

# COMS W4701: Artificial Intelligence

## Lecture 6a: RL Generalization and Applications

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# Today

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- Function approximation
- Stochastic gradient descent
- Reinforcement learning applications

# Function Approximation

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- In RL, we are trying to learn *functions* over states or state-actions:  $V, Q, \pi$
- So far these have specified values for *every* individual state!
- Not possible in problems with too many states or continuous states
- We need to do *function approximation*—instead of learning  $V^\pi(s)$ , we learn a (smaller) set of weights  $\mathbf{w}$  describing a function  $\hat{V}(s, \mathbf{w})$
- Like the evaluation functions used in game trees,  $\hat{V}$  may be a linear feature combination, neural network, decision tree, etc.

# Objective Functions

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- Suppose we've chosen a function parameterization for  $\hat{V}$  with weights  $\mathbf{w}$
- How do we *update*  $\mathbf{w}$  given a sample transition sequence?

- Specify an underlying *objective*, e.g., minimizing *squared error* in  $\hat{V}$ :

$$L(\mathbf{w}) = \frac{1}{2} \left( V^\pi(s) - \hat{V}(s, \mathbf{w}) \right)^2$$

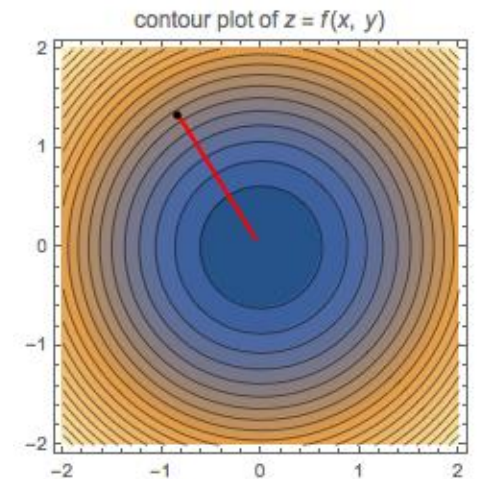
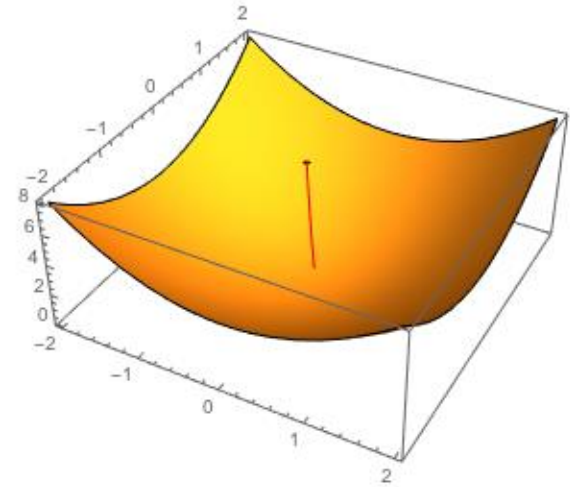
- Ideally we can find a global optimum  $\mathbf{w}^*$  that minimizes  $L$ , but many problems will have many local optima as well

# Gradient Descent

- Suppose that  $L$  is a differentiable function of the weight vector  $\mathbf{w}$
- The **gradient** of a multivariate function  $L(w_1, \dots, w_n)$  is a *vector* of partial derivatives wrt all  $w_i$

$$\frac{\partial L}{\partial \mathbf{w}} = \left[ \frac{\partial L}{\partial w_1} \quad \dots \quad \frac{\partial L}{\partial w_n} \right]$$

- Indicates magnitude and *direction* of largest change in  $L$
- The value of  $L$  *decreases* fastest along direction of  $-\frac{\partial L}{\partial \mathbf{w}}$
- **Gradient descent:** Initialize the configuration  $\mathbf{w}$ , and repeatedly update it as  $\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial L}{\partial \mathbf{w}}$  until convergence

