Cosmic Inflation

Superviser:
Prof. Koushik Dutta

Raj Gaurav Tripathi
(22MS215)

Age of Universe

$$H^2(a) = \frac{8\pi G}{3}\rho(a). \qquad \Omega_m = \frac{\rho_{m0}}{\rho_{c0}}, \qquad \Omega_\Lambda = \frac{\rho_{\Lambda 0}}{\rho_{c0}}, \qquad \rho_{c0} = \frac{3H_0^2}{8\pi G}.$$

$$H(a) = H_0\sqrt{\Omega_m a^{-3} + \Omega_\Lambda}.$$

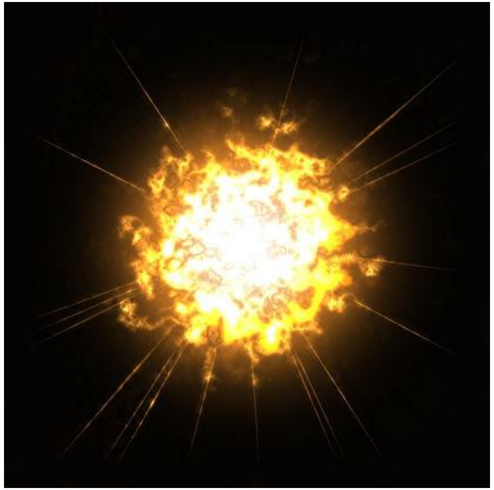
$$\boxed{t_0 = \int_0^1 \frac{da}{a\,H(a)} = \frac{1}{H_0}\int_0^1 \frac{da}{\sqrt{\Omega_m a^{-1} + \Omega_\Lambda a^2}}.}$$

$H_0 = 67.4\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m = 0.315$. For a flat model $\Omega_\Lambda = 1 - \Omega_m = 0.685$.

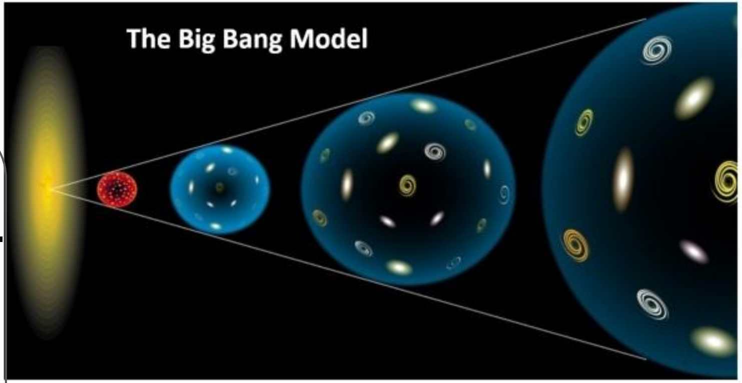
$$\boxed{t_0 \approx 13.80 \text{ billion years } (\approx 13.8 \text{ Gyr}).}$$

The Standard Big Bang

From the very beginning: matter filled space uniformly



- Theory that the universe as we know it began 13.82 ± 0.05 billion years!
- Initial state was a hot, dense, uniform soup of particles that filled space uniformly, and was expanding rapidly.

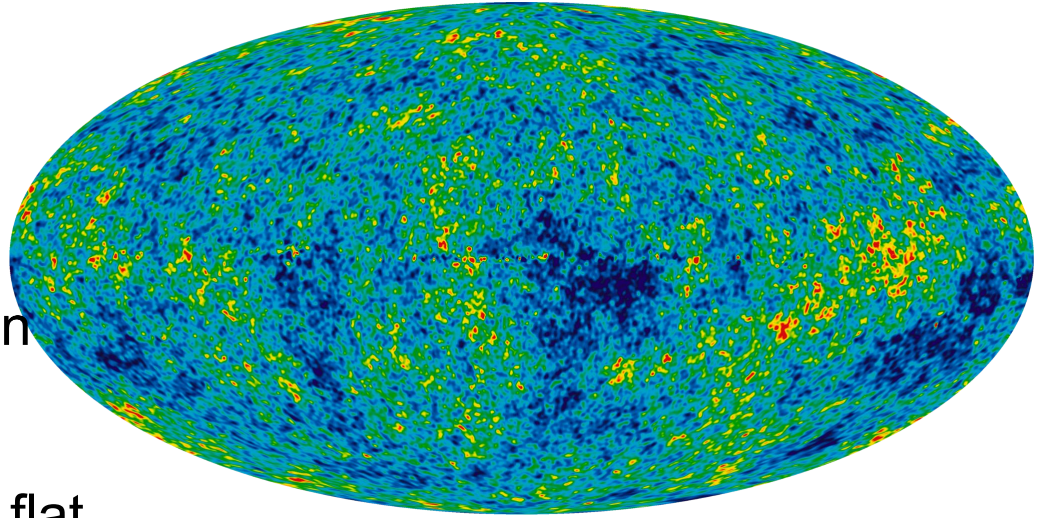


What it Describes:

- How the early universe expanded and cooled.
- How the light chemical elements formed.
- How the matter congealed to form stars, galaxies, and clusters of galaxies..

