IISER KOLKATA

Term Project - PH3203

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Delayed Choice Quantum Eraser

(Kim et al. 1999)

Flow

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Motivation

N. Bohr(1920s) –> wave & particle behaviour can't be observed in the same setup.

Scully & Druhl(1982)—> using entangled photons they proved this wrong.

Kim et al.(1999)—> Experimental realisation of Scully's proposal

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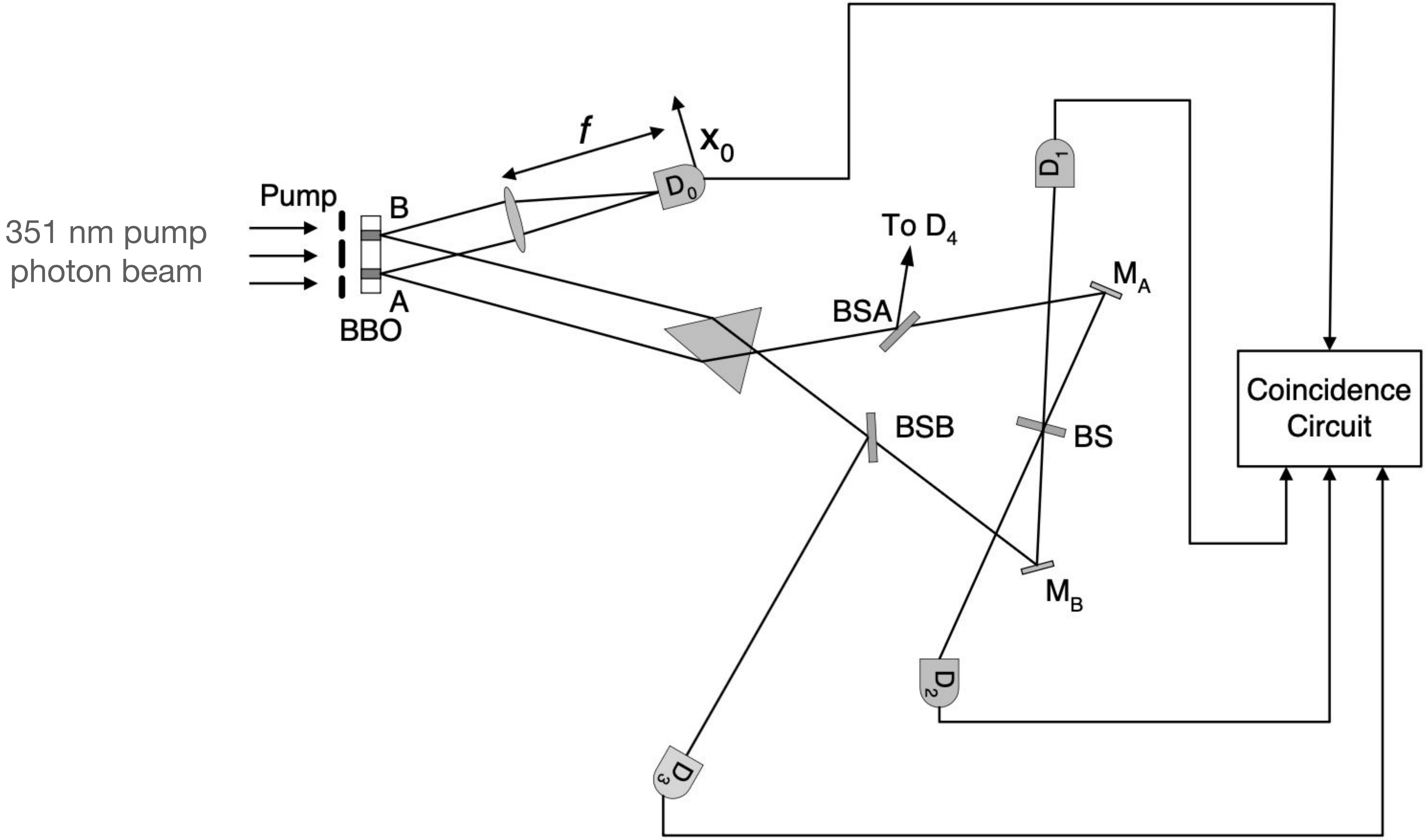
Modifying further by introducing delayed choice

