2 CHAPTER

DESCRIPTIVE STATISTICS

1. Compute mean

Arrange data in ascending order

35, 35, 37, 39, 40, 41, 45, 55, 55, 55, 55, 55, 60, 65

$$\bar{x} = \frac{\Sigma x}{n} = \frac{35+35+37+39+40+41+45+55+55+55+55+55+60+65}{14} = 48$$

$$Md = \frac{n-1}{2}^{th} \text{ item} = \frac{14+1}{2} = 7.5^{th} \text{ item}$$

$$\text{Median value} = \frac{45+55}{2} = 50$$

M_O = Most repeated value = 55

2. The number of telephone calls

No. of calls (x)	0	1	2	3	4	5	6	
Frequency	15	22	28	35	42	34	24	$N = \sum f = 200$
c.f.	15	37	65	100	142	176	200	
fx	0	22	56	105	168	170	144	$\sum fx = 665$

$$\bar{x} = \frac{\sum fx}{N} = \frac{665}{200} = 3.325$$

$$M_d = \frac{N+1}{2} = \frac{200+1}{2} = 100.5^{th} \text{ items} = 4$$

M_0 = No. of calls with maximum frequency = 4

3. The length in hours of 100 VCA cable

Length on inch	3.80 – 3.89	3.90 – 3.99	4.00 – 4.09	4.10 – 4.19	4.20 – 4.29	4.30 – 4.39	4.40 – 4.49	5.50 – 5.59
Frequency (f)	3	8	14	19	28	18	10	8
c.f.	3	11	25	44	72	90	100	108

For mode,

Model class = 4.20 – 4.29

It is inclusive class, hence adjusted model class = (4.195 – 4.295)

$f_0 = 19, f_1 = 28, f_2 = 18$

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 4.195 + \frac{f_1 - f_0}{f_1 - f_0 + f_1 - f_2} \times h$$

$$= 4.195 + \frac{28-19}{28-19+28-18} \times 0.1 = 4.24$$

For Median $\frac{N}{2} = \frac{108}{2} = 54$

Median class = (4.20 – 4.29)

Adjusted median class = (4.195 – 4.295)

$$M_d = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 4.195 + \frac{45-44}{28} \times 0.1 = 4.23$$

4. Compute the mean, median and mode

IQ	Frequency (f)	Mid-value (x)	$d' = \frac{x - 104.5}{10}$	fd'	c.f.
50 – 59	2	54.5	– 5	– 10	2
60 – 69	4	64.5	– 4	– 16	6
70 – 79	8	74.5	– 3	– 24	14
80 – 89	15	84.5	– 2	– 30	29
90 – 99	21	94.5	– 1	– 21	50
100 – 109	25	104.5	0	0	75
110 – 119	20	114.5	1	20	95
119 – 129	2	124.5	2	4	97
130 – 139	2	134.5	3	6	99
140 – 149	1	144.5	4	4	100
				$\sum fd' = -67$	

$$\bar{x} = A + \frac{\sum fd'}{N} \times h = 104.5 + \frac{(-67)}{100} \times 10 = 97.8 \qquad \frac{N}{2} = \frac{100}{2} = 50,$$

$$Md = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 89.5 + \frac{50-25}{21} \times 10 = 99.5$$

Hence M_d class = (90 – 99)

Adjusted M_d class = 89.5 – 99.5

Model class = (100 – 109)

Adjusted model class = (99.5 – 109.5)

$f_0 = 21, f_1 = 25, f_2 = 20, \Delta_1 = f_1 - f_0 = 4, \Delta_2 = f_1 - f_2 = 5$

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 99.5 + \frac{4}{4+5} \times 10 = 103.94$$

5. The data relating to

Increase in wt	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
No. of animals (f)	3	7	18	35	20	8	5	4
c.f.	3	10	28	63	83	91	96	100

$\frac{N}{4} = \frac{100}{4} = 25 \quad \therefore Q_1$ class = (10 – 15)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 20 + \frac{25-10}{18} \times 5 = 14.166$$

$\frac{3N}{4} = 3 \times 25 = 75 \quad \therefore Q_3$ class = (20 – 25)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 20 + \frac{75-63}{20} \times 5 = 23$$

$\frac{9N}{10} = \frac{9 \times 100}{10} = 90 \quad \therefore D_9$ class = (25 – 30)

$$D_9 = L + \frac{\frac{9N}{10} - c.f.}{f} \times h = 25 + \frac{90-83}{8} \times 5 = 29.375$$

$$\frac{15\,N}{100} = \frac{15 \times 100}{100} = 15 \quad \therefore P_{15} \text{ class} = (10 - 15)$$

$$P_{15} = L + \frac{\frac{15\,N}{100} - c.f.}{f} \times h = 10 + \frac{15-10}{18} \times 5 = 11.388$$

$$\frac{90\,N}{100} = \frac{90 \times 100}{100} = 90 \quad \therefore P_{90} \text{ class} = (25 - 30)$$

$$P_{90} = L + \frac{\frac{90\,N}{100} - c.f.}{f} \times h = 25 + \frac{90-83}{8} \times 5 = 29.375$$

6. Random sample

Size of pen drive	No. of user (f)
4 – 5	10
5 – 6	35
6 – 7	70
7 – 8	35
8 – 9	12
9 – 10	6

Measure of central tendency in Mode.

Here modal class is (6 – 7)

$L = 6$, $f_0 = 35$, $f_1 = 70$, $f_2 = 35$, $\Delta_{12} = f_1 - f_0 = 70 - 35 = 35$

$\Delta_2 = f_1 - f_2 = 70 - 35 = 35$

$$M_O = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 6 + \frac{35}{35+35} \times 1 = 6.5$$

7. A software

$x_1 = 50, x_2 = 35, x_3 = 40, x_4 = 25, \bar{x}_1 = 30{,}000; \bar{x}_2 = 35{,}000, \bar{x}_3 = 40{,}000,$
$\bar{x}_4 = 50{,}000$

$$\bar{x}_{1234} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3 + n_4\bar{x}_4}{n_1 + n_2 + n_3 + n_4}$$

$$= \frac{50 \times 30000 + 35 \times 35000 + 40 \times 40000 + 25 \times 50000}{50 + 35 + 40 + 25} = 37166.67$$

8. The percentage age

Age	0-4	5-9	10–14	15–19	20–24	25–29	30–34
Male popⁿ (f)	11.8	12.9	12.5	11.2	10.7	8.9	7.2
c.f.	11.8	24.7	37.2	48.4	59.1	68.0	75.2

35-39	40-44	45-49	50-54	55-59	60 and above
6.2	4.7	4.0	2.9	2.3	4.7
81.4	86.1	90.1	93.0	95.3	100

$$\frac{N}{4} = \frac{100}{4} = 25; \text{ Hence } Q_1 \text{ class} = (10 - 14)$$

Adjusted Q_1 class = (9.5 – 14.5)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 9.5 + \frac{25 - 24.7}{12.5} \times 5 = 9.62$$

$$\frac{3\,N}{4} = 3 \times 25 = 75; \text{ Hence } Q_3 \text{ class} = (30 - 34)$$

Adjusted Q_3 class = (29.5 – 34.5)

$$Q_3 = L + \frac{\frac{3\,N}{4} - c.f.}{f} \times h = 29.5 + \frac{75-68}{7.2} \times 5 = 34.36$$

$$\frac{8N}{10} = \frac{8 \times 100}{10} = 80$$

D_8 Class = (35 – 39)

Adj. D_{80} class = (34.5 – 39.5)

$$D_8 = L + \frac{\frac{8\,N}{10} - cf}{f} \times h = 34.5 + \frac{80 - 75.2}{6.2} \times 5 = 38.37$$

$$\frac{70N}{100} = \frac{70 \times 100}{100} = 70$$

P_{70} class (30 – 34)

Adj. P_{70} class = (29.5 – 34.5)

$$P_{70} = L + \frac{\frac{70\,N}{100} - cf}{f} \times h = 29.5 + \frac{70-68}{7.2} \times 5 = 30.88$$

9.

Marks	No. of students (f)	c.f.
10 – 20	15	15
20 – 40	20	35
40 – 70	30	65
70 – 90	20	85
90 – 100	15	100
	$N = \Sigma f = 100$	

Highest marks of weakest 30% students is P_{30}

$$P_{30} = \frac{30\,N}{100}^{th} \text{ item} = \frac{30 \times 100}{100} = 30$$

P_{30} class = (20 – 40)

$$P_{30} = L + \frac{\frac{30\,N}{100} - c.f.}{f} \times h = 20 + \frac{30-15}{20} \times 20 = 35$$

Lowest marks of top 40% student is P_{60}

$$P_{60} = \frac{60\,N}{100} \text{ item} = \frac{60 \times 100}{100} = 60$$

P_{60} class = (40 – 70)

$$P_{60} = L + \frac{\frac{60\,N}{100} - c.f.}{f} \times h = 40 + \frac{60 - 35}{30} \times 30 = 65$$

Lowest marks of top 20% students P_{80}

$$P_{80} = \frac{80\,N}{100} \text{ item} = \frac{80 \times 100}{100} = 80$$

P_{80} class = (70 – 90)

$$P_{80} = L + \frac{\frac{80\,N}{100} - c.f.}{f} \times h = 70 + \frac{80 - 65}{20} \times 20 = 85$$

Limit of middle 50% student is P_{25} to P_{75}.

$$P_{25} = \frac{25\,N}{100}^{th} \text{ item} = \frac{25 \times 100}{100} = 25$$

P_{25} class = (20 – 40)

$$P_{25} = L + \frac{\frac{25\,N}{100} - c.f.}{f} \times h = 20 + \frac{25 - 15}{20} \times 20 = 30$$

$$P_{75} = \frac{75\,N}{100} = L + \frac{\frac{75\,N}{100} - cf}{f} \times h = 70 + \frac{75 - 65}{20} \times 20 = 80$$

Range = $P_{75} - P_{25}$ = 80 – 30 = 50

10. Compute the quartile deviation

Size of screen	No. of laptop (f)	c.f.
9.5	1	1
10.0	8	9
10.5	20	29
11.0	30	59
11.5	50	109
12.0	95	204
12.5	110	314
13.0	150	464
13.5	200	664
14.0	250	914
14.5	280	1194
15.0	245	1439
15.5	80	1519
16.0	40	1559
16.5	35	1594
17.0	5	1599
	N = ∑f = 1599	

$$Q_1 = \left(\frac{N+1}{4}\right)^{th} \text{ item} = \frac{1599+1}{4} = \frac{16000}{4} = 400 \quad \therefore Q_1 = 13,$$

$$Q_3 = \left(\frac{3\,(N+1)}{4}\right)^{th} \text{ item} = 3 \times 400 = 1200 \quad \therefore Q_3 = 15$$

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{15 - 13}{2} = 1$$

11. The following frequency

Weight	4–5	5–6	6–7	7–8	8–9	9–10	10-11	11-12
Freq. (f)	20	24	35	48	32	24	8	2
c.f.	20	44	79	127	159	183	191	193

$$\frac{N}{4} = \frac{193}{4} = 48.25 \quad \therefore Q_1 \text{ class} = (6 - 7)$$

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 6 + \frac{48.25 - 44}{35} \times 1 = 6.12$$

$$\frac{2\,N}{4} = 2 \times 48.25 = 96.5 \quad \therefore Q_2 \text{ class} = (7 - 8)$$

$$Q_2 = L + \frac{\frac{2\,N}{4} - c.f.}{f} \times h = 7 + \frac{96.5 - 79}{48} \times 1 = 7.36$$

$$\frac{3\,N}{4} = 3 \times 48.25 = 144.75 \quad \therefore Q_3 \text{ class} = (8 - 9)$$

$$Q_3 = L + \frac{\frac{3\,N}{4} - c.f.}{f} \times h = 8 + \frac{144.75 - 127}{32} \times 1 = 8.55$$

QD = $(Q_3 - Q_1)/2$ = (8.55 – 6.12)/2 = 1.215

12. The scores obtained

L = 65, S = 35

Range = L – S = 65 – 35 = 30

Score (x)	x^2
55	3025
35	1225
60	3600
55	3025
55	3025
65	4225
40	1600
45	2025
35	1225
42	1764
Σx = 487	Σx² = 24739

$$S.D. = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{24739}{10} - \left(\frac{487}{10}\right)^2} = 10.109$$

There is collision vacant of 7i.5 O

13. Arrange data in ascending order
32, 34, 34, 35, 35, 35, 36
Range = L – S = 36 – 32 = 4 $\quad$ n = 7

$Q_1 = \frac{n+1}{4}$ th item $= \frac{7+1}{4}$ = 2nd item = 34

$Q_3 = \frac{3(n+1)}{4}$ th item = 3 × 2 = 6th item = 35

$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{35-34}{1} = 0.5$

14. The time in minutes

x	10	12	11	15	18	20	24	23	26	16
$\lvert x-\bar{x}\rvert$	7.5	5.5	6.5	2.5	0.5	2.5	6.5	5.5	8.5	1.5
x^2	100	144	121	225	324	400	576	529	676	256

$\sum x = 175, \sum \lvert x-\bar{x}\rvert = 47, \sum x^2 = 3351$

$\bar{x} = \frac{\sum x}{n} = \frac{175}{10} = 17.5$

M.D. from $\bar{x} = \frac{\sum\lvert x-\bar{x}\rvert}{n} = \frac{47}{10} = 4.7$

S.D. $= \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{3351}{10} - (17.5)^2} = 5.37$

15. To obtain information

Customer service time	fd'^2	No. of customers (f)	Mid-value (x)	$d' = \frac{x-17.5}{5}$	fd'
0 – 5	18	2	2.5	– 3	– 6
5 – 10	32	3	7.5	– 2	– 16
10 – 15	26	26	12.5	– 1	– 26
15 – 20	0	30	17.5	0	0
20 – 25	28	28	22.5	1	28
25 – 30	24	6	27.5	2	12
	$\sum fd'^2 = 128$	$N = \sum f = 100$			$\sum fd' = -8$

$\sigma = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times h = \sqrt{\frac{128}{100} - \left(\frac{-8}{100}\right)^2} \times 5 = 5.64$

16. The frequency distribution

Time in sec	No. of customers (f)	Mid-value (x)	$d' = \frac{x-17}{5}$	fd'	fd'^2
0 - 4	2	2	- 3	- 6	18
5 - 9	20	7	- 2	- 40	80
10 - 14	35	12	- 1	- 35	35
15 - 19	40	17	0	0	0
20 - 24	28	22	1	28	28
25 - 29	32	27	2	64	128
30 - 34	8	32	3	24	72
35 - 39	5	37	4	20	80
	170			55	441

$N = \sum f = 170, \sum fd' = 55, \sum fd'^2 = 441$

S.D $(\sigma) = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{fd'}{N}\right)^2} \times h = \sqrt{\frac{441}{170} - \left(\frac{55}{70}\right)^2} \times 5 = \sqrt{2.594 - 0.104} \times 5 = 7.88$

17. The average marks secured by

$\bar{x}_A = 78, \ \bar{x}_B = 80, \ \sigma_A^2 = 100, \ \sigma_B^2 = 81, \ x_A = 100, \ x_B = 150$

$\bar{x}_{AB} = \frac{n_A \bar{x}_A + n_B \bar{x}_B}{n_A + n_B} = \frac{100 \times 78 + 150 \times 80}{100 + 150} = 79.2$

$d_A = \bar{x}_A - \bar{x}_{AB} = 78 - 79.2 = -1.2$

$d_B = \bar{x}_B - \bar{x}_{AB} = 80 - 79.2 = 0.8$

$\sigma_{AB} = \sqrt{\frac{n_A(\sigma_A^2 + d_A^2) + n_B(\sigma_B^2 + d_B^2)}{n_A + n_B}} = \sqrt{\frac{100(100 + 1.44) + 150(81 + 0.64)}{100 + 150}} = 9.46$

$\sigma_{AB}^2 = 89.5$

18. The number of runs

Group A (x_A)	Group B (x_B)	x_A^2	x_B^2
10	120	100	14400
25	15	625	225
85	30	7225	900
72	35	5184	1225
115	42	13225	1764
80	65	6400	4225
52	80	2704	6400
45	34	2025	1156
30	25	900	625
10	15	100	225
$\sum x_A = 524$	$\sum x_B = 461$	$\sum x_A^2 = 38488$	$\sum x_B^2 = 29929$

$\bar{x}_A = \frac{\sum x_A}{n} = \frac{524}{10} = 52.4$

$\sigma_A = \sqrt{\frac{\sum x_A}{n} - \left(\frac{\sum x_A}{n}\right)^2} = \sqrt{\frac{38488}{10} - \left(\frac{524}{10}\right)^2} = 33.212$

$\bar{x}_B = \frac{\sum x_B}{n} = \frac{461}{10} = 46.1$

$\sigma_B = \sqrt{\frac{\sum x_B}{n} - \left(\frac{\sum x_B}{n}\right)^2} = \sqrt{\frac{29920}{10} - \left(\frac{5161}{10}\right)^2} = 29.441$

$C.V._A = \frac{\sigma_A}{\bar{x}_A} \times 100\% = \frac{33.212}{52.4} \times 100\% = 63.38\%$

$C.V._B = \frac{\sigma_B}{\bar{x}_B} \times 100\% = \frac{29.441}{46.1} \times 100\% = 63.86\%$

$C.V._A < C.V._B$; Hence group A is more consistent.

x_B^2
14400
225
900
1225
1764
4225
6400
1156
625
225
$\sum x_B^2 = 29929$

19. The following data

Students	Section A (x_A)	Section B (x_B)	x_A^2	x_B^2
1	9	10	81	100
2	8	8	64	64
3	10	6	100	36
4	6	8	36	64
5	7	9	49	81
6	8	8	64	64
7	5	7	25	49
8	6	8	36	64
9	7	5	49	25
10	8	8	64	64
	$\sum x_A = 74$	$\sum x_B = 77$	$\sum x_A^2 = 568$	$\sum x_B^2 = 611$

$$\bar{x}_A = \frac{\sum x_A}{n} = \frac{74}{10} = 7.4$$

$$\sigma_A = \sqrt{\frac{\sum x_A^2}{n} - \left(\frac{\sum x_A}{n}\right)^2} = \sqrt{\frac{568}{10} - (7.4)^2} = 1.42$$

$$\bar{x}_B = \frac{\sum x_B}{n} = \frac{77}{10} = 7.7$$

$$\sigma_B = \sqrt{\frac{\sum x_B^2}{n} - \left(\frac{\sum x_B}{n}\right)^2} = \sqrt{\frac{611}{10} - (7.7)^2} = 1.34$$

$$C.V._A = \frac{\sigma_A}{\bar{x}_A} \times 100\% = \frac{1.42}{7.4} \times 100\% = 19.18$$

$$C.V._B = \frac{\sigma_B}{\bar{x}_B} \times 100\% = \frac{1.34}{7.7} \times 100\% = 17.4\%$$

$C.V._B < C.V._A$. Hence, section B is more consistent.

20. The following data

Time for male (x_A)	Time for female (x_B)	x_A^2	x_B^2
5.70	7.52	32.49	56.55
6.80	8.20	46.24	67.24
7.25	8.32	52.56	69.22
8.20	6.90	67.24	47.61
8.10	6.80	65.61	46.24
7.20	8.30	51.84	68.89
6.88	7.45	47.33	55.50
7.20	9.00	51.84	81.00
7.35	10.50	54.02	110.25
7.45	7.20	55.50	51.84
6.90	10.20	47.61	104.04
7.22	8.26	52.12	68.22
6.85	8.50	46.92	72.25
6.40	8.32	40.96	69.22
6.20	10.00	38.44	100
$\sum x_A = 105.7$	$\sum x_B = 125.47$	$\sum x_A^2 = 750.72$	$\sum x_B = 1068.07$

$$\bar{x}_A = \frac{\sum x_A}{n} = \frac{105.7}{15} = 7.046$$

$$\sigma_A = \sqrt{\frac{\sum x_A^2}{n} - \left(\frac{\sum x_A}{n}\right)^2} = \sqrt{\frac{750.72}{15} - \left(\frac{105.7}{15}\right)^2} = 0.673$$

$$\bar{x}_B = \frac{\sum x_B}{n} = \frac{125.47}{15} = 8.364$$

$$\sigma_B = \sqrt{\frac{\sum x_B^2}{n} - \left(\frac{\sum x_B}{n}\right)^2} = \sqrt{\frac{1068.07}{15} - \left(\frac{125.47}{15}\right)^2} = 1.117$$

$$C.V._A = \frac{\sigma_A}{\bar{x}_A} \times 100\% = \frac{0.633}{7.046} \times 100\% = 8.98\%$$

$$C.V._B = \frac{\sigma_B}{\bar{x}_B} \times 100\% = \frac{1.117}{8.364} \times 100\% = 13.35\%$$

$CV_A < CV_B$. Hence male students is more consistent.

21. $$\bar{x} = \frac{\Sigma x}{n} = \frac{552}{10} = 55.20$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{31700}{10} - \left(\frac{552}{10}\right)^2} = \sqrt{122.96} = 11.08$$

$$S_{KP} = \frac{\bar{x} - M_o}{\sigma} = \frac{55.2 - 46.2}{11.08} = 0.81$$

22. $\bar{x} = 65$, Md = 70, $S_{KP} = -0.6$, $M_o = ?$, C.V. = ?

$M_O = 3Md - 2\bar{x} = 3 \times 70 - 2 \times 65 = 80$

$$S_{KP} = \frac{\bar{x} - M_O}{\sigma}$$

or $$-0.6 = \frac{65 - 80}{\sigma}$$

or $$-0.6 = \frac{-15}{\sigma}$$

or $$\sigma = \frac{15}{0.6} = \frac{150}{6} = 25$$

$$CV = \frac{\sigma}{\bar{x}} \times 100\% = \frac{25}{65} \times 100\% = 38.40\%$$

23.

Size (x)	Frequency (f)	fx	fx^2
6	7	42	252
9	12	108	972
12	19	228	2736
15	10	150	2250
18	2	36	648
	N = 50	$\Sigma fx = 564$	$\Sigma fx^2 = 6858$

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{564}{50} = 11.28$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{6858}{50} - \left(\frac{564}{50}\right)^2} = 3.149 \qquad M_O = 12$$

$$S_{KP} = \frac{\bar{x} - M_O}{\sigma} = \frac{11.28 - 12}{3.149} = -0.23$$

24. $S_{KB} = 0.6$

$Q_3 + Q_1 = 100$ .... (1)

$Md = 38$

$Q_3 = ?$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1}$$

or $0.6 = \dfrac{100 - 2 \times 38}{Q_3 - Q_1}$

or $Q_3 - Q_1 = \dfrac{100 - 76}{0.6}$

or $Q_3 - Q_1 = 40$ .... (2)

Add (1) and (2)

$2Q_3 = 140$

$\therefore$ $Q_3 = 70$

25.

x	f	c.f.
20	3	3
23	6	9
24	10	19
26	12	31
28	11	42
30	9	51
40	4	55
	$N = \Sigma f = 55$	

$$\sigma_1 = \frac{N+1}{4}^{th} \text{ item} = \frac{55+1}{4} = 14^{th} \text{ item} = 24$$

$$Md = \frac{2(N+1)}{4}^{th} \text{ item} = 2 \times 14 = 28^{th} \text{ item} = 26$$

$$Q_3 = \frac{3(N+1)}{4}^{th} \text{ item} = 3 \times 14 = 42^{th} \text{ item} = 28$$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1} = \frac{28 + 24 - 26}{28 - 24} = 0$$

26.

