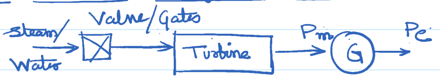
06 January 2022 13:44

Load frequency Control (LFC)

Model of Power System for LFC



$$W_{kin} = W_{kin}^{0}\left(\frac{f}{f_0}\right)^2$$

$$P_m - P_e = \frac{d}{dt}(W_{kin}) = \frac{d}{dt}\left[W_{kin}^{0}\left(\frac{f}{f_0}\right)^2\right]$$

. Initially at steady state: $P_m = P_{mo}$ $P_e = P_{eo}$
& $P_{mo} = P_{eo}$

Load frequency sensitive

$D = \frac{\partial P_D}{\partial f}$

ΔP_m ΔP_e Δf

$$\Rightarrow (\cancel{P_{mo}} + \Delta P_m) - (\cancel{P_{eo}} + \Delta P_e + D\Delta f) = W_{kin}^{0}\frac{d}{dt}\left[\left(\frac{f_0+\Delta f}{f_0}\right)^2\right]$$

Neglecting

$$\Rightarrow \Delta P_m - \Delta P_e - D\Delta f = W_{kin}^{0}\frac{d}{dt}\left[\frac{f_0^2}{f_0^2} + \frac{2\Delta f f_0}{f_0^2} + \cancel{\frac{\Delta f^2}{f_0^2}}\right]$$

$$\Rightarrow \Delta P_m - \Delta P_e - D\Delta f \simeq W_{kin}^{0}\frac{d}{dt}\left[1 + \frac{2\Delta f}{f_0}\right]$$

$$\Rightarrow \Delta P_m - \Delta P_e - D\Delta f \simeq \frac{W_{kin}^{0}\,2}{f_0}\left(\frac{d\Delta f}{dt}\right) \quad \text{MW}$$

Let, H = Inertia Constant $= \frac{W_{kin}^{0}}{P_r}$ P_r: Rating of the Machine
MJ/MW = s

$\Rightarrow W_{kin}^{0} = H P_r$

$$\Rightarrow \Delta P_m - \Delta P_e - D\Delta f \simeq \frac{2HP_r}{f_0}\left(\frac{d\Delta f}{dt}\right) \quad \text{MW}$$

$$\Rightarrow \Delta P_m - \Delta P_e - D\Delta f \simeq \frac{2H}{f_0}\left(\frac{d\Delta f}{dt}\right) \quad \text{pu MW} \quad [\text{Dividing by } P_r]$$

$$\Rightarrow \Delta P_m - \Delta P_e = \frac{2H}{f_0}\frac{d\Delta f}{dt} + D\Delta f$$

Taking Laplace Transform on both sides:

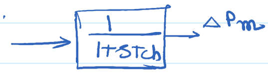
$$\Rightarrow \Delta P_m(s) - \Delta P_e(s) = \left[\frac{2H}{f_0}s + D\right]\Delta F(s)$$

$$\Rightarrow \Delta F(s) = \frac{\Delta P_m(s) - \Delta P_e(s)}{\frac{2H}{f_0}s + D} = \frac{\Delta P_m(s) - \Delta P_e(s)}{Ms + D} \qquad M = \frac{2H}{f_0}$$

$$\Rightarrow \Delta F(s) = \frac{\frac{1}{D}\left(\Delta P_m(s) - \Delta P_e(s)\right)}{1 + s\frac{2H}{f_0 D}} \qquad K_p = 1/D \qquad T_p = \frac{2H}{f_0 D} = \frac{M}{D}$$

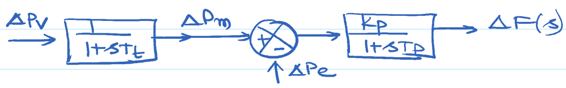
$$\Rightarrow \Delta F(s) = \frac{K_p}{1+sT_p}\left(\Delta P_m(s) - \Delta P_e(s)\right)$$

Representation of Non-Reheat type Steam Turbine:

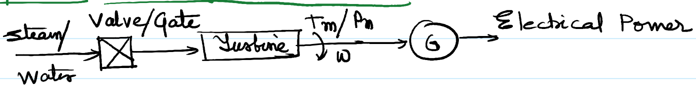


Tch: Charging Time Constant

$T_{ch} = T_t$



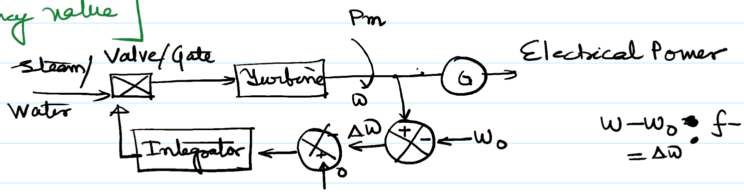
Representation of a Governor:



Isochronous Governor:

Iso — same
Chronous — Speed.

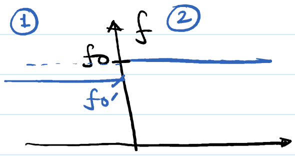
[Ensures that there is no steady-state frequency excursion from the nominal frequency value]



$\omega - \omega_0 = \Delta\omega \; : \; f - f_0 = \Delta f$

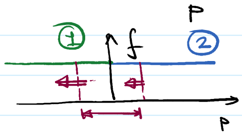
Isochronous

Isochronous Governor can be applied 'iff' you have a single generator feeding the load

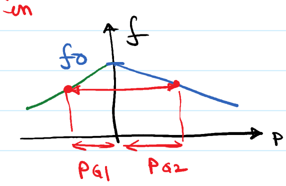


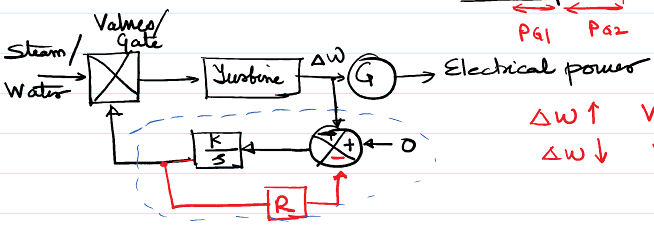
i) All Governors should have precisely the same speed setting

ii) No Precise operating point for load sharing

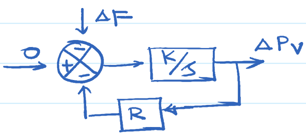


Introduce a 'droop characteristic' in the Governor





Δω ↑ Valve opening ↓
Δω ↓ Valve opeing ↑



$$\Delta P_V = \frac{K}{s}\left[-\Delta F - \Delta P_V \cdot R\right] \Rightarrow \Delta P_V\left[1+\frac{KR}{s}\right] = -\frac{\Delta F\, K}{s}$$

