Variance Risk Premia in the Interest Rate Swap market

Desi Volker *

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Abstract

In this paper I analyze the time series and cross-sectional properties of variance risk premia in the interest rate swap market. The results presented show that the term structure of variance risk premia displays non-negligible differences in a low interest rate environment, compared to normal times. Variance risk premia have on average been negative and economically significant during the sample. In a low interest rate environment, the variance risk premium tends to display more frequent episodes where it switches sign.

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* *Contact*: Department of Finance, Copenhagen Business School, Solbjerg Plads 3, A5, 2000 Frederiksberg, dv.fi@cbs.dk. I am particularly grateful to Lasse H. Pedersen, Carsten Sørensen and seminar participants at the Nordic Finance Network meeting in Stockholm 2014 and lunch seminars at the Finance Department, Copenhagen Business School.

1 Introduction

Swaption implied volatilities reflect the market participants expectations of future realized volatility, adjusted by a premium to compensate them for the risk associated with the fact that interest rate volatility is stochastic. The variance risk premium is time-varying and economically significant, and tends to rise in absolute terms, in periods of market turmoil, where uncertainty about the economy and/or investor risk aversion is high. Given the current and protracted low interest rate environment it is important to study whether the properties of variance risk premia embedded in swaptions, differ in a low rate regime compared to normal times. In this paper I analyze the historical behavior of the variance risk premium embedded in interest rate swaptions and assess its characteristics during periods of low and high/normal interest rate levels.

In loose terms the variance risk premium reflects the amount investors are willing to pay during normal times in order to insure against high realized interest rate volatility during periods of market turmoil, while from the option sellers' perspective, it reflects the compensation demanded for taking the risk of incuring significant losses in periods when realized volatility increases significantly and unexpectedly. [1] The variance risk premium will be affected by imbalances in the supply and demand for swaptions, the price associated to event risk, liquidity risk, credit risk, etc. Variance risk premia depend on the whole distribution of the underlying asset returns, that is, not just on the interest rate level and volatility, but also on the higher moments. Bakshi and Madan (2006) for example, show that under a number of assumptions, the variance risk premium is a function of the skewness and kurtosis of the underlying assets' returns. When interest rates are low and close to the zero lower bound, the distribution of interest rates is more closely approximated by a lognormal distribution, with a fat right tail, since the probability associated with an increase in interest rates is higher than it would be under the normal (assuming non-negative rates). Given that short-term nominal interest rates in large part of the developed world are at or near zero and there are no prospects for the situation to change in the near future given inflation

[1] However it is important to mention that the results presented in this paper do not correspond to the returns on tradable strategies to exploit the premium, such as at the money straddles or variance swaps.

expectations are revised downwards, it is relevant to study how this affects variance risk premia in fixed-income markets.

In analogy with the equity literature, I define the variance risk premium as the difference between expected realized future variances and risk neutral variances. Both of these components are not directly observable and a number of approaches are available for measuring them. One approach is to rely on sophisticated dynamic option-pricing models. Alternatively, one can take a model-free approach and measure realized volatilities from high frequency data as proposed by Andersen, Bollerslev, Diebold, and Ebens (2001); Barndorff-Nielsen (2002) and risk neutral volatilities using a panel of option prices as proposed by Britten-Jones and Neuberger (2000). Lastly, one can use simple parametric models. I follow (Fornari, 2010) and use Black-implied ATM swaption volatilities as measures of the unobservable risk neutral volatilities and volatility forecasts based on an asymmetric GARCH model (and conditioning for the information available at each point in time), as a measure of the unobservable expected realized volatilities. The variance risk premia obtained span various terms, going from three to twenty-four months and tenors going from two to ten years.

I analyze the time-series and cross-sectional properties of variance risk premia, over the full sample, as well as on periods of high/low interest rates and document the following results. Firstly, as expected, variance risk premia have been negative and economically significant during the full sample and on all subsamples. This suggests that volatility risk in the interest rate swap market has been largely priced. There have been however brief periods where variance risk premia have switched sign. These short lived, but consequential episodes reflect periods in which unexpected realized volatility shocks have occurred. Variance risk premia display a high co-movement across terms and tenors and generally tend to spike and fall abruptly in unison, however there are important exceptions. Most of the spikes and abrupt falls coincide with important events and crisis episodes in financial markets and the overall economy. Variance risk premia are increasing in tenor, that is the variance risk premium of shorter tenors is more negative than that of longer tenors. Along the term dimension, the term structure of variance risk premia is increasing with the term. Variance risk premia are quite persistent and the persistence increases with the term. Looking at episodes where the variance risk premium spikes or falls abruptly, one observes that the slope of the term structure in the term dimension switches it's sign. Secondly, the main determinants of the time-variation in variance risk premia are, as expected, the interest level and past volatility, which explain most of its variation. In particular an increase in both the short rate and realized volatilities is associated with an increase in the variance risk premium on the full sample. Other measures, such as the interest rate slope, the slope of the volatility curve, swap spreads, credit spreads and the stock market volatility index are significant predictors. An increase in the interest rate slope, the slope of the volatility curve or the swap spreads, is associated with an increase in the variance risk premium. The VIX and corporate credit spread have the opposite effect, reflecting the fact that equity and fixed income volatility risks are distinct and therefore display time-varying correlations. Lastly, the variance risk premium process displays structural breaks dividing the data into periods belonging to one of two distinctive regimes. The first, with high (negative) level and high dispersion, corresponding to periods where the interest rate level is relatively low, and the second, with a nearly zero level and small dispersion, corresponding to periods where the level of interest rates is high. In the low interest rate subsample, the term structure of variance risk premia across the tenor dimension is upward sloping. While in the high interest rate subsample, it is downward sloping, with the variance risk premium on shorter tenors being less negative than on longer ones. In the low interest rate regime, a change in the short rate will have differential effects on variance risk premia across tenors. In particular an increase in the short rate is associated with an increase in the the variance risk premium of the 2 year tenor and a decrease in that of the 10 year tenor, with the overall effect of flattening the term structure of variance risk premia across the tenor dimension in the period when the later has a high slope. In the high interest rate regime, an increase in the short rate increases the variance risk premium across all tenors for a given term. Similar results are found for realized volatility.