Download speed (Mbps)	Time in min.	c.f.
Below 100	10	10
100 – 150	25	35
150 – 200	145	180
200 – 250	220	400
250 – 300	70	470
300 and above	30	500

$\dfrac{N}{4} = \dfrac{500}{4} = 125$ $\qquad$ Q_1 class = (150 – 200)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 150 + \frac{125 - 35}{145} \times 50 = 181.034$$

$\dfrac{2N}{4} = 2 \times 125 = 250$ $\qquad \therefore$ Md class = (200 – 250)

$$Md = L + \frac{\frac{2N}{4} - c.f.}{f} \times h = 200 + \frac{250 - 180}{220} \times 50 = 215.90$$

$\dfrac{3N}{4} = 3 \times 125 = 375$

Q_3 class = (200 – 250)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 200 + \frac{375 - 180}{220} \times 50 = 244.32$$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1} = \frac{244.32 + 181.03 - 2 \times 215.90}{244.32 - 181.03} = -0.102$$

27.

x	f	c.f.
2	1	1
4	5	6
6	10	16
8	3	19
10	1	20
	$N = \Sigma f = 20$	

For P_{10}, $\dfrac{10(N+1)}{100} = \dfrac{10 \times 21}{100} = 2.1$ $\qquad \therefore P_{10} = 4$

For P_{90}, $\dfrac{90(N+1)}{100} = \dfrac{90 \times 21}{100} = 18.9$ $\qquad \therefore P_{90} = 8$

For Q_1, $\dfrac{N+1}{4} = \dfrac{20+1}{4} = 5.25$ $\qquad \therefore Q_1 = 4$

For Q_3, $\dfrac{3(N+1)}{4} = 3 \times 5.25 = 15.75$ $\qquad \therefore Q_3 = 6$

$$K = \frac{Q_3 - Q_1}{2(P_{90} - P_{10})} = \frac{6 - 4}{2(8 - 4)} = \frac{2}{2 \times 4} = 0.25$$

28.

Hourly remuneration	No. of person (f)	c.f.
100 - 110	10	10
110 - 120	14	24
120 - 130	18	42
130 - 140	24	66
140 - 150	16	82
150 - 160	12	94
160 - 170	6	100
	$N = \Sigma f = 100$	

$Q_1 = \frac{N+1}{4} = \frac{35+1}{4} = 9^{th}$ item

$\therefore \quad Q_1 = 22$

$Q_3 = \frac{3(N+1)^{th}}{4}$ item $= 3 \times 9 = 27^{th}$ itme

$\therefore \quad Q_3 = 25$

$$k = \frac{Q_3 - Q_1}{2(P_{90} - P_{10})} = \frac{25-22}{2(25-22)} = \frac{3}{2 \times 3} = \frac{3}{6} = 0.25$$

30.

Wage (Rs)	Mid value(m)	No of Students (f)	Cf
Below 97	95	2	2
98 - 102	100	5	7
103 - 107	105	12	19
108 - 112	110	17	36
113 - 117	115	14	50
118 - 122	120	6	56
123 - 127	125	3	59
128 above	130	1	60
		$N = \Sigma f = 60$	

Here,

It is open end class; hence we have to use median to find out central tendency

Position of Median = $N/2^{th}$ item = 60/2 = 30th item

Cf just greater than N/2 is 30 for which corresponding class is (108-112), hence corrected median class is (107.5 - 112.5)

L = 107.5, h = 5, f = 17 , cf =19

$$\text{Median} = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 107.5 + \frac{30-19}{17} \times 5 = 110.74$$

$P_{25} = 25N / 100^{th}$ item $= 25 \times \frac{60}{100} = 15$

Cf just greater than 25N/100 is 19 for which corresponding class is (103 - 107), hence corrected P_{25} class is (102.5 - 107.5)

L = 102.5, h = 5 f = 12 , cf =7

$$Q_1 = P_{25} = L + \frac{\frac{25N}{100} - c.f.}{f} \times h = 102.5 + \frac{15-7}{12} \times 5 = 105.83$$

$P_{75} = 75N/100^{th}$ item $= 75 \times \frac{60}{100} = 45$

Cf just greater than 75N/100 is 50 for which corresponding class is (113 - 117), hence correct P_{75} class is (112.5 - 117.5)

L = 112.50, h = 5 f = 14, cf = 36

$$Q_3 = P_{75} = L + \frac{\frac{75N}{100} - c.f.}{f} \times h = 112.5 + \frac{45-36}{14} \times 5 = 115.71$$

Quartile Deviation = $QD = \frac{Q_3 - Q_1}{2} = \frac{115.71 - 105.83}{2} = 4.94$

Now; $S_{KB} = \frac{Q_3 + Q_1 - 2Md}{Q_3 - Q_1} = \frac{115.71 + 105.83 - 2 \times 110.74}{115.71 - 105.83} = 0.006$

$S_{KB} = 0.006$, hence the distribution is positively skewed

Here we have to use the percentile coefficient of kurtosis

Here,

$P_{10} = 10N/100^{th}$ item $= 10 \times \frac{60}{100} = 6$

Cf just greater than 10N/100 is 6 for which corresponding class is (98 - 102), hence P_{10} class is (97.5 - 102.5)

L = 97.5, h = 5, f = 5, cf = 2

$$P_{10} = L + \frac{\frac{10N}{100} - c.f.}{f} \times h = 97.5 + \frac{6-2}{5} \times 5 = 101.5$$

$P_{90} = 90N/100^{th}$ item $= 90 \times \frac{60}{100} = 54$

Cf just greater than 90N/100 is 56 for which corresponding class is (118 - 122), hence P_{75} class is (117.5 - 122.5)

L = 117.5, h = 5 f = 6 , cf =50

$$P_{90} = L + \frac{\frac{90N}{100} - c.f.}{f} \times h = 117.5 + \frac{54-50}{6} \times 5 = 120.83$$

Percentile coefficient of kurtosis (K) $= \frac{P_{75} - P_{25}}{2(P_{90} - P_{10})} = \frac{115.71 - 105.83}{2(120.83 - 101.50)} = 0.255$

K = 0.255< 0.263, hence the distribution is leptokurtic

□□□

ester)

(98 - 102), hence

ass is (118 - 122),

$\frac{83}{.50)} = 0.255$

3 CHAPTER

CORRELATION AND REGRESSION ANALYSIS

1. If the covariance

$Cov.(x, y) = 36, \sigma_x^2 = 36, \sigma_y^2 = 100$

$$r = \frac{Cov\,(x, y)}{\sigma_x\,\sigma_y} = \frac{36}{6 \times 10} = 0.6$$

2. Find the r

$n = 10, \sigma_x = 2.05, \sigma_y = 2.06, \sum(x - \bar{x})\,(y - \bar{y}) = 40$

$$r = \frac{\frac{1}{n}\sum(x - \bar{x})\,(y - \bar{y})}{\sigma_x\,\sigma_y} = \frac{\frac{40}{10}}{2.05 \times 2.06} = 0.947$$

3. For 10 obs

$\sum x = 666, \sum y = 663, \sum x^2 = 44490, \sum y^2 = 44061, \sum xy = 44224, n = 10$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$= \frac{10 \times 44224 - 666 \times 663}{\sqrt{10 \times 44490 - (666)^2}\sqrt{10 \times 44061 - (663)^2}} = 0.576$$

4. $r = 0.8 \qquad \Sigma(x - \bar{x})\,(y - \bar{y}) = 60 \qquad \sigma_y = 61 \qquad \Sigma(x - \bar{x})^2 = 90$

$$\text{or, } 0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{\Sigma(x - \bar{x})^2}{x}} \times 61} \qquad \text{or, } 0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{90}{n}} \times 61}$$

$$\text{or, } 0.8 = \frac{60}{n} \times \sqrt{\frac{n}{90}} \times \frac{1}{61} \text{ squaring both side}$$

$$\text{or, } 0.64 \times 3721 = 40 \times \frac{4}{90}$$

$$\text{or, } n = \frac{0.64 \times 3721}{40 \times 40} = 57.5 \simeq 60$$

5.

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
8	16	64	256	128
9	15	81	225	135
$\Sigma x = 45$	$\Sigma y = 108$	$\Sigma x^2 = 285$	$\Sigma y^2 = 1356$	$\Sigma xy = 597$

$$r = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{n\,\Sigma x^2 - (\Sigma x)^2}\sqrt{n\,\Sigma y^2 - (\Sigma y)^2}} = \frac{9 \times 597 - 45 \times 108}{\sqrt{9 \times 285 - (45)^2}\sqrt{9 \times 1356 - (108)^2}} = 0.95$$

6.

size	No. of items	No. of defective items	% defective (y)	mid-size (x)	$u' = \frac{x - 27.5}{5}$	$v' = \frac{y - 50}{5}$	u'^2	v'^2	$u'v'$
15–20	200	150	75	17.5	-2	5	4	25	-10
20-25	270	162	60	22.5	-1	2	1	4	-2
25-30	340	170	50	27.5	0	0	0	0	0
30-35	360	180	50	32.5	1	0	1	0	0
35-40	400	180	45	37.5	2	-1	4	1	-2
40-45	300	120	40	42.5	3	-2	9	4	-6
					$\Sigma u' = 3$	$\Sigma v' = 4$	$\Sigma u'^2 = 19$	$\Sigma v'^2 = 34$	$\Sigma u'v' = -20$

$$r = \frac{n\,\Sigma u'v' - \Sigma u'\,\Sigma v'}{\sqrt{n\,\Sigma u'^2 - (\Sigma u')^2}\sqrt{n\,\Sigma v'^2 - (\Sigma v')^2}} = \frac{6 \times (-20) - 3 \times 4}{\sqrt{6 \times 19 - 3^2}\sqrt{6 \times 34 - 4^2}} = -0.939$$

7.

X	Y	xy	x^2	y^2
25	17	425	625	289
26	18	468	676	324
27	19	513	729	361
25	17	425	625	289
26	19	494	676	361
28	20	560	784	400
25	17	425	625	289
25	17	425	625	289
24	18	432	576	324
26	18	468	676	324
26	20	520	676	400
27	18	486	729	324
27	19	513	729	361
28	19	532	784	361
25	18	450	625	324
25	19	475	625	361
26	18	468	676	324
25	18	450	625	324
27	20	540	729	400
493	349	9069	12815	6429

Here n= 19, $\Sigma X = 493$, $\Sigma Y = 349$,

$\Sigma XY = 9069$ $\Sigma X^2 = 12815$, $\Sigma Y^2 = 6429$

Karl Pearson's Coefficient of correlation

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}} = \frac{19 \times 9069 - 493 \times 349}{\sqrt{13 \times 12815 - (493)^2}\sqrt{19 \times 6429 - (349)^2}}$$

$= 0.654$

For P_{10}, $\frac{10\,N}{100} = \frac{10 \times 100}{100} = 10$ $\therefore$ P_{10} class = 100 – 110

$$P_{10} = L + \frac{\frac{10N}{100} - c.f.}{f} \times h = 100 + \frac{10-0}{10} \times 10 = 110$$

For P_{90}, $\frac{90\,N}{100} = \frac{90 \times 100}{100} = 90$ $\therefore$ P_{90} class = 150 – 160

$$P_{90} = L + \frac{\frac{90\,N}{100} - c.f.}{f} \times h = 150 + \frac{90-82}{12} \times 10 = 156.66$$

For Q_1, $\frac{N}{4} = \frac{100}{4} = 25$ $\therefore$ Q_1 class = (120 – 130)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 120 + \frac{25-24}{18} \times 10 = 120.55$$

For Q_3, $\frac{3N}{4} = 3 \times 25 = 75$

$\therefore$ Q_3 class = (140 – 150)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 140 + \frac{75-66}{16} \times 10 = 145.625$$

$$K = \frac{145.625 - 120.55}{2\,(156.66 - 110)} = \frac{25.075}{93.32} = 0.268$$

29.

For B Sc CSIT Students

Project conducted (x)	No of Students (f)	fX	$(X-\bar{X})$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$	fx^2	cf
20	5	100	-3.26	-16.3	53.14	-173.23	564.73	2000	5
22	7	154	-1.26	-8.82	11.11	-14.00	17.64	3388	12
23	10	230	-0.26	-2.6	0.68	-0.18	0.05	5290	22
25	8	200	1.74	13.92	24.22	42.14	73.33	5000	30
26	5	130	2.74	13.7	37.54	102.85	281.82	3380	35
	$N = \Sigma f = 35$	$\Sigma fX = 818$		$\Sigma f(X-\bar{X}) = 0$	$\Sigma f(X-\bar{X})^2 = 126.69$	$\Sigma f(X-\bar{X})^4 = 42.41$	$\Sigma f(X-\bar{X})^4 = 937.57$	19058	

$$\text{Mean Value } (\bar{X}) = \frac{\Sigma fx}{N} = \frac{814}{35} = 23.25$$

$$\text{Variance} = \frac{\Sigma fX^2}{N} = \left(\frac{\Sigma fX^2}{N}\right) = \frac{19058}{35} - \left(\frac{818}{35}\right)$$

$$SD = \sqrt{\text{variance}} = \sqrt{3.94} = 1.98$$

$$CV = \frac{SD}{\bar{X}} \times 100\% = \frac{1.98}{23.25} \times 100 = 8.51\%$$

$M_o = 23$

$$Sk\,(P) = \frac{\bar{x} - M_o}{\sigma} = \frac{23.25 - 23}{1.98} = 0.12$$

$$P_{10} = \frac{10(N+1)}{100} = \frac{10 \times 36}{100} = 3.6^{th} \text{ item}$$

$$P_{90} = \frac{90\,(N+1)}{100} = \frac{90 \times 36}{100} = 32.4^{th} \text{ item}$$

$P_{90} = 26$

$$Q_1 = \frac{(N+1)}{4} = \frac{36}{4} = 9^{th} \text{ item } Q_1 = 22$$

$$Q_3 = \frac{3(N+1)}{4} = 9 \times 3 = 27^{th} \text{ item } Q_3 = 25$$

$$k = \frac{Q_3 - Q_1}{2(P_{90} - P_{10})} = \frac{25-22}{2(26-20)} = \frac{3}{2 \times 6} = 0.25$$

For BCA Students

Project conducted (x)	No of Students (f)	fX	$(X-\bar{X})$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$	fx^2	cf
20	3	60	-3.17	-9.51	30.15	-95.57	302.94	1200	3
22	7	154	-1.17	-8.19	9.58	-11.21	13.12	3388	10
23	15	345	-0.17	-2.55	0.43	-0.07	0.01	7935	25
25	8	200	1.83	14.64	26.79	49.03	89.72	5000	33
26	2	52	2.83	5.66	16.02	45.33	128.28	1352	35
	$N = \Sigma f = 35$	$\Sigma fX = 811$		$\Sigma f(X-\bar{X}) = 0$	$\Sigma f(X-\bar{X})^2 = 82.97$	$\Sigma f(X-\bar{X})^3 = -12.49$	$\Sigma f(X-\bar{X})^4 = 534.08$	18875	

$$\text{Mean Value } (\bar{X}) = \frac{\Sigma fx}{N} = \frac{811}{35} = 23.17$$

$$\text{Variance} = \mu_2 = \frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2 = \frac{18875}{35} = \left(\frac{811}{35}\right)^2 = 535.28 - 536.88 = 2.39$$

$$SD = \sqrt{\text{variance}} = \sqrt{2.37} = 1.54$$

$$CV = \frac{SD.}{\bar{X}} \times 100\% = \frac{1.54}{23.17} = 6.65\%$$

$M_o = 23$

$$S_k\,(P) = \frac{\bar{x} - M_o}{\sigma} = \frac{23.17 - 23}{1.54} = 0.11$$

$$P_{90} = \frac{10\,(N+1)}{100} = \frac{10\,(35+1)}{100} = \frac{360}{100} = 3.6^{th} \text{ item}$$

$\therefore$ $P_{10} = 25$

$$P_{30} = \frac{90(N+1)}{100} = \frac{90\,(35+1)}{100} = 32.4^{th} \text{ item}$$

$\therefore$ $P_{90} = 26$

For P_{10}, $\frac{10N}{100} = \frac{10 \times 100}{100} = 10$ $\therefore$ P_{10} class = 100 – 110

$$P_{10} = L + \frac{\frac{10N}{100} - c.f.}{f} \times h = 100 + \frac{10 - 0}{10} \times 10 = 110$$

For P_{90}, $\frac{90N}{100} = \frac{90 \times 100}{100} = 90$ $\therefore$ P_{90} class = 150 – 160

$$P_{90} = L + \frac{\frac{90N}{100} - c.f.}{f} \times h = 150 + \frac{90 - 82}{12} \times 10 = 156.66$$

For Q_1, $\frac{N}{4} = \frac{100}{4} = 25$ $\therefore$ Q_1 class = (120 – 130)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 120 + \frac{25 - 24}{18} \times 10 = 120.55$$

For Q_3, $\frac{3N}{4} = 3 \times 25 = 75$

$\therefore$ Q_3 class = (140 – 150)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 140 + \frac{75 - 66}{16} \times 10 = 145.625$$

$$K = \frac{145.625 - 120.55}{2(156.66 - 110)} = \frac{25.075}{93.32} = 0.268$$

29.

For B Sc CSIT Students

Project conducted (x)	No of Students (f)	fX	$(X-\bar{X})$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$	fx^2	cf
20	5	100	-3.26	-16.3	53.14	-173.23	564.73	2000	5
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25	8	200	1.74	13.92	24.22	42.14	73.33	5000	30
26	5	130	2.74	13.7	37.54	102.85	281.82	3380	35
	$N = \Sigma f = 35$	$\Sigma fX = 818$		$\Sigma f(X-\bar{X}) = 0$	$\Sigma f(X-\bar{X})^2 = 126.69$	$\Sigma f(X-\bar{X})^4 = 42.41$	$\Sigma f(X-\bar{X})^4 = 937.57$	19058	

Mean Value $(\bar{X}) = \frac{\Sigma fx}{N} = \frac{814}{35} = 23.25$

Variance $= \frac{\Sigma fX^2}{N} = \left(\frac{\Sigma fX^2}{N}\right) = \frac{19058}{35} - \left(\frac{818}{35}\right)$

SD $= \sqrt{\text{variance}} = \sqrt{3.94} = 1.98$

$CV = \frac{SD}{\bar{X}} \times 100\% = \frac{1.98}{23.25} \times 100 = 8.51\%$

$M_o = 23$

$Sk(P) = \frac{\bar{x} - M_o}{\sigma} = \frac{23.25 - 23}{1.98} = 0.12$

$P_{10} = \frac{10(N+1)}{100} = \frac{10 \times 36}{100} = 3.6^{th}$ item

$P_{90} = \frac{90(N+1)}{100} = \frac{90 \times 36}{100} = 32.4^{th}$ item

$P_{90} = 26$

$Q_1 = \frac{(N+1)}{4} = \frac{36}{4} = 9^{th}$ item $Q_1 = 22$

$Q_3 = \frac{3(N+1)}{4} = 9 \times 3 = 27^{th}$ item $Q_3 = 25$

$k = \frac{Q_3 - Q_1}{2(P_{90} - P_{10})} = \frac{25 - 22}{2(26 - 20)} = \frac{3}{2 \times 6} = 0.25$

For BCA Students

Project conducted (x)	No of Students (f)	fX	$(X-\bar{X})$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$	fx^2	cf
20	3	60	-3.17	-9.51	30.15	-95.57	302.94	1200	3
22	7	154	-1.17	-8.19	9.58	-11.21	13.12	3388	10
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25	8	200	1.83	14.64	26.79	49.03	89.72	5000	33
26	2	52	2.83	5.66	16.02	45.33	128.28	1352	35
	$N = \Sigma f = 35$	$\Sigma fX = 811$		$\Sigma f(X-\bar{X}) = 0$	$\Sigma f(X-\bar{X})^2 = 82.97$	$\Sigma f(X-\bar{X})^3 = -12.49$	$\Sigma f(X-\bar{X})^4 = 534.08$	18875	

Mean Value $(\bar{X}) = \frac{\Sigma fx}{N} = \frac{811}{35} = 23.17$

Variance $= \mu_2 = \frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2 = \frac{18875}{35} = \left(\frac{811}{35}\right)^2 = 535.28 - 536.88 = 2.39$

SD $= \sqrt{\text{variance}} = \sqrt{2.37} = 1.54$

$CV = \frac{SD.}{\bar{X}} \times 100\% = \frac{1.54}{23.17} = 6.65\%$

$M_o = 23$

$S_k(P) = \frac{\bar{x} - M_o}{\sigma} = \frac{23.17 - 23}{1.54} = 0.11$

$P_{90} = \frac{10(N+1)}{100} = \frac{10(35+1)}{100} = \frac{360}{100} = 3.6^{th}$ item

$\therefore$ $P_{10} = 25$

$P_{30} = \frac{90(N+1)}{100} = \frac{90(35+1)}{100} = 32.4^{th}$ item

$\therefore$ $P_{90} = 26$

Scanned with CamScanner

$$P.E. = 0.6745 \times \frac{1-r^2}{\sqrt{n}} = 0.6745 \times \frac{1-(0.654)^2}{\sqrt{19}} = 0.088$$

$6 \times P.E. = 0.53$

Since $6 \times P.E. < r$. So the correlation coefficient is significant

8. When the coefficient

$\sum x = 127, \sum y = 100, \sum x^2 = 860, \sum y^2 = 549, \sum xy = 674, n = 20$

Wrong (x, y) = (10, 14) and (8, 6)

Correct (x, y) = (8, 12) and (6, 8)

Correct r = ?

Correct values

$\sum x = 127 - 10 - 8 + 8 + 6 = 123$

$\sum y = 100 - 14 - 6 + 12 + 18 = 100$

$\sum x^2 = 860 - 10^2 - 8^2 + 8^2 + 6^2 = 796$

$\sum y^2 = 549 - 14^2 - 6^2 + 12^2 + 8^2 = 525$

$\sum xy = 674 - 10 \times 14 - 8 \times 6 + 8 \times 12 + 6 \times 8 = 630$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$= \frac{20 \times 630 - 123 \times 100}{\sqrt{20 \times 796 - (123)^2}\sqrt{20 \times 525 - (100)^2}} = \frac{300}{28.124 \times 22.36} = 0.47$$

9. A student

$r = 0.795, \quad n = 100$

$$PE(r) = 0.6745 \times \frac{1-r^2}{\sqrt{n}} = 0.6745\left\{\frac{1-(0.795)^2}{\sqrt{100}}\right\} = 0.0248$$

Here, $r \neq PE(r)$

$6\ PE(r) = 6 \times 0.0248 = 0.1489$

Here, $r > 6\ PE(r)$. Hence, r is significant. So conclusion is correct.