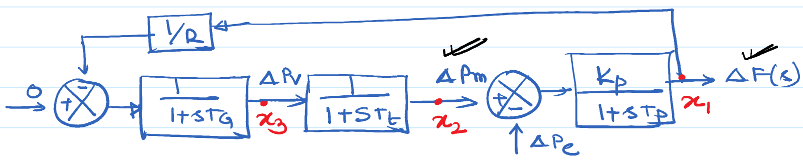
$$\Rightarrow \Delta P_V\left[\frac{s+K\cdot R}{\cancel{s}}\right] = -\frac{\Delta F\; K}{\cancel{s}}$$

$$\Rightarrow \Delta P_V = \frac{K}{s+KR}(-\Delta F) = \frac{1}{R}\cdot\left(\frac{1}{1+\frac{s}{KR}}\right) - \Delta F$$

$$T_G = \frac{1}{KR}$$

↳ Governor Time Constant

$$\Rightarrow \Delta P_V = -\left(\frac{1}{R}\right)\left(\frac{1}{1+sT_G}\right)\Delta F$$



Block-Diagram of PS for LFC study

Ref $=0 \Rightarrow$ Free Governor Action.

$$x_1 = \frac{K_p}{1+sT_p}\left[x_2 - \Delta P_e\right]$$

$$\Rightarrow x_1 + T_p \dot{x}_1 = K_p x_2 - K_p \Delta P_e$$

$$\Rightarrow \dot{x}_1 = -\frac{x_1}{T_p} + \frac{K_p}{T_p} x_2 - \frac{K_p}{T_p}\Delta P_e \qquad (1)$$

$$x_2 = \frac{1}{1+sT_t} x_3$$

$$\Rightarrow x_2 + T_t \dot{x}_2 = x_3 \Rightarrow \dot{x}_2 = -\frac{x_2}{T_t} + \frac{x_3}{T_t} \qquad (2)$$

$$x_3 = \frac{1}{1+sT_G}\left(0 - \frac{x_1}{R}\right)$$

$$\Rightarrow x_3 + T_G \dot{x}_3 = -\frac{x_1}{R}$$

$$\Rightarrow \dot{x}_3 = -\frac{x_1}{RT_G} - \frac{x_3}{T_G} \qquad (3)$$

Steady-state Response: All derivatives should be set to '0'.

$$(1) \Rightarrow 0 = -\frac{x_1}{T_p} + \frac{K_p}{T_p} x_2 - \frac{K_p}{T_p}\Delta P_e \qquad (4)$$

$$(2) \Rightarrow 0 = -\frac{x_2}{T_t} + \frac{x_3}{T_t} \Rightarrow x_2 = x_3 \qquad (5)$$

$$(3) \Rightarrow 0 = -\frac{x_1}{RT_G} - \frac{x_3}{T_G} \Rightarrow x_1 = -R x_3 \Rightarrow x_2 = -\frac{x_1}{R}$$

$$\Rightarrow x_1 = -R x_2 \qquad (6)$$

$$-x_1 + K_p x_2 - K_p \Delta P_e = 0$$

$$\Rightarrow -\left(x_1 + K_p \frac{x_1}{R}\right) = K_p \Delta P_e$$

$$\Rightarrow x_{1ss}\left(1 + \frac{K_p}{R}\right) = -K_p \Delta P_e$$

$$\Rightarrow x_{1ss} = \frac{-K_p}{1 + K_p/R}\Delta P_e =$$

$$= \frac{-\Delta P_e}{1/K_p + 1/R} = \frac{-\Delta P_e}{(D + 1/R)}$$

$\beta = (D + 1/R)$: Frequency Response characteristic

$$\boxed{\Delta f^{ss} = -\frac{\Delta P_e}{\beta}} \qquad \boxed{\Delta P_m = -\frac{\Delta f_{ss}}{R} = \frac{\Delta P_e}{\beta R}}$$

Example: $T_p = 20s$, $T_{ch} = T_t = 0.5s$, $T_G = 0.4s$

$K_p = 100$, $R = 3$

$\Delta P_e = 1\% = 0.01$ pu MW. $\qquad \beta = \frac{1}{100} + \frac{1}{3}$

$$\Delta f^{ss} = -\frac{0.01}{\left(\frac{1}{100} + \frac{1}{3}\right)} = -0.029 \text{ Hz}$$

$$\Delta P_m = \frac{+0.029}{3} = 0.0097 \text{ pu MW}$$

$$\Delta P_e(f) = \frac{1}{100} \times (-0.029) = -0.00029 \text{ pu MW}.$$

$$0.0097 \qquad (0.01 - 0.00029)$$

Without Governing loop: $R = \infty \Rightarrow 1/R = 0$

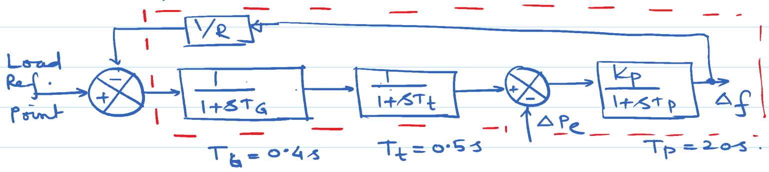
Without Governing loop: $R = \infty \Rightarrow 1/R = 0$

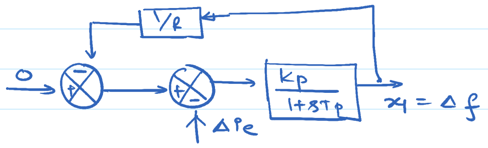
$$\Delta f^{ss} = \frac{-0.01}{\frac{1}{100}} = -1\ \text{Hz}$$

$$\Delta P_m = 0.$$

$$\Delta P_e(f) = D\,\Delta f^{ss} = -\frac{1}{100} \times 1 = -0.01\ \text{pu MW}.$$

Simplified / Approximate analysis of the loop Response





Approximate Block diagram

$$x_1 = \frac{K_p}{1+sT_p}\left[-\Delta P_e - \frac{x_1}{R}\right] \Rightarrow x_1 + T_p\dot{x}_1 = -K_p\Delta P_e - \frac{K_p}{R}x_1$$

$$\Rightarrow \dot{x}_1 T_p + x_1\left(1+\frac{K_p}{R}\right) = -K_p\Delta P_e \qquad \tau = \frac{T_p}{(1+K_p/R)}$$

$$x_1(t) = \frac{-K_p\,\Delta P_e}{(1+K_p/R)} + A e^{-\frac{t(1+K_p/R)}{T_p}}$$

At $t=0$, $x_1(0^+) = x_1(0^-) = 0 \Rightarrow A = \frac{K_p\,\Delta P_e}{1+K_p/R}$

$$\boxed{x_1(t) = \frac{K_p\,\Delta P_e}{1+K_p/R}\left[e^{-t/\tau} - 1\right]} \qquad \frac{-K_p\,\Delta P_e}{1+(K_p/R)}$$

i) **Steady state:** Frequency excursion is lower due to Governor feedback loop

ii) **Transient Performance:** $\tau = \frac{20}{1+\frac{100}{3}} = 0.583\ s.$ $(2.3-2.4)s$

Without the Governor feedback loop $\tau = \frac{20}{1} = 20s.$

$T_s \simeq 80s.$

Transient performance of the system is improved (faster response) due to the speed Governing loop.