The Experimental Setup



Calculation

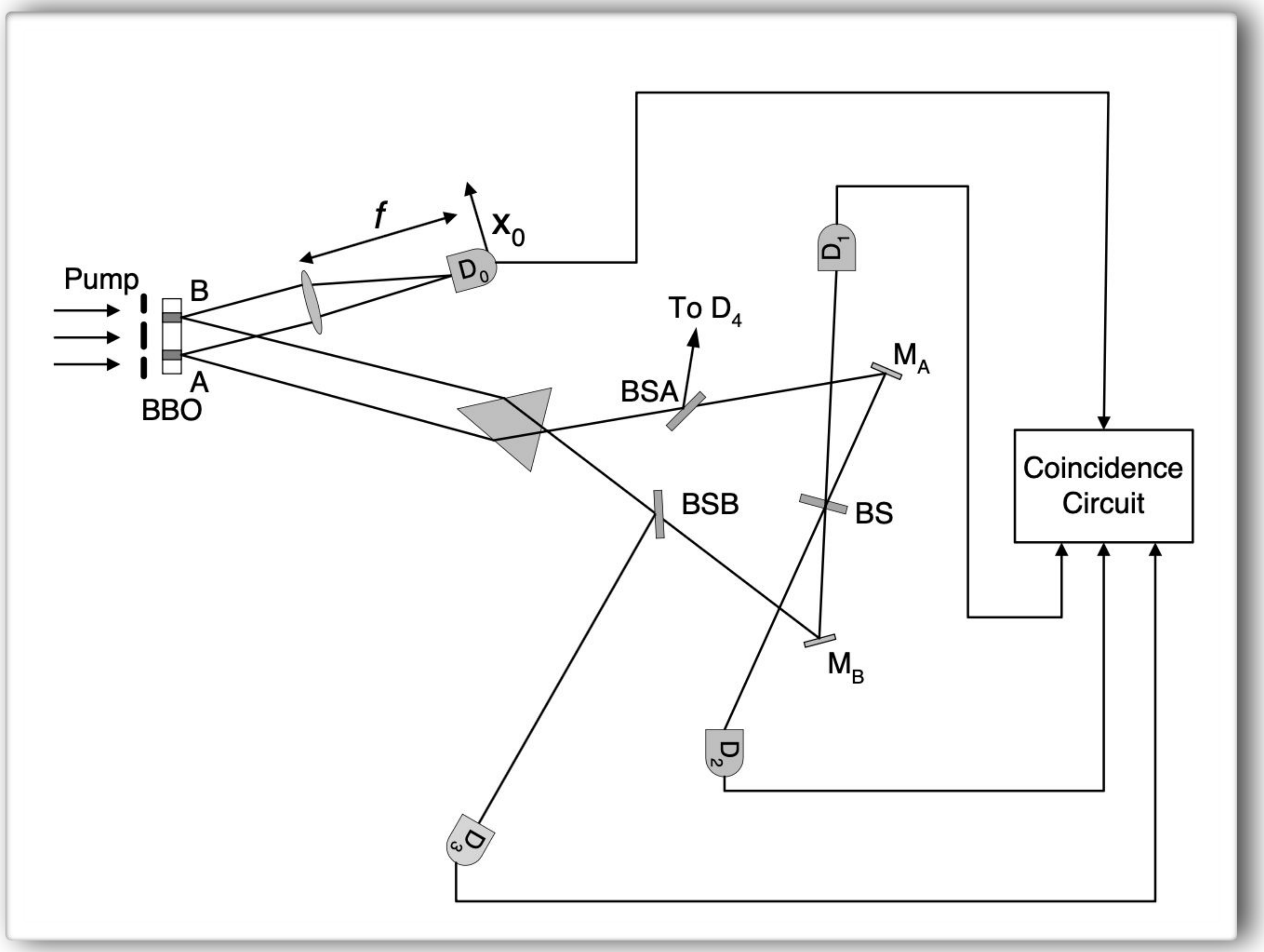
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(a^\dagger_{sA}a^\dagger_{iA} + a^\dagger_{sB}a^\dagger_{iB}\right)|0\rangle$$

$$R_{0j} \propto \left|\langle 0|E_j^{(+)}, E_0^{(+)}|\Psi\rangle\right|^2$$

Define the two-photon amplitude:

$$\Psi(x, t_j) = \langle 0|E_j^{(+)}, E_0^{(+)}|\Psi\rangle$$

So, $R_{0j} \propto |\Psi(x, t_j)|^2$.



$$E_0^{(+)} \propto a_{sA}\phi_A(x) + a_{sB}\phi_B(x)$$

Since D_0 is in the lens's focal plane, $\phi_A(x)$ and $\phi_B(x)$ are Fourier transforms of the slit functions (Fraunhofer diffraction). For a slit of width a:

$$\phi(\xi) = \begin{cases} 1 & |\xi| \leq a/2 \\ 0 & \text{otherwise} \end{cases}$$