10. $n = 10, r = 0.81$

$$PE(r) = 0.6745 \times \frac{1-r^2}{\sqrt{n}} = 0.6745 \times \frac{1-(0.81)^2}{\sqrt{10}} = 0.6745 \times 0.1087 = 0.0733$$

Here, $r = 0.81 \nless PE(r) = 0.0733$

$6PE(r) = 6 \times 0.0733 = 0.4398$

$r = 0.81 > 6\ PE(r) = 0.4398$

Hence, r is significant.

Limit of population correlation $\rho = r \pm PE(r) = 0.81 \pm 0.0733$

Take –,	Take +
$0.81 - 0.0733 = 0.7367$	$0.81 + 0.0733 = 0.8833$

11. $$r = \frac{N\Sigma fxy - \Sigma fx\,\Sigma gy}{\sqrt{N\Sigma fx^2 - (\Sigma fx)^2}\sqrt{N\Sigma fy^2 - (\Sigma fy)^2}}$$

$$= \frac{72 \times 172000 - 3560 \times 3260}{\sqrt{72 \times 196800 - (3560)^2}\sqrt{72 \times 168400 - (3260)^2}} = 0.52$$

$$PE(r) = 0.6745\frac{1-r^2}{\sqrt{N}} = 0.6745 \times \frac{1-(0.52)^2}{\sqrt{72}} = 0.057$$

$r = 0.52 \nless PE(r) = 0.057$

$6PE(r) = 6 \times 0.057 = 0.342$

$r = 0.52 > 6\ PE(r) = 0.342$

Hence, r is significant.

12. Here Given

$\sum y = 26953, \sum x = 950, \sum y^2 = 35528893, \sum x^2 = 49250, \sum xy = 1263940$

To fit $Y = a + bX$

$\Sigma y = na + b\Sigma x$

or $26953 = 22a + 950b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $1263940 = 950a + 49250b$ ......(ii)

on solving Equation (i) and (ii)

Constant	Coeff of a	Coeff of b
26953	22	950
1263940	950	49250

D = 181000; D1= 126692250 D2 = 2201330

b = 12.162 and a = 699.957

Regression equation of y on x is y = a + bx

y = 699.957 + 12.162 x

$$r^2 = \frac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)(n\Sigma Y^2 - (\Sigma Y)^2)} = \frac{(22 \times 1263940 - 950 \times 26953)^2}{(22 \times 49250 - (950)^2)(22 \times 35528893 - (26953)^2)} = 0.4852$$

13.

	Father(x)	son(y)	xy	x2	y2
	67	68	4556	4489	4624
	63	66	4158	3969	4356
	66	65	4290	4356	4225
	71	70	4970	5041	4900
	69	67	4623	4761	4489
	65	67	4355	4225	4489
	62	64	3968	3844	4096
	70	71	4970	4900	5041
	61	62	3782	3721	3844
	72	63	4536	5184	3969
Sum	666	663	44208	44490	44033

To fit $Y = a + bX$

$\Sigma y = na + b\Sigma x$

or $663 = 10a + 666b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $44208 = 666a + 44490b$ ....(ii)

on solving Equation (i) and (ii) we will get

b = 0.388 and a = 40.43

Regression equation of y on x is y = a + bx

y = 40.43 + 0.388x

When fathers height (x) = 70 inches,

140

953)²) = 0.4852

2
4624
4356
4225
4900
4489
4489
4096
5041
3844
3969
44033

Son's age $y = 40.43 + 0.388x = 40.43 + 0.388 \times 70 = 67..62$ inches

Coefficient of Determination (r^2) $= \dfrac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)(n\Sigma Y^2 - (\Sigma Y)^2)}$

$= \dfrac{(10 \times 44208 - 666 \times 663)^2}{(10 \times 44490 - (666)^2)(10 \times 44033 - (663)^2)}$

$= 0.266$

14.

Operator	Experience(x)	Performance (y)	xy	x2	y2
I	16	87	1392	256	7569
II	12	88	1056	144	7744
III	18	89	1602	324	7921
IV	4	68	272	16	4624
V	3	78	234	9	6084
VI	10	80	800	100	6400
VII	5	75	375	25	5625
VIII	12	83	996	144	6889
Sum	80	648	6727	1018	52856

To fit $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $648 = 8a + 80b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $6727 = 80a + 1018b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = 1.133$ and $a = 69.669$

Regression equation of y on x is $y = a + bx$

$y = 69.669 + 1.133x$

When Experience (x) = 8 years,

Performance $y = 69.669 + 1.133x = 69.669 + 1.133x\ 8 = 78.73$

15.

X(football games)	Y(minor accidents)	xy	x2	y2
20	6	120	400	36
30	9	270	900	81
10	4	40	100	16
12	5	60	144	25
15	7	105	225	49
25	8	200	625	64
34	9	306	1156	81
sum 146	48	1101	3550	352

To fit $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $48 = 7a + 146b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $1101 = 146a + 3550b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = 0.198$ and $a = 2.732$

Regression equation of y on x is $y = a + bx$

$y = 2.732 + 0.198x$

When nos. of football game (x) = 30,

Nos of minor accidenty $= 2.732 + 0.198x = 2.732 + 0.198x30 = 8.66 = 9$

Coefficient of Determination (r^2) $= \dfrac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)(n\Sigma Y^2 - (\Sigma Y)^2)}$

$= \dfrac{(7 \times 1101 - 146 \times 48)^2}{(7 \times 3550 - (146)^2)(7 \times 352 - (48)^2)} = 0.87$

16.

	Extraction time in minute(X)	Extraction efficiency in %(Y)	xy	x2	y2
	27	57	1539	729	3249
	45	64	2880	2025	4096
	41	80	3280	1681	6400
	19	46	874	361	2116
	35	62	2170	1225	3844
	39	72	2808	1521	5184
	19	52	988	361	2704
Total	225	433	14539	7903	27593

To fit $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $443 = 7a + 225b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $14539 = 225a + 7903b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = 0.926$ and $a = 32.096$

Regression equation of y on x is $y = a + bx$

$y = 32.096 + 0.926x$

When Extraction time (x) = 35 minutes,

Extraction Efficiency $= 32.096 + 0.926x = 32.096 + 0.926x35 = 64.5$

Coefficient of Determination =

$r^2 = \dfrac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)(n\Sigma Y^2 - (\Sigma Y)^2)} = \dfrac{(7 \times 14539 - 225 \times 433)^2}{(7 \times 7903 - (225)^2)(7 \times 27593 - (433)^2)} = 0.711$

Which indicate that 71.1% of extraction efficiency is affected by extraction tim

Son's age $y = 40.43 + 0.388x = 40.43 + 0.388 \times 70 = 67..62$ inches

Coefficient of Determination (r^2) $= \dfrac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)\,(n\Sigma Y^2 - (\Sigma Y)^2)}$

$= \dfrac{(10 \times 44208 - 666 \times 663)^2}{(10 \times 44490 - (666)^2)\,(10 \times 44033 - (663)^2)}$

$= 0.266$

14.

Operator	Experience(x)	Performance (y)	xy	x2	y2
I	16	87	1392	256	7569
II	12	88	1056	144	7744
III	18	89	1602	324	7921
IV	4	68	272	16	4624
V	3	78	234	9	6084
VI	10	80	800	100	6400
VII	5	75	375	25	5625
VIII	12	83	996	144	6889
Sum	80	648	6727	1018	52856

To fit Y = a+bX

$\Sigma y = na + b\Sigma x$

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$\Sigma xy = a\Sigma x + b\Sigma x^2$

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on solving Equation (i) and (ii) we will get

b = 1.133 and a = 69.669

Regression equation of y on x is $y = a + bx$

$y = 69.669 + 1.133x$

When Experience (x) = 8 years,

Performance $y = 69.669 + 1.133x = 69.669 + 1.133 \times 8 = 78.73$

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X(football games)	Y(minor accidents)	xy	x2	y2
20	6	120	400	36
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12	5	60	144	25
15	7	105	225	49
25	8	200	625	64
34	9	306	1156	81
sum 146	48	1101	3550	352

To fit Y = a+bX

$\Sigma y = na + b\Sigma x$

or $48 = 7a + 146b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $1101 = 146a + 3550b$ ......(ii)

on solving Equation (i) and (ii) we will get

b= 0.198 and a = 2.732

Regression equation of y on x is $y = a + bx$

$y = 2.732 + 0.198x$

When nos. of football game (x) = 30,

Nos of minor accidenty $= 2.732 + 0.198x = 2.732 + 0.198 \times 30 = 8.66 = 9$

Coefficient of Determination (r^2) $= \dfrac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)\,(n\Sigma Y^2 - (\Sigma Y)^2)}$

$= \dfrac{(7 \times 1101 - 146 \times 48)^2}{(7 \times 3550 - (146)^2)\,(7 \times 352 - (48)^2)} = 0.87$

16.

	Extraction time in minute(X)	Extraction efficiency in %(Y)	xy	x2	y2
	27	57	1539	729	3249
	45	64	2880	2025	4096
	41	80	3280	1681	6400
	19	46	874	361	2116
	35	62	2170	1225	3844
	39	72	2808	1521	5184
	19	52	988	361	2704
Total	225	433	14539	7903	27593

To fit Y = a+bX

$\Sigma y = na + b\Sigma x$

or $443 = 7a + 225b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $14539 = 225a + 7903b$ ......(ii)

on solving Equation (i) and (ii) we will get

b = 0.926 and a = 32.096

Regression equation of y on x is $y = a + bx$

$y = 32.096 + 0.926x$

When Extraction time (x) = 35 minutes,

Extraction Efficiency $= 32.096 + 0.926x = 32.096 + 0.926 \times 35 = 64.5$

Coefficient of Determination =

$r^2 = \dfrac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)\,(n\Sigma Y^2 - (\Sigma Y)^2)} = \dfrac{(7 \times 14539 - 225 \times 433)^2}{(7 \times 7903 - (225)^2)\,(7 \times 27593 - (433)^2)} = 0.711$

Which indicate that 71.1% of extraction efficiency is affected by extraction time.

17.

	X	Y	xy	x2	y2
	0	12	0	0	144
	5	15	75	25	225
	10	17	170	100	289
	15	22	330	225	484
	20	24	480	400	576
	25	30	750	625	900
Sum	75	120	1805	1375	2618

To fit Y = a+bX

$\Sigma y = na + b\Sigma x$

or $120 = 6a + 75b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $1805 = 75a + 1375\,b$ ......(ii)

on solving Equation (i) and (ii) we will get

b = 0.697 and a = 11.29

Regression equation of y on x is y = a + bx

y = 11.29 + 0.697x

Coefficient of Determination =

$$r^2 = \frac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)\,(n\Sigma Y^2 - (\Sigma Y)^2)} = \frac{(6 \times 1805 - 75 \times 120)^2}{(6 \times 1375 - (75)^2)\,(6 \times 2618 - (120)^2)} = 0.975$$

Which indicate that 97.5% of variation in y is due to x.

18.

	Percentage change in GNP (Y)	Government deficit in lakh Rs (x)	xy	y2	x2
	3	50	150	9	2500
	1	200	200	1	40000
	4	70	280	16	4900
	1	100	100	1	10000
	2	90	180	4	8100
	3	40	120	9	1600
Sum	14	550	1030	40	67100

To fit Y = a+bX

$\Sigma y = na + b\Sigma x$

or $14 = 6a + 550b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $1030 = 550a + 67100b$ ......(ii)

on solving Equation (i) and (ii) we will get

b = - 0.015 and a = 3.725

Regression coefficient b=-0.015, indicates that percentage change in GNP is - 0.015 if there is 1 lakh government deficit is observed.

Regression equation of y on x is y = a + bx

y = 3.725- 0.015x

When Government deficit (x) =110 lakhs,

Percentage change in GNP y = y = 3.725- 0.015x = y = 3.725- 0.015x110 = 2.055 if there is Rs 110 lakhs government deficit is observed.

$$\text{Coefficient of Determination } r^2 = \frac{(n\Sigma XY - \Sigma X\Sigma Y)^2}{(n\Sigma X^2 - (\Sigma X)^2)\,(n\Sigma Y^2 - (\Sigma Y)^2)}$$

$$= \frac{(6 \times 1030 - 14 \times 550)^2}{(6 \times 67100 - (550)^2)\,(6 \times 40 - (14)^2)} = 0.524$$

Which indicate that 52.4% of variation in percentage change in GNP is due to government deficit.

19.

Year	Annual advertising expenditure(x)	Annual sales revenue(Y)	xy	x_2	y_2
1	10	20	200	100	400
2	12	30	360	144	900
3	14	37	518	196	1369
4	16	50	800	256	2500
5	18	56	1008	324	3136
6	20	78	1560	400	6084
7	22	89	1958	484	7921
8	24	100	2400	576	10000
9	26	120	3120	676	14400
10	28	110	3080	784	12100
Sum	190	690	15004	3940	58810

$$\text{Correlation coefficient (r)} = \frac{n\Sigma XY - \Sigma X\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\,\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{10 \times 15004 - 190 \times 690}{\sqrt{10 \times 3940 - (190)2}\,\sqrt{10 \times 58810 - (690)2}} = 0.985$$

To fit regression model of sales as a function of advertisement expenditure

Y = a+bX

$\Sigma y = na + b\Sigma x$

Or $690 = 10a + 190b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $15004 = 190a + 3940b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = 5.739$ and $a = -40.048$

Regression equation of y on x is $y = a + bx$

$y = -40.048 + 5.739x$

When advertisement expenditure (x) =27 lakhs,

Annual sales $y = y = -40.048+5.739x = y = -40.048+5.739 \times 27 = 114.915$ (crore)

20.

	Speed x	Noise level y	xy	x2	y2
	250	83	20750	62500	6889
	340	89	30260	115600	7921
	320	88	28160	102400	7744
	330	89	29370	108900	7921
	346	92	31832	119716	8464
	260	85	22100	67600	7225
	280	84	23520	78400	7056
	395	92	36340	156025	8464
	380	93	35340	144400	8649
	400	96	38400	160000	9216
total	3301	891	296072	1115541	79549

$$\text{Correlation coefficient (r)} = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{10 \times 296072 - 3301 \times 891}{\sqrt{10 \times 1115541 - (3301)2}\sqrt{10 \times 79549 - (891)2}} = 0.957$$

To fit regression model of sales as a function of advertisement expenditure $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $891 = 10a + 3301b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $296072 = 3301a + 1115541\,b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = 0.075$ and $a = 64.191$

Regression equation of y on x is $y = a + bx$

$y = 64.191 + 0.075x$

The variation of noise level is 0.075 if there is 1 unit change in air speed.

21.

Data size(gigabytes) x	Processed requests y	xy	x2	y2
6	40	240	36	1600
7	55	385	49	3025
7	50	350	49	2500

	x	y	xy	x2	y2
	8	41	328	64	1681
	10	17	170	100	289
	10	26	260	100	676
	15	16	240	225	256
Total	63	245	1973	623	10027

To fit $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $245 = 7a + 63b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $1973 = 63a + 623\,b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = -4.142$ and $a = 72.278$

Regression equation of y on x is $y = a + bx$

$y = 72.278 - 4.142x$

Regression coefficient b=-4.142, indicates that change in process request is -0.015 if there is 1 Gb change in data size.

When data size is 12 Gb then efficiency (process request) $y = 72.278 - 4.142x = 72.278 - 4.142 \times 12 = 22.57$

When data size is 12 Gb then efficiency (process request) $y = 72.278 - 4.142x = 72.278 - 4.142 \times 30 = -52$ (which is impossible) so the data size 12 Gb is not possible.

22.

Month	Building Permits (Y)	Interest rate (X)	xy	y2	x2
1	786	10.2	8017.2	617796	104.04
2	494	12.6	6224.4	244036	158.76
3	289	13.5	3901.5	83521	182.25
4	892	9.7	8652.4	795664	94.09
5	343	10.8	3704.4	117649	116.64
6	888	9.5	8436	788544	90.25
7	509	10.9	5548.1	259081	118.81
8	987	9.2	9080.4	974169	84.64
9	187	14.2	2655.4	34969	201.64
Sum	5375	100.6	56219.8	3915429	1151.12

Correlation coefficient

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}} = \frac{9 \times 56219.8 - 5375 \times 100.6}{\sqrt{9 \times 3915429 - (5375)2}\sqrt{9 \times 1151.12 - (100.6)2}} = -0.89$$

Regression equation $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $5375 = 9a + 100.6b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $56219.8 = 100.6a + 1151.12b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = -144.947$ and $a = 2217.41$

Regression equation of y on x is $y = a + bx$

$y = 2217.41 - 144.947x$

Residual for month 9: for month 9, $x=14.2$

So, $y=2217.41 - 144.947 \times 14.2 = 159.16$

Residual = 187-159.16 = 27.84

Coefficient of determination $r^2=0.79$ indicates that 79% change in building permits is due to interest rate.

When interest rate increases 9.7% then the building permits

$y=2217.41 - 144.947 \times 9.7 = 811.42$

23.

Customer	Number of cases x	Delivery times (minutes) y	xy	x2	y2
1	52	32.1	1669.2	2704	1030.41
2	64	34.8	2227.2	4096	1211.04
3	95	37.8	3591	9025	1428.84
4	116	38.5	4466	13456	1482.25
5	143	44.2	6320.6	20449	1953.64
6	161	43	6923	25921	1849
7	184	49.4	9089.6	33856	2440.36
8	218	56.8	12382.4	47524	3226.24
9	254	61.2	15544.8	64516	3745.44
10	267	58.2	15539.4	71289	3387.24
Sum	1554	456	77753.2	292836	21754.46

i. Find the correlation coefficient between delivery times and the number of cases delivered.

ii. Develop a regression model to predict delivery time, based on the number of cases delivered.

iii. Interpret the meaning of slope in this problem.

iv. Predict the delivery time for 150 cases of soft drink.

Correlation coefficient

$$r = \frac{n\Sigma XY - \Sigma X\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}} = \frac{10\times77753.2 - 1554\times456}{\sqrt{10\times292836 - (1554)2}\sqrt{10\times21754.46 - (456)2}} = 0.981$$

Regression equation $Y = a+bX$

$\Sigma y = na + b\Sigma x$

or $456 = 10a + 1554b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $77753.2 = 456a + 292836b$ ......(ii)

on solving Equation (i) and (ii) we will get

$b = 0.134$ and $a = 24.744$

Regression equation of y on x is $y = a + bx$

$y = 24.744 + 0.134x$

Regression Coefficient 0.134 indicates that when number of cases is increased by 1 unit delivery time will increase by 0.134 minute

When number of cases of soft drink is 150

Delivery time in minutes $y = 24.744 + 0.134 \times 150 = 44.875$ minutes

Coefficient of determination $r^2 = 0.962$ indicates that 96.2% of changes in number of cases is due to delivery time.

□□□

or $56219.8 = 100.6a + 1151.12\,b$ ......(ii)

on solving Equation (i) and (ii) we will get

b = -144.947 and a = 2217.41

Regression equation of y on x is y = a + bx

y = 2217.41 - 144.947 x

Residual for month 9: for month 9, x=14.2

So, y=2217.41 - 144.947 x14.2 = 159.16

Residual = 187-159.16 = 27.84

Coefficient of determination r^2=0.79 indicates that 79% change in building permits is due to interest rate.

When interest rate increases 9.7% then the building permits

y=2217.41 - 144.947 ×9.7= 811.42

23.

Customer	Number of cases x	Delivery times (minutes) y	xy	x2	y2
1	52	32.1	1669.2	2704	1030.41
2	64	34.8	2227.2	4096	1211.04
3	95	37.8	3591	9025	1428.84
4	116	38.5	4466	13456	1482.25
5	143	44.2	6320.6	20449	1953.64
6	161	43	6923	25921	1849
7	184	49.4	9089.6	33856	2440.36
8	218	56.8	12382.4	47524	3226.24
9	254	61.2	15544.8	64516	3745.44
10	267	58.2	15539.4	71289	3387.24
Sum	1554	456	77753.2	292836	21754.46

i. Find the correlation coefficient between delivery times and the number of cases delivered.

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iv. Predict the delivery time for 150 cases of soft drink.

Correlation coefficient

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}} = \frac{10\times 77753.2 - 1554\times 456}{\sqrt{10\times 292836 - (1554)2}\sqrt{10\times 21754.46 - (456)2}} = 0.981$$

Regression equation Y = a+bX

$\Sigma y = na + b\Sigma x$

or $456 = 10a + 1554b$ .....(i)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

or $77753.2 = 456a + 292836\,b$ ......(ii)

on solving Equation (i) and (ii) we will get

b = 0.134 and a = 24.744

Regression equation of y on x is y = a + bx

y = 24.744 + 0.134 x

Regression Coefficient 0.134 indicates that when number of cases is increased by 1 unit delivery time will increase by 0.134 minute

When number of cases of soft drink is 150

Delivery time in minutes y = 24.744 + 0.134 x150 = 44.875 minutes

Coefficient of determination r^2 = 0.962 indicates that 96.2% of changes in number of cases is due to delivery time.