Control system Requirements:

i) stable

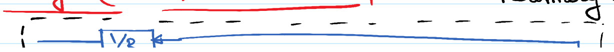
ii) Δf → steady state: $\Delta f^{ss} = 0$
→ transient state: $|\Delta f| \downarrow$

iii) $\int |\Delta f|\,dt \downarrow$

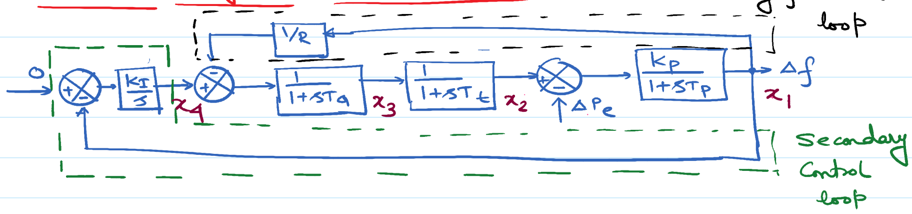
iv) Load sharing to be economically distributed among various generators present.

The Secondary (Reset) ALFC loop

Primary feedback loop



The Secondary (Reset) ALFC loop



$$x_1 = \frac{K_p}{1+sT_p}\left[x_2 - \Delta P_e\right] \Rightarrow x_1 + T_p\dot{x}_1 = K_p x_2 - K_p \Delta P_e$$

$$\Rightarrow \dot{x}_1 = -\frac{x_1}{T_p} + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}\Delta P_e \quad (1)$$

$$x_2 = \frac{1}{1+sT_t}x_3 \Rightarrow x_2 + T_t\dot{x}_2 = x_3 \Rightarrow \dot{x}_2 = -\frac{x_2}{T_t} + \frac{x_3}{T_t} \quad (2)$$

$$x_3 = \frac{1}{1+sT_G}\left[x_4 - \frac{x_1}{R}\right] \Rightarrow x_3 + T_G\dot{x}_3 = x_4 - \frac{x_1}{R}$$

$$\Rightarrow \dot{x}_3 = \frac{-x_1}{RT_G} - \frac{x_3}{T_G} + \frac{x_4}{T_G} \quad (3)$$

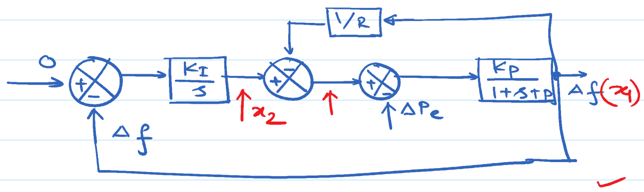
$$x_4 = \frac{-K_I}{s}x_1 \Rightarrow \dot{x}_4 = -K_I x_1 \quad (4)$$

Under steady state: $\Rightarrow x_1^{ss} = 0 \Rightarrow \boxed{\Delta f^{ss} = 0}$

$$0 = 0 + \frac{K_p}{T_p}x_2$$

$$\boxed{\Delta P_m^{ss} = \Delta P_e}$$

Approximate Analysis of the Loop Response



$$x_1 = \frac{K_p}{1+sT_p}\left[x_2 - \frac{x_1}{R} - \Delta P_e\right] \Rightarrow x_1 + T_p\dot{x}_1 = K_p x_2 - \frac{K_p x_1}{R} - K_p\Delta P_e$$

$$\Rightarrow \dot{x}_1 = +x_1\left(\frac{-K_p}{RT_p} - \frac{1}{T_p}\right) + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}\Delta P_e \quad (1)$$

$$x_2 = \frac{K_I}{s}\left[-x_1\right] \Rightarrow \dot{x}_2 = -K_I x_1 \quad (2)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{(K_p+R)}{RT_p} & \frac{K_p}{T_p}\\ -K_I & 0\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-\frac{K_p}{T_p}\\ 0\end{bmatrix}\Delta P_e$$

$$A = \begin{bmatrix} -\left(\frac{K_p+R}{RT_p}\right) & \frac{K_p}{T_p}\\ -K_I & 0\end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} \left(s + \frac{K_p+R}{RT_p}\right) & -\frac{K_p}{T_p}\\ K_I & s\end{bmatrix}$$

Charac. Eqn: $|sI - A| = s^2 + s\left(\frac{K_p+R}{RT_p}\right) + \frac{K_p K_I}{T_p} = 0$

$$s_1 = \frac{-\frac{K_p+R}{RT_p} \pm \sqrt{\left(\frac{K_p+R}{RT_p}\right)^2 - 4\frac{K_p K_I}{T_p}}}{}$$

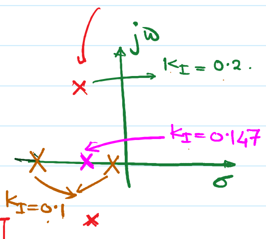
Charac. Eqn. $|sI-A|$

$$s_{1,2}=\frac{-\frac{K_p+R}{RT_p}\pm\sqrt{\left(\frac{K_p+R}{RT_p}\right)^2-4\frac{K_pK_I}{T_p}}}{2}$$

$$\Rightarrow s_{1,2}=-\frac{K_p+R}{2RT_p}\pm\sqrt{\frac{1}{4}\left(\frac{K_p+R}{RT_p}\right)^2-\frac{K_pK_I}{T_p}}$$

Case I: Complex Conjugate Poles (underdamped system)

$K_I > K_{cr}$



$$\frac{1}{4}\left(\frac{K_p+R}{RT_p}\right)^2-\frac{K_pK_I}{T_p}<0$$

$$\Rightarrow \frac{K_pK_I}{T_p}>\frac{1}{4}\frac{(K_p+R)^2}{R^2T_p^2}$$

$$\Rightarrow K_I>\frac{1}{4}\frac{(K_p+R)^2}{R^2T_pK_p}$$

$$\Rightarrow \boxed{K_I>\frac{1}{4T_pK_p}\left(1+K_p/R\right)^2}$$

Roots: $-\alpha_0\pm jw_0 \rightarrow x(t)=\underbrace{e^{-\alpha_0 t}\cos(w_0t+\theta)}$.

$T_p=20s$, $K_p=100$, $R=3$.

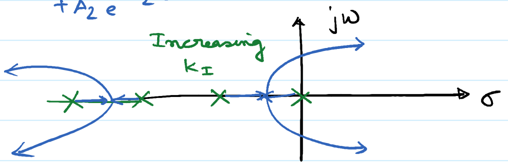
$$\frac{1}{4\times 20\times 100}\left(1+\frac{100}{3}\right)^2=0.147$$

Case II: $K_I=\frac{1}{4T_pK_p}\left(1+\frac{K_p}{R}\right)^2$ $\Rightarrow$ Repeated real poles
$\rightarrow$ Critically damped Response

$x(t)=(A_1+A_2t)e^{-\alpha_0 t}$ $\quad K_I=0.147$

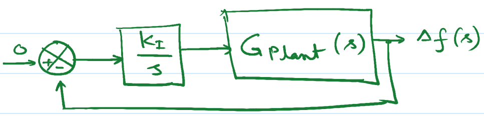
Case III: $K_I<\frac{1}{4T_pK_p}\left(1+\frac{K_p}{R}\right)^2$ $\Rightarrow$ Distinct Real poles
$\Rightarrow$ overdamped system

$x(t)=A_1e^{-\alpha_1 t}+A_2e^{-\alpha_2 t}$



$K_I\uparrow \rightarrow$ The system damping reduces, and may become unstable for very large values of K_I.