The paper is related to a small but increasing literature dealing with the measurement and analysis of variance risk premia in fixed income markets. [2] The most closely related paper is

[2]More broadly the paper is related to the large literature on equity variance risk premium literature

Fornari (2010), which analyzes the compensation for volatility risk across different countries. The sample considered however goes from 1997 to 2006 and therefore does not capture the financial crisis period and the near zero interest rate period that has prevailed since then. Other related papers are Choi, Mueller, and Vedolin (2015) and Mueller, Vedolin, and Zhou (2011) which use options on bond futures to construct model-free expected realized volatility measures and variance risk premia and exploit the information in the latter to forecast real activity and term premia. Mele and Obayashi (2013) use a similar methodology to construct a treasury implied volatility index. Mele, Obayashi, and Shalen (2015) analyze the relation between the VIX and the SRVX, the swap rate volatility index and find significant differences in their behavior, especially during periods of distress in bond markets.

The remainder of the paper is organized as follows. Section 2 discusses the methodology used for the measurement of the unobservable variance risk premia. Section 3 analyzes the time series and cross-sectional properties of variance risk premia over the full sample and across subsamples corresponding to periods of high and low interest rates. Section 4 provides predictive regression results for variance risk premia on a set of likely predictors and Section 5 concludes.

2 Constructing Variance Risk Premia

The variance risk premium is defined as the difference between (squared) expected future realized volatilities and (squared) implied volatilities for a given tenor τ and term h:

$$VRP_t^{\tau,h} = E_t^{\mathbb{P}}\Big(\int_t^{t+\tau} (\sigma_s^h)^2\, ds\Big) - E_t^{\mathbb{Q}}\Big(\int_t^{t+\tau} (\sigma_s^h)^2\, ds\Big)$$

For expected volatilities under the risk neutral measure, I use at-the-money swaption implied volatilities with tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months. The data is taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016. To estimate expectations of realized volatility under the

(Carr and Wu, 2009; Chernov, 2007; Trolle and Schwartz, 2009, 2014; Bollerslev, Gibson, and Zhou, 2011; Dew-Becker, Giglio, Le, and Rodriguez, 2015) physical measure one has to rely in a particular model-free methodology (Bollerslev, Gibson, and Zhou, 2011; Choi, Mueller, and Vedolin, 2015), in an option pricing model (Chernov, 2007; Trolle and Schwartz, 2009, 2014), or in forecasts based on parameter estimates of an assumed model for the historical volatility process (Fornari, 2010). In this paper I follow the latter option and use the filtered historical simulation approach developed by Barone-Adesi, Engle, and Mancini (2008) and used in Fornari (2010). Specifically the methodology works as follows. I model realized volatilities as a GARCH process and produce forecasts of realized volatility based on the model's parameter estimates conditioning on the information available at each point in time. I assume that the historical volatility process for daily log swap returns follows an assymetric GARCH(1,1) model:

$$
\begin{aligned}
r_t &= \alpha_0 + \alpha_1 \, r_{t-1} + \sigma_t \, \nu_t; \qquad \nu_t \sim i.i.d.\ \mathcal{N}(0,1) \\
\sigma_t^2 &= \beta_0 + \beta_1 \, \sigma_{t-1}^2 + \beta_2 \, \epsilon_{t-1}^2 + \beta_3 \, \max(0, -\epsilon_{t-1})^2 \\
\epsilon_t &= \sigma_t \, \nu_t | I_{t-1} \sim \mathcal{N}(0, \sigma_t^2)
\end{aligned}
$$

with r_t denoting daily logarithmic swap returns with a given maturity τ ($r_t = log\Big(sr_{t+1}^{\tau} / sr_t^{\tau})$), σ_t denotes the conditional volatility for the swap returns, I_{t-1} denotes the information set available at time t and ϵ_t denote the forecast errors. Since when simulating future paths of conditional volatilities, ν_t is bootstrapped from the set of realized forecast errors, the forecasts of future realized volatilities will not suffer from model specification due to the specific choice of the model within the ARCH family. Using daily swap rate data that start a decade earlier to the desired starting date for the expected realized volatilities, I estimate a GARCH(1,1) model on expanding windows. At each day t, I use the parameter estimates, filtered volatilities and bootstrapped standardized forecast errors from the model estimated on a sample that end at t, to forecast future realized volatilities. In this way expectations are formed conditioning on the information available at each point in time. In particular, for each day t and each swap rate, I simulate 10000 paths of future volatility over different horizons (corresponding to the terms of the ATM swaption implied volatilities employed above) and then take the averages over the options terms and the simulations as the measure of the expected volatility for a particular tenor and term. The series are then annualized to be made comparable to the swaption implied volatilities, and variance risk premia are computed as the difference between the squared expected realized volatility and squared implied volatility for a particular tenor and term. The variance risk premia, $VRP_t^{\tau,h}$, obtained from the procedure have a daily frequency spanning the period June 2, 1997 to o May 19, 2016 and include tenors of 2, 5, and 10 years and terms of 3, 6, 12 and 24 months.

3 The Time Series of Swaption Implied Volatilities and Variance Risk Pemia

Figure 1 displays swaption implied volatilities for tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months along with the one year Treasury rate, while Table **I** reports their summary statistics. The figures are in percent, annualized.