❑❑❑

r)

reased by 1

in number

4 CHAPTER

PROBABILITY

1. A Card id drawn

Let king = K

Heart = H

Face Card = F

N = 52

m (k) = 4

m (H) = 13

m(f) =- 12

$P (K) = \frac{m(k)}{N} = \frac{4}{52} = \frac{1}{13}$

$P(H) = \frac{m(H)}{N} = \frac{13}{52} = \frac{1}{4}$

$P(F) = \frac{m(F)}{N} = \frac{12}{52} = \frac{3}{13}$

2. Let Boy = B

Girl = G

Total = 15B + 8G = 23

(N) = m(b) = 15, m (G) = 8

$P(B) = \frac{m(B)}{N} = \frac{15}{23}$

3. On rolling a die sample space = {1, 2, 3, 4, 5, 6}

Let even no. = A

N = C

m (A) = 3

$P (A) = \frac{m(A)}{N} = \frac{3}{6} = \frac{1}{2}$

4. On throwing dice

Sample space {(1, 1), (1, 2), (1, 3), (1, 4}, (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Let A = sum greater than 9, B = Neither 8 nor 10

N = 3C

m(A) = 6

m(B) = 36 – 7 = 28

$P(A) = \frac{m(A)}{N} = \frac{6}{36} = \frac{1}{6}$; $P(B) = \frac{m(B)}{N} = \frac{28}{36} = \frac{7}{9}$

5. A secretary

$N = {}^5P_5 = 5\,! = 120$

Let event of putting letters in correct envelop = A

m(A) = 1

$P(\bar{A}) = 1 - P(A) = 1 - \frac{m(A)}{N} = 1 - \frac{1}{120} = \frac{119}{120}$

6. Leap year contains 366 days

A year has 52 complete weeks = 52 × 7 = 364 days

Remaining days = 2

∴ Sample space of remaining 2 days

∴ (Sun – Mon, Mon– Tue, Tue – Wed, Wed – Thu, Thu – Fri, Fri – Sat, Sat – Sun)

Favourable no. of cases for Saturday (m) = 2

Total no. of cases (N) = 7

$P = \frac{m}{N} = \frac{2}{7}$

7. From a pack of

Total cards = 52, selected = 2

Let king = K, Queen = Q, Face card = F, Spade = S, Club = C

$N = {}^{52}C_2 = 1326$

$P(KQ) = \frac{m\,(KQ)}{N} = \frac{{}^4C_1 \times {}^4C_1}{1326} = \frac{4 \times 4}{1326} = 0.012$

$P(2F) = \frac{m\,(2F)}{N} = \frac{{}^{12}C_2}{1326} = \frac{66}{1326} = 0.049$

$P(SC) = \frac{m\,(SC)}{N} = \frac{{}^{13}C_1 \times {}^{13}C_1}{1326} = \frac{13 \times 13}{1326} = 0.127$

8. Thirty Tickets

Let ticket no. multiple of 4 = A

9 = B

3 = C

7 = D

m (A) = 7

m (B) = 3

m (c) = 10

m (D) = 4

m (C ∩ D) = 1

$P (A \cup B) = P (A) + P (B) = \frac{m(A)}{N} + \frac{m(B)}{N} = \frac{7}{30} + \frac{3}{30} = 10\frac{10}{30} = \frac{1}{3}$

$P (C \cup D) = P (C) + P (D) - P (C \cap D) = \frac{m(C)}{N} + \frac{m(D)}{N} - \frac{C \cap D}{N}$

$= \frac{10}{30} + \frac{4}{30} - \frac{1}{30} = \frac{13}{30}$

9. A card is drawn

Total cards = 52, Cards selected = 1

Let, King = k, Queen = Q, Jack = J, Club = C

N = 52, m(k) = 4, m(Q) = 4, m(J) = 4, m(C) = 13, m(J∩C) = 1

i. $P(k \cup Q) = P(k) + P(Q)$

$$= \frac{m(k)}{N} + \frac{m(Q)}{N} = \frac{4}{52} + \frac{4}{52} = \frac{2}{13}$$

ii. $P(J \cup C) = P(J) + P(C) - P(J \cap C)$

$$= \frac{m(J)}{N} + \frac{m(C)}{N} - \frac{m(J \cap C)}{N} = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

10. A problem in

$$P(A) = \frac{1}{5},\ P(B) = \frac{2}{5},\ P(C) = \frac{3}{5}$$

$$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$$

$$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$$

$$= 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$$

$$= 1 - [1 - P(A)]\,[1 - P(B)]\,[1 - P(C)]$$

$$= 1 - \left[1 - \frac{1}{5}\right]\left[1 - \frac{2}{5}\right]\left[1 - \frac{3}{5}\right]$$

$$= 1 - \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5} = 1 - \frac{24}{125} = \frac{101}{125}$$

11. Probability that a

Let Numerical method = M, Computer graphic = S

P(M) = 0.75, P(S) = 0.85

$P(M \cap S) = P(M)\,P(S) = 0.75 \times 0.85 = 0.6375$

$P(M \cup S) = P(M) + P(S) - P(M \cap S) = 0.75 + 0.85 - 0.6375 = 0.962$

12. $P(A) = 0.4,\ P(B) = 0.3,\ P(A \cup B) = 0.58$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Or, $0.58 = 0.4 + 0.3 - P(A \cap B)$

Or, $P(A \cap B) = 0.7 - 0.58 = 0.12$

Here, $P(A)\,P(B) = 0.4 \times 0.3 = 0.12$

$P(A \cap B) = P(A)\,P(B)$

Hence, A and B are independent.

13. The probability that

Let defective itehing = A, crack defect = B

$P(A) = 0.12,\ P(B) = 0.29,\ P(A \cap B) = 0.07$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.12 + 0.29 - 0.07 = 0.34$

14. The probability that

Let 50 yrs old man alive at 60 = A

45 yrs woman alive at 55 = B

P(A) = 0.83, P(B) = 0.87

i. $P(A \cap B) = P(A)\,P(B) = 0.83 \times 0.87 = 0.722$

ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.83 + 0.87 - 0.722 = 0.978$

15. The odds against

For B

$$\frac{N - m}{m} = \frac{4}{3}$$

Or, $P(B) = \frac{m}{N} = \frac{3}{3 + 4} = \frac{3}{7}$

For A

$$\frac{m}{N - m} = \frac{7}{6}$$

$$P(A) = \frac{m}{N} = \frac{7}{7 + 6} = \frac{7}{13}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3}{7} + \frac{7}{13} - \frac{3}{7} \times \frac{7}{13} = \frac{39 + 49 - 21}{91} = \frac{67}{91}$$

16. Two cards are

Total cards = 52, Black cards = 26, Red cards = 26, Ace card = 4

Selected cards = 2 in which are after other

Let Black card = B, red card = R, Ace = A

i. $P((B_I \cap R_{II}) = P(B_I)\,P(R_{II}/B_I) = \frac{26}{52} \times \frac{26}{51} = \frac{13}{51}$

ii. $P(A_I \cap A_{II}) = P(A_I)\,P(A_{II}/A_I) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

17. A book case contains

Total books = 6 DL + 9 MP = 15 books

Books selected = 4

i. $P(4DL_I \cap 4MP_{II}) = P(4\,DL_I)\,P(4\,MP_{II}) = \frac{^6C_4}{^{15}C_4} \times \frac{^9C_4}{^{15}C_4} = \frac{15 \times 126}{1365 \times 1365} = 0.001$

ii. $P(4DL_I \cap 4MP_{II}) = P(4DL_I) \times P(4MP_{II}/4DL_I) = \frac{^6C_4}{^{15}C_4} \times \frac{^9C_4}{^{11}C_4} = \frac{15 \times 125}{1365 \times 330} = 0.004$

18. The student body

Let girls = G, Boys = B, Interest in sports = S

P(G) = 60% = 0.6, P(B) = 40% = 0.4

$P(G \cap S)$ = 40% of 60% = 0.4 × 0.6 = 0.24

$P(B \cap S)$ = 60% of 40% = 0.6 × 0.4 = 0.24

$$P(S/G) = P(GS)/P(G) = \frac{0.24}{0.6} = \frac{2}{5}$$

19. A and B toss

Prob. of A's getting head $P(A) = \frac{1}{2}$

Prob. of not getting head $P(\bar{A}) = 1 - P(A) = \frac{1}{2}$

Prob. of B's getting head $P(B) = \frac{1}{2}$

Prob. of not getting $P(\bar{B}) = 1 - P(B) = \frac{1}{2}$

If A starts game

Prob. of A's winning = $P(A \cup \bar{A}\bar{B}A \cup \bar{A}\bar{B}\bar{A}\bar{B}A \cup \ldots)$

$= P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$

$= \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \ldots\ldots$

$= \dfrac{\frac{1}{2}}{1 - \frac{1}{2} \times \frac{1}{2}} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

Prob. of B's winning = 1 – Prob. of A's winning = $1 - \frac{2}{3} = \frac{1}{3}$

20. A person is known

Let a person A and another B

Here $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$

$P(A \cap B) = P(A) \times P(B) = \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

21. A six faced dice

Let Odd = O and even = E

$P(E) = 2/3$, $P(O) = 1/3$

P(Even) = P(EE or OO) = $P(E)P(E) + P(O)P(O) = \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{4}{9} + \frac{1}{9} = \frac{5}{9}$

22. In tossing a coin

Let head = H, tail = T

Sample space = {HHH, HHT, HTH, THH, HTT, THT, TTH,TTT}

N = 8

P(No. of heads) = $\frac{1}{8}$

P(Two heads) = $\frac{3}{8}$

P(At least two heads) = $\frac{4}{8} = \frac{1}{2}$

23. There are three traffic

$P(G) = 0.7$, $P(R) = 1 - P(G) = 1 - 0.7 = 0.3$

Sample space = {RRR, RRG, GRR, GRG, RGG, GGR, GGG}

P(stop no. more than one) = $P(GRG \cup RGG \cup GGR \cup GGG)$

$= P(G)P(R)P(G) + P(R)P(G)P(G) + P(G)P(G)P(R) + P(G)P(G)P(G)$

$= 0.7 \times 0.3 \times 0.7 + 0.3 \times 0.7 \times 0.7 + 0.7 \times 0.7 \times 0.3 + 0.7 \times 0.7 \times 0.7$

$= 0.441 + 0.343 = 0.784$

24. The odds that

Let first critics A, second B and third C.

For A, $\frac{m}{N - m} = \frac{3}{2}$ For B, $\frac{m}{N - m} = \frac{4}{3}$

$P(A) = \frac{m}{N} = \frac{3}{3 + 2} = \frac{3}{5}$ $P(A) = \frac{m}{N} = \frac{4}{3} = \frac{4}{7}$

For C, $\frac{m}{N - m} = \frac{2}{3}$ $P(C) = \frac{m}{N} = \frac{2}{2 + 3} = \frac{2}{15}$

P(majority will be favourable) = $P(AB\bar{C} \cup \bar{A}BC \cup A\bar{B}C \cup ABC)$

$= P(A)P(B)P(\bar{C}) + P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C) + P(A)P(B)P(C)$

$= \frac{3}{5} \times \frac{4}{7} \times \left(1 - \frac{2}{5}\right) + \left(1 - \frac{3}{5}\right) \times \frac{4}{7} \times \frac{2}{5} + \frac{3}{5} \times \left(1 - \frac{4}{7}\right) \times \frac{2}{5} + \frac{3}{5} \times \frac{4}{7} \times \frac{2}{5}$

$= \frac{36}{175} + \frac{16}{175} + \frac{18}{175} + \frac{24}{175} = \frac{94}{175} = 0.537$

25.

Bank Credit card	Travel & Entertainment Credit card		
	Yes	No	Total
Yes	60	60	120
No	15	65	80
Total	75	125	200

a. P(the student has a bank credit card) = $\frac{120}{200} = \frac{3}{5} = 0.6$

b. P(the student has a bank credit card and a travel and entertainment card)

$= \frac{60}{200} = 0.3$

c. P(the student has a bank credit card or a travel and entertainment card)

$= P(B \text{ or } T) = P(B) + P(T) - P(B\&T) = \frac{120}{200} + \frac{75}{200} - \frac{60}{200} = \frac{135}{200} = 0.675$

d. P(he or she has a travel and entertainment card) = $\frac{60}{120} = 0.5$

26. a.

Gender	Health Club Facility		Total
	Used	Not used	
Male	65	105	170
Female	45	35	80
total	110	140	250

b. P(an employee chosen at random is a female and has used the health club facilities) $= \frac{45}{250} = 0.18$

c. P(an employee chosen at random is a male) $= \frac{170}{250} = 0.68$

d. P(an employee chosen at random is a male or has not used the health club facilities)

P(M or NH) = P(M) + P(NH) − P(M&NH)

$= \frac{170}{250} + \frac{140}{250} - \frac{105}{250} = \frac{100}{250} = 0.82$

27. Comment on

Let $x \sim B(n, p)$

$E(x) = 7$

$V(x) = 11$

Now,

$E(x) = np = 7$ .... (i)

$V(x) = n\,p\,q = 11$ .... (ii)

Substitute the value of np from (i) in (ii)

$7q = 11$

Or, $q = \frac{11}{7} = 1.57 > 1$, Which is impossible.

Hence, given information is incorrect.

28. Find p if

$n = 6,\ 9\,p\,(x = 4) = p(x = 2),\ p = ?$

Here, $9p\,(x = 4) = p(x = 2)$

or, $9\,c(6, 4)\,p^4 q^2 = c\,(6, 2)\,p^2 q^4$

or, $9 \times 15\,p^4 q^2 = 15\,p^2 q^4$

or, $9p^2 = q^2$

or $9p^2 - q^2 = 0$

or, $9p^2 - (1 - p)^2 = 0$

or, $9p^2 - (1 - 2p + p^2) = 0$

or, $9p^2 - 1 + 2p - p^2 = 0$

or, $8p^2 + 2p - 1 = 0$

or, $8p^2 + 4p - 2p - 1 = 0$

or, $4p\,(2p + 1) - 1\,(2p + 1) = 0$

or, $(2p + 1)(4p - 1) = 0$

$\therefore\ p = -\frac{1}{2}, \frac{1}{4}$

$p = -\frac{1}{2}$ is not possible $\therefore\ p = \frac{1}{4}$

Let defective itenng ...

$P(A) = 0.12,\ P(B) = 0.29,\ P(A \cap B) = 0.07$

$P(A \cup B) = 0.12 + 0.29 - 0.07 = 0.34$

29. In a binomial distribution

Let $x \sim B(n, p)$ where x = no of success

$n = 6$

$p\,(x = 3) = 0.2457$

$p\,(x = 4) = 0.0818$

Now,

$p(x = 3) = c(6, 3)\,p^3 q^3 = 0.2457$ .... (i)

$p(x = 4) = c(6, 4)\,p^4 q^2 = 0.0818$ .... (ii)

Divide (i) by (ii)

$\frac{C(6, 3)\,p^3 q^3}{C(6, 4)\,p^4 q^2} = \frac{0.2457}{0.0818}$

or, $\frac{20\,q}{15\,p} = 3.003$

or, $4q = 3\,p \times 3.003$

or, $4\,(1 - p) = 9.009\,p$

or, $4 - 4p = 9.009\,p$

or, $4 = 13.009\,p$

or, $p = \frac{4}{13.009}$

$\therefore\ p = 0.307$

$q = 1 - p = 1 - 0.307 = 0.619$

Mean $= n\,p = 6 \times 0.307 = 1.842$

Variance $= n\,p\,q = 6 \times 0.307 \times 0.619 = 1.14$

30. The mean and variance

Let $x \sim B(n, p)$

$E(x) = np = 3$ (i)

$V(x) = npq = 2$ (ii)

Substitute value of np from (i) in (ii)

$3q = 2$ Or, $q = \frac{2}{3}$

$p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

substitute the value of p in (i)

$n \times \frac{1}{3} = 3$ Or, $n = 9$

(i) $p(x \le 2) = p(x = 0) + p(x = 1) + p(x = 2)$

$= C(9, 0)\,p^0 q^9 + C(9, 1)\,p^1 q^8 + C(9, 2)\,p^2 q^7$

$= 1 \times \left(\frac{2}{3}\right)^9 + 9 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^8 + 3\,C \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^7$

31.

32.

33

34

$= 0.026 + 0.117 + 0.234 = 0.377$

(ii) $p(x \geq 7) = p(x = 7) + p(x = 8) + p(x = 9)$

$= C(9, 7)\, p^7 q^2 + C(9, 8)\, p^8 q^1 + C(9, 9)\, p^9 q^0$

$= 36 \times \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^2 + 9 \times \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right) + 1 \times \left(\frac{1}{3}\right)^9$

$= \left(\frac{1}{3}\right)^7 \left[16 + 2 + \frac{1}{9}\right] = \frac{18.111}{2187} = 0.0082$

31. 12% of the items

Let, defective items = x

Prob. of defective $p = 12\% = 0.12$, $q = 1 - p = 0.88$

$n = 20$

$p(x = 5) = c(20, 5)\, p^5 q^{15} = c(20, 5)\, (0.12)^5 (0.88)^{15} = 0.056$

32. The average no.

Let x = defective pieces

Prob. Of defective pieces (p) $= \frac{1}{10}$, $q = \frac{9}{10}$

$n = 10$

$p(x = 3) = c(10, 3)\, p^3 q^7 = c(10, 3) \left(\frac{1}{10}\right)^3 \left(\frac{1}{10}\right)^9 = 120 \times 0.001 \times 0.4782 = 0.057$

33. Let x = defective computers

$P = 5\% = 0.05$, $q = 0.95$

$n = 20$,

$P(X = 3) = {}^{20}C_3\, (0.05)^3 (0.95)^{17} = 1140 \times 0.0000522 = 0.059$

34. The probability that

Let, x = No. of students graduate

Prob. of students graduate $p = 0.4$, $q = 1 - p = 1 - 0.4 = 0.6$

(i) $P(x = 0) = C(n, x)\, p^x q^{n-x} = C(5, 0)\, (0.4)^0 (0.6)^{5-0} = 0.077$

(ii) $P(x = 1) = C(n, x)\, p^x q^{n-x} = C(5, 1)\, (0.4)^1 (0.6)^{5-1} = 0.25$

(iii) $P(x \geq 1) = 1 - P(x < 1) = 1 - P(x = 0) = 1 - 0.077 = 0.923$

(iv) $P(x = 5) = C(n, x)\, p^x q^{n-x} = C(5, 5)\, (0.4)^5 (0.6)^0 = 0.0102$

35. Let x = No. of times success

p = probability of developing software

$p = \frac{1}{3}$, $q = \frac{2}{3}$

$P(x \geq 1) > 90\%$

Or, $1 - P(X < 1) > 0.9$

Or, $0.1 > C(n, 0) \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n$

Or, $0.1 > \left(\frac{2}{3}\right)^n$

By trial method $n = 6$

36. At a particular university

Let x = No. of students with draw without computing course

p = prob. of students withdraw without computing course

$p = 20\% = 0.2$, $q = 1 - p = 0.8$, $n = 18$

i. $P(x = 0) = C(18, 0)\, (0.2)^0 (0.8)^{18-0} = (0.8)^{18} = 0.01$

ii. $P(x \geq 1) = 1 - P(X < 1) = 1 - P(x = 0) = 1 - 0.01 = 0.99$

iii. $P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$

$= C(18, 0)\, (0.2)^0 (0.8)^{18} + C(18, 1)\, (0.2)^1 (0.8)^{17} + C(18,2)\, (0.2)^2 (0.8)^{16}$

$= (0.8)^{16}\{0.64 + 2.88 + 6.12\} = (0.8)^{16} \times 9.64 = 0.271$

37. Solution.

When no. of success = 2

No. of failure = 1

Total trial = 2 + 1 = 3

Probability of success (p) $= \frac{2}{3}$ $\therefore q = \frac{1}{3}$

Let x = no. of trial,

$n = 6$

$P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6)$

$= c(6, 4) \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + c(6, 5) \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + c(6, 6) \left(\frac{2}{3}\right)^6$

$= 15 \times 0.0219 + 6 \times 0.0438 + 1 \times 0.087741$

$= 0.6742$

38. From the past experience

Let, x = no. of telephone calls which are ordered

Prob. of ordered telephone calls (p) = 70% = 0.7

$n = 8$

(i) $P(x = 5) = C(n, x)\, p^x q^{n-x} = C(8, 5)\, (0.7)^5 (0.3)^3 = 0.254$

(ii) $P(x \geq 6) = P(x = 6) + P(x = 7) + P(x = 8)$

$= C(8, 6)\, (0.7)^6 (0.3)^2 + C(8, 7)\, (0.7)^7 (0.3)^1 + C(8, 8)\, (0.7)^8$

$= (0.7)^6\, [28 \times 0.09 + 8 \times 0.21 + 0.49] = 0.551$

39. A discrete

Let $x \sim B(n, p)$

$E(x) = np = 6$ .... (i)

$V(x) = n\,pq = 2$ .... (ii)

Substitute value of np from (i) in (ii)

$6q = 2$

Or, $q = \frac{1}{3}$ $\therefore p = \frac{2}{3}$

Substitute p in (i)

$n \times \frac{2}{3} = 6$

or, $n = \frac{18}{2} = 9$

$P(5 < x < 7) = P(x = 6) = C(n, x)\, p^x q^{n-x}$

$= C(9, 6) \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^{9-6} = 84 \times 0.087 \times 0.037 = 0.27$

40. X = No. of program must be upgrade

Prob. of upgrading program (p) = $\frac{5}{12}$; $q = 1 - \frac{5}{12} = \frac{7}{12}$

n = 4

(i) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(x = 0) + P(x = 1)]$

$= 1 - \left[{}^4C_0 \left(\frac{5}{12}\right)^0 \left(\frac{7}{12}\right)^4 + {}^4C_1 \left(\frac{5}{12}\right)^1 \left(\frac{7}{12}\right)^3 \right]$

$= 1 - \left[\left(\frac{7}{12}\right)^4 + 4 \frac{5}{12} \times \left(\frac{7}{12}\right)^3 \right]$

$= 1 - \left(\frac{7}{12}\right)^3 \times \frac{27}{12} = 1 - (0.583)^3 = 0.554$

$E(x) = np = 4 \times 0.554 = 2.216$

41. Let X = No. of player buy advanced version of game

Prob. of player buy advanced version game (b) = 40% of 50% = $\frac{40}{100} \times \frac{50}{100} = 0.2$

$q = 0.8, n = 12$

$E(x) = np = 12 \times 0.2 = 2.4$

$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(x = 0) + P(X = 1)]$

$= 1 - [{}^{12}C_0(0.2)^0 (0.8)^{12} + 12C_1(0.2)^1 (0.8)^{11}$

$= 1 - (0.8)^{11} \times 3.2 = 0.725$

$P(X \leq 3) = P(X = 0) + P(X = 11) + P(X = 2) + P(X = 3)$

$= {}^{12}C_0 (0.2)^0 (0.8)^{12} + {}^{12}C_1 (0.2)^1 (0.8)^{11} + {}^{12}C_2 (0.2)^2 (0.8)^{10} + {}^{12}C_3 (0.2)^3 (0.8)^9$

$= (0.8)^{12} + 12 \times 0.2 \times (0.8)^{11} + 66 \times 0.04 \times (0.8)^{10} + 220 \times 0.08 \times (0.8)^9$

$= 0.794$

42. An Electronic device

$p = 3\% = 0.03, q = 0.97$

$x = 20$

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0) = 1 - c(20, 0) (0.30)^0 (0.97)^{20} = 0.45$

Again,

$p = 0.45, q = 0.55$

$n = 10$

$p(X = 3) = c(10, 3) (0.45)^3 (0.55)^7 = 0.104$

3. An automatic machine

Let, x = no. of defectives

Prob. of defective (p) = $\frac{1}{400}$

$n = 100$

Average defective (λ) = $n p = 100 \times \frac{1}{400} = 0.778$

i. $P(x = 0) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.25} (0.25)^0}{0!} = 0.778$

ii. $P(x \geq 1) = 1 - P(x < 1) = 1 - P(x = 0) = 1 - 0.778 = 0.221$

iii. $P(x < 2) = P(x \leq 1) = P(x = 0) + P(x = 1)$

$= 0.778 + \frac{e^{-0.25} (0.25)^1}{1!} = 0.778 + 0.194 = 0.972$

44. The chance of

Let x = No. of traffic accidents

Probability of traffic accident in a street (p) = 0.0005

No. of street (n) = 1000

Now,

$\lambda = n p = 1000 \times 0.0005 = 0.5$

i. $P(x = 0) = \frac{e^{-0.5} (0.5)^0}{0!} = 0.606$

Consider in a year

No. of days with no accidents = $365 \times P(x = 0)$

$= 365 \times 0.606 = 221.38 = 221$

iii. $P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$= 1 - \left\{ \left[\frac{e^{-0.5} (0.5)^0}{0!} + \frac{e^{-0.5} (0.5)^1}{1!} + \frac{e^{-0.5} (0.5)^2}{2!} + e^{-0.5}(0.5)^3/3! \right] \right\}$