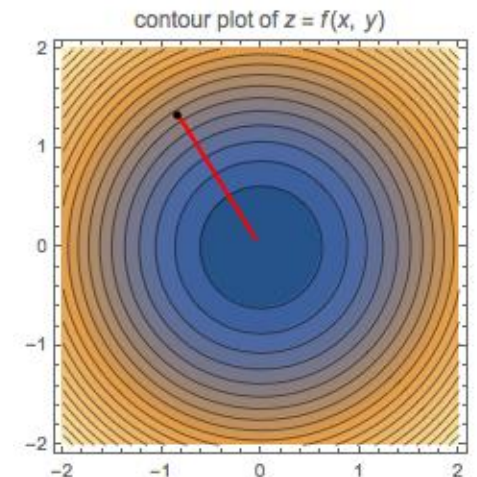
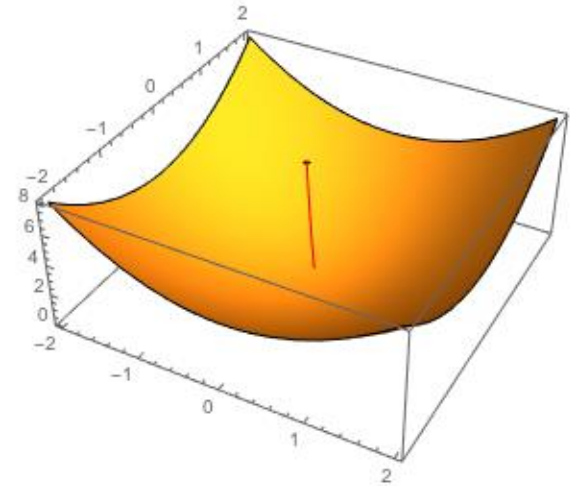


# Gradient Descent

- We are *searching* for a weight vector  $\mathbf{w}$  that minimizes  $L$
- The **gradient** of  $L$  is a *vector* of partial derivatives wrt all  $w_j$

$$\frac{\partial L}{\partial \mathbf{w}} = \left( \frac{\partial L}{\partial w_0}, \frac{\partial L}{\partial w_1}, \dots, \frac{\partial L}{\partial w_p} \right)$$

- Indicates magnitude and *direction* of largest increase in  $L$
- The value of  $L$  *decreases* fastest along direction of  $-\frac{\partial L}{\partial \mathbf{w}}$
- **Gradient descent:** Initialize the configuration  $\mathbf{w}$ , and repeatedly update it as  $\mathbf{w} \leftarrow \mathbf{w} - \alpha \frac{\partial L}{\partial \mathbf{w}}$  until convergence



# Updating the Weights

- The gradient descent update for the mean squared error of  $\hat{V}$  is thus

$$\begin{aligned}\mathbf{w}_{t+1} &= \mathbf{w}_t - \alpha \frac{\partial}{\partial \mathbf{w}_t} \frac{1}{2} \left( V^\pi(s_t) - \hat{V}(s_t, \mathbf{w}_t) \right)^2 \\ &= \mathbf{w}_t + \alpha \left( V^\pi(s_t) - \hat{V}(s_t, \mathbf{w}_t) \right) \frac{\partial \hat{V}(s_t, \mathbf{w}_t)}{\partial \mathbf{w}_t}\end{aligned}$$

- Ex: Linear combination of features

$$\hat{V}(s, \mathbf{w}) = w_1 x_1(s) + w_2 x_2(s) + \dots = \sum_i w_i x_i(s) = \mathbf{w}^\top \mathbf{x}(s)$$

- Gradient is simply  $\frac{\partial \hat{V}(s_t, \mathbf{w}_t)}{\partial \mathbf{w}_t} = (x_1, x_2, \dots, x_n) = \mathbf{x}(s)$