[Insert Figure 1 and Table **I** here.]

The first feature of the data shown in the figure is that there is significant time variation in swaption implied volatilities and a high correlation across the different tenors and terms. Secondly the series appears to display two different regimes, one with a low level and low dispersion, present between 1997 and 2001 and between 2005 and 2008, and one with a high level and dispersion, present between 2001 and 2005 and after 2008. The first, corresponds to a period where the interest rate level is relatively high (given the sample) and one where interest rates are low. During the high interest rate regime, the cross-section across tenors for a given term is almost flat, while in the low interest rate regime it appears to be significantly downward sloping. All series peak during the financial crisis of 2008.

[Insert Figure 2 here.]

Figure 2 shows the cross section of swaption implied volatilities for the overall sample and extending the tenors to include 1, 2, 3, 4, 5 and 10 years, and the terms to include 6, 12, 60 and 120 months. Again the swaption implied volatilities are downward sloping in the tenor dimension and decreasing with the term. The steepness of the slope decreases with the term, with longer terms of 60 and 120 months having an almost flat curve.

Figure 3 plots swaption implied volatilities $E^{\mathbb{Q}}[\sigma_t^{\tau,h}]$ and expected realized volatility forecasts $E^{\mathbb{P}}[\sigma_t^{\tau,h}]$ along with their 95% confidence bounds, for the 2 years tenor, and terms going from 3m to 24 months. The expected realized volatility forecasts and their confidence bounds are computed at each point in time from simulations based on the methodology described above. The figures are in percent and annualized.

[Insert Figure 3 here.]

Overall, expected realized volatilities have been lower than risk neutral volatilities, with periods where they overlap corresponding to a high interest rate level, and periods where the gap widens significantly, in 2003 and after 2009. There are however, brief and sudden periods where expected realized volatilities have surpassed risk-neutral volatilities. For longer terms (and tenors, not reported here for brevity) swaption implied volatilities lie well within the 95% confidence bounds. For the shorter tenor and term however, there are brief periods, occuring in the low interest rate regime, where the swaption implied volatility lies beyond the upper bound. This can be explained by the fact that the data displays discernible breaks, with periods of distinct volatility levels and dispersion, while the simulations were based on parameters and forecast errors from a GARCH(1,1) model estimated on the historical data up to that point in time. This suggests that the large spikes in implied volatility were largely unexpected.

Figure 4 plots the variance risk premia computed as the difference between squared expected realized volatilities and squared risk neutral volatilities for different tenors and terms.

[Insert Figure 4 here.]

As already glimpsed in Figure 3, the plots in Figure 4 confirm that variance risk premia as defined here have been negative on average, implying a negative premium on average for the investor who hedges against volatility risk and a positive compensation on average for the option seller. There are brief periods, especially after 2008 where the variance risk premia has switched sign, implying that interest rate option sellers, have incured on average higher and more frequent losses after 2008, and have demanded a higher compensation to account for the increased risk. Option sellers were therefore exposed to sharp losses for brief periods, but these periods were followed with a quick recovery, since risk neutral volatilities reacted more sharply to the shocks than expected realized volatility and remained higher for next few periods, reflecting heightened risk aversion. Variance risk premia are increasing with the tenor and with term, with shorter tenor and terms displaying also more pronounced and more frequent spikes (both positive and negative). Variance risk premia are quite persistent (see Figure 5) and the persistence increases for longer tenor and terms.

Figure 6 displays minus the variance risk premia series for the 5 year tenor and 3 month term, $VRP_t^{5y,3m}$, and the stock market volatility index, VIX.

[Insert Figure 5 and Figure 6 here.]

The two series have a correlation of -40% and seem to folllow similar overall trends, with spikes in crisis periods, such as the Russian debt and currency crisis of 1998 and the global financial crisis of 2008.

A first glance at the time series of variance risk premia suggest there might be changes in the data generating process in periods where interest rates move from a high to a low level and vice-versa. it is important therefore to test more formally for the existence of potential structural breaks. Since there is a suspicion of multiple breaks in the data and I do not want to take a stance on which particular dates the structural breaks occur, I use the methods developed and applied in Bai and Perron (1998, 2003a,b). The authors devise various tests to not only determine the presence of structural change but also the number of breaks and their location along with confidence bounds. Figure 7 plots swaption implied volatilities and variance risk premia along with the structural break points determined by the tests.

[Insert Figure 7 here.]

Confirming the original suspicion, the tests find three major structural breaks, the first one corresponding to the 9/11 attacks, the second in the end of 2004 and the third to the beginning of the 2008 financial crisis. A fourth break is found for the swaption implied volatilities in the beginning of 2012, however it is not present for the variance risk premia series. Dividing the data according to the structural breaks, one obtains fundamentally two regimes for variance risk premia, one with an almost zero level and very low dispersion, corresponding to a high interest rate environment, and one with a high (negative) level and high dispersion, corresponding to a low interest rate environment. These two regimes present interesting differences in the cross-section of variance risk premia in the tenor dimension, as well as in the term structure of variance risk premia (see Figure 8 and Figure 9). Specifically in the high interest rate regime, variance risk premia are slightly decreasing and close to flat in the tenor dimension with very small differences between he 2 year, 5 year and 10 year tenors for a given term, while in the low interest rate subsample, variance risk premia are strictly and significantly increasing with the tenor. Looking at the term structure of variance risk premia, they are slightly increasing and almost flat in both regimes for longer tenors. For the shortest tenor however, we observe differences in the two regimes, with variance risk premia being linearly increasing with the term (i.e. the variance risk premium with a 3 month term is more negative than that with a 24 month term) for the high interest rate subsample, and it is hump-shaped and increasing for the low interest rate subsample.