$= 1 - e^{-0.5}\{1 + 0.5 + 0.125 + 0.0208)) = 1 - 1.645\, e^{-0.5} = 0.007$

No. of days with more then three accidents = $365 \times P(x > 3)$

$= 365 \times 0.0017 = 0.64 = 1$

45. Let λ = No. of telephone calls

$\lambda = 3$ per minute

$P(X = 0) = \frac{e^{-3}\, 3^0}{0!} = 0.049$

$\lambda = 3 \times 3 = 9$ per three minute

$P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= \frac{e^{-9}\, 9^0}{0!} + \frac{e^{-9}\, 9^1}{1!} + \frac{e^{-9}\, 9^2}{2!} + \frac{e^{-9}\, 9^3}{3!} + \frac{e^{-9}\, 9^4}{4!} = 0.945$

46. The average

Let x = No. of network error

Average no. network error (λ) = 2.4

i. $P(x = 0) = \frac{e^{-2.4} (2.4)^0}{0!} = 0.09$

ii. $P(x \geq 1) = 1 - P(x < 1) = 1 - P(x = 0) = 0.91$

iii. $P(x = 1) = \frac{e^{-2.4} (2.4)^1}{1!} = 0.21$

47. Solution.

Let x = No. of messages arises

$\lambda = 9$ per hour

(a) $P(x \geq 3) = 1 - P(X < 3) = 1 - \left[\frac{e^{-9} 9^0}{0!} + \frac{e^{-9} 9^1}{1!} + \frac{e^{-9} 9^2}{2!}\right]$

$= 0.997$

(b) $\lambda = 9 \times 2 = 18$ per hour

$P(x = 5) = \frac{e^{-18} 18^5}{5!} = 0.066$

48. If A random variable

$x \sim P(\lambda)$

Here, $P(x = 1) = P(x = 2)$

Or, $\frac{e^{-\lambda} \lambda^1}{1!} = \frac{e^{-\lambda} \lambda^2}{2!}$

Or, $\lambda = \frac{\lambda^2}{2}$

Or, $\lambda = 2$

$\therefore$ Mean = $E(x) = \lambda = 2$

Variance = $V(x) = \lambda = 2$

49. Calculate mean and

$x \sim P(x)$

$P(x = 4) = P(x = 5)$

Or, $\frac{e^{-\lambda} \lambda^4}{4!} = \frac{e^{-\lambda} \lambda^5}{5!}$

Or, $\lambda = 5$

$\therefore$ Mean = $E(x) = \lambda = 5$

Variance = $V(x) = \lambda = 5$

50. Let λ = no. of file affected by virus

$n = 250$

$p = 0.032$

$\lambda = np = 250 \times 0.032 = 8$

$P(X > 5) = ?$

$P(X > 5) = 1 - P(X \leq 5)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)]$

$= 1 - \left[\frac{e^{-8} 8^0}{0!} + \frac{e^{-8} 8^1}{1!} + \frac{e^{-8} 8^2}{2!} + \frac{e^{-8} 8^3}{3!} + \frac{e^{-8} 8^4}{4!} + \frac{e^{-8} 8^5}{5!}\right] = 0.808$

51. A manufacturer produces IC chips

Let x = No. of IC chips defective

Probability of IC chips is defective (p) = 1% = 0.01

No. of boxes (n) = 100

$\lambda = n\,p = 100 \times 0.01 = 1$

$P(x = 0) = \frac{e^{-1} 1^0}{0!} = 0.36$

52. The probability of

Let x = No. of error in transmission of a bit

Probability of error in transmission of a bit (p) = 0.001

No. of bit (n) = 1000

$\lambda = n\,p = 1000 \times 0.001 = 1$

$P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$= 1 - \left[\frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!} + \frac{e^{-1} 1^2}{2!} + \frac{e^{-1} 1^3}{3!}\right]$

$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right] = 1 - 0.982 \times 2.66 = 0.018$

53. X = No. of electronic message; $\lambda = 9$

$P(X \geq 5) = 1 - P(X < 5)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]\ PCx = 3 + P(x = 4)$

$= 1 - \left[\frac{e^{-9} 9^0}{0!} + \frac{e^{-9} 9^1}{1!} + \frac{e^{-9} 9^2}{2!} + \frac{e^{-9} 9^3}{3!} + \frac{e^{-9} 9^4}{4!}\right]$

$= 1 - e^{-9} [1 + 9 + 9 + 121.5 + 273.375)$

$= 1 - e^{-9} \times 413.875 = 0.948$

$P(x = 7) = \frac{e^{-9} e^7}{7!} = 0.117$

54. Prob. that computer crash (p) $= \frac{1}{1000}$

$n = 5000;\ \lambda = n\,p = 5000 \times \frac{1}{1000} = 5$

a. $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = \left[\frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!}\right] = 0.124$

b. $P(\lambda = 10) = \frac{e^{-5} 5^{10}}{10!} = 0.018$

55. Suppose that the

Let, x = no. of calls between 10 a. m. and 11 a. m.

y = no. of calls between 11 a. m. and 12 noon.

$x \sim P(\lambda);\ \lambda = 2$

$y \sim P(\lambda);\ \lambda = 4$

$z = x + y$ = no. of calls between 10 a. m. and 12 noon

$z \sim P(\lambda)$ $\quad\quad \lambda = 2 + 4 = 6$ for $\geq$ hr.

$P(z \geq 5) = 1 - P(z < 5)$

$= 1 - [P(z = 0) + P(z = 1) + P(z = 2) + P(z = 3) + P(z = 4)]$

$= 1 - \left[\frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} + \frac{e^{-6}6^3}{3!} + \frac{e^{-6}6^4}{4!}\right]$

$= 1 - e^{-6}[1 + 6 + 18 + 36 + 54] = 1 - e^{-6} \times 115$

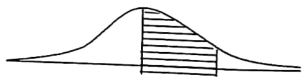
$= 1 - 0.285$

$= 0.715$

56. If Z is standard

$Z \sim N(0, 1)$

a. $P(0 < Z < 1.34) = 0.4099$

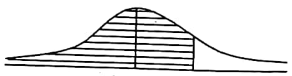


z = 0 z = 1.34

b. $P(Z < 1.34) = 0.5 + P(0 < 2 < 1.34)$

$= 0.5 + 0.4099$

$= 0.9099$



z = 0 z = 2.34

c. $P(Z > 1.96) = 0.5 - P(0 < Z < 1.96)$

$= 0.5 - 0.4750$

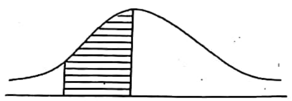
$= 0.025$

z = 0 z = 1.96

d. $P(1.34 < Z < 0)$

$= P(0 < Z < 1.34)$

$= 0.4099$



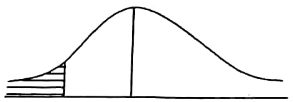
z = – 1.34 z = 0

e. $P(Z < -1.34)$

$= 0.5 - P(-1.34 < Z < 0)$

$= 0.5 - P(0 < Z < 1.34)$

$= 0.5 - 0.4099 = 0.0901$



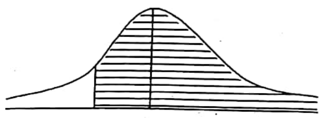
z = – 1.34 z = 0

f. $P(Z > -2.04)$

$= P(-2.04 < Z < 0) + 0.5$

$= P(0 < Z < 2.04) + 0.5$

$= 0.4793 + 0.5 = 0.9793$



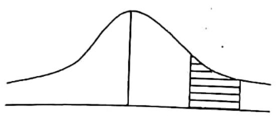
z = –2.04 z = 0

g. $P(1.96 < Z < 2.84)$

$= P(0 < Z < 2.84) - P(0 < Z < 1.96)$

$= 0.4977 - 0.4750$

$= 0.0227$



z = 0 Z=1.96 Z=2.84

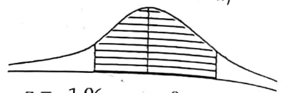
h. $P(-1.96 < Z < 2)$

$= P(-1.96 < Z < 0) + P(0 < Z < 2)$

$= P(0 < Z < 1.96) + P(0 < Z < 2)$

$= 0.4750 + 0.4772$

$= 0.9522$



z = –1.96 z = 0 z = 2

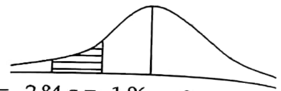
i. $P(-2.48 < Z < -1.96)$

$= P(-2.48 < Z < 0) - P(-1.96 < Z < 0)$

$= P(0 < Z < 2.84) - P(0 < Z < 1.96)$

$= 0.4977 - 0.4750$

$= 0.0227$



z = –2.84 z = –1.96 z = 0

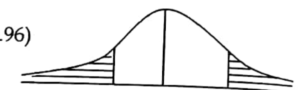
j. $P(Z < -1.96 \text{ or } Z > 1.96)$

$= 0.5 - P(-1.96 < Z < 0) + 0.5 - P(0 < Z < 1.96)$

$= 1 - 2P(0 < Z < 1.96)$

$= 1 - 2 \times 0.4750$

$= 1 - 0.95 = 0.05$



z = –1.96 z = 0 z = 1.96

57. Suppose X follows the

$\mu = 100, \sigma = 10$

$X \sim N(\mu, \sigma^2)$

a. $P(100 < X < 110) = ?$

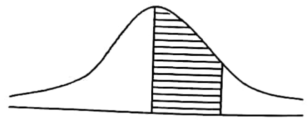
Define $Z = \frac{X - \mu}{\sigma} = \frac{X - 100}{10}$

When $X = 100$, $Z = \frac{100 - 100}{10} = 0$

When $X = 110$, $Z = \frac{110 - 100}{10} = 1$

$P(100 < X < 110) = P(0 < Z < 1)$

$= 0.3413$



z = 0 Z = 1

b. $P(X > 120)$

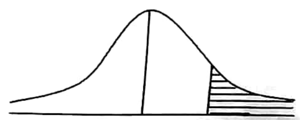
When X = 120,

$Z = \frac{120 - 100}{10} = 2$

$P(X > 120) = P(Z > 2)$

$= 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772 = 0.0228$



z = 0 z = 2

c. $P(X < 115)$

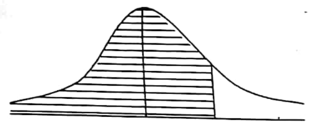
When, X = 115

$Z = \frac{115 - 100}{10} = 1.5$

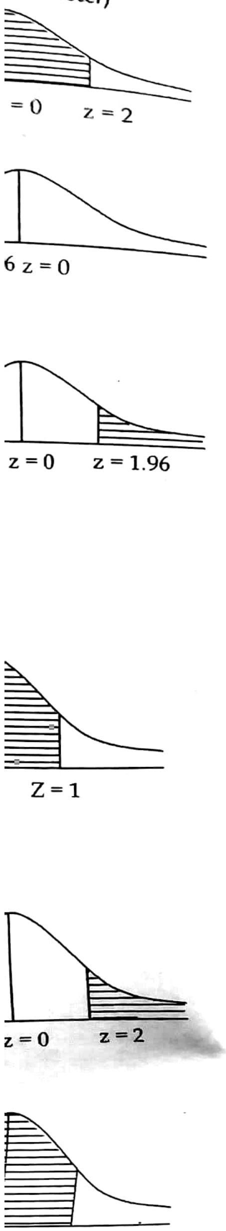
$P(X < 115) = P(Z < 1.5)$

$= 0.5 + P(0 < Z < 1.5)$

$= 0.5 + 0.4332 = 0.8332$



z = 0 Z = 1.5



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d. P(80 < X < 100)

When X = 80,

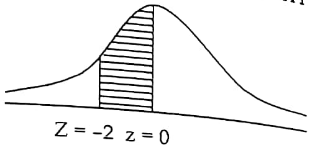
$Z = \dfrac{80-100}{10} = 2$

When X = 100

$Z = \dfrac{X-100}{100} = \dfrac{100-100}{100} = 0$

$P(80 < X < 100) = P(-Z < Z < 0)$

$= P(0 < Z < 2) = 0.4772$

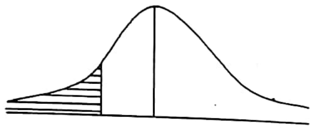


e. $P(X < 80) = P(Z < -2)$

$= 0.5 - P(-2 < Z < 0)$

$= 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772 = 0.028$



f. P(X > 90)

When X = 90

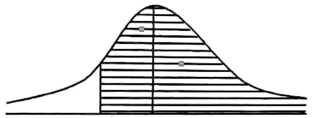
$Z = \dfrac{90-100}{10} = -1$

$P(X > 90) = P(Z > -1)$

$= P(-1 < Z < 0) + 0.5$

$= P(0 < 2 < 1) + 0.5$

$= 0.3413 + 0.5 = 0.8413$



g. P(112 < X < 130)

When X = 112, $Z = \dfrac{112-100}{10} = 1.2$

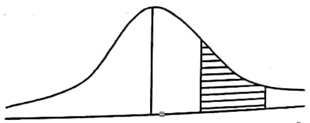
When X = 130, $Z = \dfrac{130-100}{10} = 3.0$

$P(112 < X < 130) = P(1.2 < Z < 3)$

$= P(0 < Z < 3) - P(0 < Z < 1.2)$

$= 0.49865 - 0.3849$

$= 0.11375$



h. P(85 < X < 125)

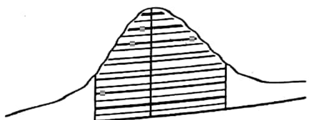
When X = 85, $Z = \dfrac{85-100}{10} = -1.5$

When X = 125, $Z = \dfrac{125-100}{10} = 2.5$

$P(85 < X < 125) = P(-1.5 < Z < 2.5)$

$= P(-1.5 < Z < 0) + P(0 < Z < 2.5)$

$= 0.4332 + 0.4938 = 0.9270$



i. P(75 < X < 95)

When X = 75, $Z = \dfrac{75-100}{10} = -2.5$

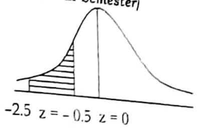
When X = 95, $Z = \dfrac{95-100}{10} = -0.5$

$P(75 < X < 95) = P(-2.5 < Z < -0.5)$

$= P(-2.5 < Z < 0) - P(-0 < Z < 0)$

$= P(0 < Z < 2.5) - P(0 < Z < 0.5)$

$= 0.4938 - 0.1915 = 0.3023$



58. Solution.

Let x = monthly production of computer parts $x \sim N(\mu, \sigma^2)$

$\mu = 100{,}000$

$\sigma = 20{,}000$

(i) P(x > 120,000)

Define $Z = \dfrac{X-\mu}{\sigma} = \dfrac{X-100000}{20000}$

When X = 120,000, $Z = \dfrac{120000-100000}{20000} = 1$

∴ $P(X > 120{,}000) = P(Z > 1)$

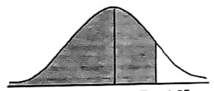
$= 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413 = 0.1587$

(ii) P(x < 125000)

When X = 125,000

$Z = \dfrac{125000-100000}{20000} = 1.25$



∴ P(X < 125000)

$= P(Z < 1.25)$

$= 0.5 + P(0 < Z < 1.25)$

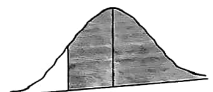
$= 0.5 + 0.3944$

$= 0.8944$

(iii) P(x > 60,000)

When X = 60,000

$Z = \dfrac{60000-100000}{20000} = -2$



P(X > 60,000)

$= P(Z > -2)$

$= P(-2 < Z < 0) + 0.5$

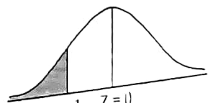
$= 0.4772 + 0.5$

$= 0.9772$

(iv) P(X < 80,000)

When X = 80,000

$Z = \dfrac{80000-100000}{20000} = -1$

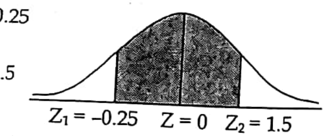


$\therefore$ $P(X < 80,000)$
$= P(Z < -1)$
$= 0.5 - P(-1 < Z < 0)$
$= 0.5 - P(0 < Z < 1)$
$= 0.5 - 0.3413 = 0.1587$

(v) $P(105000 < X < 130000)$

When $X = 1050,00$ $Z_1 = \dfrac{105000 - 100000}{20000} = 0.25$

When $X = 130000$, $Z_2 = \dfrac{130000 - 100000}{20000} = 1.5$



$P(105000 < X < 130000)$
$= P(-0.25 < Z < 1.5) = P(0 < Z < 1.5) - P(0 < Z < 0.25)$
$= P(0 < Z < 1.5) - P(0 < Z < 0.25)$
$= 0.4332 - 0.0987$
$= 0.3345$

59. The mean yield

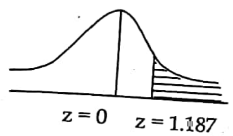
Let, x = yield in kilos

$x \sim N(\mu, \sigma^2)$, $\mu = 662$, $\sigma = 32$, $N = 1000$

i. $P(x > 700)$

Define, $Z = \dfrac{x - \mu}{\sigma} = \dfrac{x - 662}{32}$

When, $x = 700$, $z = \dfrac{700 - 662}{32} = 1.87$

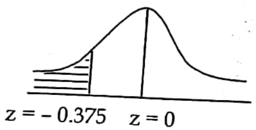


$P(x > 700) = P(z > 1.187)$
$= 0.5 - P(0 < z < 1.187)$
$= 0.5 - 0.381 = 0.119$

$\therefore$ No. of plot $= N\,P(x > 700) = 1000 \times 0.119 = 119$

ii. $P(x < 650)$

When, $x = 650$, $z = \dfrac{x - 662}{32} = \dfrac{650 - 662}{32} = -0.375$



$P(x < 650) = P(z < -0.375)$
$= 0.5 - P(0 < z < 0.375)$
$= 0.5 - 0.144$
$= 0.356$

$\therefore$ No. of puts $= N\,P(x < 650) = 1000 \times 0.356 = 356$

ii. Let, lowest yield $= x_1$

$P(x \geq x_1) = \dfrac{100}{1000}$

Or, $P(x \geq x_1) = 0.1$

When $x = x_1$

$z = \dfrac{x_1 - 662}{32} = z_1$ (say) .... (i)

Then,

$P(x \geq x_1) = 0.1$

$\Rightarrow$ $P(z \geq z_1) = 0.1$

$\Rightarrow$ $0.5 - P(0 \leq z \leq z_1) = 0.1$

$\Rightarrow$ $0.5 - 0.1 = P(0 \leq z \leq z_1)$

$\Rightarrow$ $P(0 \leq z \leq z_1) = 0.4$

Then $z_1 = 1.28$

Substitute z_1 in (i)

$\dfrac{x_1 - 662}{32} = 1.28$

Or, $x_1 = 32 \times 1.28 + 662 = 702.96 \approx 703$

Hence, lowest yield f best 100 plots is 703 kilos.

60. Let x = time taken to download, n = 95

For one file

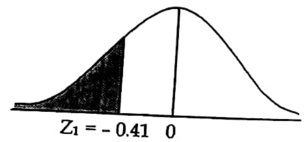
$\mu = 16$, $\sigma = 5$, $\sigma^2 = 25$

For 95 file

$\mu = 16 \times 95 = 1520$

$\sigma^2 = 25 \times 95 = 2375$ $\quad\therefore \sigma = 48.73$

$\mu = 1520$, $\sigma = 48.73$



$P(x < 1500) = P\left(z \leq \dfrac{1500 - 1520}{48.73}\right)$
$= P(Z < -0.41)$
$= 6.5 - P(-0.41 < z < 0)$
$= 0.5 - 0.1591$
$= 0.3409$

61. The local authorities

No. of lamps (N) = 1000

Average life of lamp (μ) = 1000 hrs

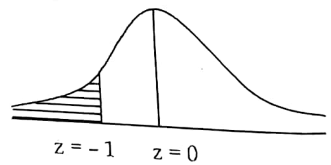
S.D. (σ) = 200 hrs

Let, X = no. of burning hrs of lamp

Define $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 1000}{200}$

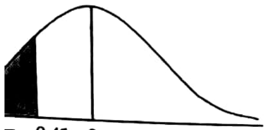
i. $P(X < 800)$

When, $X = 800$, $Z = \dfrac{800 - 1000}{200} = -1$

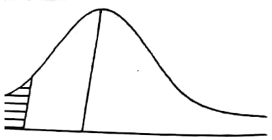


$P(X < 800) = P(Z < -1)$
$= 0.5 - P(-1 < Z < 0)$
$= 0.5 - 0.3413 = 0.1587$

No. of bulb burn fail in 800 hrs $= N\,P(X < 800) = 1000 \times 0.1587 = 158.7 \approx 159$



$= -0.41 \quad 0$



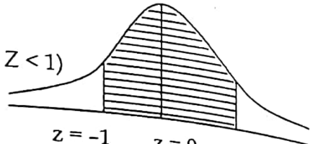
$-1 \quad z = 0$

$0 \times 0.1587 = 158.7 \approx 159$

ii. $P(800 < X < 1200)$

When $X = 1200$, $Z = \dfrac{1200 - 1000}{200} = 1$

$P(800 < X < 1200) = P(-1 < Z < 1)$
$= P(-1 < Z < 0) + P(0 < Z < 1)$
$= 2\,P(0 < Z < 1)$
$= 2 \times 0.3413$
$= 0.6826$



$z = -1 \quad z = 0 \quad z = 1$

No. of bulb burn between 800 and 1200 hrs

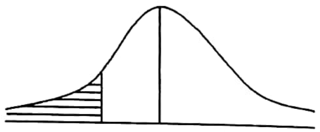
$= N\,P(800 < X < 1200) = 1000 \times 0.6826 = 682.6 \approx 683$

Let, after $x = x_1$ hrs. 10% lamp fail

$P(x < x_1) = 10\%$

When, $x = x_1$

$Z = \dfrac{x_1 - 1000}{200} = -Z_1$ (say) .... (i)



$z = -z_1 \quad z = 0$

$P(x < x_1) = 0.1$

Or, $P(z < -z_1) = 0.1$

Or, $0.5 - P(-z_1 < z < 0) = 0.1$

Or, $0.5 - 0.1 = P(0 < z < z_1)$

Or, $P(0 < Z < Z_1) = 0.4$

$Z_1 = 1.28$

Substitute z_1 in (i)

$\dfrac{x_1 - 1000}{200} = -1.28$

Or, $x_1 - 1000 = -256$

Or, $x_1 = 1000 - 256 = 744$

∴ 10% lamps fails on burning 744 hrs

Let after $x = x_2$ hrs. 10% lamp continue burning

$P(x > x_2) = 10\%$

When $x = x_2$, $z = \dfrac{x_2 - 1000}{200} = z_2$ (say) .... (ii)

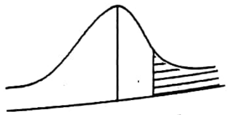
$P(x > x_2) = 0.1$

Or, $P(z > z_2) = 0.1$

Or, $0.5 - 0.1 = P(0 < z < z_2)$

Or, $P(0 < z < z_2) = 0.4$

$Z_2 = 1.28$



$z = 0 \quad z = z_2$

Substitute z_2 in (ii)

$\dfrac{x_2 - 1000}{200} = 1.28$

Or, $x_2 - 1000 = 256$

Or, $x_2 = 1000 + 256 = 1256$

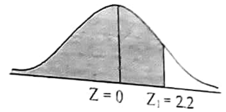
10% of lamps continue burning after 1256 hrs.