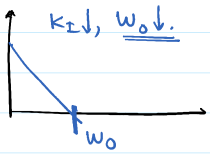
$K_I\downarrow\rightarrow$



$$G_I(s)=\frac{K_I}{s}=\frac{1}{sT_I}=\frac{1}{s/w_0}$$

$T_I=1/K_I$, $w_0=K_I$. $\rightarrow K_I\downarrow \rightarrow T_I\uparrow \Rightarrow$ Slow acting

$K_I\downarrow$, $w_0\downarrow$.



Higher archaical Control

Control level	Function	Speed of Response	Comments
1. Primary	Coarse freq adjustment and load tracking	2.0 to 20s	Local measurement
2. Secondary	Fine adjustment	1 minute	Local measurement

of freq/ freq Restoration

3. Tertiary — Economic operation — ~5 minutes. — Communication channel reqd.

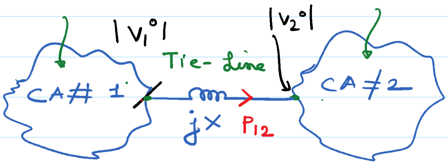
Concept of Control Area:

Large system ⟶ National Grid.
↳ break into smaller systems (Regional grid/ utilities)

Control area: All generating units in a control area are tightly coupled and swing in unision to a load change or other disturbances. In other words, all generators in a control area are characterised by a common frequency both in steady state and transient condition.

You can use a single lumped Generator-turbine-Governor model to represent all units present in a single Control area.

Pool operation:



$$P_{12} = \frac{|V_1||V_2|}{X}\sin(\delta_1-\delta_2) \qquad P_{12}^{\circ} \longrightarrow \delta_1^{\circ}, \delta_2^{\circ}, |V_1^{\circ}|, |V_2^{\circ}|$$

$$\Rightarrow P_{12}^{\circ} + \Delta P_{12} = \frac{|V_1^{\circ}||V_2^{\circ}|}{X}\sin(\delta_1^{\circ}+\Delta\delta_1-\delta_2^{\circ}-\Delta\delta_2)$$

$$\Rightarrow P_{12}^{\circ} + \Delta P_{12} = \frac{|V_1^{\circ}||V_2^{\circ}|}{X}\Big[\sin(\delta_1^{\circ}-\delta_2^{\circ})\underbrace{\cos(\Delta\delta_1-\Delta\delta_2)}_{\simeq 1} + \cos(\delta_1^{\circ}-\delta_2^{\circ})\underbrace{\sin(\Delta\delta_1-\Delta\delta_2)}_{\simeq(\Delta\delta_1-\Delta\delta_2)}\Big]$$

$$\Rightarrow \cancel{P_{12}^{\circ}} + \Delta P_{12} = \cancel{\frac{|V_1^{\circ}||V_2^{\circ}|}{X}\sin(\delta_1^{\circ}-\delta_2^{\circ})} + \frac{|V_1^{\circ}||V_2^{\circ}|}{X}\cos(\delta_1^{\circ}-\delta_2^{\circ})(\Delta\delta_1-\Delta\delta_2)$$

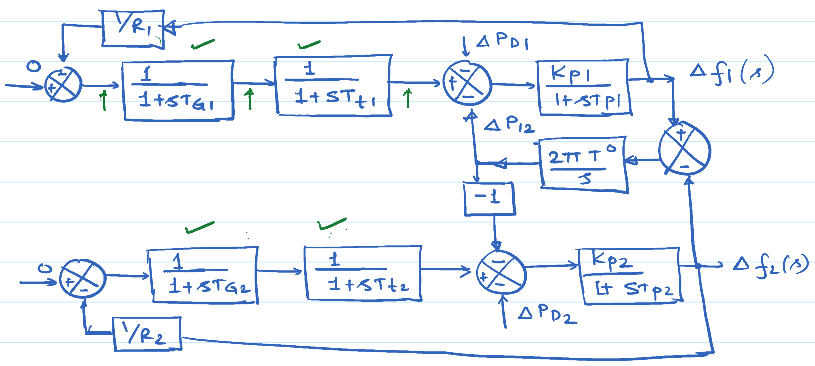
$$\Rightarrow \Delta P_{12} = \frac{|V_1^{\circ}||V_2^{\circ}|}{X}\cos(\delta_1^{\circ}-\delta_2^{\circ})(\Delta\delta_1-\Delta\delta_2)$$

T° = synchronizing coefficient of the tie-line
= Stiffness coefficient of the tie-line
$\triangleq \frac{|V_1^{\circ}||V_2^{\circ}|}{X}\cos(\delta_1^{\circ}-\delta_2^{\circ})$

$$\Rightarrow \Delta P_{12} = T^{\circ}(\Delta\delta_1-\Delta\delta_2)$$

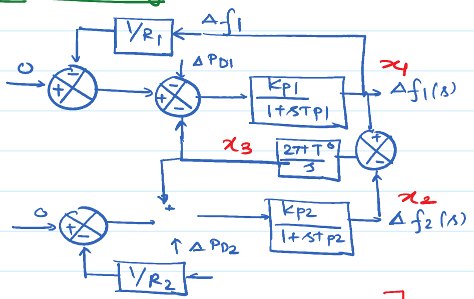
$$\Delta\delta = \int 2\pi\,\Delta f\,dt \Rightarrow \Delta\delta(s) = \frac{2\pi}{s}\Delta f(s)$$

$$\therefore \boxed{\Delta P_{12}(s) = \frac{2\pi T^{\circ}}{s}\left(\Delta f_1(s) - \Delta f_2(s)\right)}$$



Block - Diagram of two Control area System

Approximate Analysis



$$x_1 = \frac{K_{P1}}{1+sT_{P1}}\left[-\Delta P_{D1} - x_3 - \frac{x_1}{R_1}\right]$$

$$\Rightarrow x_1 + T_{P1}\dot{x}_1 = -K_{P1}\Delta P_{D1} - \frac{K_{P1}}{R_1}x_1 - K_{P1}x_3$$

$$\Rightarrow \dot{x}_1 = -\frac{1}{T_{P1}}\left(1+\frac{K_{P1}}{R_1}\right)x_1 - \frac{K_{P1}}{T_{P1}}x_3 - \frac{K_{P1}}{T_{P1}}\Delta P_{D1} \quad (1)$$

$$\dot{x}_2 = -\frac{1}{T_{P2}}\left(1+\frac{K_{P2}}{R_2}\right)x_2 + \frac{K_{P2}}{T_{P2}}x_3 - \frac{K_{P2}}{T_{P2}}\Delta P_{D2} \quad (2)$$