[Insert Figure 8 and Figure 9 here.]

Performing the tests for variance risk premia of various tenors and terms one finds largely similar results, however for longer tenors and terms the level and dispersion in the variance risk premia on the low interest rate regime decreases substantially. In order to see this more clearly, Figure 10 depicts principal components of changes in variance risk premia across tenors, for the terms 3, 6, 12 and 24 months.

[Insert Figure 10 and Figure 11 here.]

It is evident form the figure that the series display very similar patterns, with almost the same spikes and drops. Figure 11 relates the most significant and largest changes in variance risk premia to major financial and economic events, such as the Russian debt crisis, the collapse of Bear Sterns and Lehman Brothers, S&P's downgrade of the US credit rating to AA+, etc.

Figure 12 displays the term structure of variance risk premia (i.e. in the term dimension) in the dates where the variance risk premium for the 2 year tenor and 3 month term, $VRP_t^{2y,3m}$, takes its highest negative values and its highest positive values.

[Insert Figure 12 here.]

In days where variance risk premium on the 2 year tenor and 3 month term, $VRP_t^{2y,3m}$, the term structure of variance risk premia changes shape and becomes downward sloping (i.e. decreasing with the term), suggesting expectations of a near term event risk. Furthermore during these days, the variance risk premia are decreasing with the tenor.

4 Predictor Variables of Variance Risk Pemia

By construction, the main determinants of variance risk premia are the interest rate level and volatility. The underlying state variables driving the interest rate level and volatility processes will also drive the time-variation in variance risk premia.

Given the tests for structural changes imply the existence of three breaks, I start by running predictive regressions of variance risk premia on the interest rate level, realized volatility, a binary variable indicating the subsamples associated with the breaks (which largely correspond to a high and a low interest rate regime) and interaction terms between the regressors and the binary variable:

$$\begin{aligned} VRP_t^{\tau,h} &= c + \beta_1 r_{t-1} + \beta_2 E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] + \beta_3 D_{t-1}^i \\ &\quad + \gamma_1 \left(r_{t-1} \times D_{t-1}^i\right) + \gamma_2 \left(E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] \times D_{t-1}^i\right) + \epsilon_t^{\tau,h} \end{aligned}$$

Since the breaks are quite distinct, inference on the regression coefficients should not be compromised. Tables **II** reports the regression results for variance risk premia with a term of 3 months and 6 months respectively and for tenors of 2, 5 and 10 years.

[Insert Table **II** here.]

In the low interest rate regime, a change in the short rate will have differential effects on variance risk premia with short versus long tenors for a given term. In particular an increase in the short rate is associated with an increase in the the variance risk premium for the 2 year tenor, has no significant effect on the 5 year tenor and decreases the variance risk premium on the 10 year tenor, with the overall effect of flattening the term structure of variance risk premia across the tenor dimension. Recall that the cross-section of variance risk premia in the tenors dimension is upward sloping and the slope is higher in the low interest rate regime. Therefore an increase in the interest rate will have the effect of reducing the slope in times where the latter is high.

In the high interest rate regime, an increase in the short rate increases the variance risk premium across different tenors, for a given term. In particular a one standard deviation increase in the short rate will increase variance risk premia by 0.1 to 0.3 standard deviations. The increase in the variance risk premium for the 10 year tenor is larger than the increase in the variance risk premium for the 2 year tenor, with the overall effect of increasing the slope of the cross-section of variance risk premia in the tenor dimension in times when the latter is less steep.

Similar results are found for realized volatility. In the low interest rate regime, an increase in the realized volatility increases the variance risk premium of the 2 year tenor, has no significant effect on the 5 year tenor and decreases the variance risk premium on the 10 year tenor, for a given term. The overall effect is again a flattening of the term structure of variance risk premia across the tenor dimension. In the high interest rate regime, an increase in variance risk premia will increase the variance risk premium of the 2 year tenor, have no significant effect on the 5 year tenor and decrease the variance risk premium on the 10 year tenor, for a given term. The magnitudes of the effects are in the order of 0.15 to 0.3 standard deviations change in variance risk premia for a one standard deviation change in realized volatility.

Next I turn to exploring, what other financial variables are able to explain the time variation in variance risk premia, controlling for the interest rate level and volatility. From an empirical standpoint, even in the absence of causality, understanding which financial and economic variables explain the time variation in variance risk premia is important for forecasting purposes.

Firstly, in order to draw reliable conclusions about the explanatory variables, it is important to examine whether variance risk premia have a unit root. The departure from stationarity due to the presence of either trends or breaks compromises statistical inference and forecasts made based on time series regressions. At first inspection the series exhibits long-run swings consistent with a process with a stochastic trend and that it might suffer from time-instability in the form of structural breaks. The sample autocorrelation function (ACF) suggests that the series is quite persistent (see Figure 5). Testing for unit root nonstationarity, the Augmented Dickey-Fuller (ADF) test rejects the unit-root null in favor of the alternative. However, a variance ratio test rejects the hypothesis that the series is a random walk, compromising the results from the Dickey-Fuller test, which applies to homoschedastic series. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test rejects the hypothesis that the variance risk premia series are trend stationary. The overall evidence about the presence of a unit root is inconclusive and it remains unclear whether the data has a unit root as it is common with many macro and financial time series.

Secondly, since the main financial variables likely to explain the time-variation in variance risk premia tend to be highly correlated with each other and with the interest level and volatility, it is important to rule out problems arising from multicollinearity. Even imperfect multicollinearity can be problematic, as it inflates the variance of the regression coefficients. This can result in parameter instability and therefore complicates the interpretation of the coefficients.

[Insert Table **III** here.]