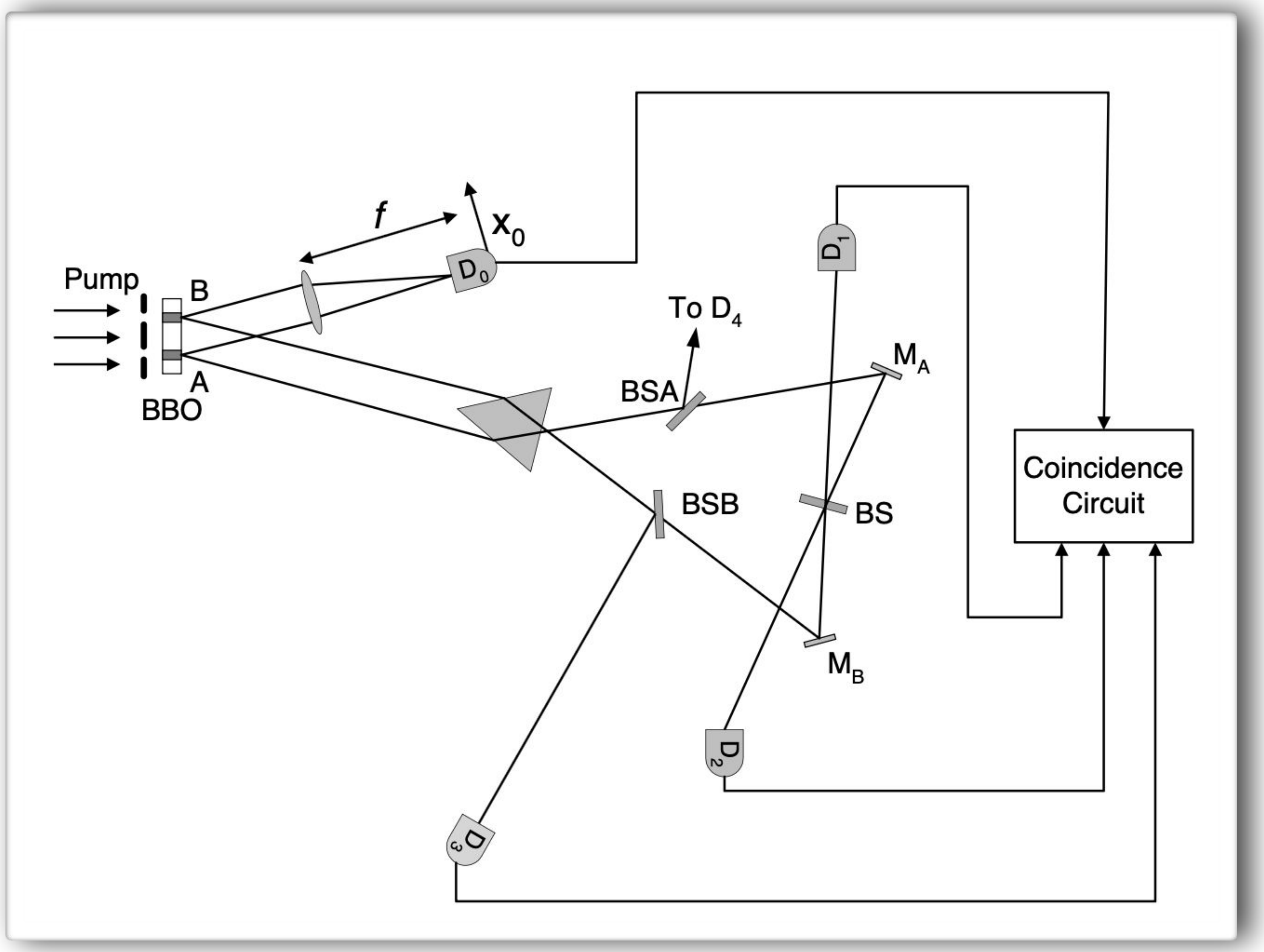
Centered at $\xi = -d/2$ (slit A) or $d/2$ (slit B), the amplitude at x is:

$$\phi_A(x) \propto \int_{-d/2-a/2}^{-d/2+a/2} e^{-i\frac{2\pi}{\lambda f}x\xi} d\xi$$

Assuming equal amplitudes:

$$\phi_A(x) = Ce^{i\theta}s(x), \quad \phi_B(x) = Ce^{-i\theta}s(x)$$

$$s(x) = \sin c\left(\frac{\pi a x}{\lambda f}\right), \quad \theta = \frac{\pi d x}{\lambda f}$$



For D_1:

$$\Psi(x,t_1) \propto \langle 0|(a_{iA}+a_{iB})\cdot \frac{1}{\sqrt{2}}\left(\phi_A a_{iA}^\dagger + \phi_B a_{iB}^\dagger\right)|0\rangle$$

$$= \frac{1}{\sqrt{2}}(\phi_A+\phi_B)$$

For D_2:

$$\Psi(x,t_2) \propto \frac{1}{\sqrt{2}}(\phi_A-\phi_B)$$

Derivation for D_3:

1. Apply $E_0^{(+)}$ to $|\Psi\rangle$:

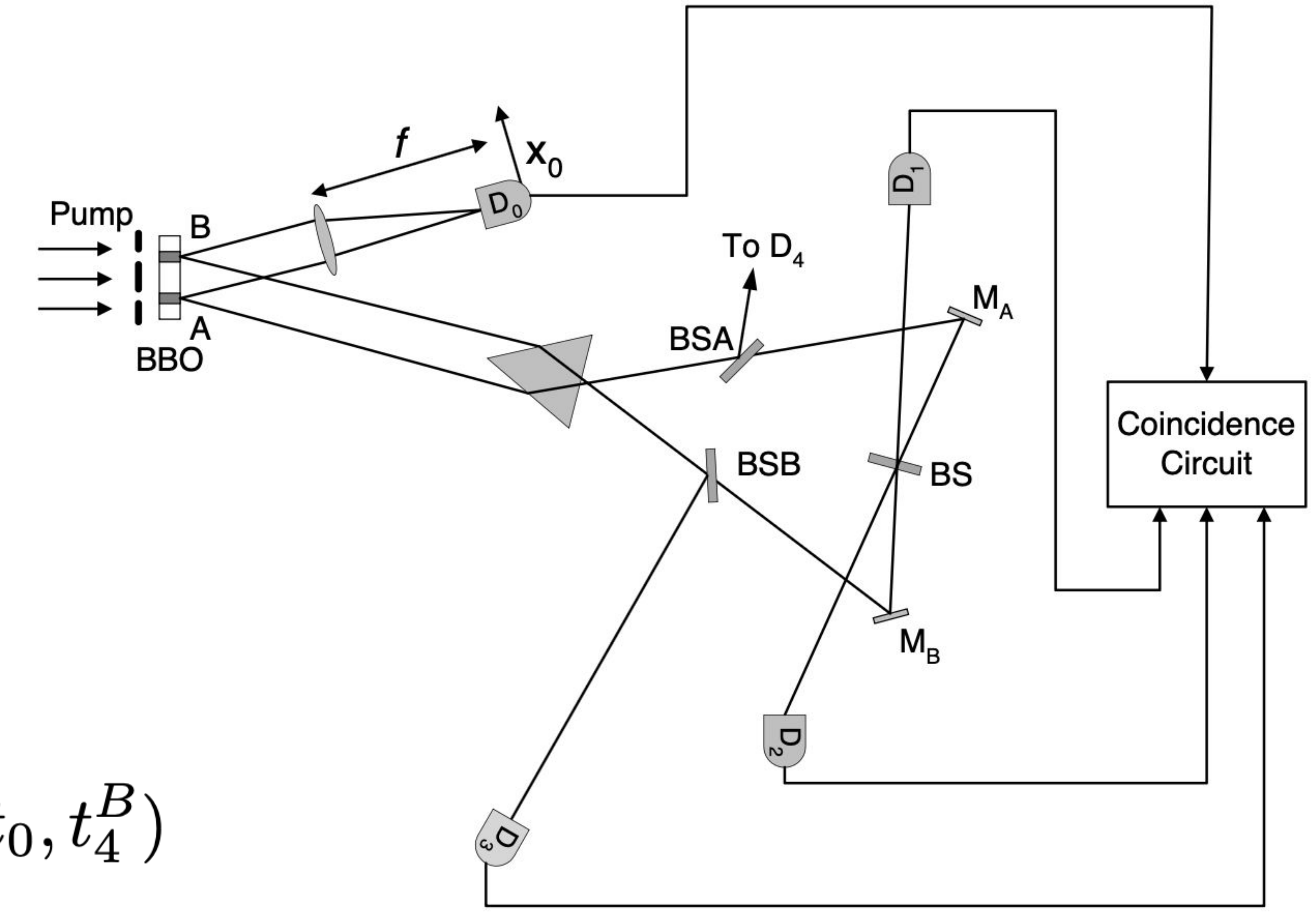
$$E_0^{(+)}|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\phi_A(x)a_{iA}^\dagger + \phi_B(x)a_{iB}^\dagger\right)|0\rangle$$

2. Apply $E_3^{(+)}$:

$$E_3^{(+)}E_0^{(+)}|\Psi\rangle = \frac{1}{\sqrt{2}}\phi_A(x)|0\rangle$$

3. Vacuum projection:

$$\Psi(x,t_3) = \frac{1}{\sqrt{2}}\phi_A(x) = A(t_0,t_3^A)$$



$$\Psi(x,t_4) = \frac{1}{\sqrt{2}}\phi_B(x) = A(t_0,t_4^B)$$

- Amplitudes depend solely on $\phi_A(x)$ or $\phi_B(x)$.