$$\dot{x}_3 = 2\pi T^0\left[x_1 - x_2\right] \quad (3)$$

$$D_1 + 1/R_1 = \beta_1$$

Steady - state: $(3) \Rightarrow x_1^{ss} = x_2^{ss}$

$$0 = +\frac{1}{T_{P1}}\left(1+\frac{K_{P1}}{R_1}\right)x_1^{ss} + \frac{K_{P1}}{T_{P1}}x_3^{ss} + \frac{K_{P1}}{T_{P1}}\Delta P_{D1}$$

$$\Rightarrow \left(\frac{1}{K_{P1}} + \frac{1}{R_1}\right)x_1^{ss} + x_3^{ss} + \Delta P_{D1} = 0$$

$$\Rightarrow \left(D_1 + \frac{1}{R_1}\right)x_1^{ss} + x_3^{ss} + \Delta P_{D1} = 0$$

$$\Rightarrow \beta_1 x_1^{ss} + x_3^{ss} + \Delta P_{D1} = 0 \quad (4)$$

$$\beta_2 x_1^{ss} - x_3^{ss} + \Delta P_{D2} = 0 \quad (5)$$

$$(\beta_1+\beta_2)x_1^{ss} = -(\Delta P_{D1} + \Delta P_{D2})$$

$$\Rightarrow x_1^{ss} = \frac{-(\Delta P_{D1}+\Delta P_{D2})}{\beta_1+\beta_2}$$

$$\boxed{\Delta f^{ss} = \frac{-(\Delta P_{D1}+\Delta P_{D2})}{\beta_1+\beta_2}}$$

$$(4)-(5) \Rightarrow \beta_1 x_1^{ss} + x_3^{ss} + \Delta P_{D1} - \beta_2 x_1^{ss} + x_3^{ss} - \Delta P_{D2} = 0$$

$$\Rightarrow 2x_3^{ss} = -\beta_1 x_1^{ss} + \beta_2 x_1^{ss} - \Delta P_{D1} + \Delta P_{D2}$$

$$2x_3^{ss} = -\frac{(\beta_2-\beta_1)(\Delta P_{D1}+\Delta P_{D2})}{\beta_1+\beta_2} - \Delta P_{D1} + \Delta P_{D2}$$

$$\Rightarrow 2x_3^{ss} = \frac{-\beta_2\Delta P_{D1} - \cancel{\beta_2\Delta P_{D2}} + \beta_1\cancel{\Delta P_{D1}} + \beta_1\Delta P_{D2} - \cancel{\Delta P_{D1}\beta_1} - \Delta P_{D1}\beta_2 + \Delta P_{D2}\beta_1 + \cancel{\Delta P_{D2}\beta_2}}{(\beta_1+\beta_2)}$$

$$\Rightarrow 2x_3^{ss} = \frac{2(-\Delta P_{D1}\beta_2 + \Delta P_{D2}\beta_1)}{\beta_1+\beta_2} \Rightarrow x_3^{ss} = \frac{-\Delta P_{D1}\beta_2 + \Delta P_{D2}\beta_1}{\beta_1+\beta_2}$$

Summary: $\Delta f^{ss} = \frac{-(\Delta P_{D1}+\Delta P_{D2})}{\beta_1+\beta_2}$ $\quad \Delta P_{12}^{ss} = \frac{-\Delta P_{D1}\beta_2 + \Delta P_{D2}\beta_1}{\beta_1+\beta_2}$

Problem: Two Control area system:

Control Area 1
Base : 2GW

Control area 2
Base: 10GW

Problem: Two Control area system:

Control Area 1

Base: 2 GW

$R = 2.40\ Hz/pu\ MW$

$D = 8.33\times10^{-3}\ pu\ MW/Hz$

Control area 2

Base: 10 GW

$R = 2.40\ Hz/pu\ MW$

$D = 8.33\times10^{-3}\ pu\ MW/Hz$

Load increase of 20 MW in Control area 1:

Δf^{ss}, $\Delta P_{12}^{ss} = ?$

Solution: $\Delta P_{D1} = \dfrac{20}{2\times10^3} = 0.01$ pu MW on a 2 GW base

$\Delta P_{D2} = 0$ pu MW on a 2 GW Base

$\beta_1 = D_1 + 1/R_1 = \left(8.33\times10^{-3} + \dfrac{1}{2.40}\right)$

$= 0.425$ pu MW/Hz. on a 2 GW Base

$\beta_2 = D_2 + 1/R_2 = 0.425$ pu MW/Hz on a 10 GW base

$= (5\times 0.425)$ pu MW/Hz on a 2 GW base $= 2.125$ pu MW/Hz on a 2 GW base.

$\Delta f^{ss} = \dfrac{-0.01}{0.425+2.125}\ Hz = -0.00392\ Hz$

$\Delta P_{12}^{ss} = -\dfrac{(0.01\times 2.125)}{0.425+2.125}$ pu MW on a 2 GW base

$= -8.33\times10^{-3}$ pu MW on a 2 GW base

$= -8.33\times10^{-3}\times 2\times10^{3}\ MW = \underline{-16.67}\ MW.$

Single Area operation: $\Delta f^{ss} = \dfrac{-0.01}{0.425}\ Hz = -0.023\ Hz.$

Control Area-2: Very very large: (1000 GW) Infinite Bus

$\beta_2 = (0.425\times 500) = 212.5$ pu MW/Hz on a 2 GW base

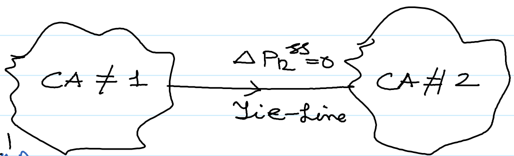
$\Delta f^{ss} = \dfrac{-0.01}{0.425+212.5} = -4.69\times10^{-5}\ Hz.$

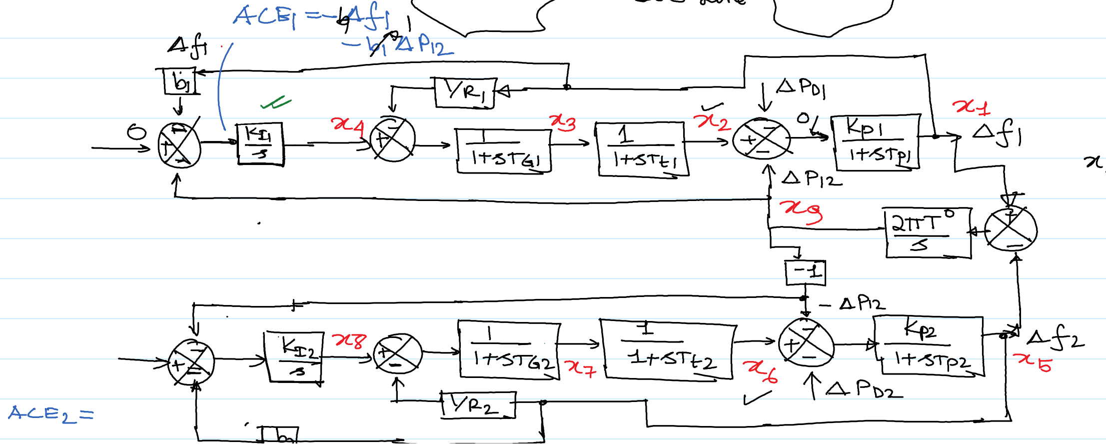
Objectives/Goals of Pool operation:

i) $\Delta f^{ss} = 0$

ii) $\Delta P_{12}^{ss} = 0$

iii) Economic operation.