Table **III** shows the correlation correlation matrix for the explanatory variables of interest. These include, the three month interest rate, the variance risk premium for the 2 years tenor, 3 months term, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's seasoned Baa corporate bond yield relative to the yield on 10-year treasury constant maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the Ted Spread. The data has been taken from Bloomberg and FRED (the Federal Reserve Economic Data of the St. Louis Fed), it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016. As we can see, many of the variables of interest are highly correlated with the level of interest rates.

I conduct several multicollinearity tests in order to determine whether this is an issue and single out other problematic regressors. The variance inflation factor (VIF), helps assess whether multicollinearity is problematic for a set of regressors, by evaluating the increase in the variance of an estimated regression coefficient due to the correlation among the regressors. In the case where all explanatory variables are uncorrelated, the variance inflation factor for all coefficients will be one.

[Insert Table **IV** here.]

Table IV shows the results of the test and confirms that the high correlation of the interest rate level with the other regressors has the potential to inflate the variance of the estimated coefficients. Given that the stationarity tests were inconclusive I consider an Augmented Distributed Lag (ADL) regression model both on the variance risk premium level:

$$VRP_t^{\tau,h} = c + \alpha VRP_{t-1}^{\tau,h} + \beta_1 r_{t-1} + \beta_2 E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] + \sum_{i=1}^{N} \gamma_i F_{t-1}^i + \epsilon_t^{\tau,h}$$

and on its difference (assuming the series is difference stationary):

$$\Delta VRP_t^{\tau,h} = c + \alpha VRP_{t-1}^{\tau,h} + \beta_1 r_{t-1} + \beta_2 E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] + \sum_{i=1}^{N} \gamma_i F_{t-1}^i + \epsilon_t^{\tau,h}$$

where F_{t-1}^i is a vector containing the financial variables of interest. Table V shows results of predictive regressions of the variance risk premium for the 2 year tenor and 3 month term, $VRP_t^{2y,3m}$, on these variables.

[Insert Table V here.]

Since variance risk premia are highly persistent, the lagged variance risk premium explain almost 87% of its variation, with a one standard deviation increase in $VRP_{t-1}^{2y,3m}$ being associated with a 0.93 standard deviations increase in $VRP_t^{2y,3m}$. The short term interest rate is also a significant explanatory variable for the variance risk premium, with a one standard deviation increase in the short rate being associated with a 0.8 standard deviations increase in the variance risk premium. Realized volatility is also highly significant, with a one standard deviation increase in the Overall, an increase in either of the two main determinants of variance risk premia, the interest rate level and realized volatility, is associated with an increase in the variance risk premium, implying a higher compensation demanded by option sellers for taking volatility risk, and symmetrically a higher (negative) premium that investors are willing to pay in order to insure against sudden increases in volatility.

A one standard deviation increase in the slope of the yield curve, implies a 0.47 standard deviation increase in the variance risk premium. The effect is consistent with that of the short rate, since a steeper yield curve implies rising expected future short rates. Similarly, an increase in the slope of the volatility curve, is also associated with an increase in the variance risk premium, since it implies a rise in expected future interest rate volatilities.

The 10-year swap spread is also a significant explanatory variable for variance risk premia. A one standard deviation increase in the swap spread is associated with a 0.14 standard deviation increase in the variance risk premium with a two year tenor and three month term. Since the spread between swap rates and Treasury yields largely reflects a premium demanded for liquidity and default risk, the sign of the coefficient follows economic intuition. The higher the liquidity and default risk in swap markets, the higher will be the premium that investors will have to pay in order to insure themselves against sudden rises in realized volatility.

An increase in the stock market volatility index VIX and corporate credit spread, as measured by Moody's seasoned Baa 10-year corporate bond yield relative to the yield on 10-year treasury constant maturity, is associated with a decrease in the variance risk premium. The VIX and the swaption variance risk premia have time-varying correlations, displaying both periods of divergence and co-movement (in the full sample analyzed here they are overall negatively correlated for most tenors and terms). The volatility risk and therefore the compensation demanded for this risk is different in equity and government debt markets. Mele, Obayashi, and Shalen (2015) show, for example, that the swap rate volatility index they construct on data going from 2007 to 2013, reacts in an opposite direction to the VIX in periods of distress in bond markets. This also explains the negative sign of the coefficient for the corporate bond spread.

As expected an increase in the TED spread, is associated with an increase in the variance risk premium, since it reflects the default risk in the interbank loan market. The magnitude of the coefficient is economically more modest, with a one standard deviation increase in the TED spread corresponding to a 0.08 standard deviation in $VRP_t^{2y,3m}$.

[Insert Table **VI** here.]

Table VI reports regression results of variance risk premia for various tenors and terms on the same predictors, where the data has been divided into two subsamples corresponding to the structural break points determined by the tests. The first subsample coincides with a period where interest rate are relatively high and the second with a period where interest rate are low.

The interest rate level has a positive effect on variance risk premia across all tenors and terms in both regimes. The size of the effect however is larger in the regime where interest rates are high. Similarly, an increase in realized volatility is associated with an increase in the variance risk premium in both regimes. For the longest tenors and terms however, the coefficient is insignificant.

Similarly to the level of interest rates, an increase in the slope of the yield curve has a positive effect on the variance risk premium across all tenors and terms, both when interest rates are high and when they are low. The magnitude of the effect is however significantly larger (more than twice as large) for the low interest rate period. The slope of the volatility curve retains the significance and positive effect for most terms and tenors in the high interest rate regime, but it switches the sign of the effect for longer terms and tenors in the low interest rate period.

The swap spread has a significant and positive effect only on the variance risk premium of the longer tenors and terms, with the effect being stronger in the low interest rate period. The VIX and corporate credit spread retain their negative effect on the variance risk premia across all terms and tenors and for both subsamples. The effect of the VIX is considerably stronger in the subsample corresponding to the low interest rate period.