DOES BIG BANG EXPLAIN EVERYTHING?

Cosmic Microwave Background Radiation



Faint background.

- CMB was released during decoupling
- Variations of only about 1 part in 100,000

It feels good that our universe is flat..

Isn't it?

NO!

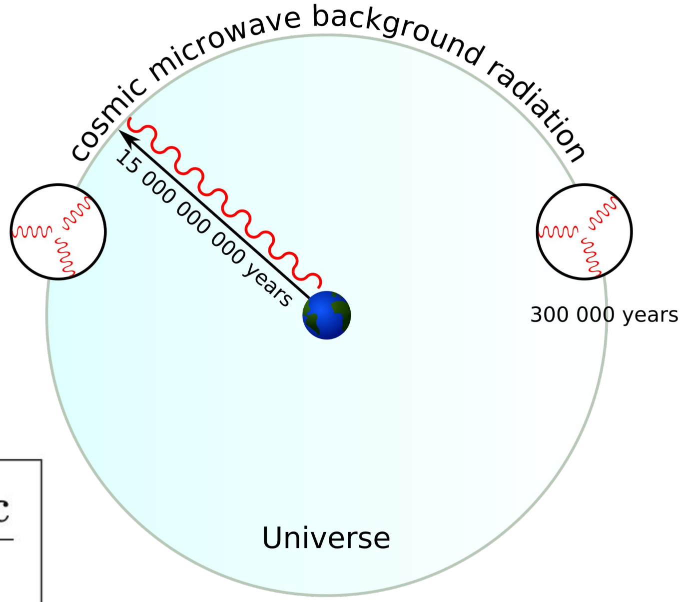
$$\tau(t) = \int_0^t \frac{dt'}{a(t')},$$

$$\chi_p(t) = \tau(t).$$

$$\chi_{\rm LSS} = \tau(t_0) - \tau(t_{\rm dec}) \simeq \tau_0 - \tau_{\rm dec}$$

$$\theta \simeq \frac{L}{\chi_{\rm LSS}}.$$

$$\boxed{\theta_{\rm hor} \simeq \frac{\tau_{\rm dec}}{\tau_0 - \tau_{\rm dec}} \approx \frac{\tau_{\rm dec}}{\tau_0}}$$



$$\theta_{\rm hor} \simeq \frac{\tau_{\rm dec}}{\tau_0 - \tau_{\rm dec}} \approx \frac{\tau_{\rm dec}}{\tau_0} \sim 0.02\ \text{rad}.$$

$$\boxed{0.02\ \mathrm{rad} \times \frac{180^\circ}{\pi} \approx 1.1^\circ.}$$

Causally connected region on the last-scattering surface $\sim 1^\circ$ across on the CMB sky.

$$\boxed{N \sim \left(\frac{180^\circ}{1^\circ}\right)^2 \sim 3 \times 10^4}$$

Still, the CMB is uniform to one part in 10^5. That's the **horizon (homogeneity) problem**