# Stochastic Gradient Descent

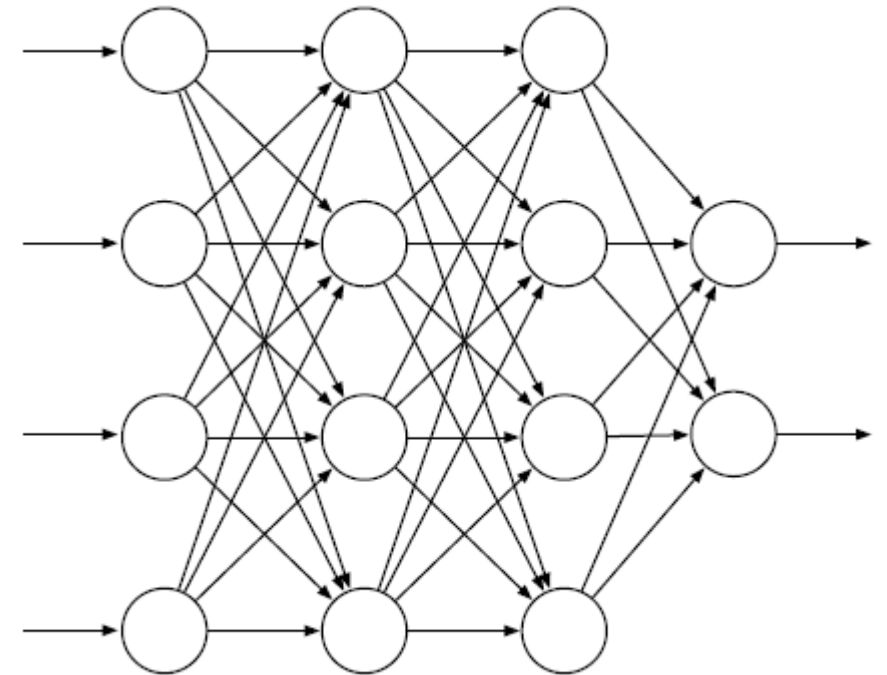
$$\mathbf{w}_{t+1} = \mathbf{w}_t + \alpha \left( V^\pi(s_t) - \hat{V}(s_t, \mathbf{w}_t) \right) \frac{\partial \hat{V}(s_t, \mathbf{w}_t)}{\partial \mathbf{w}_t}$$

- Last issue: We don't know the “true” or *target* value  $V^\pi(s_t)$
- **Stochastic gradient descent:** *Approximate* the gradient using samples, just like in classical RL
- Gradient MC prediction:  $\mathbf{w}_{t+1} = \mathbf{w}_t + \alpha \left( G_t - \hat{V}(s_t, \mathbf{w}_t) \right) \frac{\partial \hat{V}(s_t, \mathbf{w}_t)}{\partial \mathbf{w}_t}$
- Semi-gradient TD(0):  $\mathbf{w}_{t+1} = \mathbf{w}_t + \alpha \left( r + \gamma \hat{V}(s_{t+1}, \mathbf{w}_t) - \hat{V}(s_t, \mathbf{w}_t) \right) \frac{\partial \hat{V}(s_t, \mathbf{w}_t)}{\partial \mathbf{w}_t}$