62. Let x = Time spent on training $X \sim N(\mu, \sigma^2)$

$\mu = 500$, $\sigma = 100$

$P(X < 420) = ?$

Let $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 200}{100}$



$Z = 0 \quad Z_1 = 2.2$

When $X = 4200$, $Z = \dfrac{420 - 200}{100} = \dfrac{220}{100} = 2.2$

$P(X < 420) = P(Z < 2.2) = 0.5 + P(0 < Z < 2.2) = 0.5 + 0.4861 = 0.9861$

63. **Solution.**

Let x = life time of electronic component

$x \sim N(\mu, \sigma^2)$

$\mu = 5000$, $\sigma = 100$

$P(X < 5012) = ?$

$P(4000 < X < 6000) = ?$

$P(X > 7000) = ?$

Define $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 5000}{100}$

When $X = 5012$

$Z = \dfrac{5012 - 5000}{100} = 0.12$

$P(X < 5012) = P(Z < 0.12) = 0.452$

When $X = 4000$

$Z = \dfrac{4000 - 5000}{100} = -10$

When $X = 6000$

$Z = \dfrac{6000 - 5000}{100} = 10$

$P(4000 < X < 6000) = P(-10 < Z < 10) = 0.998$

When $X = 7000$

$Z = \dfrac{7000 - 5000}{100} = 20$

$P(X > 7000) = P(Z > 20) = 0$

64. Let X = Life of battery

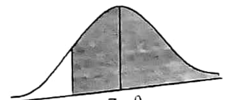
$X \sim N(\mu, \sigma^2)$

$\mu = 40$, $\sigma = 5$

$N = 1000$

$P(X > 35) = ?$

Let $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 40}{5}$



$Z_1 = -1 \quad Z = 0$

When $X = 35$, $Z = \dfrac{35 - 40}{5} = -1$

$P(X > 35)$
$= P(Z > -1)$
$= P(-1 < Z < 0) + 0.5$
$= P(0 < Z < 1) + 0.5$
$= 0.3413 + 0.5$
$= 0.8413$

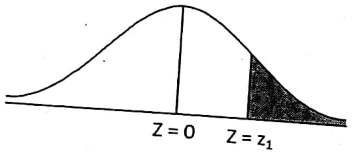
No. of cylinder need replacement after 35 days $= N\,P(X > 35) = 1000 \times 0.8413$
$= 841.3 \approx 841$

65. Let x = marks secured y students
$N = 5000$
$X \sim N(65, 400)$
Let lowest mark of top 10% student $= x_1$
$P(X \geq x_1) = 10\% = 0.1$

Define $Z = \dfrac{X-\mu}{\sigma}$

When $X = x_1$, $Z = \dfrac{x_1 - 65}{\sqrt{400}} = z_1(\text{say})$ .....(i)

$P(X \geq x_1) = 0.1$
$\Rightarrow P(Z \geq Z_1) = 0.1$
$\Rightarrow 0.5 - P(0 \leq Z \leq Z_1) = 0.1$
$\Rightarrow 0.5 - 0.1 = P(0 \leq Z \leq Z_1)$
$\Rightarrow 0.4 = P(0 \leq Z \leq Z_1)$
$\Rightarrow Z_1 = 1.28$



Substitute Z_1 in (i),

$\dfrac{x_1 - 65}{\sqrt{400}} = 1.28$

$\Rightarrow \dfrac{x_1 - 65}{20} = 1.28$

$\Rightarrow x_1 = 20 \times 1.28 + 65 = 90.6$

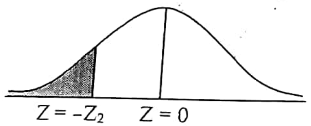
Hence lowest marks of top 10% student is 90.6
Let highest marks by poorest 500 students $= x_2$.

$P(X \leq x_2) = \dfrac{500}{5000}$

$\Rightarrow P(X \leq x_2) = 0.1$

When $X = x_2$, $Z = \dfrac{x_2 - 65}{\sqrt{40}} = -Z_2$ (say) ...(ii)

$P(X \leq x_2) = 0.1$
$\Rightarrow P(Z \leq -z_2) = 0.1$
$\Rightarrow 0.5 - P(-Z_2 \leq Z < 0) = 0.1$
$\Rightarrow 0.5 - 0.1 = P(0 \leq Z \leq Z_2)$
$\Rightarrow P(0 \leq Z \leq Z_2) = 0.4$



$\Rightarrow Z_2 = 1.28$
Substitute Z_2 in (ii)

$\dfrac{x_2 - 65}{\sqrt{400}} = -1.28$

$\Rightarrow \dfrac{x_2 - 65}{20} = -1.28$

$\Rightarrow x_2 = 20 \times (-1.28) + 65 = 39.4$

Hence, highest marks of poorest 500 students is 39.4
Let limit of marks of middle 80% students be x_3 and x_4
$P(x_3 \leq x \leq x_4) = 80\%$
$\Rightarrow P(X_3 \leq X \leq X_4) = 0.8$

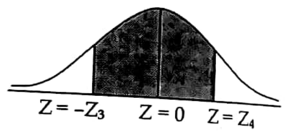
When $X = X_3$;

$Z = \dfrac{X_3 - 65}{20} = -Z_3$ (Say) .... (2)

When $X = X_4$;

$Z = \dfrac{X_4 - 65}{20} = Z_4$ (Say) ... (3)

$P(X_3 \leq X \leq X_4) = 0.8$
$\Rightarrow P(-Z_3 \leq Z \leq Z_4) = 0.8$
$\Rightarrow 2P(0 \leq Z \leq Z_4) = 0.8$ (Since it is middle)
$\Rightarrow P(0 \leq Z \leq Z_4) = 0.4$
$\Rightarrow Z_4 = 1.28,\ Z_3 = 1.28$



Substitute values

$\dfrac{X_3 - 65}{6.324} = -1.28$

$\Rightarrow X_3 = 39.4$

$\dfrac{X_4 - 65}{20} = 1.28$

$\Rightarrow X_4 = 90.6$

Limit = 39.4 to 90.6

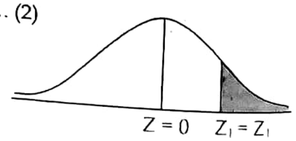
66. **Let x = marks secured & full marks = 100**
$P(X > 80) = 5\%$
$P(X < 30) = 10\%$

$Z = \dfrac{X-\mu}{\sigma}$

When $X = 80$, $Z = \dfrac{80-\mu}{\sigma} = Z_1$ (Say) .... (1)

When $X = 30$, $Z = \dfrac{30-\mu}{\sigma} = -Z_2$ (say) .... (2)

$P(X > 80) = 5\%$
Or $P(Z > Z_1) = 0.05$
Or $0.5 - P(0 < Z < Z_1) = 0.05$



$P(X > 35)$
$= P(Z > -1)$
$= P(-1 < Z < 0) + 0.5$
$= P(0 < Z < 1) + 0.5$
$= 0.3413 + 0.5$
$= 0.8413$

No. of cylinder need replacement after 35 days = $N\,P(X > 35) = 1000 \times 0.8413$
$= 841.3 \approx 841$

65. Let x = marks secured y students

$N = 5000$

$X \sim N(65, 400)$

Let lowest mark of top 10% student = x_1

$P(X \geq x_1) = 10\% = 0.1$

Define $Z = \dfrac{X - \mu}{\sigma}$

When $X = x_1$, $Z = \dfrac{x_1 - 65}{\sqrt{400}} = z_1(\text{say})$ .....(i)

$P(X \geq x_1) = 0.1$

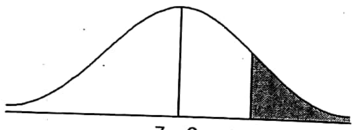
$\Rightarrow$ $P(Z \geq Z_1) = 0.1$

$\Rightarrow$ $0.5 - P(0 \leq Z \leq Z_1) = 0.1$

$\Rightarrow$ $0.5 - 0.1 = P(0 \leq Z \leq Z_1)$

$\Rightarrow$ $0.4 = P(0 \leq Z \leq Z_1)$

$\Rightarrow$ $Z_1 = 1.28$



$Z = 0$ $Z = z_1$

Substitute Z_1 in (i),

$\dfrac{x_1 - 65}{\sqrt{400}} = 1.28$

$\Rightarrow$ $\dfrac{x_1 - 65}{20} = 1.28$

$\Rightarrow$ $x_1 = 20 \times 1.28 + 65 = 90.6$

Hence lowest marks of top 10% student is 90.6

Let highest marks by poorest 500 students = x_2.

$P(X \leq x_2) = \dfrac{500}{5000}$

$\Rightarrow$ $P(X \leq x_2) = 0.1$

When $X = x_2$, $Z = \dfrac{x_2 - 65}{\sqrt{40}} = -Z_2$ (say) ...(ii)

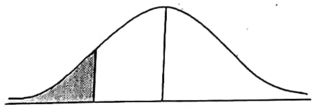
$P(X \leq x_2) = 0.1$

$\Rightarrow$ $P(Z \leq -z_2) = 0.1$

$\Rightarrow$ $0.5 - P(-Z_2 \leq Z < 0) = 0.1$

$\Rightarrow$ $0.5 - 0.1 = P(0 \leq Z \leq Z_2)$

$\Rightarrow$ $P(0 \leq Z \leq Z_2) = 0.4$



$Z = -Z_2$ $Z = 0$

$\Rightarrow$ $Z_2 = 1.28$

Substitute Z_2 in (ii)

$\dfrac{x_2 - 65}{\sqrt{400}} = -1.28$

$\Rightarrow$ $\dfrac{x_2 - 65}{20} = -1.28$

$\Rightarrow$ $x_2 = 20 \times (-1.28) + 65 = 39.4$

Hence, highest marks of poorest 500 students is 39.4

Let limit of marks of middle 80% students be x_3 and x_4

$P(x_3 \leq x \leq x_4) = 80\%$

$\Rightarrow$ $P(X_3 \leq X \leq X_4) = 0.8$

When $X = X_3$;

$Z = \dfrac{X_3 - 65}{20} = -Z_3$ (Say) .... (2)

When $X = X_4$;

$Z = \dfrac{X_4 - 65}{20} = Z_4$ (Say) ... (3)

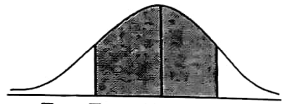
$P(X_3 \leq X \leq X_4) = 0.8$

$\Rightarrow$ $P(-Z_3 \leq Z \leq Z_4) = 0.8$

$\Rightarrow$ $2P(0 \leq Z \leq Z_4) = 0.8$ (Since it is middle)

$\Rightarrow$ $P(0 \leq Z \leq Z_4) = 0.4$

$\Rightarrow$ $Z_4 = 1.28$, $Z_3 = 1.28$



$Z = -Z_3$ $Z = 0$ $Z = Z_4$

Substitute values

$\dfrac{X_3 - 65}{6.324} = -1.28$

$\Rightarrow$ $X_3 = 39.4$

$\dfrac{X_4 - 65}{20} = 1.28$

$\Rightarrow$ $X_4 = 90.6$

Limit = 39.4 to 90.6

66. Let x = marks secured & full marks = 100

$P(X > 80) = 5\%$

$P(X < 30) = 10\%$

$Z = \dfrac{X - \mu}{\sigma}$

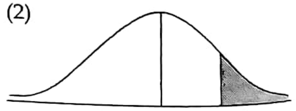
When $X = 80$, $Z = \dfrac{80 - \mu}{\sigma} = Z_1$ (Say) .... (1)

When $X = 30$, $Z = \dfrac{30 - \mu}{\sigma} = -Z_2$ (say) .... (2)

$P(X > 80) = 5\%$

Or $P(Z > Z_1) = 0.05$

Or $0.5 - P(0 < Z < Z_1) = 0.05$



$Z = 0$ $Z_1 = Z_1$

Or $P(0 < Z < Z_1) = 0.45$

Or $Z_1 = 1.64$

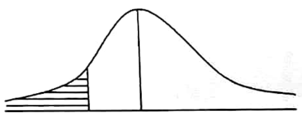
$P(X < 30) = 10\%$

Or $P(Z < -Z_2) = 0.1$

Or $0.5 - P(-Z_2 < Z < 0) = 0.1$

Or $0.4 = P(0 < Z < Z_2)$

Or $Z_2 = 1.28$

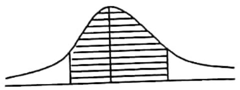


$z = Z_2$ $z = 0$

Substitute Z_1 in equation (1) $\frac{80-\mu}{\sigma} = 1.64$ .... (3)

Substitute Z_2 in equation (2) $\frac{30-\mu}{\sigma} = -1.28$ .... (4)

Divide $\frac{80-\mu}{\sigma} = -1.3125$



$z = -0.21$ $z = 0$ $z = 0.58$

Or $\mu = 51.92$

Substitute μ in (3); $\sigma = 17.11$

$P(45 < X < 60)$

When $X = 45$, $Z = \frac{45 - 49.02}{18.88} = -0.21$

When $X = 60$, $Z = \frac{60 - 49.02}{18.88} = 0.58$

$P(45 < X < 60) = P(-0.21 < Z < 0.58) = P(-0.21 < Z < 0) + P(0 < Z < 0.58)$

$= 0.0832 + 0.219 = 0.3022 = 30.22\%$

67. The life of

Let x = life of printer

$\mu = 4, \sigma = 200$

$P(x < 400) = 10\%$

Or, $P(x < 400) = 0.1$

Define $Z = \frac{X-\mu}{\sigma}$

When $X = 400$, $Z = \frac{400-\mu}{200} = -z_1$ (say) .... (i)

$P(X < 400) = 0.1$

$\Rightarrow$ $P(Z < -Z_1) = 0.1$

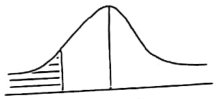
$\Rightarrow$ $0.5 - P(-Z_1 < Z < 0) = 0.1$

$\Rightarrow$ $0.5 - 0.1 = P(-Z_1 < Z < 0)$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.4$

$\therefore Z_1 = 1.28$

Substitute Z_1 in (i)



$z = -z_1$ $z = 0$

$\frac{400-\mu}{200} = -1.28$

Or, $400 - \mu = -256$

Or, $400 + 256 = \mu$

$\therefore$ $\mu = 656$

Hence mean life of bulb is 656 hrs.

68. Sacks of grain

Let, x = weight of bag

$\mu = 114$, $X \sim N(\mu, \sigma^2)$, $\sigma = ?$

Here $P(X > 115) = 15\%$

Or, $P(X > 115) = 0.15$

Define $Z = \frac{X-\mu}{\sigma}$

When $X = 115$, $Z = \frac{115-114}{\sigma} = \frac{1}{\sigma} = z_1$ (say) .... (i)

$P(X > 115) = 0.15$

$\Rightarrow$ $P(Z > Z_1) = 0.15$

$\Rightarrow$ $0.5 - P(0 < Z < Z_1) = 0.15$

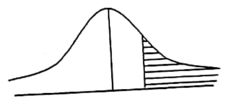
$\Rightarrow$ $0.5 - 0.15 = P(0 < Z < Z_1)$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.35$

$\therefore$ $Z_1 = 1.04$

Substitute Z_1 in (i)

$\frac{1}{\sigma} = 1.04$



$z = 0$ $z = z_1$

Or, $\sigma = \frac{1}{1.04} = 0.96$

Hence, S.D. = 0.96.

69. **Solution.**

Let marks secured = X

$X \sim N(\mu, \sigma^2)$

$P(X > 60) = 10\%$; $P(X < 40) = 30\%$

Define $Z = \frac{X-\mu}{\sigma}$

When $X = 60$, $Z = \frac{60-\mu}{\sigma} = Z_1$ (say) .... (1)

When $X = 40$, $Z = \frac{40-\mu}{\sigma} = -Z_2$ (say) .... (2)

$P(X > 60) = 10\%$

$\Rightarrow$ $P(Z > Z_1) = 0.1$

$\Rightarrow$ $0.5 - P(0 < Z < Z_1) = 0.1$

$\Rightarrow$ $0.5 - 0.1 = P(0 < Z < Z_1)$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.4$

$\Rightarrow$ $Z_1 = 1.28$

Substitute Z_1 in equation (1)

$Z = 0$ $Z = Z_1$

$$\frac{60-\mu}{\sigma} = 1.28 \quad \ldots (3)$$

$P(X < 40) = 30\%$

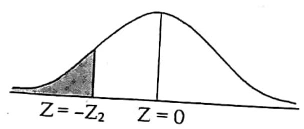
$\Rightarrow P(Z < -Z_2) = 0.3$

$\Rightarrow 0.5 - P(-Z_2 < Z < 0) = 0.3$

$\Rightarrow 0.5 - 0.3 = P(-Z_2 < Z < 0)$

$\Rightarrow P(0 < Z < Z_2) = 0.2$

$\Rightarrow Z_2 = 0.52$



Substitute Z_2 in equation (2)

$$\frac{40-\mu}{\sigma} = -0.52 \quad \ldots (4)$$

Divide (3) by (4)

$$\frac{\frac{60-\mu}{\sigma}}{\frac{40-\mu}{\sigma}} = \frac{1.28}{-0.52}$$

$\Rightarrow \frac{60-\mu}{40-\mu} = -2.461$

$\Rightarrow 60 - \mu = -98.46 + 2.461\mu$

$\Rightarrow 60 + 98.46 = 2.461\mu + \mu$

$\Rightarrow \mu = \frac{158.46}{3.461} = 45.78$

Substitute μ in equation (3)

$$\frac{60 - 45.78}{\sigma} = 1.28$$

$\Rightarrow \sigma = \frac{14.22}{1.28} = 11.109$

70. Fit the binomial

x	0	1	2	3	4	5	6	Total
f	7	6	19	35	23	7	1	$\sum f = 98$
fx	0	6	38	105	92	35	6	$\sum fx = 282$

$\bar{x} = \frac{\sum fx}{N} = \frac{282}{98} = 2.877$

Here, n = 6

$\bar{x} = np$

Or, $2.877 = 6p$

Or, $p = \frac{2.877}{6} = 0.479,\ q = 0.52$

x	$P(x) = C(n, x)\, p^x q^{n-x}$	Expected frequency = N P(x)
0	$C(6, 0)\,(0.479)^0\,(0.52)^6 = 0.019$	$1.86 \approx 2$
1	$C(6, 1)\,(0.479)^1\,(0.52)^5 = 0.1092$	$10.70 \approx 11$
2	$C(6, 2)\,(0.479)^2\,(0.52)^4 = 0.2516$	$24.65 \approx = 25$
3	$C(6, 3)\,(0.479)^3\,(0.52)^3 = 0.3090$	$30.28 \approx 30$
4	$C(6, 4)\,(0.479)^4\,(0.52)^2 = 0.2153$	$21.1 \approx 21$
5	$C(6, 5)\,(0.479)^5\,(0.52)^1 = 0.078$	$7.64 \approx 8$
6	$C(6, 6)\,(0.479)^6 = 0.012$	$1.176 \approx 1$

71. Five fair coin

No. of heads (x)	Frequency (f)	fx
0	2	0
1	10	10
2	24	48
3	38	114
4	18	72
5	8	40
	$N = \sum f = 100$	$\sum fx = 284$

$\bar{x} = \frac{\sum fx}{N} = \frac{284}{100} = 2.84$ n = 5

$\bar{x} = n\,p$

Or, $2.84 = 5p$

Or, $p = \frac{2.84}{5} = 0.568;\ q = 1 - p = 0.432$

$x \sim B(n, p)$

x	$P(x) = C(n, x)\, p^x q^{n-x}$	Expected frequency = N P(x)
0	$C(5, 0)\,(0.568)^0\,(0.432)^5 = 0.015$	$1.5 \approx 2$
1	$C(5, 1)\,(0.568)^1\,(0.432)^4 = 0.098$	$9.8 \approx 10$
2	$C(5, 2)\,(0.568)^2\,(0.432)^3 = 0.2601$	$26.01 \approx 26$
3	$C(5, 3)\,(0.568)^3\,(0.432)^2 = 0.341$	$34.10 \approx 34$
4	$C(5, 4)\,(0.568)^4\,(0.432)^1 = 0.224$	$22.4 \approx 22$
5	$C(5, 5)\,(0.568)^2\,(0.432)^3 = 0.059$	$5.9 \approx 6$

72. 192 Families

No. of girl child (x)	No. of families (f)	fx
0	77	0
1	90	90
2	20	40
3	5	15
	$N = \sum f = 192$	$\sum fx = 145$

$\bar{x} = \frac{\sum fx}{N} = \frac{145}{192} = 0.755$

n = 3

(i) When both sexes are equally probable

$p = q = \frac{1}{2}$

n = 3

No. of girl child (x)	No. of families (f)	fx
0	77	0
1	90	90
2	20	40
3	5	15
	$N = \sum f = 192$	$\sum fx = 145$

$\bar{x} = \frac{\sum fx}{N} = \frac{145}{192} = 0.755;$ n = 3

i. When both sexes are equally probable

$p = q = \frac{1}{2}$ n = 3

5 CHAPTER

SAMPLE SURVEY

1. $\sigma = 500$ $\quad d = 600$ $\quad \propto = 5\%$

$$n = \frac{z\propto_{12}^{2}\,\sigma^2}{d^2} = \frac{(1.96)^2 \times (5000)^2}{(600)^2} = 266.77 \simeq 267$$

2. $S = 0.95$ $\quad n = ?$ $\quad \propto = 5\%$ $\quad d = 0.01$

$$n = \frac{z\propto_{12}^{2}\,s^2}{d^2} = \frac{(1.96)^2 \times (0.95)^2}{(0.01)^2} = \frac{3.467}{0.0001} = 34670$$

3. $P = 0.3$

$Q = 0.7$

$d = 10\% = 0.1$

$$n = \frac{2\propto_{12}^{2}\,pq}{d^2} = \frac{3^2 \times 0.3^2 \times 0.7}{(0.1)^2} = 189$$

4. $p = 0.2,$ $\quad q = 0.8$ $\quad d = 0.05$

$$n = \frac{2\propto_{12}^{2}\,pq}{d^2} = \frac{2^2 \times 0.2^2 \times 0.8}{(0.05)^2} = 256$$