Lastly, since the stationarity tests for variance risk premia are largely inconclusive about the presence of a unit root, I run predictive regressions of first differences of variance risk premia on the set of predictive variables. Table VII presents the regression results for the variance risk premium with a tenor of 2 years and term of 3 months $VRP_t^{2y,3m}$, and on the variance risk premium with a tenor of 5 years and term of 12 months $VRP_t^{5y,12m}$.

[Insert Table **VII** here.]

As expected most coefficients switch their sign given that the variance risk premia are negative on average and we are assessing first differences. An increase in the interest rate level has no significant effect on the $VRP_t^{2y,3m}$, and it is associated with a decrease in the change on the variance risk premium with a tenor of 5 years and term of 12 months $VRP_t^{5y,12m}$. Changes in the interest rate level have no significant effect. The significance of realized volatility as an explanatory variable for variance risk premia persists across all terms and tenors, with the effect being stronger for shorter terms and tenors. Similarly, the significance of the slope of the yield curve on variance risk premia persists across all tenors and terms. The slope of the volatility curve on the other hand, is only significant for the shorter tenors and terms. VIX, the corporate credit spread and the TED spread become insignificant.

5 Conclusions

The variance risk premium embedded in swaptions, defined as the difference between expected realized variances and risk neutral variances, reflects the compensation demanded for holding interest rate variance risk. Its time-variation depends on the distribution of interest rates and their volatilities, as well as on other variables that affect the demand and supply for swaptions. Given the current near zero interest rate environment it becomes relevant to study whether any differences in the properties of variance risk premia are observed compared to normal times. The variance risk premium is unobservable, and to measure it I rely on Black-implied swaption volatilities, as proxies of risk neutral volatilities and forecasts of realized volatility based on a GARCH specification for the volatility process, as estimates of expected realized volatilities. To produce the forecasts, I condition on each period's information set.

Analyzing the time-series properties of variance risk premia, I find that the compensation for volatility risk has been economically significant and, as expected, negative on average during the sample, reflecting the fact that investors are willing to pay a premium during normal times in order to insure against high realized volatility during periods of market turmoil (if we think in terms of a variance swap). The series displays however brief periods where it switches sign, implying unexpected shocks in realized variance. Variance risk premia are highly correlated across terms and tenors.

The process fluctuates around two distinct regimes, one with high (negative) level and high dispersion, corresponding to periods where the interest rate level is low, and the second with a nearly zero level and no dispersion, corresponding to periods where the interest rate level is relatively high. During these two regimes, the slope of the term structure of variance risk premia along the tenor dimension displays significant differences. In the low interest rate period, more frequent and severe episodes where the variance risk premium switches sign are observed. Looking at the episodes where the variance risk premum spikes or falls abrubtly, we observe that the term structure of variance risk premia in the term dimension, display a switch in the sign of the slope. For the spike episodes, the term structure is significantly upward sloping, while for the abrupt fall episodes it is downward sloping.

Predictive regression results suggest that the main determinants of the variation in variance risk premia are as expected the interest level and past volatility, which explain most of its variation. Other measures, such as credit spreads, swap spreads, the interest rate slope and the stock market volatility index are significant predictors.

References

ANDERSEN, T. G., T. BOLLERSLEV, F. X. DIEBOLD, AND H. EBENS (2001): "The distribution of realized stock return volatility," *Journal of financial economics*, 61(1), 43–76.

BAI, J., AND P. PERRON (1998): "Estimating and testing linear models with multiple structural changes," *Econometrica*, pp. 47–78.

——— (2003a): "Computation and analysis of multiple structural change models," *Journal of applied econometrics*, 18(1), 1–22.

——— (2003b): "Critical values for multiple structural change tests," *The Econometrics Journal*, 6(1), 72–78.

BAKER, S. R., N. BLOOM, AND S. J. DAVIS (2013): "Measuring economic policy uncertainty," *Chicago Booth research paper*, (13-02).

BAKSHI, G., AND D. MADAN (2006): "A theory of volatility spreads," *Management Science*, 52(12), 1945–1956.

BARNDORFF-NIELSEN, O. E. (2002): "Econometric analysis of realized volatility and its use in estimating stochastic volatility models," *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64(2), 253–280.

BARONE-ADESI, G., R. F. ENGLE, AND L. MANCINI (2008): "A GARCH option pricing model with filtered historical simulation," *Review of Financial Studies*, 21(3), 1223–1258.

BOLLERSLEV, T., M. GIBSON, AND H. ZHOU (2011): "Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities," *Journal of econometrics*, 160(1), 235–245.

BRITTEN-JONES, M., AND A. NEUBERGER (2000): "Option prices, implied price processes, and stochastic volatility," *The Journal of Finance*, 55(2), 839–866.

CARR, P., AND L. WU (2009): "Variance risk premiums," *Review of Financial Studies*, 22(3), 1311–1341.

CHERNOV, M. (2007): "On the role of risk premia in volatility forecasting," *Journal of Business & Economic Statistics*, 25(4), 411–426.

CHOI, H., P. MUELLER, AND A. VEDOLIN (2015): "Bond variance risk premiums," *Available at SSRN 1787478.*

DEW-BECKER, I., S. GIGLIO, A. LE, AND M. RODRIGUEZ (2015): "The price of variance risk," Discussion paper, National Bureau of Economic Research.

FORNARI, F. (2010): "Assessing the compensation for volatility risk implicit in interest rate derivatives," *Journal of Empirical Finance*, 17(4), 722–743.

HANSEN, L. P., AND R. J. HODRICK (1983): "Risk averse speculation in the forward foreign exchange market: An econometric analysis of linear models," in *Exchange rates and international macroeconomics*, pp. 113–152. University of Chicago Press.

