APMA 2070 Deep Learning for Scientists and Engineers Homework 01

Due Date: 02-03-2025, 11:59 pm (E.T.)

Warm up exercises: Numerical Method: 101

Numerical Differentiation

- 1. First Derivative using Forward Difference: Given the function $f(x) = x^2 + 2x + 1$ at the point x = 2, use the forward difference method with a step size h = 0.01 to estimate f'(2).
- 2. Second Derivative using Central Difference: For the function $g(x) = \sin(x)$, compute the second derivative at $x = \pi/4$ using the central difference method. Use a step size h = 0.1.
- 3. Error Analysis in Differentiation: Compare the numerical and analytical derivatives of $h(x) = e^x$ at x = 1. Use the forward difference method with different step sizes (h = 0.1, 0.01, 0.001) and discuss how the error changes with the step size.

Numerical Integration

- 1. Trapezoidal Rule: Estimate the integral of $f(x) = x^3$ from x = 1 to x = 2 using the trapezoidal rule with 4 equal subdivisions.
- 2. Simpson's 1/3 Rule: Apply Simpson's 1/3 Rule to approximate the integral of $g(x) = \sqrt{x}$ from x = 1 to x = 4. Use 6 subdivisions and compare the result with the exact integral.
- 3. Calculate the integral of $h(x) = \ln(x)$ from x = 1 to x = 2 using both the trapezoidal rule and Simpson's 1/3 Rule with the same number of subdivisions (e.g., 4). Discuss the differences in the results.