Toy Standard Model using FeynCalc

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August 10, 2022

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1 Single Scalar (Phi34)

1.1 Lagrangian

$$\mathcal{L}_{\text{Phi34}} = (Z_P)\frac{1}{2}(\partial\phi)^2 - (Z_P Z m_P^2)\frac{1}{2}m_P^2\ \phi^2 - (Z_P^2 Z g_3)\frac{1}{3!}g_3\ \phi^3 - (Z_P^2 Z g_4)\frac{1}{4!}g_4\ \phi^4$$
$$- \delta_{\texttt{vev}} m_P^2\ \phi - \frac{1}{2}\delta_{\texttt{vev}} g_3\ \phi^2 - \frac{1}{3!}\delta_{\texttt{vev}} g_4 \phi^3 \tag{1}$$

where,

$$\phi \longrightarrow \delta_{\texttt{vev}} + \phi$$

1.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{g_3}{16\pi^2} \tag{2}$$

$$\delta P = 0 \qquad \delta m_P = \frac{1}{\epsilon}\frac{g_4}{32\pi^2} \tag{3}$$

$$\delta g_3 = \frac{1}{\epsilon}\frac{g_4}{8\pi^2} \qquad \delta g_4 = \frac{1}{\epsilon}\frac{3g_4}{16\pi^2} \tag{4}$$

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2 Linear sigma model (LSM)

2.1 Lagrangian

$$\mathcal{L}_{\text{LSM}} = \partial_\mu \bar{\Phi} \partial^\mu \Phi - \frac{g_4}{4}\left(\bar{\Phi}\Phi - \left(\frac{\texttt{vev}^2}{2}\right)\right)^2 \tag{5}$$

where,

$$\Phi \longrightarrow \frac{1}{\sqrt{2}}\Big((Z_{\texttt{vev}})\texttt{vev} + (\sqrt{Z_H})H + i(\sqrt{Z_b})b\Big)$$

which gives us,

$$\begin{aligned}\mathcal{L}_{\text{LSM}} = &+ (Z_b)\ \partial_\mu b \partial_\mu b + (Z_H)\ \partial_\mu H \partial_\mu H - (Z m_H^2 Z_H Z_{\texttt{vev}}^2)\ \frac{1}{4} g_4 \texttt{vev}^2\ H^2 \\ &- (Z_{bbbb} Z_b^2)\ \frac{1}{16} g_4\ b^4 - (Z_{bbHH} Z_b Z_H)\ \frac{1}{8} g_4\ b^2 H^2 - (Z_{HHHH} Z_H^2)\ \frac{1}{16} g_4\ H^4 \\ &- (Z_{bbH} Z_b \sqrt{Z_H} Z_{\texttt{vev}})\ \frac{1}{4} g_4 \texttt{vev}\ b^2 H - (Z_{HHH} Z_H \sqrt{Z_H} Z_{\texttt{vev}})\ \frac{1}{4} g_4 \texttt{vev}\ H^3 \\ &- (Z_{\texttt{vev}}^2 - 1)\ \left((Z_H)\frac{1}{8} g_4 \texttt{vev}^2\ H^2 + (Z_{\texttt{vev}}\sqrt{Z_H})\frac{1}{4} g_4 \texttt{vev}^3 H + (Z_b)\frac{1}{8} g_4 \texttt{vev}^2\ b^2\right) \\ &- \left((Z_{\texttt{vev}}^2 - 1)\texttt{vev}^2\right)^2 \frac{g_4}{16}\end{aligned} \tag{6}$$

where the classical values of the masses is given by,

$$m_H = \frac{\sqrt{g_4}\ \texttt{vev}}{\sqrt{2}} \tag{7}$$

$$m_b = 0 \tag{8}$$

2.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{3g_4}{32\pi^2} \tag{9}$$

$$\delta H = 0 \qquad \delta m_H = \frac{1}{\epsilon}\frac{g_4}{16\pi^2} \tag{10}$$

$$\delta b = 0 \qquad \delta m_b = 0 \tag{11}$$

$$\delta_{HHH} = \frac{1}{\epsilon}\frac{7g_4}{32\pi^2} \qquad \delta_{bbH} = \frac{1}{\epsilon}\frac{7g_4}{32\pi^2} \tag{12}$$

$$\delta_{HHHH} = \frac{1}{\epsilon}\frac{5g_4}{16\pi^2} \qquad \delta_{bbbb} = \frac{1}{\epsilon}\frac{5g_4}{16\pi^2} \tag{13}$$