- **Idler photon at D_1:**$E_1^{(+)} \propto a_{iA}+a_{iB}$ (for D_1).

- **Idler photon at D_3:**$E_2^{(+)} \propto a_{iA}-a_{iB}$ (for D_2).

This arises from BS combining paths A and B with a phase difference (e.g., transmission vs. reflection).

- **Idler photon at D_3:**

$$E_3^{(+)} \propto a_{iA} \quad \text{(transmitted through BSA)}$$

- **Idler photon at D_4:**

$$E_4^{(+)} \propto a_{iB} \quad \text{(transmitted through BSB)}$$

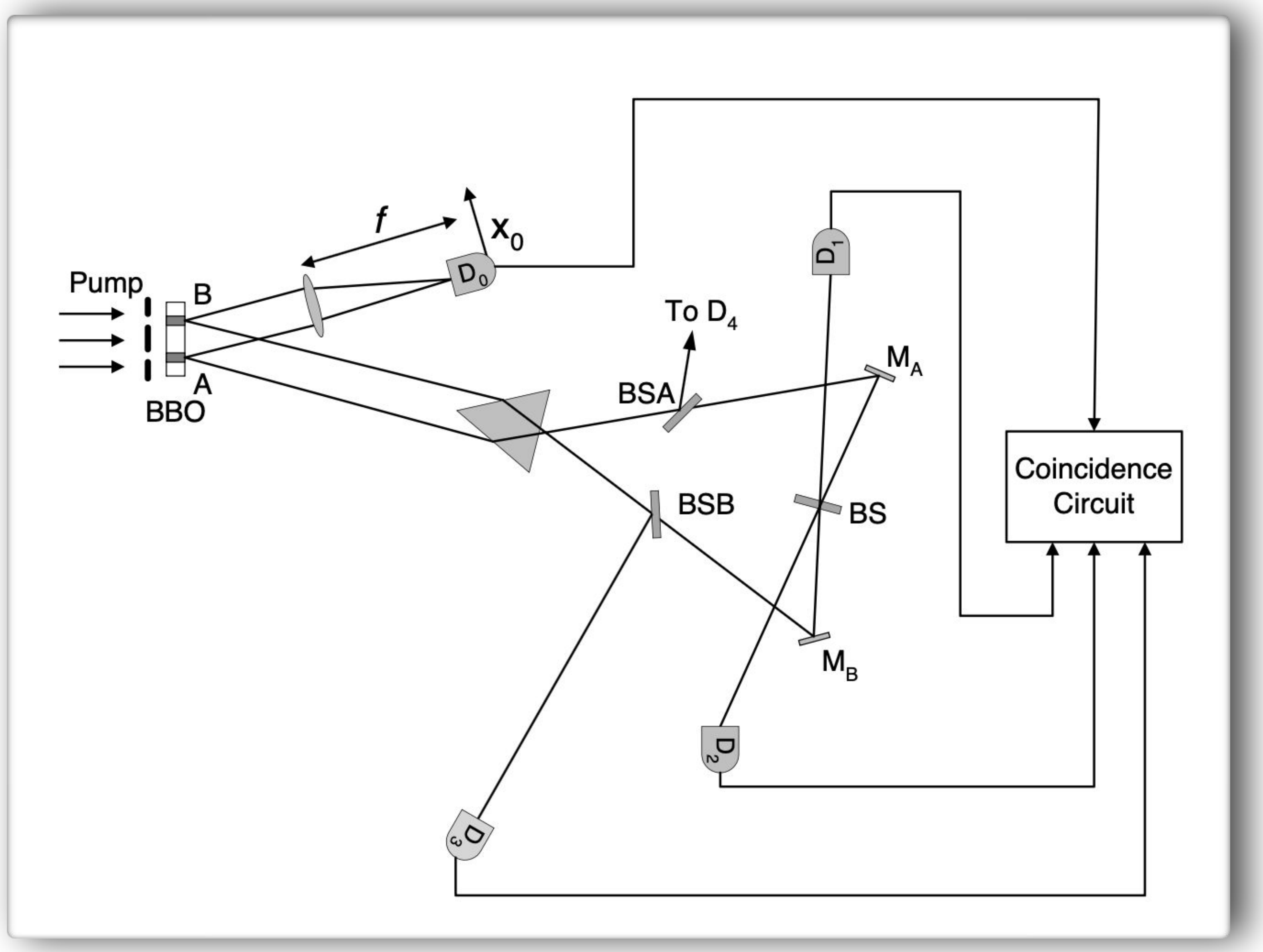
For R_{01}:

$$R_{01} \propto |\phi_A + \phi_B|^2 = |Cs(e^{i\theta} + e^{-i\theta})|^2 = |2Cs\cos\theta|^2 = 4C^2s^2\cos^2\theta$$