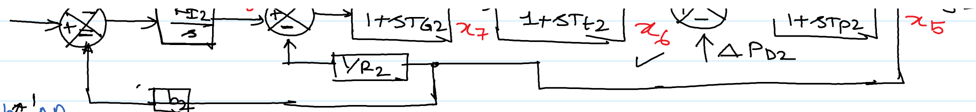
$D\Delta f^{ss} = 0.$

ΔP_{D1}

$x_2 - \Delta P_{12}^{ss} - \Delta P_{D1} = 0$

$\Rightarrow \boxed{x_2^{ss} = \Delta P_{D1}}$

$\boxed{x_6^{ss} = \Delta P_{D2}}$



$ACE_2 = -b_2\Delta f_2 - b_2'\Delta P_{21}$ $\Delta f_2(s)$

Block Diagram

$$x_1 = \frac{K_{P1}}{1+sT_{P1}}\left[x_2 - \Delta P_{D1} - x_9\right] \Rightarrow x_1 + T_{P1}\dot{x}_1 = K_{P1}x_2 - K_{P1}\Delta P_{D1} - K_{P1}x_9$$

$$\Rightarrow \dot{x}_1 = -\frac{x_1}{T_{P1}} + \frac{K_{P1}}{T_{P1}}x_2 - \frac{K_{P1}}{T_{P1}}x_9 - \frac{K_{P1}}{T_{P1}}\Delta P_{D1} \quad (1)$$

$$\dot{x}_5 = -\frac{x_5}{T_{P2}} + \frac{K_{P2}}{T_{P2}}x_6 + \frac{K_{P2}}{T_{P2}}x_9 - \frac{K_{P2}}{T_{P2}}\Delta P_{D2} \quad (2)$$

$$x_2 = \frac{1}{1+sT_{t1}}x_3 \Rightarrow x_2 + T_{t1}\dot{x}_2 = x_3 \Rightarrow \dot{x}_2 = -\frac{x_2}{T_{t1}} + \frac{x_3}{T_{t1}} \quad (3)$$

$$\Rightarrow \dot{x}_6 = -\frac{x_6}{T_{t2}} \quad (4)$$

$$x_3 = \frac{1}{1+sT_{G1}}\left[x_4 - \frac{x_1}{R_1}\right] \Rightarrow x_3 + T_{G1}\dot{x}_3 = x_4 - \frac{x_1}{R_1}$$

$$\Rightarrow \quad = -\frac{x_1}{R_1T_{G1}} - \frac{}{T_{G1}} + \frac{x_4}{T_{G1}} \quad (5)$$

$$\dot{x}_7 = -\frac{x_5}{R_2T_{G2}} - \frac{x_7}{T_{G2}} + \frac{x_8}{T_{G2}} \quad (6)$$

$$x_4 = \frac{K_{I1}}{s}\left[0 - b_1x_1 - x_9\right] \Rightarrow \dot{x}_4 = -K_{I1}b_1x_1 - K_{I1}x_9 \quad (7)$$

$$\dot{x}_8 = -K_{I2}b_2x_5 + K_{I2}x_9 \quad (8)$$

$$x_9 = \frac{2\pi T^0}{s}(x_1 - x_5) \Rightarrow \dot{x}_9 = 2\pi T^0(x_1 - x_5) \quad (9)$$

Steady-state: $\circlearrowleft^{ss} \rightarrow$ drop. $\quad \dot{x}_9 = 0 \Rightarrow x_1 = x_5$

$\therefore$ Under steady state frequencies of the two systems are the same.

$$\left.\begin{aligned} K_{I1}x_1 + K_{I1}b_1x_9 &= 0 \\ K_{I2}x_5 - K_{I2}b_2x_9 &= 0 \end{aligned}\right\}$$

$$\Rightarrow K_{I2}x_1 - K_{I2}b_2x_9 = 0$$

$$\Rightarrow \begin{aligned} x_1 + b_1x_9 &= 0 \quad (A) \\ x_1 - b_2x_9 &= 0 \quad (B) \end{aligned}$$

$$(A) - (B) \Rightarrow \cancel{x_1} + b_1x_9 - \cancel{x_1} + b_2x_9 = 0$$

$$\Rightarrow (b_1 + b_2)x_9 = 0 \quad \forall (b_1 \text{ and } b_2) \neq 0$$

$$x_9 = 0 \Rightarrow \left.\begin{aligned} \Delta P_{12}^{ss} &= 0. \\ \Rightarrow \Delta f^{ss} &= 0 \end{aligned}\right\}$$ objectives (i) and (ii) have been met.

$(A) \Rightarrow x_1 = 0$

Historically, Increase in Generation in each Control Area to meet objectives (i) and (ii) is known as Area Control Error (ACE)

When: $\left.b_1 = \beta_1 \quad b_2 = \beta_2\right\}$ $\quad P = Bf. \quad B = D + 1/R$

$$ACE_1 = -b_1\Delta f_1 - \Delta P_{12}$$

$$\Rightarrow + b_1\frac{(\Delta P_{D1} + \Delta P_{D2})}{\beta_1 + \beta_2} - \frac{(-\Delta P_{D1}\beta_2 + \Delta P_{D2}\beta_1)}{\beta_1 + \beta_2} = \Delta P_{D1}$$

$$\Rightarrow \frac{b_1\Delta P_{D1} + b_1\Delta P_{D2} + \Delta P_{D1}\beta_2 - \Delta P_{D2}\beta_1}{} = \Delta P_{D1}$$

$$\Rightarrow \frac{b_1 \Delta P_{D1} + b_1 \Delta P_{D2} + \Delta P_{D1}\beta_2 - \Delta P_{D2}\beta_1}{\beta_1+\beta_2} = \Delta P_{D1}$$

$$\Rightarrow b_1 \Delta P_{D1} + b_1 \Delta P_{D2} + \cancel{\Delta P_{D1}\beta_2} - \Delta P_{D2}\beta_1 = \Delta P_{D1}\beta_1 + \cancel{\Delta P_{D1}\beta_2}$$

$$\Rightarrow \Delta P_{D1}(b_1 - \beta_1) + \Delta P_{D2}(b_1-\beta_1) = 0$$

$$\boxed{b_1 = \beta_1}$$

$$ACE_2 = -b_2 \Delta f_2 + \Delta P_{12} = \Delta P_{D2}$$

$$= +\frac{b_2(\Delta P_{D1} + \Delta P_{D2})}{\beta_1+\beta_2} + \frac{-\Delta P_{D1}\beta_2 + \Delta P_{D2}\beta_1}{\beta_1+\beta_2} = \Delta P_{D2}$$