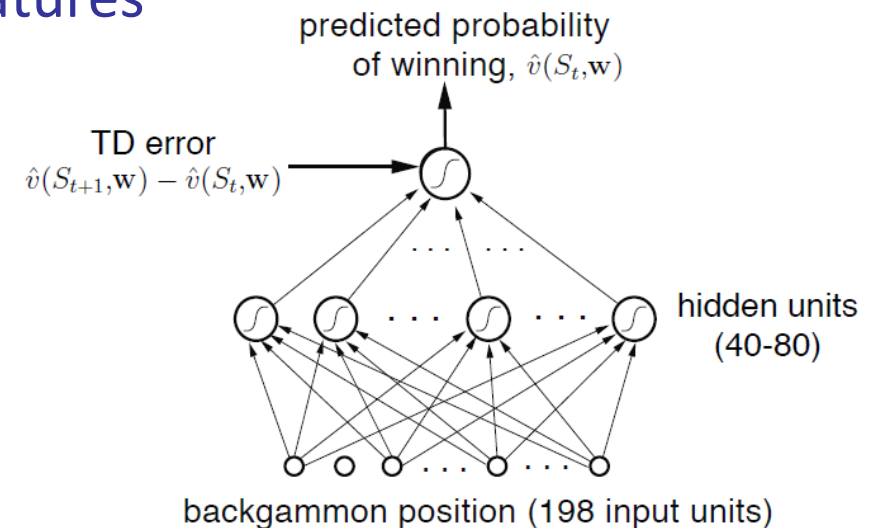
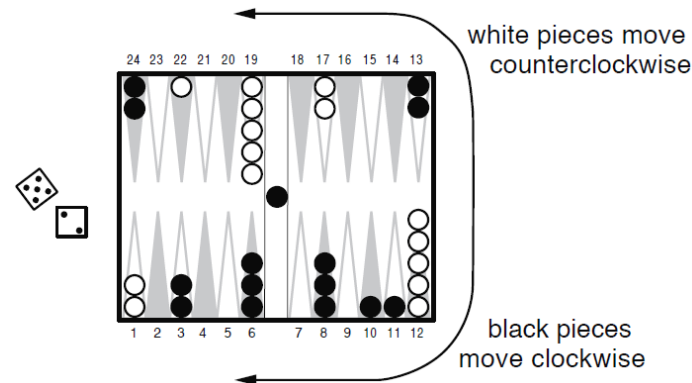
# Neural Networks

- **Neural networks** are function approximators composed of *layers of neurons*
- Each neuron applies a nonlinear *activation function* to a weighted sum of inputs
- Hidden (internal) layers are “feature” transformations of raw inputs
- More neurons yield greater representative power, but at the expense of training efficiency
- Goal: Learn the weights of the neuron connections
- Most methods like *backpropagation* run some kind of stochastic gradient descent