For R_{02}:

$$R_{02} \propto |\phi_A - \phi_B|^2 = |Cs(e^{i\theta} - e^{-i\theta})|^2 = |2iCs\sin\theta|^2 = 4C^2s^2\sin^2\theta$$

$$R_{03} \propto |\phi_A(x)|^2 = |C|^2\mathrm{sinc}^2\left(\frac{\pi a x}{\lambda f}\right)$$



$$R_{04} \propto |\phi_B(x)|^2 = |C|^2\mathrm{sinc}^2\left(\frac{\pi a x}{\lambda f}\right)$$

After Substitutions:

$$R_{01} \propto \mathrm{sinc}^2\left(\frac{\pi a x}{\lambda f}\right)\cos^2\left(\frac{\pi d x}{\lambda f}\right)$$

$$R_{02} \propto \mathrm{sinc}^2\left(\frac{\pi a x}{\lambda f}\right)\sin^2\left(\frac{\pi d x}{\lambda f}\right)$$

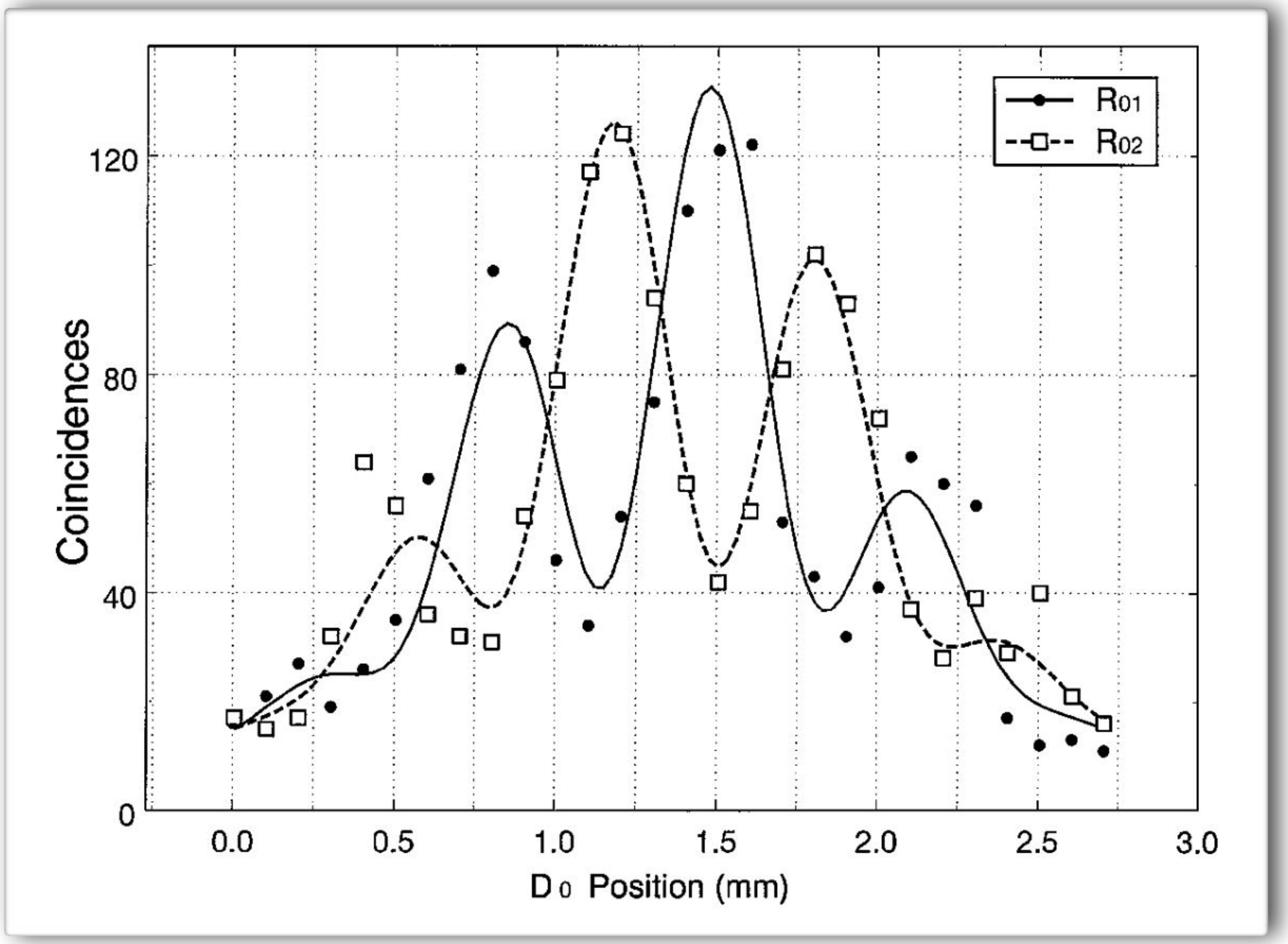
The sinc^2 term is the single-slit diffraction envelope, and $\cos^2$ or $\sin^2$ are the double-slit interference terms, with a π phase shift between them. The detection rates

exhibit only single-slit diffraction patterns:

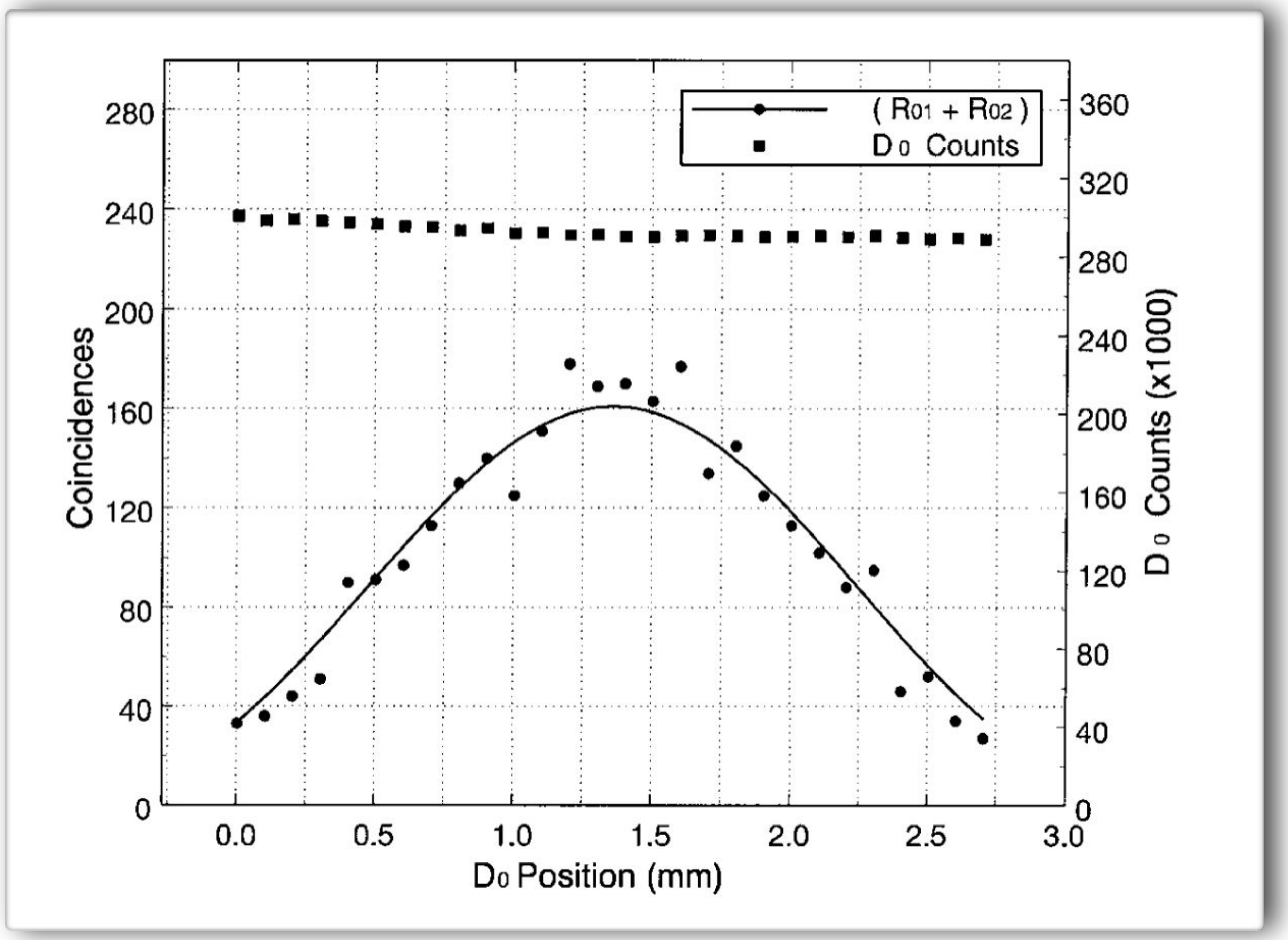
$$R_{03} \propto \mathrm{sinc}^2\left(\frac{\pi a x}{\lambda f}\right), \quad R_{04} \propto \mathrm{sinc}^2\left(\frac{\pi a x}{\lambda f}\right)$$

- No $\cos^2 / \sin^2$ terms appear because which-path information is preserved.

