

Toy Standard Model using FeynCalc

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1 Single Scalar (Phi34)

1.1 Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{Phi34}} = & (Z_P)\frac{1}{2}(\partial\phi)^2 - (Z_P Z m_P^2)\frac{1}{2}m_P^2 \phi^2 - (Z_P^2 Z g_3)\frac{1}{3!}g_3 \phi^3 - (Z_P^2 Z g_4)\frac{1}{4!}g_4 \phi^4 \\ & - \delta_{\text{vev}}m_P^2 \phi - \frac{1}{2}\delta_{\text{vev}}g_3 \phi^2 - \frac{1}{3!}\delta_{\text{vev}}g_4\phi^3\end{aligned}\quad (1)$$

where,

$$\phi \longrightarrow \delta_{\text{vev}} + \phi$$

1.2 Answers

$$\delta_{\text{vev}} = \frac{1}{\epsilon} \frac{g_3}{16\pi^2} \quad (2)$$

$$\delta P = 0 \quad \delta m_P = \frac{1}{\epsilon} \frac{g_4}{32\pi^2} \quad (3)$$

$$\delta g_3 = \frac{1}{\epsilon} \frac{g_4}{8\pi^2} \quad \delta g_4 = \frac{1}{\epsilon} \frac{3g_4}{16\pi^2} \quad (4)$$

2 Linear sigma model (LSM)

2.1 Lagrangian

$$\mathcal{L}_{\text{LSM}} = \partial_\mu \bar{\Phi} \partial^\mu \Phi - \frac{g_4}{4} \left(\bar{\Phi} \Phi - \left(\frac{\text{vev}^2}{2} \right) \right)^2 \quad (5)$$

where,

$$\Phi \longrightarrow \frac{1}{\sqrt{2}} \left((Z_{\text{vev}}) \text{vev} + (\sqrt{Z_H}) H + i(\sqrt{Z_b}) b \right)$$

which gives us,

$$\begin{aligned} \mathcal{L}_{\text{LSM}} = & + (Z_b) \partial_\mu b \partial^\mu b + (Z_H) \partial_\mu H \partial^\mu H - (Z m_H^2 Z_H Z_{\text{vev}}^2) \frac{1}{4} g_4 \text{vev}^2 H^2 \\ & - (Z_{bbbb} Z_b^2) \frac{1}{16} g_4 b^4 - (Z_{bbHH} Z_b Z_H) \frac{1}{8} g_4 b^2 H^2 - (Z_{HHHH} Z_H^2) \frac{1}{16} g_4 H^4 \\ & - (Z_{bbH} Z_b \sqrt{Z_H} Z_{\text{vev}}) \frac{1}{4} g_4 \text{vev} b^2 H - (Z_{HHH} Z_H \sqrt{Z_H} Z_{\text{vev}}) \frac{1}{4} g_4 \text{vev} H^3 \\ & - (Z_{\text{vev}}^2 - 1) \left((Z_H) \frac{1}{8} g_4 \text{vev}^2 H^2 + (Z_{\text{vev}} \sqrt{Z_H}) \frac{1}{4} g_4 \text{vev}^3 H + (Z_b) \frac{1}{8} g_4 \text{vev}^2 b^2 \right) \\ & - ((Z_{\text{vev}}^2 - 1) \text{vev}^2) \frac{2 g_4}{16} \end{aligned} \quad (6)$$

where the classical values of the masses is given by,

$$m_H = \frac{\sqrt{g_4} \text{vev}}{\sqrt{2}} \quad (7)$$

$$m_b = 0 \quad (8)$$

2.2 Answers

$$\delta_{\text{vev}} = \frac{1}{\epsilon} \frac{3g_4}{32\pi^2} \quad (9)$$

$$\delta H = 0 \quad \delta m_H = \frac{1}{\epsilon} \frac{g_4}{16\pi^2} \quad (10)$$

$$\delta b = 0 \quad \delta m_b = 0 \quad (11)$$

$$\delta_{HHH} = \frac{1}{\epsilon} \frac{7g_4}{32\pi^2} \quad \delta_{bbH} = \frac{1}{\epsilon} \frac{7g_4}{32\pi^2} \quad (12)$$

$$\delta_{HHHH} = \frac{1}{\epsilon} \frac{5g_4}{16\pi^2} \quad \delta_{bbbb} = \frac{1}{\epsilon} \frac{5g_4}{16\pi^2} \quad (13)$$