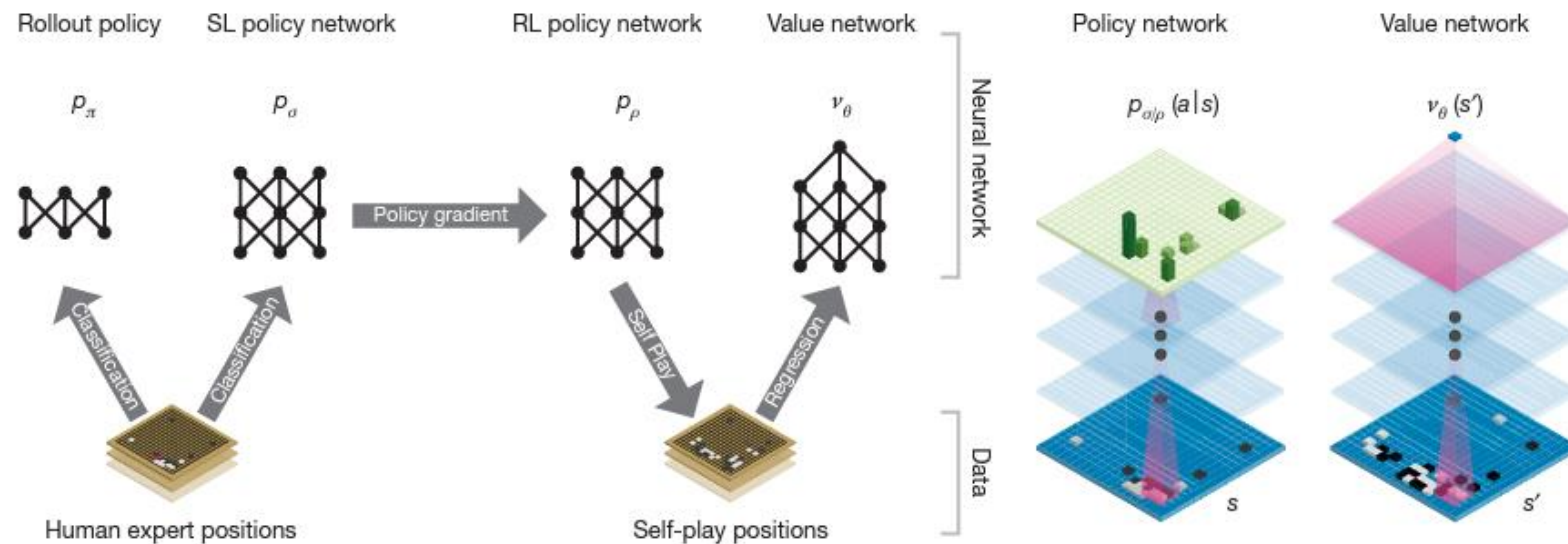
# TD-Gammon

- Backgammon: High branching factor ( $\sim 400$ ), highly stochastic game
- Can approach using depth-limited tree search with evaluation function
- TD-Gammon (Tesauro, 1992) *approximated* the eval function using a neural net
- Weights were learned via semi-gradient TD learning on self-play
- Incorporated human expert data for hand-designed features



# AlphaGo

- Go has much larger game tree than chess, no high-performance eval function
- AlphaGo used MCTS with rollout and selection policies trained on human expert data
- SL network improved using RL, then evaluated using self play to obtain value network
- MCTS values updated using combination of rollout result and value network output