$$\delta_{bbHH} = \frac{1}{\epsilon}\frac{5g_4}{16\pi^2} \tag{14}$$

2.3 Consistancy checks

$$\delta_{HHHH} = \delta_{bbbb} = \delta_{bbHH} = \delta g_4 \tag{15}$$

$$\delta m_H = \frac{1}{2}\delta g_4 - \delta_{\texttt{vev}} \tag{16}$$

$$\delta_{HHH} = \delta_{bbH} = \delta g_4 - \delta_{\texttt{vev}} \tag{17}$$

3 Yukawa

3.1 Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Y}} = &+(Z_P - 1)\ i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - (Z_P Z m_P - 1)\ m_P\bar{\Psi}\Psi \\
&+ (Z_P)\frac{1}{2}(\partial\phi)^2 - (Z_P Z m_P^2)\frac{1}{2}m_P^2\ \phi^2 - (Z_P^2 Z g_3)\frac{1}{3!}g_3\ \phi^3 - (Z_P^2 Z g_4)\frac{1}{4!}g_4\ \phi^4 \\
&- (Z_F\sqrt{Z_P}Z_{FFP})\ y\ \phi\bar{\Psi}\Psi - (Z_F\sqrt{Z_P}Z_{FG5FP})\ iy_5\ \phi\bar{\Psi}\gamma^5\Psi \\
&- (\delta_{\texttt{vev}})m_P^2\ \phi - \frac{1}{2}(\delta_{\texttt{vev}})g_3\ \phi^2 - \frac{1}{3!}(\delta_{\texttt{vev}})g_4\phi^3 - (\delta_{\texttt{vev}})\ y\ \bar{\Psi}\Psi - (\delta_{\texttt{vev}})\ iy_5\ \bar{\Psi}\gamma^5\Psi \\
&- (\delta_\theta)\ im_P\bar{\Psi}\gamma^5\Psi - (\delta_\theta)\ iy\ \phi\bar{\Psi}\gamma^5\Psi + (\delta_\theta)\ y_5\ \phi\bar{\Psi}\Psi
\end{aligned}
\tag{18}
$$

where,

$$
\begin{aligned}
\phi &\longrightarrow \delta_{\texttt{vev}} + \phi \\
\psi &\longrightarrow e^{i\gamma^5\theta/2}\psi
\end{aligned}
$$

3.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{\left(g_3 m_P^2 - 8y m_F^3\right)}{16\pi^2 m_P^2} \tag{19}$$

$$\delta P = -\frac{1}{\epsilon}\frac{\left(y^2 + y_5^2\right)}{4\pi^2} \tag{20}$$

$$\delta m_P = \frac{1}{\epsilon}\frac{\left(8g_3 y m_F^3 + m_P^2\left(g_4 m_P^2 + 4y^2\left(m_P^2 - 6m_F^2\right) + y_5^2\left(4m_P^2 - 8m_F^2\right)\right)\right)}{32\pi^2 m_P^4} \tag{21}$$

$$\delta F = -\frac{1}{\epsilon}\frac{\left(y^2 + y_5^2\right)}{16\pi^2} \tag{22}$$

$$\delta m_F = \frac{1}{\epsilon}\frac{\left(-g_3 y m_P^2 + y^2\left(8m_F^3 + 3m_F m_P^2\right) - y_5^2 m_F m_P^2\right)}{16\pi^2 m_F m_P^2} \tag{23}$$

$$\delta_\theta = \frac{1}{\epsilon}\frac{y_5\left(4y m_F\left(2m_F^2 + m_P^2\right) - g_3 m_P^2\right)}{16\pi^2 m_F m_P^2} \tag{24}$$

$$\delta g_3 = \frac{1}{\epsilon}\frac{\left(g_4\left(g_3 m_P^2 + 4y m_F^3\right) + 3m_P^2\left(y^2 + y_5^2\right)\left(g_3 - 8y m_F\right)\right)}{8\pi^2 g_3 m_P^2} \tag{25}$$

$$\delta y = \frac{1}{\epsilon}\frac{\left(-g_3 y_5^2 m_P^2 + 5y^3 m_F m_P^2 + y y_5^2 m_F\left(8m_F^2 + 9m_P^2\right)\right)}{16\pi^2 y m_F m_P^2} \tag{26}$$

$$\delta y_5 = \frac{1}{\epsilon}\frac{\left(g_3 y m_P^2 + y^2\left(m_F m_P^2 - 8m_F^3\right) + 5y_5^2 m_F m_P^2\right)}{16\pi^2 m_F m_P^2} \tag{27}$$

$$\delta g_4 = \frac{1}{\epsilon}\frac{\left(3g_4^2 + 8g_4\left(y^2 + y_5^2\right) - 48\left(y^2 + y_5^2\right)^2\right)}{16\pi^2 g_4} \tag{28}$$