$$\delta_{bbHH} = \frac{1}{\epsilon} \frac{5g_4}{16\pi^2} \quad (14)$$

2.3 Consistency checks

$$\delta_{HHHH} = \delta_{bbbb} = \delta_{bbHH} = \delta g_4 \quad (15)$$

$$\delta m_H = \frac{1}{2} \delta g_4 - \delta_{\text{vev}} \quad (16)$$

$$\delta_{HHH} = \delta_{bbH} = \delta g_4 - \delta_{\text{vev}} \quad (17)$$

3 Yukawa

3.1 Lagrangian

$$\begin{aligned}
\mathcal{L}_Y = & +(Z_P - 1) i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - (Z_P Z m_P - 1) m_P \bar{\Psi} \Psi \\
& + (Z_P) \frac{1}{2} (\partial \phi)^2 - (Z_P Z m_P^2) \frac{1}{2} m_P^2 \phi^2 - (Z_P^2 Z g_3) \frac{1}{3!} g_3 \phi^3 - (Z_P^2 Z g_4) \frac{1}{4!} g_4 \phi^4 \\
& - (Z_F \sqrt{Z_P} Z_{FFP}) y \phi \bar{\Psi} \Psi - (Z_F \sqrt{Z_P} Z_{FG5FP}) i y_5 \phi \bar{\Psi} \gamma^5 \Psi \\
& - (\delta_{\text{vev}}) m_P^2 \phi - \frac{1}{2} (\delta_{\text{vev}}) g_3 \phi^2 - \frac{1}{3!} (\delta_{\text{vev}}) g_4 \phi^3 - (\delta_{\text{vev}}) y \bar{\Psi} \Psi - (\delta_{\text{vev}}) i y_5 \bar{\Psi} \gamma^5 \Psi \\
& - (\delta_\theta) i m_P \bar{\Psi} \gamma^5 \Psi - (\delta_\theta) i y \phi \bar{\Psi} \gamma^5 \Psi + (\delta_\theta) y_5 \phi \bar{\Psi} \Psi
\end{aligned} \tag{18}$$

where,

$$\begin{aligned}
\phi & \longrightarrow \delta_{\text{vev}} + \phi \\
\psi & \longrightarrow e^{i\gamma^5 \theta/2} \psi
\end{aligned}$$

3.2 Answers

$$\delta_{\text{vev}} = \frac{1}{\epsilon} \frac{(g_3 m_P^2 - 8 y m_F^3)}{16 \pi^2 m_P^2} \tag{19}$$

$$\delta P = -\frac{1}{\epsilon} \frac{(y^2 + y_5^2)}{4 \pi^2} \tag{20}$$

$$\delta m_P = \frac{1}{\epsilon} \frac{(8 g_3 y m_F^3 + m_P^2 (g_4 m_P^2 + 4 y^2 (m_P^2 - 6 m_F^2) + y_5^2 (4 m_P^2 - 8 m_F^2)))}{32 \pi^2 m_P^4} \tag{21}$$

$$\delta F = -\frac{1}{\epsilon} \frac{(y^2 + y_5^2)}{16 \pi^2} \tag{22}$$

$$\delta m_F = \frac{1}{\epsilon} \frac{(-g_3 y m_P^2 + y^2 (8 m_F^3 + 3 m_F m_P^2) - y_5^2 m_F m_P^2)}{16 \pi^2 m_F m_P^2} \tag{23}$$

$$\delta_\theta = \frac{1}{\epsilon} \frac{y_5 (4 y m_F (2 m_F^2 + m_P^2) - g_3 m_P^2)}{16 \pi^2 m_F m_P^2} \tag{24}$$

$$\delta g_3 = \frac{1}{\epsilon} \frac{(g_4 (g_3 m_P^2 + 4 y m_F^3) + 3 m_P^2 (y^2 + y_5^2) (g_3 - 8 y m_F))}{8 \pi^2 g_3 m_P^2} \tag{25}$$

$$\delta y = \frac{1}{\epsilon} \frac{(-g_3 y_5^2 m_P^2 + 5 y^3 m_F m_P^2 + y y_5^2 m_F (8 m_F^2 + 9 m_P^2))}{16 \pi^2 y m_F m_P^2} \tag{26}$$

$$\delta y_5 = \frac{1}{\epsilon} \frac{(g_3 y m_P^2 + y^2 (m_F m_P^2 - 8 m_F^3) + 5 y_5^2 m_F m_P^2)}{16 \pi^2 m_F m_P^2} \tag{27}$$

$$\delta g_4 = \frac{1}{\epsilon} \frac{(3 g_4^2 + 8 g_4 (y^2 + y_5^2) - 48 (y^2 + y_5^2)^2)}{16 \pi^2 g_4} \tag{28}$$

4 Scalar QED

4.1 Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{SQED}} = & -(Z_A) \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (Z_P) \partial_\mu \bar{\Phi} \partial^\mu \Phi - (Z_P Z m_P^2) m_P^2 \bar{\Phi} \Phi - (Z_P^2 Z_{PPPP}) \frac{g_4}{4} (\bar{\Phi} \Phi)^2 \\ & + (Z_{APP} Z_P \sqrt{Z_A}) i e (-\bar{\Phi} \partial_\mu \Phi A^\mu + \Phi \partial_\mu \bar{\Phi} A^\mu) + (Z_{AAPP} Z_P Z_A) e^2 \bar{\Phi} \Phi A_\mu A^\mu \end{aligned} \quad (29)$$

4.2 Answers

$$\delta P = -\frac{1}{\epsilon} \frac{e^2 (\xi_A - 3)}{8\pi^2} \quad \delta m_P = \frac{1}{\epsilon} \frac{(g_4 - 3e^2)}{16\pi^2} \quad (30)$$

$$\delta A = -\frac{1}{\epsilon} \frac{e^2}{24\pi^2} \quad (31)$$

$$\delta_{APP} = \frac{1}{\epsilon} \frac{e^2}{48\pi^2} \quad (32)$$

$$\delta_{PPPP} = \frac{1}{\epsilon} \frac{(24e^4 - 12e^2 g_4 + 5g_4^2)}{16\pi^2 g_4} \quad \delta_{AAPP} = \frac{1}{\epsilon} \frac{e^2}{24\pi^2} \quad (33)$$

4.3 Implications of BRST

1. $\delta_{APP} + \frac{1}{2} \delta A = 0$
2. $\delta_{AAPP} + \delta A = 0$