Problem 1: Horizon Problem

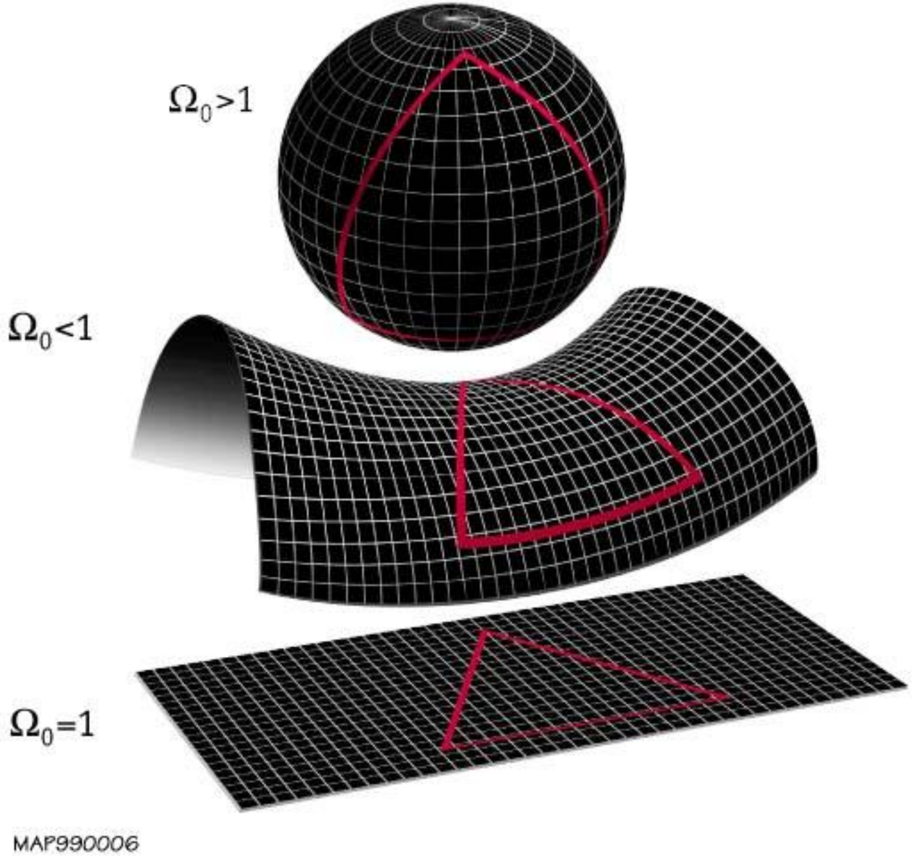
Flat Universe

Plank: $\Omega_k = 0 \pm 0.002.$

It feels good that our universe is flat..

Isn't it?

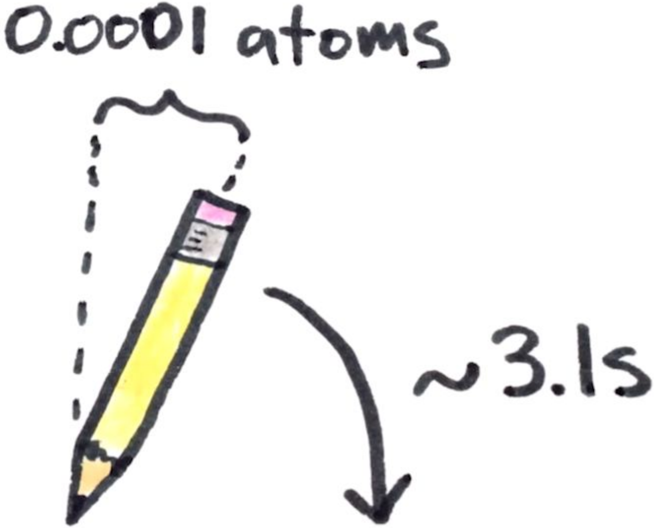
NO!



$$H^2 = \frac{1}{3}\rho(a) - \frac{k}{a^2}$$

$$\Omega(a) \equiv \frac{\rho(a)}{\rho_{\text{crit}}(a)} \qquad \rho_{\text{crit}}(a) \equiv 3H(a)^2$$

$$\boxed{1 - \Omega(a) = \frac{-k}{(aH)^2}}$$



Radiation dominated: $\rho \propto a^{-4}$ $\boxed{|\Omega - 1| \propto a^2}$

Matter dominated: $\rho \propto a^{-3}$ $\boxed{|\Omega - 1| \propto a}$

Today $\Omega \leq 0.01$ require $\Omega \approx 10^{-60}$ at planks time this is some fine tuned thing!!!

Problem 2: Flatness Problem

What is Needed?

A Period of Accelerated Expansion!

INFLATION

A period of accelerated expansion.

$$\rho \propto a^0 \implies |\Omega - 1| \propto \frac{1}{a^2} = \frac{1}{(e^{Ht})^2} = e^{-2Ht} \implies |\Omega - 1| \propto e^{-2Ht}$$

Miracle of Physics #1: Gravity can be repulsive!

Miracle of Physics #2: Energy of Gravitational field is negative!

$$\boxed{\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \quad \Rightarrow \quad \frac{d^2a}{dt^2} > 0 \quad \Rightarrow \quad \rho + 3p < 0}$$

Scalar Field Dynamics of Inflation

The dynamics of a scalar field (minimally) coupled to gravity is governed by the action

$$\boxed{S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]} = S_{\mathrm{EH}} + S_\phi$$

$$\frac{\delta S_\phi}{\delta\phi} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) + V_{,\phi} = 0\,,$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

Friedmann-Klein-Gordon equation for homogeneous scalar field in an expanding universe.

$$T^{(\phi)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right)$$

The energy-momentum tensor for the scalar field.

