# Toy Standard Model using FeynCalc

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August 24, 2022

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## Useful links

1. FeynCalc: https://feyncalc.github.io/
2. FeynHelpers: https://feyncalc.github.io/referenceFeynHelpersDev
3. Package-X: https://www.sciencedirect.com/science/article/abs/pii/S0010465517301297

## Contents

# 1 Single Scalar (Phi34)

## 1.1 Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{Phi34}} = (Z_P)\frac{1}{2}(\partial\phi)^2 - (Z_P Z m_P^2)\frac{1}{2}m_P^2\ \phi^2 - (Z_P^2 Z g_3)\frac{1}{3!}g_3\ \phi^3 - (Z_P^2 Z g_4)\frac{1}{4!}g_4\ \phi^4 \\ - \delta_{\texttt{vev}} m_P^2\ \phi - \frac{1}{2}\delta_{\texttt{vev}} g_3\ \phi^2 - \frac{1}{3!}\delta_{\texttt{vev}} g_4 \phi^3\end{aligned} \tag{1}$$

where,

$$\phi \longrightarrow \delta_{\texttt{vev}} + \phi$$

## 1.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{g_3}{16\pi^2} \tag{2}$$

$$\delta P = 0 \qquad \delta m_P = \frac{1}{\epsilon}\frac{g_4}{32\pi^2} \tag{3}$$

$$\delta g_3 = \frac{1}{\epsilon}\frac{g_4}{8\pi^2} \qquad \delta g_4 = \frac{1}{\epsilon}\frac{3g_4}{16\pi^2} \tag{4}$$

## 2 Linear sigma model (LSM)

### 2.1 Lagrangian

$$\mathcal{L}_{\text{LSM}} = \partial_\mu \bar{\Phi} \partial^\mu \Phi - \frac{g_4}{4}\left(\bar{\Phi}\Phi - \left(\frac{\texttt{vev}^2}{2}\right)\right)^2 \tag{5}$$

where,

$$\Phi \longrightarrow \frac{1}{\sqrt{2}}\Big((Z_{\texttt{vev}})\texttt{vev} + (\sqrt{Z_H})H + i(\sqrt{Z_b})b\Big)$$

which gives us,

$$\begin{aligned}
\mathcal{L}_{\text{LSM}} = &+ (Z_b)\ \partial_\mu b \partial_\mu b + (Z_H)\ \partial_\mu H \partial_\mu H - (Zm_H^2 Z_H Z_{\texttt{vev}}^2)\ \frac{1}{4} g_4 \texttt{vev}^2\ H^2 \\
&- (Z_{bbbb} Z_b^2)\ \frac{1}{16} g_4\ b^4 - (Z_{bbHH} Z_b Z_H)\ \frac{1}{8} g_4\ b^2 H^2 - (Z_{HHHH} Z_H^2)\ \frac{1}{16} g_4\ H^4 \\
&- (Z_{bbH} Z_b \sqrt{Z_H} Z_{\texttt{vev}})\ \frac{1}{4} g_4 \texttt{vev}\ b^2 H - (Z_{HHH} Z_H \sqrt{Z_H} Z_{\texttt{vev}})\ \frac{1}{4} g_4 \texttt{vev}\ H^3 \\
&- (Z_{\texttt{vev}}^2 - 1)\ \left((Z_H)\frac{1}{8} g_4 \texttt{vev}^2\ H^2 + (Z_{\texttt{vev}}\sqrt{Z_H})\frac{1}{4} g_4 \texttt{vev}^3 H + (Z_b)\frac{1}{8} g_4 \texttt{vev}^2\ b^2\right) \\
&- \big((Z_{\texttt{vev}}^2 - 1)\texttt{vev}^2\big)^2 \frac{g_4}{16}
\end{aligned} \tag{6}$$

where the classical values of the masses is given by,

$$m_H = \frac{\sqrt{g_4}\ \texttt{vev}}{\sqrt{2}} \tag{7}$$

$$m_b = 0 \tag{8}$$

### 2.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{3g_4}{32\pi^2} \tag{9}$$

$$\delta H = 0 \qquad \delta m_H = \frac{1}{\epsilon}\frac{g_4}{16\pi^2} \tag{10}$$