Mele, A., and Y. Obayashi (2013): "Volatility Indexes and Contracts for Government Bonds and Time Deposits," *Swiss Finance Institute Research Paper*, (13-26).

Mele, A., Y. Obayashi, and C. Shalen (2015): "Rate fears gauges and the dynamics of fixed income and equity volatilities," *Journal of Banking & Finance*, 52, 256–265.

Mueller, P., A. Vedolin, and H. Zhou (2011): "Short-run bond risk premia," in *AFA 2013 San Diego Meetings Paper*.

Trolle, A. B., and E. S. Schwartz (2009): "A general stochastic volatility model for the pricing of interest rate derivatives," *Review of Financial Studies*, 22(5), 2007–2057.

——— (2014): "The swaption cube," *Review of Financial Studies*, 27(8), 2307–2353.

Tables

Table I
Descriptive Statistics of Swaption Implied Volatilities

The table reports descriptive statistics for Swaption Implied Volatilities with tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

Swaption Implied Volatility (% p.a)

	Term 3m			Term 6m		
	2 y	**5 y**	**10 y**	**2 y**	**5 y**	**10 y**
Mean	41.36	33.05	26.45	40.21	31.97	25.99
St. Dev.	22.11	15.44	11.44	20.86	14.25	10.52
Skewness	00.01	00.23	00.81	00.00	00.18	00.63
Kurtosis	01.57	02.06	03.62	01.49	01.87	02.81

	Term 12m			Term 24m		
	2 y	**5 y**	**10 y**	**2 y**	**5 y**	**10 y**
Mean	37.43	29.86	24.98	32.23	26.78	23.33
St. Dev.	18.46	12.48	09.44	14.54	10.06	08.11
Skewness	00.11	00.24	00.55	00.43	00.44	00.64
Kurtosis	01.52	01.88	02.40	02.00	02.19	02.52

Table II
Predictive Regressions of Variance Risk Premia

The table reports predictive regression results of variance risk premia, computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$, on their main lagged explanatory variables, i.e. the short term interest rate (the 3 month Treasury rate) and the forecasts of interest rate realized volatility obtained assuming a GARCH(1,1) model for the volatility process. Due to the structural breaks in the data, determined by the methods developed in (Bai and Perron, 1998), a dummy variable $D_{t-1}^{High\,r}$ for the high interest rate regime and interaction terms are included in the regression. The regression results are reported for VRP with tenors of 2, 5 and 10 years and a term of 3 months. t-statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

	$VRP_t^{\tau=2,5,10\,y\,;\,h=3m}$			$VRP_t^{\tau=2,5,10\,y\,;\,h=6m}$		
	2 years	**5 years**	**10 years**	**2 years**	**5 years**	**10 years**
r_{t-1}^{3m}	1.12	0.11	-0.83	1.98	0.35	-0.70
	[7.25]	[0.66]	[-6.11]	[13.28]	[1.90]	[-5.43]
$E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$	0.39	0.04	-0.58	0.61	0.02	-0.60
	[5.08]	[0.47]	[-7.30]	[9.32]	[0.22]	[-9.95]
$D_{t-1}^{High\,r}$	1.07	0.60	-0.46	1.49	0.56	-0.62
	[8.30]	[3.98]	[-3.23]	[12.59]	[3.66]	[-4.61]
$r_{t-1}^{3m} \times D_{t-1}^{High\,r}$	-1.01	0.12	1.11	-1.90	-0.06	1.04
	[-6.00]	[0.63]	[7.56]	[-11.71]	[-0.28]	[7.50]
$E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}] \times D_{t-1}^{High\,r}$	-0.22	-0.27	0.26	-0.36	-0.17	0.45
	[-3.32]	[-3.05]	[2.81]	[-5.65]	[-1.87]	[4.95]
Adj R^2	42.96	31.63	34.51	56.08	44.87	47.62

Table III
Correlation Matrix of Variables of Interest

The table reports the correlation matrix for the explanatory variables of interest. These include in the order reported, the three month interest rate, the variance risk premia computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ (and in this case with tenor 2 years, term 3 months), the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the Ted Spread. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

Correlation of Explanatory Variables

	r	**VRP**	**YSlope**	**VSlope**	**SS**	**VIX**	**BAA**	**EPU**	**TS**
r	1	-	-	-	-	-	-	-	-
VRP	-0.61	1	-	-	-	-	-	-	-
YSlope	-0.83	0.47	1	-	-	-	-	-	-
VSlope	0.84	-0.67	-0.77	1	-	-	-	-	-
SS	0.82	-0.49	-0.55	0.67	1	-	-	-	-
VIX	-0.02	0.30	0.14	-0.05	0.18	1	-	-	-
BAA	-0.57	0.49	0.48	-0.42	-0.27	0.65	1	-	-
EPU	-0.29	0.30	0.25	-0.27	-0.16	0.42	0.46	1	-
TS	0.33	-0.15	-0.29	0.32	0.40	0.50	0.21	0.10	1

Table IV
Variance Inflation Factor to assess Multicollinearity

The table reports results from the variance inflation factor, a measure that assesses the degree of multicollinearity in a given set of regressors, by evaluating the increase in the variance of an estimated regression coefficient that is due to the correlation among the regressors. In the case where all explanatory variables are uncorrelated, the variance inflation factor for all coefficients will be one. A VIF value larger than 10 for a particular variable, suggests that the variable is problematic. The variables of interest include (in the order reported), the variance risk premia computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ (and in this case with tenor 2 years, term 3 months) the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the Ted Spread. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