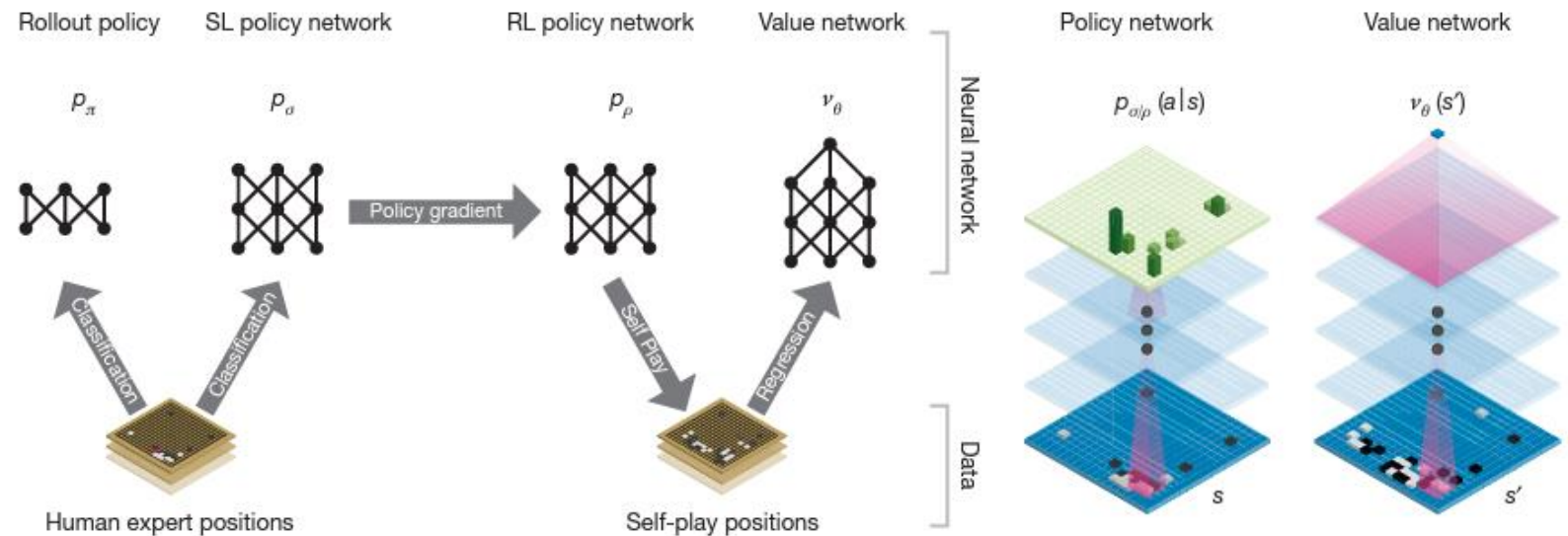


<https://deepmind.google/technologies/alphago/>

# AlphaGo

- AlphaGo used a combination of reinforcement learning with MCTS to master Go
- Human expert data was first used to train a *policy network* (board state  $\rightarrow$  action) using supervised learning (classification)
- Policy network was significantly improved via RL and self-play

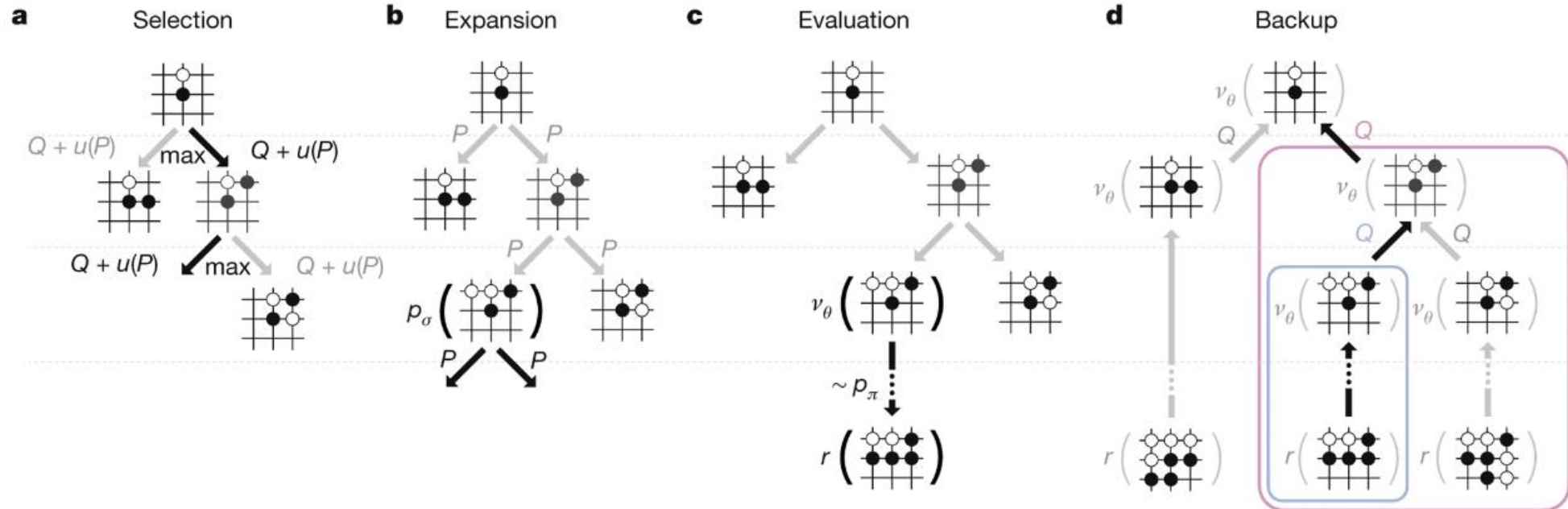
- Policy network was then used to generate *simulated* data from self-play
- Finally, regression was used to train a *value network* (board  $\rightarrow$  value)



<https://deepmind.google/technologies/alphago/>

# AlphaGo

- During actual gameplay, AlphaGo ran MCTS using its hard-trained network functions
- Node values are weighted averages of value network outputs and simulated values
- Policy network is used for encouraging exploration during the selection step as well as the rollout steps during simulation (evaluation)