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,.$$

$$\boxed{w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}}$$

shows that a scalar field can lead to negative pressure ($w_\phi < 0$) and accelerated expansion ($w_\phi < -1/3$) if the potential energy V dominates over the kinetic energy $\frac{1}{2}\dot{\phi}^2$.

$$\boxed{H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)}$$

Slow Roll Inflation

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_\phi + 3p_\phi) = H^2(1 - \varepsilon)$$

$$\varepsilon \equiv \frac{3}{2}(w_\phi + 1) = \frac{1}{2}\frac{\dot{\phi}^2}{H^2} \qquad \boxed{\varepsilon = -\frac{\dot{H}}{H^2}} = -\frac{d\ln H}{dN}, \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon - \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN}$$

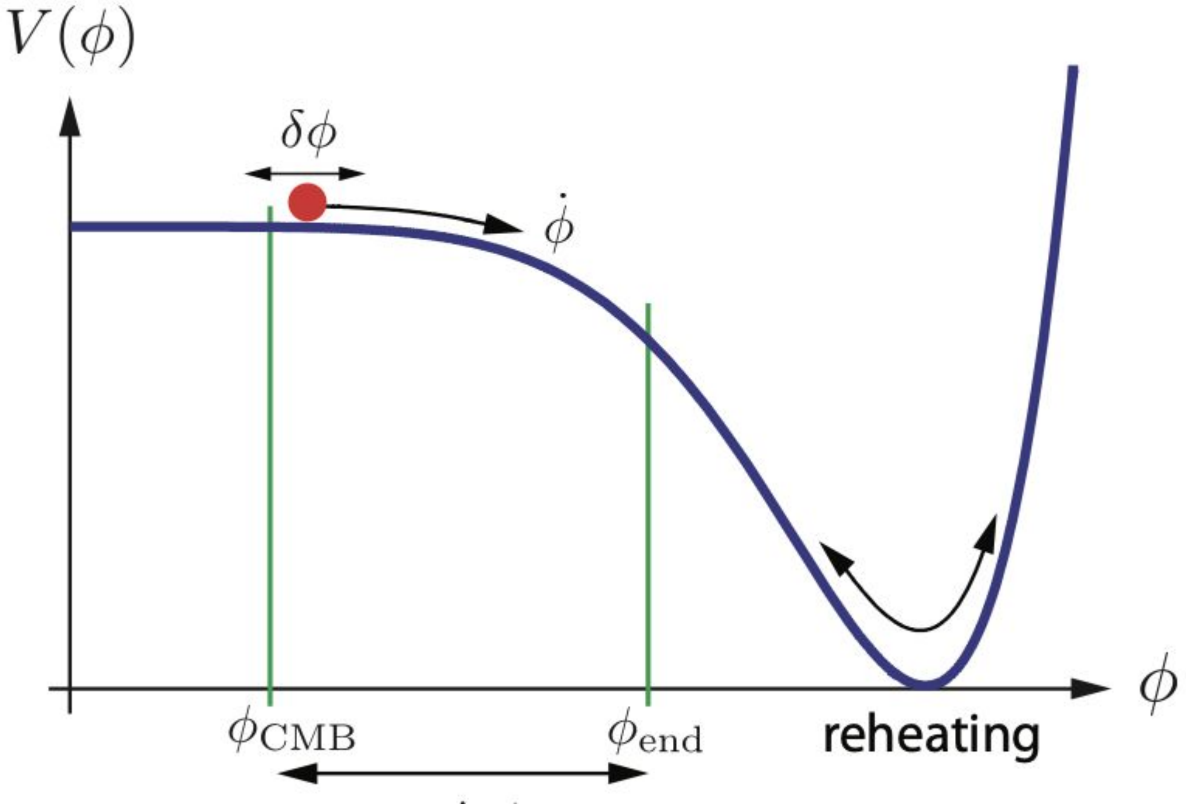
Accelerated expansion occurs if $\varepsilon < 1$.

The de Sitter limit, $p_\phi \to -\rho_\phi$, corresponds to $\varepsilon \to 0$.

$$\dot{\phi}^2 \ll V(\phi)\,. \qquad |\ddot{\phi}| \;\ll\; |3H\dot{\phi}|\,,\, |V_{,\phi}|\,.$$

$$H^2 \approx \frac{1}{3}V(\phi) \approx \text{const.}\,,$$

$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H}\,,$$



spacetime is approximately *de Sitter* $a(t) \sim e^{Ht}$.