$$\delta b = 0 \qquad \delta m_b = 0 \tag{11}$$

$$\delta_{HHH} = \frac{1}{\epsilon}\frac{7g_4}{32\pi^2} \qquad \delta_{bbH} = \frac{1}{\epsilon}\frac{7g_4}{32\pi^2} \tag{12}$$

$$\delta_{HHHH} = \frac{1}{\epsilon}\frac{5g_4}{16\pi^2} \qquad \delta_{bbbb} = \frac{1}{\epsilon}\frac{5g_4}{16\pi^2} \tag{13}$$

$$\delta_{bbHH} = \frac{1}{\epsilon}\frac{5g_4}{16\pi^2} \tag{14}$$

# 3 Yukawa

## 3.1 Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{Y}} = &+(Z_P)\ i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - (Z_P Zm_P - 1)\ m_P\bar{\Psi}\Psi \\
&+ (Z_P)\frac{1}{2}(\partial\phi)^2 - (Z_P Zm_P^2)\frac{1}{2}m_P^2\ \phi^2 - (Z_P^2 Zg_3)\frac{1}{3!}g_3\ \phi^3 - (Z_P^2 Zg_4)\frac{1}{4!}g_4\ \phi^4 \\
&- (Z_F\sqrt{Z_P}Z_{FFP})\ y\ \phi\bar{\Psi}\Psi - (Z_F\sqrt{Z_P}Z_{FG5FP})\ iy_5\ \phi\bar{\Psi}\gamma^5\Psi \\
&- (\delta_{\texttt{vev}})m_P^2\ \phi - \frac{1}{2}(\delta_{\texttt{vev}})g_3\ \phi^2 - \frac{1}{3!}(\delta_{\texttt{vev}})g_4\phi^3 - (\delta_{\texttt{vev}})\ y\ \bar{\Psi}\Psi - (\delta_{\texttt{vev}})\ iy_5\ \bar{\Psi}\gamma^5\Psi \\
&- (\delta_\theta)\ im_P\bar{\Psi}\gamma^5\Psi - (\delta_\theta)\ iy\ \phi\bar{\Psi}\gamma^5\Psi + (\delta_\theta)\ y_5\ \phi\bar{\Psi}\Psi
\end{aligned} \tag{15}$$

where,

$$\phi \longrightarrow \delta_{\texttt{vev}} + \phi$$
$$\psi \longrightarrow e^{i\gamma^5\theta/2}\psi$$

## 3.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{\left(g_3 m_P^2 - 8ym_F^3\right)}{16\pi^2 m_P^2} \tag{16}$$

$$\delta P = -\frac{1}{\epsilon}\frac{\left(y^2 + y_5^2\right)}{4\pi^2} \tag{17}$$

$$\delta m_P = \frac{1}{\epsilon}\frac{\left(8g_3 y m_F^3 + m_P^2\left(g_4 m_P^2 + 4y^2\left(m_P^2 - 6m_F^2\right) + y_5^2\left(4m_P^2 - 8m_F^2\right)\right)\right)}{32\pi^2 m_P^4} \tag{18}$$

$$\delta F = -\frac{1}{\epsilon}\frac{\left(y^2 + y_5^2\right)}{16\pi^2} \tag{19}$$

$$\delta m_F = \frac{1}{\epsilon}\frac{\left(-g_3 y m_P^2 + y^2\left(8m_F^3 + 3m_F m_P^2\right) - y_5^2 m_F m_P^2\right)}{16\pi^2 m_F m_P^2} \tag{20}$$

$$\delta_\theta = \frac{1}{\epsilon}\frac{y_5\left(4ym_F\left(2m_F^2 + m_P^2\right) - g_3 m_P^2\right)}{16\pi^2 m_F m_P^2} \tag{21}$$

$$\delta g_3 = \frac{1}{\epsilon}\frac{\left(g_4\left(g_3 m_P^2 + 4ym_F^3\right) + 3m_P^2\left(y^2 + y_5^2\right)\left(g_3 - 8ym_F\right)\right)}{8\pi^2 g_3 m_P^2} \tag{22}$$

$$\delta y = \frac{1}{\epsilon}\frac{\left(-g_3 y_5^2 m_P^2 + 5y^3 m_F m_P^2 + yy_5^2 m_F\left(8m_F^2 + 9m_P^2\right)\right)}{16\pi^2 y m_F m_P^2} \tag{23}$$