# Deep Q-Network

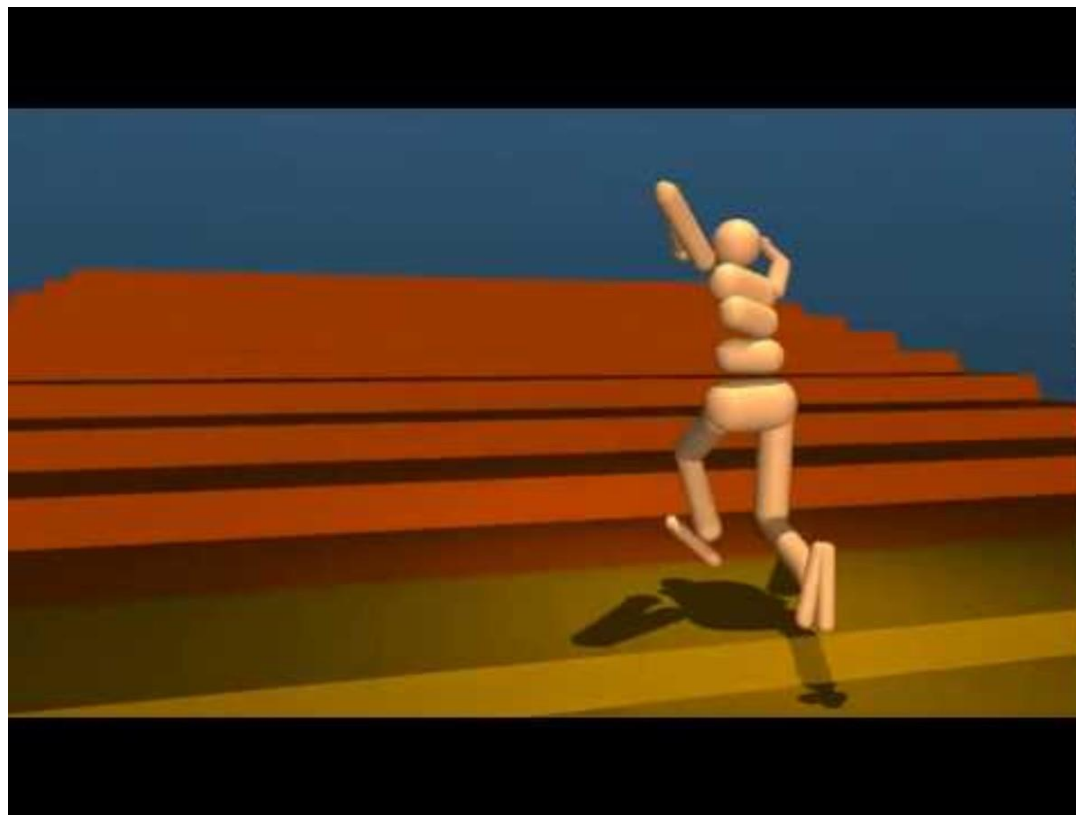
- Combination of Q-learning with deep convolutional neural networks (CNNs)
- Raw inputs in the form of video game streams: Automated feature design
- DQN achieved human level play on a few dozen different arcade video games
- Learned different policies to cope with different dynamics and reward structures
- No game-specific modifications!



<https://deepmind.com/blog/article/deep-reinforcement-learning>

# RL for Locomotion

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[https://www.youtube.com/watch?v=hx\\_bgoTF7bs](https://www.youtube.com/watch?v=hx_bgoTF7bs)



# Multi-Agent RL



<https://openai.com/blog/emergent-tool-use/>



# Summary

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- Function approximations are important for generalization when tabular representations are not feasible
- Learning is done via different forms of gradient descent
- Many modern applications of RL using deep neural networks