Where N = 1000

$$\text{Sample size} = \frac{n}{1 + \frac{n}{N}} = \frac{256}{1 + \frac{256}{1000}} = \frac{256}{1.256} = 203.8 \simeq 204$$

x	$P(x) = C(n, x)\, p^x q^{n-x}$	Expected frequency = N P(x)
0	$C(3, 0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = 0.125$	24
1	$C(3, 1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = 0.375$	72
2	$C(3, 2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = 0.375$	72
3	$C(3, 3)\left(\frac{1}{2}\right)^3 = 0.125$	24

ii. When probability vary

$\bar{x} = np$ Or, $0.755 = 3p$ Or, $p = \frac{0.755}{3} = 0.25$,

$q = 1 - p = 1 - 0.25 = 0.75$

x	$P(x) = C(n, x)\, p^x q^{n-x}$	Expected frequency = N P(x)
0	$C(3, 0)\,(0.25)^0\,(0.75)^3 = 0.421$	80.83 ≈ 81
1	$C(3, 1)\,(0.25)^1\,(0.75)^2 = 0.421$	80.83 ≈ 81
2	$C(3, 2)\,(0.25)^2\,(0.75)^3 = 0.1406$	26.99 ≈ 27
3	$C(3, 3)\,(0.25)^3 = 0.0156$	3

73. Fit the Poisson

Mistaken per page (x)	No. of page (f)	fx	$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	Expected frequency = N P(x)
0	142	0	$\frac{e^{-0.95}(0.25)^0}{0!} = 0.367$	146.8 ≈ 147
1	156	156	$\frac{e^{-0.95}(0.25)^1}{1!} = 0.367$	146.8 ≈ 147
2	69	138	$\frac{e^{-0.95}(0.25)^2}{2!} = 0.183$	73.2 ≈ 73
3	27	81	$\frac{e^{-0.95}(0.25)^3}{3!} = 0.061$	24.4≈ 24
4	5	20	$\frac{e^{-0.95}(0.25)^4}{4!} = 0.015$	6
5	1	5	$\frac{e^{-0.95}(0.25)^5}{5!} = 0.003$	1.22 = 1
	$N = \sum f = 400$	$\sum fx = 380$		

$$\lambda = \bar{x} = \frac{\sum fx}{N} = \frac{400}{400} = 1$$

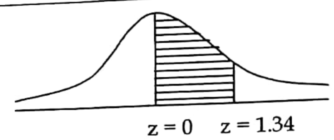
74. Fit Poisson distribution

x	f	fx	$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	Expected frequency = N P(x)
0	71	0	$\frac{e^{-1.746}(1.76)^0}{0!} = 0.172$	69.82 ≈ 70
1	112	112	$\frac{e^{-1.74}(1.74)}{1!} = 0.305$	121.695 ≈ 122
2	117	234	$\frac{e^{-1.74}(1.74)^2}{2!} = 0.265$	105.73 ≈ 106

3	57	161	$\frac{e^{-1.74}(1.74)^3}{3!} = 0.154$	61.446 ≈ 61
4	27	108	$\frac{e^{-1.74}(1.74)^4}{4!} = 0.067$	26.733 ≈ 27
5	11	55	$\frac{e^{-1.74}(1.74)^5}{5!} = 0.023$	9.177 ≈ 9
6	3	18	$\frac{e^{-1.74}(1.74)^6}{6!} = 0.0067$	2.67 ≈ 3
7	1	7	$\frac{e^{-1.74}(1.74)^7}{7!} = 0.0016$	0.67 ≈ 1
	$N = \sum f = 399$	$\sum fx = 695$		

$\lambda = \bar{x}$

$$= \frac{\sum fx}{N} = \frac{705}{399} = 1.76$$



75. Fit normal distribution

Class	Frequency (f)	Mid (x)	fx	fx^2
20-30	1	25	25	625
30-40	3	35	105	3675
40-50	16	45	720	32400
50-60	34	55	1870	102850
60-70	28	65	1820	118300
70-80	14	75	1050	78750
80-90	3	85	255	21675
90-100	1	95	95	9025
	$n = \Sigma f$		$\Sigma fx = 5940$	$\Sigma fx^2 = 367300$

$$\mu = \bar{x} = \frac{\Sigma fx}{N} = \frac{5940}{100} = 59.4$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{367300}{100} - (59.4)^2} = \sqrt{3673 - 3528.36} = \sqrt{144.64} = 12.02$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 59.4}{12.02}$$

Class	Lower limit	$z = \frac{x - M}{\sigma}$	$\frac{1}{\sqrt{2\pi}}\int_0^z e^{-\frac{1}{2}z^2}dz$	$\Delta\psi(z)$	$N\Delta\psi(z)$
Below 20	$-\infty$	$-\infty$	0	0.0005	1
20-30	20	- 3.27	0.0005	0.0068	1
30-40	30	- 2.44	0.0073	0.0464	5
40-50	40	- 1.61	0.0537	0.164	16
50-60	50	- 0.78	0.2177	0.2983	30
60-70	60	0.04	0.516	0.2946	29
70-80	70	0.88	0.8106	0.1458	15
80-90	80	1.71	0.9564	0.0381	4
90-100	90	2.54	0.9945	0.0051	1
100 and above	100	3.37	0.9996		

6 CHAPTER SAMPLESURVEY METHODS

1.

a. Here for Population Mean $\bar{Y} = \frac{\sum Y}{N} = \frac{0+1+3+8+9}{5} = 4.2$

Population Variance, $S^2 = \frac{1}{N-1}\sum(Y_i - \bar{Y})^2$

$= \frac{1}{N-1}\{\sum Y_i^2 - N\bar{Y}^2\}$

$= \frac{1}{4}\{0^2 + 1^2 + 3^2 + 8^2 + 9^2 - 5(4.2)^2\} = 16.7$

b. Here the sample of size 2 from population of size 6 can be drawn in C(5,2) ways = 10 ways

Hence the samples are;(0,1) (0,3) (0,8) (0,9) (1,3) (1,8) (1,9) (3,8) (3,9) (8,9)

Sample No.	Sample values (y_i)	Sample mean ($\bar{y}_i$)	$\{y_i - (\bar{y}_i)\}^2$	$s_i^2 = \frac{1}{n-1}\sum(y_i^2 - n\bar{y}^2)$
1	(0,1)	0.5	13.69	0.5
2	(0,3)	1.5	7.29	4.5
3	(0,8)	4	0.04	32
4	(0,9)	4.5	0.09	40.5
5	(1,3)	2	4.84	2
6	(1,8)	4.5	0.09	24.5
7	(1,9)	5	0.64	32
8	(3,8)	5.5	1.69	12.5
9	(3,9)	6	3.24	18
10	(8,9)	8.5	18.49	0.5
Total		42	50.1	167

c. $E(\bar{y}) = \frac{1}{C(N,n)}\sum_i y_i = \frac{42}{10} = 4.2.$

d. $V(\bar{y}) = \frac{1}{C(N,n)}\sum_i \bar{y}_i - E(\bar{y})^2\} = \frac{50.1}{10} = 5.01$

e. Yes, $E(\bar{y}) = \bar{Y}$ verified

Here, sample mean is unbiased estimate of the population mean

$E(s^2) = \frac{1}{C(N,n)}\sum_i S_i^2 = \frac{167}{10} = 16.7$. Hence, $E(s^2) = S^2$

f. Now, $V(\bar{y}) = \frac{(1-f)S^2}{n} = \left(1 - \frac{2}{5}\right) \times \frac{16.7}{2} = 5.01$ Hence, Formula verified.

?. Problem to test

$H_0 = \mu_1 = \mu_2 = \mu_3 = \mu_3 = \mu_5$

2. Solution.

Here, Population size (N) = 5, Sample size (n) = 2.

a. Calculation of population mean and population variance:

Y_i	$Y_i - \bar{Y}$	$(Y - \bar{Y})^2$
0	-3.2	10.24
1	-2.2	4.84
3	-0.2	0.04
5	1.8	3.24
7	3.8	14.44
$\sum Y = 16$		$\sum(Y - \bar{Y})^2 = 32.8$

Population mean (μ or $\bar{Y}$) $= \frac{\sum Y_i}{N} = \frac{16}{5} = 3.2$

Population variance (σ^2) $= \frac{\sum(Y_i - \bar{Y})^2}{N} = \frac{32.8}{5} = 6.56$

b. Number of possible samples of size 2 that can be drawn from the population of size N = 5 by using with replacement technique is: Number of possible samples = $N^n = 5^2 = 25$

Thus the possible samples are: (0,0), (0,1), (0,3), (0,5), (0,7), (1,0), (1,1), (1,3), (1,5), (1,7), (3,0), (3,1), (3,3), (3,5), (3,7), (5,0), (5,1), (5,3), (5,5), (5,7), (7,0), (7,1), (7,3), (7,5), (7,7).

c. Calculation of sample means and variance of the sampling distribution of means:

S. No.	Sample Values (Y_i)	Sample Means ($\bar{y}_i$)	$(\bar{y}_i - \bar{y})$	$(\bar{y}_i - \bar{y})^2$	$s_i^2 = \frac{1}{n-1}\{\sum y_i^2 - n\bar{y}^2\}$
1	(0,0)	0	-3.2	10.2	0
2	(0,1)	0.5	-2.7	7.29	0.5
3	(0,3)	1.5	-1.7	2.89	4.5
4	(0,5)	2.5	-0.7	0.49	12.5
5	(0,7)	3.5	0.3	0.09	24.5
6	(1,0)	0.5	-2.7	7.29	0.5
7	(1,1)	1	-2.2	4.84	0
8	(1,3)	2	-1.2	1.44	2
9	(1,5)	3	-0.2	0.04	8
10	(1,7)	4	0.8	0.64	18
11	(3,0)	1.5	-1.7	2.89	4.5
12	(3,1)	2	-1.2	1.44	2
13	(3,3)	3	-0.2	0.04	0
14	(3,5)	4	0.8	0.64	2
15	(3,7)	5	1.8	3.24	8
16	(5,0)	2.5	-0.7	0.49	12.5
17	(5,1)	3	-0.2	0.04	8
18	(5,3)	4	0.8	0.64	2
19	(5,5)	5	1.8	3.24	0
20	(5,7)	6	2.8	7.84	2
21	(7,0)	3.5	0.3	0.09	24.5
22	(7,1)	4	0.8	0.64	18
23	(7,3)	5	1.8	3.24	8
24	(7,5)	6	2.8	7.84	2
25	(7,7)	7	3.8	14.4	0
		80		82	

Mean of the sample means $(\bar{y}) = \dfrac{\sum \bar{y}_i}{\text{No. of samples } (N^n)} = \dfrac{80}{25} = 3.2$

Since the mean of the sample means $(\bar{y}) = 3.2$ is equal to the population mean $(\bar{Y}) = 3.2$, so we can conclude that the mean of the sampling distribution of the sample means is equal to the population mean.

d. Variance of the sample means is

$$V(\bar{y}) = \frac{\sum(\bar{y}_i - \bar{y})^2}{\text{No. of samples} = N^n} = \frac{82}{25} = 3.28 \text{ or}$$

$$V(\bar{y}) = \frac{\sigma^2}{n} = \frac{6.56}{2} = 3.28$$

e. variance of sample means gives the same value as calculated from (d)

3. Solution:

Here N =800, n = 120

$$Var(\bar{y}_{st})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right) \sum W_i S_i^2$$

$$= \left(\frac{1}{nN} - \frac{1}{N^2}\right) \sum N_i S_i^2 \quad \left\{W_i = \frac{N_i}{N}\right\}$$

$$= \left(\frac{1}{120 \times 800} - \frac{1}{800^2}\right) \{200\times36 + 300\times64 + 300\times144\}$$

$$= 0.616$$

$$Var(\bar{y}_{st})_{opt} = \frac{1}{n}(\sum W_i S_i)^2 - \frac{1}{N}\sum W_i S_i^2$$

$$= \frac{1}{nN^2}(\sum N_i S_i)^2 - \frac{1}{N^2}\sum N_i S_i^2$$

$$= \frac{1}{120 \times 800^2}(200 \times 6 + 300 \times 8 + 300 \times 12)^2 - \frac{1}{800^2}(200 \times 36 + 300 \times 64 + 300 \times 144)$$

$$= 0.675 - 0.1087$$

$$= 0.566$$

4. $\mu = 1800$

$\sigma = 100$

$x = 50$

$\bar{x} = 1850$

$$z = \frac{\bar{x} - M}{\frac{\sigma}{\sqrt{x}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}} = \frac{50}{\frac{100}{\sqrt{50}}} = 3.53$$

5. $x_1 = 35$

$\sigma_1 = 5.2$

$\bar{x}_1 = 81$

$x_2 = 36$

$\sigma_2 = 3.4$

$\bar{x}_2 = 76$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{x_1} + \frac{\sigma_2^2}{x_2}}} = \frac{81 - 76}{\sqrt{\frac{\sigma 5.2^2}{35} + \frac{\sigma 3.4^2}{36}}} = \frac{5}{\sqrt{0.772 + 0.0032}} = \frac{5}{\sqrt{0.775}} = 5.68$$

6. $P = 10\% = 0.1, Q = 0.9, n = 400$

$$p = \frac{34}{400} = 0.085$$

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.085 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{400}}} = \frac{-0.015}{0.015} = -1$$

7. $x_1 = 250, n_2 = 200, x_1 = 24, x_2 = 10$

$$p_1 = \frac{x_1}{n_1} = \frac{4}{250} = \frac{24}{250} = 0.096$$

$$p_2 = \frac{x_2}{n_2} = \frac{10}{200} = 0.05$$

$$p = \frac{x_1 p_1 + n_2 p_2}{x_1 + x_2} = \frac{x_1 + x_2}{x_1 + n_2} = \frac{24 + 10}{250 + 200} = 0.0755$$

$Q = 0.9244$

$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.096 - 0.05}{\sqrt{0.0755 \times 0.9244\left(\frac{1}{250} + \frac{1}{200}\right)}} = 1.84$$

8. $n = 20$

$r = 0.63$

$p = 0.55$

$$z = \frac{z - \xi}{\sqrt{\frac{1}{x-3}}} = \frac{0.741 - 0.617}{\sqrt{\frac{1}{20-3}}} = \frac{0.124}{0.2424} = 0.49$$

$$z = \frac{1}{2}\log e \frac{1+r}{1-r}$$

$$= \frac{1}{2}\log e \frac{1+0.63}{1-0.63}$$

$$= \frac{1}{2}\log e \frac{1+0.63}{1-0.67}$$

$$= \frac{1}{2}\log e\, 4.405 = 0.741$$

$$\xi = \frac{1}{2}\log e \frac{1+\rho}{1-\rho} = \frac{1}{2}\log e \frac{1+0.55}{1-0.55} = \frac{1}{2}\log e \frac{1.55}{0.45} = \frac{1}{2}\log e\, 3.44 = 0.617$$

9. $x_1 = 28, x_2 = 35, r_1 = 0.5, r_2 = 0.3$

$$z_1 = \frac{1}{2}\log e \frac{1+r_1}{1-r_1} = \frac{1}{2}\log e \frac{1+0.5}{1-0.5} = \frac{1}{2}\log \frac{1.5}{0.5} = 0.549$$

$$z_2 = \frac{1}{2}\log e \frac{1+r_2}{1-r_2} = \frac{1}{2}\log e \frac{1+0.3}{1-0.3} = \frac{1}{2}\log \frac{1.3}{0.7} = 0.309$$

$$z = \frac{z_1 - z_2}{\sqrt{\frac{1}{x_1-3} + \frac{1}{n_2-3}}} = \frac{0.549 - 0.3069}{\sqrt{\frac{1}{28-3} + \frac{1}{35-3}}} = \frac{0.240}{\sqrt{\frac{1}{25} + \frac{1}{32}}} = \frac{0.24}{\sqrt{0.04 + 0.031}} = \frac{0.24}{0.266}$$

$= 0.902$

12. Problem to test

$H_0 H = \mu_1 = \mu_2 = \mu_3 = \mu_3 = \mu_5$

... are μ_1 . . is diff.

10. $\mu = 0.05$

$x = 10$

$\bar{x} = 0.053$

$s = 0.003$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{x-1}}} = \frac{0.053 - 0.05}{\frac{0.003}{\sqrt{10-1}}} = \frac{0.003}{\frac{0.003}{3}} = 3$$

11. $\bar{x}_1 = 200$ $\quad \bar{x}_2 = 250$

$S_1 = 20$ $\quad S_2 = 25$

$x_1 = 20$ $\quad x_2 = 25$

$$S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2} = \frac{25 \times 20^2 + 25 \times 25^2}{25 + 25 - 2} = 533.85$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{200 - 250}{\sqrt{233.85\left(\frac{1}{25} + \frac{1}{25}\right)}} = \frac{-50}{6.534} = -7.6$$

$\therefore \quad |t| = 7.6$

Roll No.	Before training	After training	d = X − Y	d^2
1	12	15	−3	9
2	14	16	−2	4
3	11	10	1	1
4	9	7	1	1
5	7	5	2	4
6	10	12	−2	4
7	3	0	−7	49
8	0	2	−2	4
9	5	3	−2	
10	6	8	$\Sigma d = -12$	$\Sigma d^2 = 84$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-12}{10} = -1.2$$

$$sd = \sqrt{\left(\frac{1}{n-1}(\Sigma d^2 - n\bar{d}^2)\right)}$$

$$= \sqrt{\frac{1}{10-1}(84 - 10 \times (-1.2)^2)}$$

$$= \sqrt{\frac{1}{9}(84 - 14.4)} = 2.78$$

$$t = \frac{\bar{d}}{\frac{sd}{\sqrt{x}}} = \frac{-1.2}{\frac{2.78}{\sqrt{10}}} = \frac{-1.2\sqrt{10}}{2.78} = -1.36$$

13. $x = 27,$ $\quad r = 0.6$

$$\frac{0.6\sqrt{27-2}}{\ldots} = \frac{0.6 \times 5}{\ldots} = 3.75$$

14.

Side	O_i	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
1	8	$\frac{1}{6}$	10	
2	9	$\frac{1}{6}$	10	4/10
3	13	$\frac{1}{6}$	10	1/10
4	7	$\frac{1}{6}$	10	9/10
5	15	$\frac{1}{6}$	10	25/10
6	8	$\frac{1}{6}$	10	4/10

$$X^2 = \Sigma\frac{(O_i - E_i)^2}{E_i} = 5.2$$

15.

		Son eye color		Total
		Not light	Light	
Father eye color	Not light	230	148	378
	Light	151	471	622
Total		381	619	1000

O_{ij}	E_{ij}	$\frac{(O_i - E_i)^2}{E_i}$
230	$\frac{3 + 8 \times 381}{100} = 144$	51.36
148	$\frac{378 \times 619}{100} = 243$	37.13
151	$\frac{622 \times 381}{1000} = 237$	31.20
471	$\frac{622 \times 389}{1000} = 385$	13.21
		$\Sigma\Sigma\frac{(O_i - E_i)^2}{E_i} = 138.9$

$$X^2 = \Sigma\frac{(O_i - E_i)^2}{E_i} = 138.9$$

□□□

$\frac{}{75} = 5.68$

0.617

$= \frac{0.24}{0.266}$

12. Problem to test
$H_0 = \mu_1 = \mu_2 = \mu_3 = \mu_3 = \mu_5$
... are μ_1 ... is diff.

10. $\mu = 0.05$

$x = 10$

$\bar{x} = 0.053$

$s = 0.003$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{x-1}}} = \frac{0.053 - 0.05}{\frac{0.003}{\sqrt{10-1}}} = \frac{0.003}{\frac{0.003}{3}} = 3$$

11. $\bar{x}_1 = 200$ $\quad \bar{x}_2 = 250$

$S_1 = 20$ $\quad S_2 = 25$

$x_1 = 20$ $\quad x_2 = 25$

$$S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2} = \frac{25 \times 20^2 + 25 \times 25^2}{25 + 25 - 2} = 533.85$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{200 - 250}{\sqrt{233.85\left(\frac{1}{25} + \frac{1}{25}\right)}} = \frac{-50}{6.534} = -7.6$$

$\therefore \quad |t| = 7.6$

Roll No.	Before training	After training	d = X – Y	d²
1	12	15 ·	– 3	9
2	14	16	– 2	4
3	11	10	1	1
4	9	7	1	1
5	7	5	2	4
6	10	12	– 2	4
7	3	0	– 7	49
8	0	2	– 2	4
9	5	3	– 2	
10	6	8	Σd = – 12	Σd² = 84

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-12}{10} = -1.2$$

$$sd = \sqrt{\left(\frac{1}{n-1}(\Sigma d^2 - n\,\bar{d}^2)\right)}$$

$$= \sqrt{\frac{1}{10-1}(84 - 10 \times (-1.2)^2)}$$

$$= \sqrt{\frac{1}{9}(84 - 14.4)} = 2.78$$

$$t = \frac{\bar{d}}{\frac{sd}{\sqrt{x}}} = \frac{-1.2}{\frac{2.78}{\sqrt{10}}} = \frac{-1.2\sqrt{10}}{2.78} = -1.36$$

13. $x = 27, \quad r = 0.6$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.6\sqrt{27-2}}{\sqrt{1-0.6^2}} = \frac{0.6 \times 5}{0.8} = 3.75$$

14.

Side	O_i	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
1	8	$\frac{1}{6}$	10	
2	9	$\frac{1}{6}$	10	4/10
3	13	$\frac{1}{6}$	10	1/10
4	7	$\frac{1}{6}$	10	9/10
5	15	$\frac{1}{6}$	10	25/10
6	8	$\frac{1}{6}$	10	4/10

$$X^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 5.2$$

15.