$$\delta y_5 = \frac{1}{\epsilon}\frac{\left(g_3 y m_P^2 + y^2\left(m_F m_P^2 - 8m_F^3\right) + 5y_5^2 m_F m_P^2\right)}{16\pi^2 m_F m_P^2} \tag{24}$$

$$\delta g_4 = \frac{1}{\epsilon}\frac{\left(3g_4^2 + 8g_4\left(y^2 + y_5^2\right) - 48\left(y^2 + y_5^2\right)^2\right)}{16\pi^2 g_4} \tag{25}$$

# 4 Scalar QED

## 4.1 Lagrangian

$$\mathcal{L}_{\text{SQED}} = -(Z_A)\,\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (Z_P)\,\partial_\mu\bar{\Phi}\partial^\mu\Phi - (Z_P Z m_P^2)\,m_P^2\bar{\Phi}\Phi - (Z_P^2 Z_{PPPP})\,\frac{g_4}{4}(\bar{\Phi}\Phi)^2 \\ + (Z_{APP}Z_P\sqrt{Z_A})\,ie\,(-\bar{\Phi}\partial_\mu\Phi A^\mu + \Phi\partial_\mu\bar{\Phi}A^\mu) + (Z_{AAPP}Z_P Z_A)\,e^2\,\bar{\Phi}\Phi A_\mu A^\mu \tag{26}$$

## 4.2 Answers

$$\delta P = -\frac{1}{\epsilon}\frac{e^2(\xi_A - 3)}{8\pi^2} \qquad \delta m_P = \frac{1}{\epsilon}\frac{\left(g_4 - 3e^2\right)}{16\pi^2} \tag{27}$$

$$\delta A = -\frac{1}{\epsilon}\frac{e^2}{24\pi^2} \tag{28}$$

$$\delta_{APP} = \frac{1}{\epsilon}\frac{e^2}{48\pi^2} \tag{29}$$

$$\delta_{PPPP} = \frac{1}{\epsilon}\frac{\left(24e^4 - 12e^2 g_4 + 5g_4^2\right)}{16\pi^2 g_4} \qquad \delta_{AAPP} = \frac{1}{\epsilon}\frac{e^2}{24\pi^2} \tag{30}$$

# 5 QED

## 5.1 Lagrangian

$$\mathcal{L}_{\text{QED}} = -(Z_A)\,\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (Z_P)\; i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - (Z_P Z m_P)\; m_P\bar{\Psi}\Psi \\ + (Z_{APP}Z_P\sqrt{Z_A})\; e\;\bar{\Psi}\partial_{\mu}\Psi A^{\mu} \tag{31}$$

## 5.2 Answers

$$\delta P = -\frac{1}{\epsilon}\frac{e^2\xi_A}{8\pi^2} \qquad \delta m_P = -\frac{1}{\epsilon}\frac{3e^2}{8\pi^2} \tag{32}$$

$$\delta A = -\frac{1}{\epsilon}\frac{e^2}{6\pi^2} \tag{33}$$

$$\delta_{APP} = \frac{1}{\epsilon}\frac{e^2}{12\pi^2} \tag{34}$$

## 5.3 Lamb shift

Coulomb potential and its correction in large distance and small momentum limit,

$$V(p) = \frac{e^2}{p^2} - \frac{e^4}{60\pi^2 m_P^2} + \cdots \tag{35}$$

# 6 Yang-Mills

## 6.1 Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{YM}} = & (Z_G)\left(-\frac{1}{4}\partial_\nu G^\mu_a \partial^\mu G^a_\nu + \frac{1}{2}\partial_\nu G^\mu_a \partial^\nu G^a_\mu - \frac{1}{4}\partial_\mu G^\nu_a \partial^\nu G^a_\mu\right) \\ & + (Z_{G3} Z_G \sqrt{Z_G})\left(\frac{1}{2} g f_{abc} \partial_\nu G^\mu_a G^b_\mu G^c_\nu - \frac{1}{2} g f_{abc} \partial_\mu G^\nu_a G^b_\mu G^c_\nu\right) \\ & - (Z_{G4} Z_G^2)\frac{1}{4} g^2 f_{abc} f^{dec} G^\mu_a G^\mu_b G^d_\mu G^e_\nu \\ & - (Z_c)\bar{c}\partial_\mu \partial^\mu c - (Z_{Gcc}\sqrt{Z_G} Z_c) g f_{abc} \bar{c}_a \partial_\mu (G^\mu_b c_c)\end{aligned} \tag{36}$$