$$\Rightarrow b_2 \Delta P_{D1} + b_2 \Delta P_{D2} - \beta_2 \Delta P_{D1} + \cancel{\beta_1 \Delta P_{D2}} = \cancel{\Delta P_{D2}\beta_1} + \Delta P_{D2}\beta_2$$

$$\Rightarrow \Delta P_{D1}(b_2 - \beta_2) + \Delta P_{D2}(b_2 - \beta_2) = 0.$$

$$\boxed{b_2 = \beta_2}$$

Implementation of time domain simulation

$$r \longrightarrow \boxed{\frac{K}{1+sT}} \longrightarrow x$$

$$x = \frac{K}{1+sT} r \Rightarrow x + T\dot{x} = Kr$$

$$\Rightarrow \dot{x} = -\frac{x}{T} + \frac{K}{T} r$$

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{x}{T} + \frac{K}{T} r$$

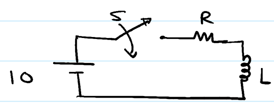
Discretization: ΔT $\quad k \quad k+1$, $\leftarrow \Delta T \rightarrow$

$$\frac{x(k+1) - x(k)}{\Delta T} = -\frac{x(k)}{T} + \frac{K}{T} r(k)$$

$$\Rightarrow \boxed{x(k+1) = x(k) + \frac{\Delta T}{T}\left[-x(k) + K\, r(k)\right]}$$

$K = 1/R$

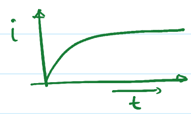
$T = L/R$.



$$V = iR + L\frac{di}{dt}$$

$$V \longrightarrow \boxed{\frac{1}{R+sL}} \longrightarrow i. \qquad V \longrightarrow \boxed{\frac{1/R}{1+sL/R}} \longrightarrow i$$

$$i(k+1) = i(k) + \frac{\Delta T}{L/R}\left[-i(k) + \frac{1}{R} V(k)\right]$$



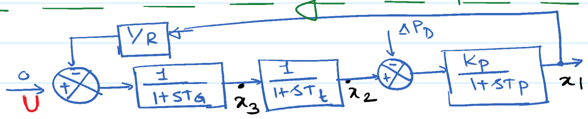
$$r \longrightarrow \boxed{\frac{K_I}{s}} \longrightarrow x$$

$$x = \frac{K_I}{s} r \Rightarrow \dot{x} = K_I r$$

$$\Rightarrow \frac{x(k+1) - x(k)}{\Delta T} = K_I r(k)$$

$$\Rightarrow x(k+1) = x(k) + \Delta T\, K_I\, r(k)$$

Time domain simulation using state-space Model



$$x_1 = \frac{K_P}{1+sT_P}\left[x_2 - \Delta P_D\right] \Rightarrow x_1 + T_P\dot{x}_1 = K_P x_2 - K_P\Delta P_D$$

$$\Rightarrow \dot{x}_1 = -\frac{x_1}{T_P} + \frac{K_P}{T_P}x_2 - \frac{K_P}{T_P}\Delta P_D \quad (1)$$

$$x_2 = \frac{1}{1+sT_t}x_3 \Rightarrow x_2 + T_t\dot{x}_2 = x_3 \Rightarrow \dot{x}_2 = -\frac{x_2}{T_t} + \frac{x_3}{T_t} \quad (2)$$

$$x_3 = \frac{1}{1+sT_G}\left[U - \frac{x_1}{R}\right] \Rightarrow x_3 + T_G\dot{x}_3 = -\frac{x_1}{R} + U$$

$$\Rightarrow \dot{x}_3 = -\frac{x_1}{RT_G} - \frac{x_3}{T_G} + \frac{U}{T_G} \quad (3)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \underbrace{\begin{bmatrix}-\frac{1}{T_P} & \frac{K_P}{T_P} & 0\\ 0 & -\frac{1}{T_t} & \frac{1}{T_t}\\ -\frac{1}{RT_G} & 0 & -\frac{1}{T_G}\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ 1/T_G\end{bmatrix}}_{B}\left[U\right] + \underbrace{\begin{bmatrix}-\frac{K_P}{T_P}\\ 0\\ 0\end{bmatrix}}_{\Gamma}\left[\Delta P_D\right]$$

$$y = \underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \underbrace{0}_{D}\cdot U$$

Approximate Discretization of the state space Equations

T:

$$\dot{X} = AX + BU + \Gamma P \qquad Y(k) = C_d X(k) \qquad C_d = C.$$

$$\Rightarrow \frac{X(k+1) - X(k)}{T} = AX(k) + BU(k) + \Gamma P(k)$$

$$\Rightarrow X(k+1) = X(k)\left[I + AT\right] + BT\,U(k) + \Gamma T\,P(k)$$

$$\Rightarrow X(k+1) = A_d X(k) + B_d U(k) + \Gamma_d P(k) \rightarrow \text{Recursively solve this}$$

$$\boxed{A_d \triangleq (I + AT) \qquad B_d \triangleq BT \qquad \Gamma_d = \Gamma T}$$

More accurate discretization Approach

$$e^{At} = \left[I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \cdots\right]$$

$$\dot{X} = AX + BU$$

$$\Rightarrow X(t) = e^{At}X(0) + \int_0^t e^{A(t-\tau)}BU(\tau)\,d\tau$$

At $t = kT$

$$\Rightarrow X(kT) = e^{AkT}X(0) + \int_0^{kT} e^{A(kT-\tau)}BU(\tau)\,d\tau$$

Assume that the control force $U(\tau)$ remains the same in the time interval kT to $(k+1)T$

kT $(k+1)T$

$$X((k+1)T) = e^{A(k+1)T}X(0) + \int_0^{kT} e^{A((k+1)T-\tau)}BU(\tau)\,d\tau + \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)}BU(kT)\,d\tau$$

$$X(k+1) \triangleq X((k+1)T) \qquad X(k) \triangleq X(kT)$$

$$\Rightarrow X(k+1) = e^{AT}\left[\underbrace{e^{AkT}X(0) + \int_0^{kT} e^{A(kT-\tau)}BU(\tau)\,d\tau}\right] + \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)}BU(kT)\,d\tau$$

$$\Rightarrow x(k+1) = \underbrace{e^{AT}}_{A_d} x(k) + I_2$$

$$I_2 = \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} \underbrace{B\,u(kT)}\, d\tau = \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)}\, d\tau \; B\,u(kT)$$

$$\alpha \triangleq (k+1)T - \tau \Rightarrow d\alpha = -d\tau$$

when $\tau = kT \Rightarrow \alpha = T$

$\tau = (k+1)T \Rightarrow \alpha = 0$

$$\Rightarrow I_2 = \int_{T}^{0} e^{A\alpha} (-d\alpha)\, B\,u(kT) = \int_{0}^{T} e^{A\alpha}\, d\alpha \; B\,u(kT)$$

$$\Rightarrow I_2 = \int_{0}^{T} \left[e^{A\tau}\, d\tau \right] B\,u(kT) = \int_{0}^{T} \left(I + A\tau + \frac{A^2\tau^2}{2!} + \frac{A^3\tau^3}{3!} + \dots \right) d\tau \; B\,u(kT)$$