		Son eye color		Total
		Not light	Light	
Father eye color	Not light	230	148	378
	Light	151	471	622
Total		381	619	1000

O_{ij}	E_{ij}	$\frac{(O_i - E_i)^2}{E_i}$
230	$\frac{3 + 8 \times 381}{100} = 144$	51.36
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151	$\frac{622 \times 381}{1000} = 237$	31.20
471	$\frac{622 \times 389}{1000} = 385$	13.21
		$\Sigma\Sigma\frac{(O_i - E_i)^2}{E_i} = 138.9$

$$X^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 138.9$$

□□□

12. Problem to test
$HoH = \mu_1 = \mu_2 = \mu_3 = \mu_3 = \mu_5$
H_1H = At least are μ_1 . . is diff.
B, C, D
II = μ . III
. . diff

7 CHAPTER

DESIGN OF EXPERIMENT

1. $x_1 = 11,$ $x_2 = 9$ $S_2 = 0.8,$ $S_2 = 0.5$

$S_1^2 = \frac{1}{x_1}\Sigma(x_1 - \bar{x}_1)^2$

$(0.8)^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{11} = \Sigma(x_1 - \bar{x}_1)^2 = 7.04$

$S_2^2 = \frac{1}{n_2}(x_2 - \bar{x}_1)^2$

$(0.9)^2 = \frac{1}{9}\Sigma(x_2 - \bar{x}_2)^2$

$\Sigma(x_2 - \bar{x}_2)^2 = = 2.25$

$S_1^2 = \frac{1}{n_{1-1}}\Sigma(x_1 - \bar{x}_1)^2 = \frac{7.04}{11-1} = 0.704$

$S_2^2 = \frac{1}{n_{1-1}}\Sigma(x_1 - \bar{x}_1)^2 = \frac{2.25}{9-1} = 0.281$

$F = \frac{S_1^2}{S_2^2} = \frac{0.704}{0.280} = 2.51$

2. $H_0 : \mu_1 = \mu_2 = \mu_3$
H_1 : At least are μ_i = is different]
i = 1, 2, 3

								T_i
Position 1	90	82	79	98	83	91		523
Position 2	105	89	93	104	89	95	86	661
Position 3	83	89	80	94				346
								$G = \mu\Sigma T_i = 1530$

$N = \Sigma x_i = 6 + 7 + 4 = 17$

$\frac{G^2}{N} = \frac{1530^2}{17} = 137700$

$TSS = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N}$

$= 90^2 + 82^2 + 79^2 + 98^2 + 83^2 + 91^2 + 105^2 + 85^2 + 93^2 + 104^2 + 85^2 + 95^2 + 86^2 + 83^2$
$+ 89^2 + 80^2\ 94^2 - 137700$

$= 938$

$SSP = \frac{T_{i.}^2}{n_i} - \frac{G^2}{N} = \frac{5>3}{6} + \frac{661^2}{7} + \frac{346^2}{4} - 1337700$

$= 234.45$

$= SSE = TSS - SSP = 703.547$

ANOVA table

SV	df	SS	MS	f_{cal}	f_{tab}
Position	2	234.45	117.22	2.33	
Error	14	703.54	50.25	3.73	
Total	16	938			

Decision
$F = 2.33 < F_{0.05}(2,14) = 3.73$
Accept H_o at ∝ 5%
Conclusion
There is no significant difference between position.

3. Problem to test
$H_oT: \mu_I = \mu_{II} = \mu_{III} = \mu_{IV}$
H_1T: At least are μ_1 is different

								Ti.
Technical	I	6	14	10	8	11	49
	II	14	9	12	10	14	59
	III	16	12	7	15	11	55
	IV	9	12	8	10	11	50
							ΣTi = 213

$N = 20, G = \Sigma T_i = 213$
$\Sigma\Sigma y_{ij}^2 = G^2 + 14^2 + 10^2\ 8^2 + 11^2 + 14^2 + 9^2 + 12^2, \ldots, + 11^2 =$

$cf = \frac{G^2}{N} = \frac{(213)^2}{20} = 2268.45$

$TSS = \Sigma\Sigma y_{ij}^2 = -\frac{G^2}{N} = 114.55$

$SST = \Sigma\frac{Ti^2}{ni} - \frac{G^2}{N} = 12.95$

$SSE = TSS - SST = 101.6$

ANOVA Table

SV	df	SS	MS	f_{cal}	f_{tab}
Technician	3	12.95	4.32	0.679	3.23
Error	16	101.6	6.35		
Total	19	114.55			

Dicision
$F = 0.979 < f(3, 16) = 3.23$
Accept H_0 at ∝ = 5%
Conclusion
There is no significant difference between 4 technicians

4. Problem to text
$H_0v: \mu.1 = \mu.2 = \mu.3$

H_1V : A+ least are Mij is diff.

T = A, B, C

	Variety of Wheat			Ti.
Plot of Land	A	B	C	
1	6	5	5	16
2	7	5	4	16
3	3	3	3	9
4	8	7	4	19

m = 4, N = 3, N = 4 × 3 = 12 G = ΣT.j = 60

$$\frac{G^2}{N} = \frac{(60)^2}{12} = 300$$

$$TSS = 6^2 + 5^2 + 5^2 + \ldots + 4^2 - 300 = 332 - 300 = 32$$

$$SST = \Sigma\Sigma\frac{Ti.^2}{n} - \frac{G^2}{N} = \frac{24^{\cdot 2} + 20^2 + 16^2}{4} - \frac{(60)^2}{12} = 8$$

$$SEE = TSS - SSP - SSV = 6^2 + 5^2 + 5^2 + \ldots + 4^2 - 300 = 332 - 300 = 32$$

ANOVA Table

SV	df	SS	MS	f_{cal}	f_{lab}
Variety of wheat	2	8	4	1.5	4.25
Error	9	24	2.66		
Total	11	32			

Decision

$F_v = 1.56 < F(2, 9) = 4.25$

Accept H_0V at ∝ = 5%

Conclusion:

There is no significant difference between variety.

5. H_0D: $\mu_A = \mu_B = \mu_C = \mu_D = \mu_E$

H_1D = A+ least and D is different D = A, B, C, D, E

	Drugs				
Patient No.	A	B	C	D	E
1	13	11	13	12	12
2	12	10	14	11	12
3	14	13	13	10	11
4	12	12	14	12	14
T.j	51	46	54	45	49

N = 4 × 5 = 20 G = ΣT.j = 245 $\frac{G^2}{N} = \frac{(245)^2}{20} = 3001.25$

$$TSS = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N} = 13^2 + 11^2 + 12^2 + \ldots + 14^2 - 3001.25 = 29.75$$

$$SSD = \Sigma\frac{T.j^2}{m} - \frac{G^2}{N} = \frac{51^{\cdot 2} + 46^2 + 54^2 + 45^2 + 45^2}{4} - 3001.25$$

SSE = TSS – SSD = 16.25

ANOVA Table

SV	df	SS	MS	f_{cal}	f_{lab}
Variety of Drug	4	13.5	3.375	3.11	3.055
Error	15	16.25	1.083		
Total	19	29.75			

Decision

$F_D = 3.11 > F_{0.05}(4, 15) = 3.055$

Reject H_0D at α = 5%

Conclusion

There is significance difference in efficiency of pain relieving drugs.

6. Problem to test

$H_{0T} = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

H_{1T} = At least are Mi is different

$H_{0F} = \mu_1 = \mu_2 = \mu_3$

H_{1F} = At least are µ.j is different i = 1, 2, 3

Types of Land	Fertilizers			Ti
	F_1	F_2	F_3	
1	20	30	25	75
2	18	25	28	71
3	16	22	18	56
4	23	15	22	60
5	13	28	17	58
T.j	90	120	110	G = 320

m = 5, n = 6, N = m × n = 15, G = ΣTi = ΣT.j = 320

$$TSS = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N} = 20^2 + 30^2 + 25^2 + \ldots + 17^2 - \frac{(320)^2}{15} = 375.33$$

$$SST = \Sigma\frac{Ti^2}{n} - \frac{G^2}{N} = \frac{75^2 + 71^2 + 56^2 + 60^2 + 58^2}{3} - \frac{(320)^2}{15} = 95.33$$

$$SSF = \Sigma\frac{T.j^2}{m} - \frac{G^2}{N} = \frac{90^2 + 120^2 + 110^2}{5} - \frac{(320)^2}{15} = 93.33$$

SSE = TSS – SST – SSF = 186.66

ANOVA Table

SV	df	SS	MS	f_{cal}	f_{lab}
Types of land	4	95.33	23.83	1.021	3.83
Type of fertilizer	2	93.33	46.66	2	4.4
Error	8	186.66	23.33		
total	14	375.33			

Decision

$F_T = 1.021 < F(4, 8) = 3.83$

There is collision ... vacant of Tis O

Accept $H_oT_{0.05}$ at $\alpha = 5\%$

$F_T = 2 < F(2,8) = 4.4$

Accept H_of at $\alpha = 5\%$

Conclusion

There is no significant different between type of land. There is no significant different between fertilizer.

7. Problem of test

H_oR: $\mu_1 = \mu_2 = \mu_3$

H_1R = At least are μi . is different

$H_oC = \mu . I = \mu . II = \mu . III$

H_1C = At least are $\mu . j$ is different

Rations	Class			Ti
	I	II	III	
R_1	4	16	19	39
R_2	14	18	19	51
R_3	3	14	7	24
T.j	21	48	45	G = 114

m = 3, n = 3

N = m × n = 3 × 3 = 9

G = 114

$$\frac{G^2}{N} = \frac{(114)^2}{9} = 1444$$

$$TSS = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N} = 4^2 + 16^2 + 19^2 + \ldots + 7^2 - 1444 = 324$$

$$SSR = \frac{\Sigma Ti^2}{n} - \frac{G^2}{N} = \frac{39^2 + 51^2 + 24^2}{3} - 1444 = 122$$

$$SSC = \frac{\Sigma T.j^2}{m} - \frac{G^2}{N} = \frac{21^2 + 48^2 + 45^2}{3} - \frac{(114)^2}{9} = 146$$

SSE = TSS – SSR – SSC = 56

ANOVA

SV	df	SS	MS	f_{cal}	f_{lab}
Rations	2	122	61	4.35	6.94
Class	2	146	73	5.21	6.94
Error	4	56	14		
Total	8	324			

Decision

$F_R = 4.35 < F(2, 4) = 6.94$

Accept H_oR at $\alpha = 5\%$

$F_c = 5.21 < F(2,4) = 6.94$

Accept H_oC = at $\alpha = 5\%$

Conclusion

There is no significant different between rations.

There is no significant different between class.

8. Problem to test

$HoF = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

H_1F = At least are μ_1 = is different

$H_oR = \mu . 1 = \mu . 2 = \mu . 3 = \mu . 4 = \mu . 5$

H_1R = At least are M.j is different

Farm Size	Class					Ti
	Eastern	Central	Western	Mid Western	Far Western	
1	140	148	119	170	134	711
2	137	133	112	165	130	677
3	144	128	110	172	123	677
4	136	139	107	167	118	657
5	122	119	106	135	111	593
T.j	679	657	554	809	616	G = 3315

m = S, n = 5 N = m × n = 5 × 5 = 25, G = 3315

$$TSS = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N} = 140^2 + 148^2 + , \ldots , + 111^2 - \frac{(3315)^2}{25} = 9238$$

$$SSF = \frac{\Sigma T_i^2}{n} - \frac{G^2}{N} = \frac{711^2 + 677^2 + 677^2 + 657^2 + 593^2}{5} - \frac{(3315)^2}{25} = 1526.4$$

$$SSR = \frac{\Sigma T_i^2}{m} - \frac{G^2}{N} = \frac{6709^2 + 657^2 + 664^2 + 809^2 + 616^2}{5} - \frac{(3315)^2}{25} = 7139.6$$

SEE = TSS – SSF – SSR = 572

ANOVA

SV	df	SS	MS	f_{cal}	f_{lab}
Farm Size	4	1526.4	381.6	10.67	3.006
Region	4	7139.6	1784.9	49.92	3.006
Error	16	572	35.75		
Total	24	9238			

Decision

$F_F = 10.67 > F_{0.05} = (4, 16) = 3.006$

Accept H_oF at $\alpha = 5\%$

$F_R = 49.92 < F_{0.05} = (4, 16)$

Rejected H_oR at $\alpha = 5\%$

Conclusion

There is no significant different in crop inter sing due to farm size.

There is significant different in crop intercity due to region.

9. Problem to text

H_ow: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

H_1w = At least are μ_i is different

i = 1, 2, 3, 4, 5

$H_oM = \mu . 1 = \mu . 2 = \mu . 3 = \mu . 4$

H_1M= At least are $\mu . j$ is different

j = 1, 2, 3, 4

Workers	Machine	T.

	1	2	3	4	
1	2	2.2	1.6	2	7.8
2	1.7	2.4	1.3	1.9	7.3
3	3	2.6	3.4	3	12
4	1.2	1.8	1.5	1.6	6.1
5	2.4	2.8	2.3	2.1	9.6
T.j	10.3	11.8	10.1	10.6	G = 42.8

m = 5, x = 4, N = m × n = 5 × 4 = 20

$$\frac{G^2}{N} = \frac{(42.8)^2}{20}$$

$$TSS = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N} = 2^2\ T\ 2.2^2 + 1.6^2 + 2^2 + \ldots + 2.1^2$$

$$SSW = \frac{\Sigma T_i^2}{n} - \frac{a^2}{N} = \frac{7.8^2 + 7.3^2 + 12^2 + 6.1^2 + 7.6^2}{4} - \frac{(42.8)^2}{20}$$

$$SSM = \frac{\Sigma T_i^2}{m} - \frac{G^2}{N} = \frac{10.3^2 + 11.8^2 + 10.1^2 + 10.6^2}{5} - \frac{(42.8)^2}{20} = 0.348$$

SSE = TSS – SSW – SSM = 1.237

ANOVA

SV	df	SS	MS	f_{cal}	f_{lab}
Worker	4	5.283	1.32	12.8	3.25
Machine	3	0.348	0.116	1.12	3.49
Error	12	1.237	0.103		
Total	19	6.868			

Decision

$F_w = 12.8 > F_{0.05} = (4, 12) = 3.25$

Reject How at ∝ = 5%

$F_m = 1.12 < F_{0.05}\ (3,12) = 3.49$

Accept H_oM at ∝ = 5%

Conclusion

There is no significant different between productivity of workers.

There is significant different between efficiency of machine.

10. Problem to test

HoF = $\mu_A = \mu_B = \mu_C = \mu_D$

H_1F = At least are μ_1 is different

i = A, B, C, D

$H_oC = \mu.1 = \mu.2 = \mu.3 = \mu.4$

H_1C: At least are μ . j is diff.

i = 1, 2, 3, 4

H_1FC: $r_{ij} = 0$

H_1FC: $r_{ij} \neq 0$

Fertilizer	Chemical Treatment				$T_{i..}$
	1	2	3	4	
A	43	41	39	35	158
B	26	27	27	27	107
C	26	30	25	27	108
D	27	27	27	27	108
T.j	122	125	118	116	G = 481

$$p = 4,\ q = 4,\ m = 2,\ N = pqm = 4 \times 4 \times 2 = 32\ \frac{G^2}{N} = \frac{(481)^2}{32}$$

$$TSS = \Sigma\Sigma\Sigma\ y_{ij}k^2 - \frac{G^2}{N}$$

$$= 22^2 + 21^2 + + 21^2 + 20^2 + 19^2 + 18^2 + 17^2 + \ldots + 12^2 + 15^2 - \frac{(481)^2}{32} = 9238$$

$$SSF = \frac{\Sigma T_{i..}^2}{qm} - \frac{G^2}{N} = \frac{158^2 + 107^2 + 108^2 + 108^2}{4 \times 2} - \frac{(481)^2}{32} = 237.59$$

$$SSR = \frac{\Sigma T_j^2}{pm} - \frac{G^2}{N} = \frac{112^2 + 125^2 + 118 + 116^2}{4 \times 2} - \frac{(481)^2}{32} = 6.09$$

$$SSFC = \frac{\Sigma\Sigma T_{ij.}^2}{m} - \frac{\Sigma Ti..^2}{qm} - \frac{\Sigma T.i..^2}{pm} + \frac{G^2}{N}$$

$$= \frac{43^2 + 41^2 + \ldots + 27^2}{2} - \frac{158^2 + 107^2 + 108^2 + 108^2}{4 \times 2} - \frac{(481)^2}{32} = 237.59$$

$$SSC = \frac{\Sigma T_j^2}{pm} - \frac{G^2}{N} = \frac{122^2 + 125^2 + 118^2 + 116^2}{4 \times 2} - \frac{(481)^2}{32} = 6.09$$

$$SSFC = \frac{\Sigma\Sigma T_{ij.}^2}{m} - \frac{\Sigma\Sigma T_{ij.}^2}{qm} - \frac{\Sigma T_{j.}^2}{pm} + \frac{G^2}{N} = \frac{43^2 + 41^2 + 27^2}{2} - \frac{158^2 + 107^2 + 108^2 + 108^2}{4 \times 2}$$

$$- \frac{122^2 + 125^2 + 118^2 + 116^2}{4 \times 2} + \frac{(481)^2}{2}$$

SSE = TSS – SSF – SSC – SSFC = 16.5

ANOVA

SV	SS	df	μS	F_{cal}	F_{tab}
Fertilizer	237.59	3	75.19	76.79	3.23
Chemical	6.09	3	2.03	1.96	3.23
Interaction	18.79	9	2.08	2.02	2.53
Error	16.5	16	1.03		
Total	278.96	32			

Decision

$F_F = 76.79 > F_{0.05} = (3, 16) = 3.23$

Reject H_oF at ∝ = 5%

$FC = 1.96 < F_{0.05}\ (3, 16) = 3.23$

Accept H_oC at ∝ = 5%

$F_{FC} = 2.02 < F_{0.05}\ (9, 16) = 2.53$

Accept H_oFC at ∝ = 5%

Conclusion

There is significant different between fertilizer

There is no significant different between chemical treatment

There is no significant different between interaction effect of fertilizer and chemical treatment.

11. Problem to test

$HoG = \mu_1 = \mu_2 = \mu_3 = \mu_3 = \mu_5$

H_1F = At least are μ_1 . . is diff.

$H_om = \mu.1 = \mu.2 = \mu.3 = \mu.4$

H_1m: At least are M.j. is diff.

j = 1, 2, 3, 4, 5

1 = (15 – 50) 3 = (25 – 30) 2 = (20 + 25) 4 = (30 – 35)

5 = (35 and over)

$H_oGm: r_{..} = 0$
$H_1Gm: r_{..} \neq 0$

Order of gravida	Mother's age					$T_{i..}$
	15-20	20-25	25-30	30-35	35 and over	
1	14.9	15.4	15.1	14.8	15	75.2
2	15.8	16.1	15.7	15.7	15.45	78.7
3	17.4	18.1	17.6	16.8	16.8	86.7
4	17.9	18.7	18.1	17.3	16.9	88.9
5 and over	18.0	18.4	17.7	17.4	17.7	89.2
						G = 418.7

$p = 5, q = 5, m = 3, N = pqm = 75$

$$\frac{G^2}{N} = \frac{(418.7)^2}{75}$$

$$TSS = \Sigma\Sigma\Sigma\, y_{ij}k^2 - \frac{G^2}{N} = 14.9^2 + 15.8^2 + 17.4^2 + \ldots + 15^2 + 15.45^2 + 16.8^2 + 16.9^2 - \frac{(418.7)^2}{75} = 13.72$$

$$SSG = \frac{\Sigma T_{i..}^2}{qm} - \frac{G^2}{N} = \frac{75.2^2 + 78.7^2 + 88.9^2 + 89.2^2}{5 \times 3} - \frac{(481.7)^2}{75} = 10.9$$

$$SSM = \frac{\Sigma T_{.j.}^2}{pm} - \frac{G^2}{N} = \frac{84^2 + 86.7^2 + 84.2 + 82^2 + 81.8^2}{5 \times 3} - \frac{(418.7)^2}{75} = 1.05$$

$$SSGM = \frac{\Sigma\Sigma T_{ij.}^2}{m} - \frac{\Sigma T_{i..}^2}{qm} - \frac{\Sigma T_{.j.}^2}{pm} + \frac{G^2}{N}$$

$$= \frac{75.2^2 + 78.7^2 + \ldots + 86.7^2}{5 \times 3} - \frac{75.5^2 + 78.7^2 + 86.7^2 + 88.9^2 + 89.2^2}{5 \times 3} - \frac{84^2 + 86.7^2 + 84.2^2 + 82.2^2 + 81.8^2}{5 \times 3} = \frac{(418.7)^2}{75} = 0.35$$

SSE = TSS – SSG – SSM – SSGM = 1.41

ANOVA

SV	SS	df	µS	F_{cal}	F_{tab}
Order of gravida	10.9	4	2.72	96.42	2.55
Mother's age	1.05	4	0.26	9.33	2.55
Interaction	0.35	16	0.022	0.788	1.85
Error	1.41	50	0.028		
Total	13.72	74			

Decision

$F_G = 96.42\ (4, 50) = 2.55$

Reject H_oG at $\propto = 5\%$

$Fm = 9.33 < F_{0.05}\,(4, 50) = 2.55$

Reject H_0M at $\propto = 5\%$

$F_{CM} = 0.78 < F_{0.05}\,(16, 50) = 1.84$

Accept H_oGM at $\propto = 5\%$

Conclusion

Order of gravida significant after birth weight age of mother significant affect birth weight Interaction effect of order of gravida and age of mother does not, significantly affect birth weight.

12. Problem to test

$HoH = \mu_1 = \mu_2 = \mu_3 = \mu_3 = \mu_5$

H_1H = At least are μ_1 . . is diff.

i. = A, B, C, D

$H_oR = \mu\,.\,I = \mu\,.\,II = \mu\,.\,III$

H_1R = At least are $\mu\,.\,j$ is diff.

i. = I, II, III

$H_0HR: r_{ij} = 0$

Hogs	Rations			Ti..
	I	II	III	
A	6.6	11	10.1	27.7
B	15.2	13.1	15.6	43.9
C	10.5	12	8.2	30.7
D	20	27	19	66
T . j .	52.3	63.1	52.9	G = 168.3

$p = 4, q = 3, m = 2, N = pqm = 24, \frac{G^2}{N} = \frac{(168.3)^2}{24}$

$$TSS = \Sigma\Sigma\Sigma\, y_{ij}k^2 - \frac{G^2}{N} = 3^2 + 3.6^2 + \ldots + 9^2 + 10^2 - \frac{(168.3)^2}{24} = 191.58$$

$$SSH = \frac{\Sigma T_{i..}^2}{qm} - \frac{G^2}{N} = \frac{27.7^2 + 43.9^2 + 30.7^2 + 66^2}{3 \times 2} - \frac{(168.3)^2}{24} = 151.96$$

$$SSR = \frac{\Sigma T_{.j.}^2}{pm} - \frac{G^2}{N} = \frac{52.3^2 + 63.1^2 + 52.9^2}{4 \times 2} - \frac{(168.3)^2}{24} = 9.21$$

$$SSHR = \frac{\Sigma\Sigma T_{ij.}^2}{m} - \frac{\Sigma T_{i..}^2}{qm} - \frac{\Sigma T_{.j.}^2}{pm} + \frac{G^2}{N}$$

$$= \frac{6.6^2 + 11^2 + \ldots + 19^2}{2} - \frac{27.7^2 + 43.9^2 + 30.7^2 + 66^2}{3 \times 2} - \frac{(52.3^2 + 63.1^2 + 52.5^2)}{5 \times 3}$$

$$= \frac{(418.7)^2}{75} = 20.66$$

SSE = TSS – SSG – SSM – SSGM = 9.755

ANOVA

SV	df	SS	µS	F_{cal}	F_{tab}
Hogs	3	151.96	50.65	62.31	3.49
Mother's age	2	9.21	4.6	5.66	3.88
Interaction	6	20.66	3.44	4.23	2.99
Error	12	9.75	0.81		
Total	23	191.58			

Decision

$F_H = 62.31 > F_{0.05}\,(3,12) = 3.49$

Reject H_oH at $\propto = 5\%$

$F_R = 5.66 < F_{0.05}\,(2, 12) = 3.88$

Reject H_0M at $\propto = 5\%$

$F_{RC} = 4.23 < F_{0.05}\,(6, 12) = 2.99$

Accept H_oRC at $\propto = 5\%$

Conclusion

There is significant difference between Hogs

There is significant difference rations interaction effect of hogs and rations.