operation and the results were plotted in a line chart in MS-Excel (Fig. 3.10). From this plot it can be seen that the pore volumes of oil produced has been increased by addition of polymer into the water, thus improving the overall recovery of the field.

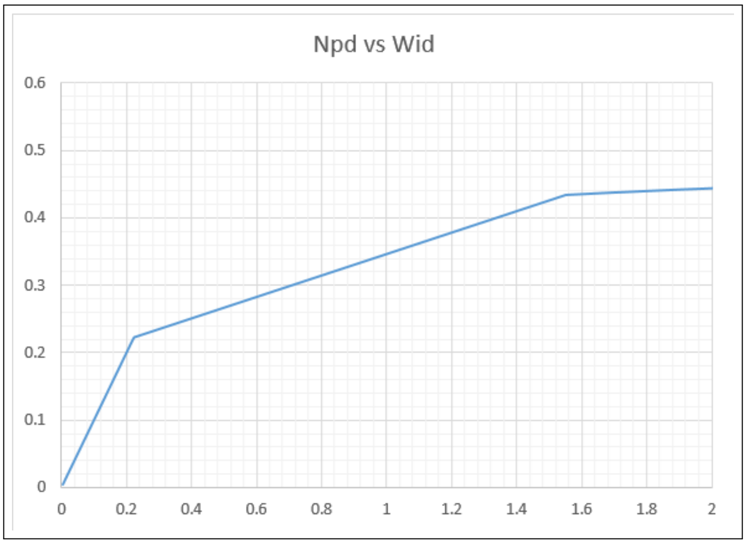


Figure 3.10: Recovery plot for Polymer flood

After this, all the steps were repeated for different values of the polymer viscosity injected into water in order to see the variations that occurs in different profiles and plots. A combined recovery plot for different values of polymer viscosity was made in a single chart to compare them with each other (Fig. 3.11). It can be seen that, as the viscosity of the polymer is increased, the mobility ratio becomes more favourable, which in turn improves the sweep efficiency, thus, finally leading to more incremental oil recovery.

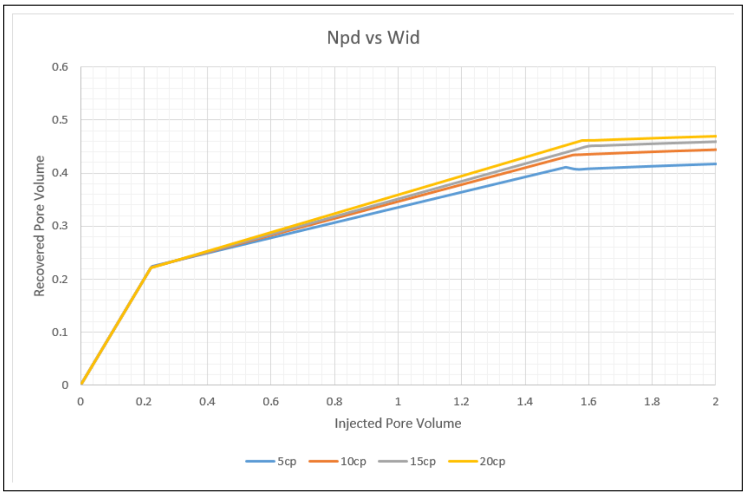


Figure 3.11: Comparison of Npd vs Wid for different polymer viscosities

# CHAPTER 4

# CONCLUSION

## 4.1 INFERENCES

Fractional flow theory and Buckley-Leverett analysis still serves as an important tool in order to understand the mechanisms of different Chemical Enhanced Oil Recovery (CEOR) process. Even though there are several assumptions required to reach the analytical solution, fractional flow and Buckley-Leverett theory can still be applied for studying various CEOR techniques.The conclusions made in this project are mentioned below:

1. Heavy oils or oil with higher viscosities leads to early breakthrough of the injected fluid (such as water). This is mainly due to unfavourable mobility ratio which in turn causes viscous fingering conformance problem in the reservoir. Furthermore, more volumes of water will be required to reach the residual oil saturation in water-flooding operation.
2. Significant increase in oil recovery is observed in polymer flood as compared to water flood for the same pore volume of water injected. Also, as the polymer viscosity increases, the increment in oil recovery decreases.
3. The introduction of polymer into the system, improves the mobility ratio, which approaches the favourable value (i.e. 1). This leads to piston-like displacement, and hence, increases the sweep efficiency. If the mobility ratio and inter-facial tension are reduced together, the displacement of the flood-front will occur uniformly at a dimensionless velocity of unity.
4. In polymer flooding, from the equation derived using the multiphase conservation equation and Buckley-Leverett theory, it was seen that a single and constant value of concentration is associated with the a saturation value. Also, it is known that only one value of saturation can be there at a particular distance or position in the reservoir. Therefore, it can be concluded that, a fixed value of concentration of polymer is there at any distance in the reservoir.

The scope and application of the Fractional Flow theory and Buckley-Leverett theory is immeasurable. Even now, it can be safely assumed that new applications of these concepts can further be discovered in the future.

## 4.2 BIBLIOGRAPHY REFERENCES

While we are on the topic, let me recommend another great book for the reference in polymer flooding: Phil. (1991). Or perhaps, some other favorites might interest you if you work in related fields: see Pope (1980), Lake (1989), Dake (1978) or Ahmed (2010) for a nice selection. It is also worth mentioning that the work of Lei Ding (2020) is well regarded to understand the importance of fractional flow theory in various CEOR processes. Also you can make use of glossaries such as that of Schlumbergers', in case you are not able to understand some concept or topic related to the field. glo

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