## 6.2 Answers

$$\delta G = \frac{1}{\epsilon}\frac{g^2 N(13 - 3\xi_G)}{48\pi^2} \qquad \delta c = -\frac{1}{\epsilon}\frac{g^2 N(\xi_G - 3)}{32\pi^2} \tag{37}$$

$$\delta_{Gcc} = -\frac{1}{\epsilon}\frac{g^2 11 N}{48\pi^2} \qquad \delta_{G3} = -\frac{1}{\epsilon}\frac{g^2 11 N}{48\pi^2} \tag{38}$$

$$\delta_{G4} = -\frac{1}{\epsilon}\frac{g^2 11 N}{24\pi^2} \tag{39}$$

# 7 Scalar QED - Higgsed

## 7.1 Lagrangian

$$\mathcal{L}_{\text{GaugeT}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{40}$$

$$\mathcal{L}_{\text{ScalarT}} = D_\mu\bar{\Phi}D^\mu\Phi - \frac{g_4}{4}\left(\bar{\Phi}\Phi - \left(\frac{\texttt{vev}^2}{2}\right)\right)^2 \tag{41}$$

$$\mathcal{L}_{\text{GaugeFixing}} = -\frac{1}{2\xi_A}\partial_\mu A_\mu \partial_\nu A_\nu - e\texttt{vev}\ A_\mu\partial_\mu b - \frac{1}{2}\xi_A e^2\texttt{vev}^2\ b^2 \tag{42}$$

$$\mathcal{L}_{\text{Ghost}} = -\bar{c}\partial_\mu\partial^\mu c - \xi_A e^2\texttt{vev}^2\ \bar{c}c - \xi_A e^2\texttt{vev}\ H\bar{c}c \tag{43}$$

where,

$$\Phi \longrightarrow \frac{1}{\sqrt{2}}(\texttt{vev} + H + ib) \tag{44}$$

$$m_H = \frac{\sqrt{g_4}\ \texttt{vev}}{\sqrt{2}} \tag{45}$$

$$m_A = e\texttt{vev} \tag{46}$$

$$m_b = e\texttt{vev}\sqrt{\xi_A} \tag{47}$$

$$m_c = e\texttt{vev}\sqrt{\xi_A} \tag{48}$$

$$\begin{aligned}
\mathcal{L}_{\text{CT}} = &+ (Z_b - 1)\ \partial_\mu b\partial_\mu b - (Z_c - 1)\ \bar{\partial}_\mu c\partial_\mu c + (Z_H - 1)\ \partial_\mu H\partial_\mu H \\
&- (Z_A - 1)\ \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi_A}(\partial_\nu A_\mu\partial_\mu A_\nu) \\
&- (Zm_H^2 Z_H Z_{\texttt{vev}}^2 - 1)\ \frac{1}{4}g_4\texttt{vev}^2\ H^2 + (Zm_A^2 Z_A Z_{\texttt{vev}}^2 - 1)\ \frac{1}{2}e^2\texttt{vev}^2\ A_\mu^2 \\
&- (Zm_b^2 Z_b Z_{\texttt{vev}}^2 - 1)\ \frac{1}{2}e^2\texttt{vev}^2\xi_A\ b^2 - (Zm_c^2 Z_c Z_{\texttt{vev}}^2 - 1)\ e^2\texttt{vev}^2\xi_A\ \bar{c}c \\
&- (Z_{bbbb}Z_b^2 - 1)\ \frac{1}{16}g_4\ b^4 - (Z_{bbHH}Z_b Z_H - 1)\ \frac{1}{8}g_4\ b^2H^2 - (Z_{HHHH}Z_H^2 - 1)\ \frac{1}{16}g_4\ H^4 \\
&- (Z_{bbH}Z_b\sqrt{Z_H}Z_{\texttt{vev}} - 1)\ \frac{1}{4}g_4\texttt{vev}\ b^2H - (Z_{HHH}Z_H\sqrt{Z_H}Z_{\texttt{vev}} - 1)\ \frac{1}{4}g_4\texttt{vev}\ H^3 \\
&+ (Z_{AAbb}Z_b Z_A - 1)\ \frac{1}{2}e^2\ b^2A_\mu^2 + (Z_{AAHH}Z_H Z_A - 1)\ \frac{1}{2}e^2\ H^2A_\mu^2 \\
&+ (Z_{AAH}Z_A\sqrt{Z_H}Z_{\texttt{vev}} - 1)\ e^2H\texttt{vev}\ A_\mu^2 + (Z_{AbH}\sqrt{Z_b Z_A Z_H} - 1)\ (+e\ HA_\mu\partial_\mu b - e\ bA_\mu\partial_\mu H) \\
&- (Z_{ccH}Z_c\sqrt{Z_H}Z_{\texttt{vev}} - 1)\ e^2\texttt{vev}\xi_A\ \bar{c}cH \\
&- (Z_{\texttt{vev}}^2 - 1)\left(\frac{1}{8}g_4\texttt{vev}^2\ H^2 + Z_{\texttt{vev}}\frac{1}{4}g_4\texttt{vev}^3H + \frac{1}{8}g_4\texttt{vev}^2\ b^2\right) - \left((Z_{\texttt{vev}}^2 - 1)\texttt{vev}^2\right)^2\frac{g_4}{16}
\end{aligned} \tag{49}$$

