Division of Applied Mathematics

Brown University

Subject : APMA 2070-Deep Learning for Scientists and Engineers Year : Spring 2026

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Homework : 1 Due Date : 02/02/2026

Numerical Differentiation

1. Given the function $f(x) = x^2 + 2x + 1$ at the point $x = 2$, use the Forward and Backward Difference methods to estimate the derivative. Determine which method is more accurate and explain why. Under what conditions do these formulas fail ? Construct the average of the two methods and check if it provides a more accurate estimate. Draw the discrete stencil and discuss the pros and cons of all three methods. Finally, enumerate the differences between your work and that generated by ChatGPT or other LLMs.
2. Second Derivative using Central Difference : For the function $g(x) = \sin(x)$, compute the second derivative at $x = \pi/4$ using the Central Difference method. Use a step size $h = 0.1$. Show numerically that this is a second order method. At which step size the method fails. Compare your own results with the answers of two of your favorite LLMs and tell us which one is more accurate and why.

Numerical Integration

1. Calculate the integral of $h(x) = \ln(x)$ from x = 0 to x = 1 using both the trapezoidal rule and Simpson's one-third rule with the same number of subdivisions (for example, eight). Discuss the differences in the results. Compare these results with those obtained using Gauss–Legendre quadrature, and determine which method is more accurate. Finally, compare the numerical results with the analytical solution and analyze the associated errors.

2. Investigate whether any of these methods converges to machine precision when using 32-bit floating-point arithmetic. Determine how many quadrature points are required for each method to reach this accuracy, and tabulate your results.
3. Investigate the effect of introducing the transformation $x = y^2$, and evaluate how many Gauss–Legendre quadrature points are required to achieve machine precision.
4. What if we apply the Simpson and Trapezoidal rules to solve the above question (achieve machine precision) ?
5. Substitute $x = y^4$ and repeat the computation from step 3.
6. What is the smallest value of p in the transformation $x = y^p$ that requires only 5 points for Gauss–Legendre quadrature ?
7. Explain why this transformation is useful.
8. Finally, include the Jupyter notebook containing the code written using your own logic, not generated by LLMs.