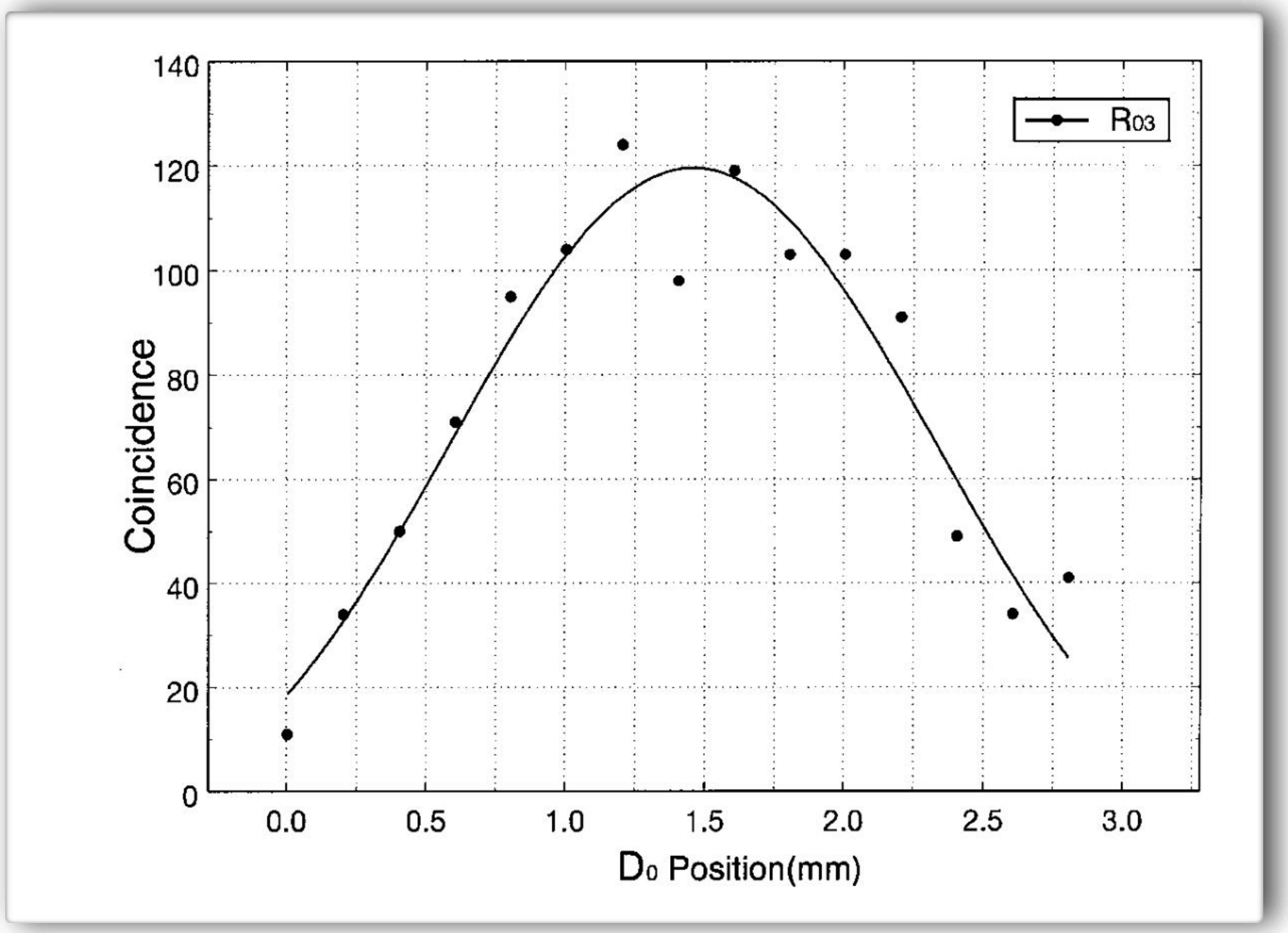
Results



R01
R02
120
80
40
0
Coincidences
0.0
0.5
1.0
1.5
2.0
2.5
3.0
D0 Position (mm)



(R01 + R02)
D 0 Counts
Coincidences
D 0 Counts (x1000)
D0 Position (mm)



R03
Coincidence
D0 Position(mm)

Conclusion

The experimental results demonstrate the possibility of determining particle-like and wave-like behaviours of a photon via quantum entanglement.

Quoting Feynman "*the double-slit experiment has in it the heart of quantum mechanics. In reality, it contains the only mystery*".