### 7.2 Answers

$$\delta_{\texttt{vev}} = \frac{1}{\epsilon}\frac{\left(3\left(8e^4+g_4^2\right)+2e^2g_4\xi_A\right)}{32\pi^2g_4} \tag{50}$$

$$\delta H = -\frac{1}{\epsilon}\frac{e^2\left(\xi_A-3\right)}{8\pi^2} \qquad \delta m_H = \frac{1}{\epsilon}\frac{\left(g_4-3e^2\right)}{16\pi^2} \tag{51}$$

$$\delta A = -\frac{1}{\epsilon}\frac{e^2}{24\pi^2} \qquad \delta m_A = -\frac{1}{\epsilon}\frac{\left(72e^4-16e^2g_4+9g_4^2\right)}{96\pi^2g_4} \tag{52}$$

$$\delta b = -\frac{1}{\epsilon}\frac{e^2\left(\xi_A-3\right)}{8\pi^2} \qquad \delta m_b = -\frac{1}{\epsilon}\frac{3\left(8e^4+2e^2g_4+g_4^2\right)}{32\pi^2g_4} \tag{53}$$

$$\delta c = 0 \qquad \delta m_c = \frac{1}{\epsilon}\frac{e^2\xi_A}{16\pi^2} \tag{54}$$

$$\delta_{HHH} = \frac{1}{\epsilon}\frac{\left(24e^4-18e^2g_4+7g_4^2\right)}{32\pi^2g_4} \qquad \delta_{AAH} = -\frac{1}{\epsilon}\frac{\left(72e^4-14e^2g_4+9g_4^2\right)}{96\pi^2g_4} \tag{55}$$

$$\delta_{bbH} = \frac{1}{\epsilon}\frac{\left(24e^4-18e^2g_4+7g_4^2\right)}{32\pi^2g_4} \qquad \delta_{AbH} = \frac{1}{\epsilon}\frac{e^2}{48\pi^2} \tag{56}$$

$$\delta_{ccH} = -\frac{1}{\epsilon}\frac{3\left(8e^4+2e^2g_4+g_4^2\right)}{32\pi^2g_4} \tag{57}$$

$$\delta_{HHHH} = \frac{1}{\epsilon}\frac{\left(24e^4-12e^2\mathrm{g4}+5\mathrm{g4}^2\right)}{16\pi^2\mathrm{g4}} \qquad \delta_{bbHH} = \frac{1}{\epsilon}\frac{\left(24e^4-12e^2\mathrm{g4}+5\mathrm{g4}^2\right)}{16\pi^2\mathrm{g4}} \tag{58}$$

$$\delta_{AAbb} = \frac{1}{\epsilon}\frac{e^2}{24\pi^2} \qquad \delta_{AAHH} = \frac{1}{\epsilon}\frac{e^2}{24\pi^2} \tag{59}$$