

Toy Standard Model using FeynCalc

Raj Patil (Analysis Note)

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Project done under the guidance of
Dr. Arnab Rudra

1 Single Scalar (Phi34)

1.1 Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{Phi34}} = & (Z_P)\frac{1}{2}(\partial\phi)^2 - (Z_P Z m_P^2)\frac{1}{2}m_P^2 \phi^2 - (Z_P^2 Z g_3)\frac{1}{3!}g_3 \phi^3 - (Z_P^2 Z g_4)\frac{1}{4!}g_4 \phi^4 \\ & - \delta_{\text{vev}}m_P^2 \phi - \frac{1}{2}\delta_{\text{vev}}g_3 \phi^2 - \frac{1}{3!}\delta_{\text{vev}}g_4\phi^3\end{aligned}\quad (1)$$

1.2 Answers

$$\delta_{\text{vev}} = \frac{1}{\epsilon} \frac{g_3}{16\pi^2} \quad (2)$$

$$\delta P = 0 \quad \delta m_P = \frac{1}{\epsilon} \frac{g_4}{32\pi^2} \quad (3)$$

$$\delta g_3 = \frac{1}{\epsilon} \frac{g_4}{8\pi^2} \quad \delta g_4 = \frac{1}{\epsilon} \frac{3g_4}{16\pi^2} \quad (4)$$

2 Linear sigma model (LSM)

2.1 Lagrangian

$$\mathcal{L}_{\text{LSM}} = \partial_\mu \bar{\Phi} \partial^\mu \Phi - \frac{g_4}{4} \left(\bar{\Phi} \Phi - \left(\frac{\text{vev}^2}{2} \right) \right)^2 \quad (5)$$

where,

$$\Phi \longrightarrow \frac{1}{\sqrt{2}} \left((Z_{\text{vev}}) \text{vev} + (\sqrt{Z_H}) H + i(\sqrt{Z_b}) b \right)$$

which gives us,

$$\begin{aligned} \mathcal{L}_{\text{LSM}} = & + (Z_b) \partial_\mu b \partial^\mu b + (Z_H) \partial_\mu H \partial^\mu H - (Z m_H^2 Z_H Z_{\text{vev}}^2) \frac{1}{4} g_4 \text{vev}^2 H^2 \\ & - (Z_{bbbb} Z_b^2) \frac{1}{16} g_4 b^4 - (Z_{bbHH} Z_b Z_H) \frac{1}{8} g_4 b^2 H^2 - (Z_{HHHH} Z_H^2) \frac{1}{16} g_4 H^4 \\ & - (Z_{bbH} Z_b \sqrt{Z_H} Z_{\text{vev}}) \frac{1}{4} g_4 \text{vev} b^2 H - (Z_{HHH} Z_H \sqrt{Z_H} Z_{\text{vev}}) \frac{1}{4} g_4 \text{vev} H^3 \\ & - (Z_{\text{vev}}^2 - 1) \left((Z_H) \frac{1}{8} g_4 \text{vev}^2 H^2 + (Z_{\text{vev}} \sqrt{Z_H}) \frac{1}{4} g_4 \text{vev}^3 H + (Z_b) \frac{1}{8} g_4 \text{vev}^2 b^2 \right) \\ & - ((Z_{\text{vev}}^2 - 1) \text{vev}^2) \frac{2 g_4}{16} \end{aligned} \quad (6)$$

where the classical values of the masses is given by,

$$m_H = \frac{\sqrt{g_4} \text{vev}}{\sqrt{2}} \quad (7)$$

$$m_b = 0 \quad (8)$$

2.2 Answers

$$\delta_{\text{vev}} = \frac{1}{\epsilon} \frac{3g_4}{32\pi^2} \quad (9)$$

$$\delta H = 0 \quad \delta m_H = \frac{1}{\epsilon} \frac{g_4}{16\pi^2} \quad (10)$$

$$\delta b = 0 \quad \delta m_b = 0 \quad (11)$$

$$\delta_{HHH} = \frac{1}{\epsilon} \frac{7g_4}{32\pi^2} \quad \delta_{bbH} = \frac{1}{\epsilon} \frac{7g_4}{32\pi^2} \quad (12)$$

$$\delta_{HHHH} = \frac{1}{\epsilon} \frac{5g_4}{16\pi^2} \quad \delta_{bbbb} = \frac{1}{\epsilon} \frac{5g_4}{16\pi^2} \quad (13)$$

$$\delta_{bbHH} = \frac{1}{\epsilon} \frac{5g_4}{16\pi^2} \quad (14)$$

2.3 Consistency checks

$$\delta_{HHHH} = \delta_{bbbb} = \delta_{bbHH} = \delta g_4 \quad (15)$$

$$\delta m_H = \frac{1}{2} \delta g_4 - \delta_{\text{vev}} \quad (16)$$

$$\delta_{HHH} = \delta_{bbH} = \delta g_4 - \delta_{\text{vev}} \quad (17)$$