How Much expansion we need?

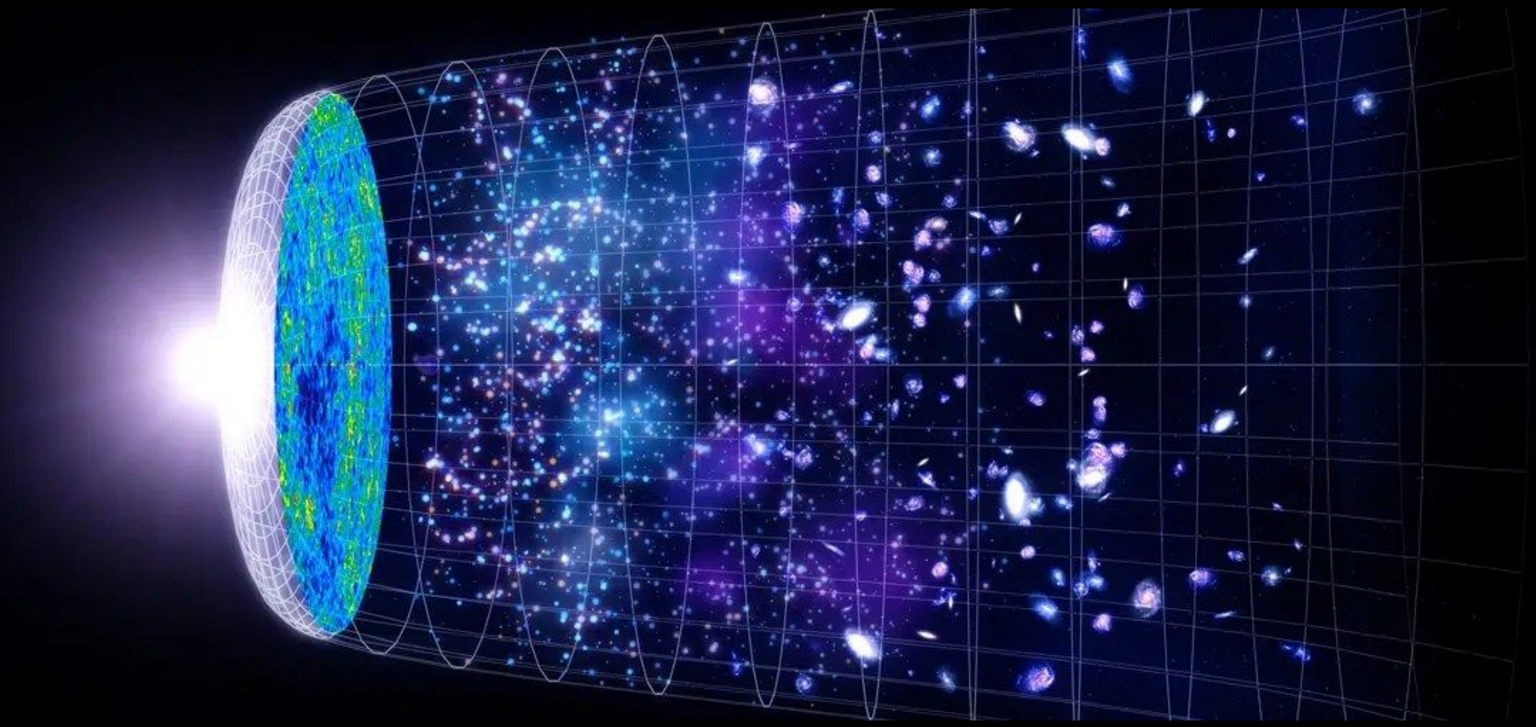
$$N(\phi) \equiv \ln \frac{a_{\text{end}}}{a} = \int_t^{t_{\text{end}}} H\mathrm{d}t = \int_\phi^{\phi_{\text{end}}} \frac{H}{\dot\phi}\mathrm{d}\phi \approx \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V_{,\phi}}\mathrm{d}\phi$$

$$\boxed{N(\phi) = \int_{\phi_{\text{end}}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\varepsilon}} \approx \int_{\phi_{\text{end}}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_{\text{v}}}}} \, .$$

$$N_{\text{tot}} \equiv \ln \frac{a_{\text{end}}}{a_{\text{start}}} \gtrsim 60 \, .$$

10^{-36} **to** 10^{-32} **seconds**

Eventually, the inflationary energy density is converted into standard model degrees of freedom and the hot Big Bang commences.



THANK YOU