$$\Rightarrow I_2 = \left[TI + \frac{AT^2}{2!} + \frac{A^2T^3}{3!} + \dots\dots \right] B\,u(kT)$$

$$\Rightarrow I_2 = A^{-1} \left[\underbrace{AT + \frac{A^2T^2}{2!} + \frac{A^3T^3}{3!} + \dots + I} - I \right] B\,u(kT)$$

$$\Rightarrow I_2 = A^{-1} \left[e^{AT} - I \right] B\,u(kT) = A^{-1} \left[A_d - I \right] B\,u(kT)$$

$$\left. \begin{aligned} x(k+1) &= A_d\, x(k) + B_d\, u(k) \\ y(k+1) &= C_d\, x(k+1) + D_d\, u(k+1) \end{aligned} \right\}$$

$$A_d = e^{AT} \qquad B_d = A^{-1}\left[A_d - I\right] B$$

$$C_d = C \qquad D_d = D \qquad \Gamma_d = A^{-1}\left[A_d - I\right] \Gamma$$

$$A_d = e^{AT} = \left[I + AT + \overbrace{\frac{A^2T^2}{2!} + \frac{A^3T^3}{3!}} + \dots \right]$$

$$\simeq (I + AT)$$

$$B_d \approx A^{-1} \left[\cancel{I} + AT - \cancel{I} \right] B = TB$$

<u>MATLAB</u>: c2d

```
sys = ss(A,B,C,D)
dis_sys = c2d(sys,T)
```

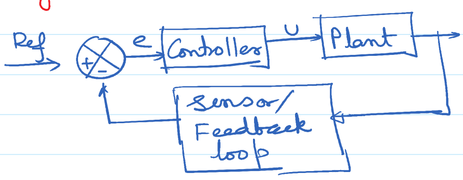
$$e^{AT} = \text{expm}(A*T) = I + AT + \frac{A^2T^2}{2!} + \frac{A^3T^3}{3!} + \dots\dots$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0\,u.$$

LQR (Linear Quadratic Regulator)

Optimal Control ?



Puts a limit on the control force (U), which in turn will minimize the wear and tear in the actuator

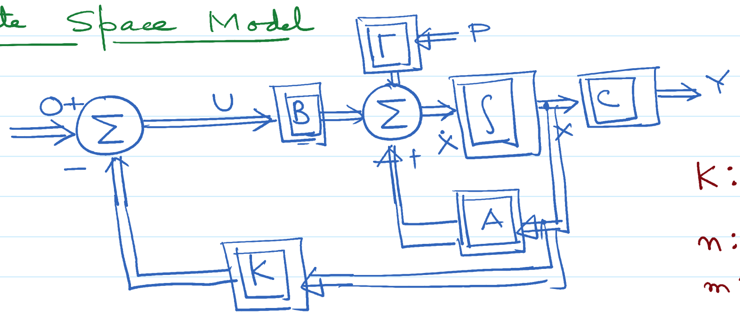
Steps of designing an optimal Controller

1. Derive the state space Model of the plant

2. Define a Control objective (J) [Integral squared error Index]

3. Find the structure of the optimal Controller.

(1) State Space Model



Block Diagram

K : Gain Matrix

n : No. of states

m : No of Control inputs

(2) Control Objective :

$$\text{Min } J = \int_0^\infty \left\{ (q_1 x_1^2 + q_2 x_2^2 + \cdots + q_n x_n^2) + (r_1 u_1^2 + r_2 u_2^2 + \cdots + r_m u_m^2) + 2 n_1 x_1 u_1 + \cdots \right\} dt$$

$q_i \rightarrow$ weight on the i^{th} state

$r_j \rightarrow$ weight on the j^{th} Control input

$q, r \rightarrow$ Positive penalty factors

$$Q \triangleq \begin{bmatrix} q_1 & & & \\ & q_2 & & \\ & & \ddots & \\ & & & q_n \end{bmatrix}_{n \times n} \qquad R \triangleq \begin{bmatrix} r_1 & & & \\ & r_2 & & \\ & & \ddots & \\ & & & r_m \end{bmatrix}$$

$$\text{Min } J = \int_0^\infty (X^T Q X + U^T R U)\, dt$$

(3) Structure of the Controller:

Control forces are expressed as a linear Combination of system states.

$$u_1 = -k_{11} x_1 - k_{12} x_2 - \cdots - k_{1n} x_n$$

$$u_2 = -k_{21} x_1 - k_{22} x_2 - \cdots - k_{2n} x_n$$

$$\vdots$$

$$u_m = -k_{m1} x_1 - k_{m2} x_2 - \cdots - k_{mn} x_n$$

$$K = \text{Gain Matrix} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & & & \vdots \end{bmatrix}$$

$$K = \text{Gain Matrix} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & & & \\ k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix}_{m \times n}$$

Solve Algebraic Ricatti Equation (ARE)

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$

$R, Q \rightarrow$ known/user defined

$A, B \rightarrow$ known (state space Model)

P (unknown) $\rightarrow$ obtained by Solving the ARE

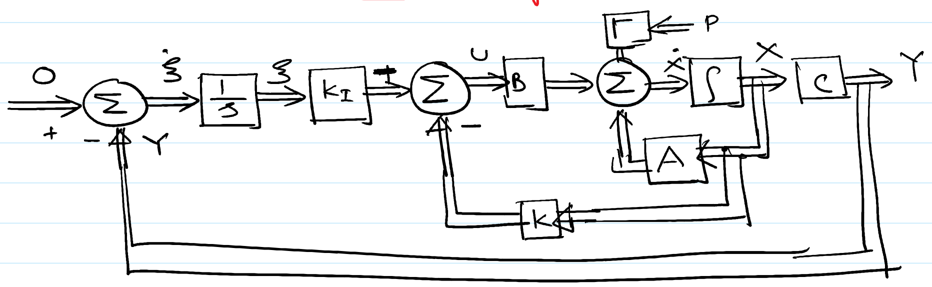
Solve for P:

$$\boxed{K = R^{-1}B^T P}$$

Limitations: i) (A, B) must be stabilizable
ii) $R > 0$
iii) No unobservable mode on imaginary axis

MATLAB: $[K, S, e] = lqr(sys, Q, R, N)$
where $sys = ss(A, B, C, D)$

LQI (Linear Quadratic Integral) Control



Block Diagram

$$\dot{X} = AX + BU + \Gamma P \qquad \bar{X} \triangleq \begin{bmatrix} X \\ \xi \end{bmatrix}$$

$$\dot{\xi} = -Y = -CX$$

$$Y = CX$$

$$\dot{\bar{X}} = \begin{bmatrix} \dot{X} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} X \\ \xi \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} P$$

$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ \xi \end{bmatrix}$$

$$\left.\begin{aligned} \dot{\bar{X}} &= \bar{A}\bar{X} + \bar{B}U + \bar{\Gamma}P \\ Y &= \bar{C}\bar{X} \end{aligned}\right\} \text{Implement LQR Control}$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \qquad \bar{\Gamma} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$