Variance Inflation Factor										
	VRP	**r**	$E^{\mathbb{P}}[\sigma_t^{\tau,h}]$	**YSlope**	**VSlope**	**SS**	**VIX**	**BAA**	**EPU**	**TS**
VIF	3.81	22.91	13.10	6.02	10.33	5.10	4.53	5.76	1.47	1.79
VIF	-	20.24	8.23	5.00	5.66	5.01	3.92	5.59	1.47	1.77
VIF	-	-	7.69	2.82	5.42	2.76	3.18	3.83	1.47	1.77
VIF	1.78	-	4.57	2.35	-	3.10	3.35	3.55	1.47	1.78

Table V
Predictive Regressions of Variance Risk Premia

The table reports predictive regression results of variance risk premia computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ (and in this case with tenor 2 years, term 3 months and tenor 5 years, term 12 months) on a set of laged explanatory variables. These include the laged variance risk premia itself, the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), and the Ted Spread. t-statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

	$VRP_t^{2y,3m}$				
	(I)	**(II)**	**(III)**	**(IV)**	**(V)**
$VRP_{t-1}^{\tau,h}$	0.93	–	-	0.87	-
	[109.70]	-	-	[67.45]	-
r_{t-1}^{3m}	-	0.60	-	-	0.80
	-	[35.83]	-	-	[10.30]
$E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$	-	-	0.88	-	1.01
	-	-	[15.71]	-	[18.93]
Yield	-	-	0.21	-0.00	0.47
Slope$_{t-1}$	-	-	[9.70]	[-0.27]	[13.20]
Vol	-	-	1.08	0.03	0.99
Slope$_{t-1}$	-	-	[26.99]	[2.04]	[25.46]
Swap	-	-	0.41	0.03	0.14
Spread$_{t-1}$	-	-	[17.28]	[4.72]	[4.40]
VIX$_{t-1}$	-	-	-0.24	-0.03	-0.39
	-	-	[-7.48]	[-2.82]	[-10.35]
Credit	-	-	-0.43	-0.02	-0.19
Spread$_{t-1}$	-	-	[-10.72]	[-1.67]	[-3.83]
Ted	-	-	0.08	0.02	0.08
Spread$_{t-1}$	-	-	[3.79]	[1.66]	[3.78]
Adj R^2	86.90	36.54	64.69	87.23	67.86

Table VI
Predictive Regressions of Variance Risk Premia by Interest Rate Regime

The table reports predictive regression results of variance risk premia computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ (and in this case with tenor 2 years, term 3 months and tenor 5 years, term 12 months) on a set of laged explanatory variables. The data has been divided into two subsets according to the structural break points determined by the methods developed in (Bai and Perron, 1998). Although there are four breaks, the periods correspond to two regimes, one where interest rates are relatively low (based on the given sample), and one where they are relatively high. The regressors include the laged variance risk premia itself, the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the Ted Spread. t-statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

	High Interest Rate Regime			**Low Interest Rate Regime**		
	$VRP_t^{2y,3m}$	$VRP_t^{5y,12m}$	$VRP_t^{10y,24m}$	$VRP_t^{2y,3m}$	$VRP_t^{5y,12m}$	$VRP_t^{10y,24m}$
r_{t-1}^{3m}	0.57	0.85	0.59	0.45	0.29	0.17
	[9.09]	[11.65]	[6.52]	[6.40]	[2.78]	[2.10]
$E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$	1.73	0.49	-0.02	0.70	0.58	-0.01
	13.05	6.31	-0.33	17.17	10.15	-0.18
Yield	0.15	0.17	0.04	0.34	0.61	0.45
Slope$_{t-1}$	4.24	4.46	0.79	11.56	19.84	17.62
Vol	1.57	0.55	-0.08	0.72	-0.07	-0.30
Slope$_{t-1}$	13.04	8.33	-1.34	22.27	-2.50	-13.20
Swap	0.03	0.00	0.18	-0.12	0.32	0.25
Spread$_{t-1}$	0.65	0.10	4.75	-2.09	4.07	4.47
VIX$_{t-1}$	-0.17	-0.04	-0.07	-0.55	-0.31	-0.39
	-5.35	-1.21	-1.99	-9.05	-5.91	-8.54
Credit	-0.17	-0.17	-0.51	-0.14	-0.13	-0.02
Spread$_{t-1}$	-2.66	-3.24	-9.49	-2.01	-2.00	-0.34
EPU$_{t-1}$	-0.03	-0.02	-0.01	0.02	-0.02	0.03
	-1.40	-1.25	-0.83	0.80	-1.04	2.00
Ted	-0.02	0.04	0.07	0.09	0.01	-0.01
Spread$_{t-1}$	-0.48	1.54	2.52	1.96	0.38	-0.25
Adj R^2	56.26	74.75	72.56	52.06	48.03	67.21

Table VII
Predictive Regressions of Changes in Variance Risk Premia

The table reports predictive regression results of first differences in variance risk premia on a set of laged explanatory variables. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$ (and in this case the results for the tenor 2 years, term 3 months are presented). These include the laged variance risk premia itself, the three month interest rate, realized volatility estimated with a GARCH(1,1) model, the yield curve slope, the volatility slope, the swap spread, the VIX, Moody's Seasoned Baa Corporate Bond Yield relative to the yield on 10-Year Treasury Constant Maturity (a measure of credit spread), the Economic Policy Uncertainty Index of (Baker, Bloom, and Davis, 2013) and the Ted Spread. t-statistics are shown in parenthesis, below the reported estimated coefficients. The standard errors are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. All variables are standardized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

	$\Delta VRP_t^{2y,3m}$		$\Delta VRP_t^{5y,12m}$	
r_{t-1}^{3m}	-0.10	-0.09	-0.18	-0.18
	-1.52	-1.32	-2.71	-2.64
Δr_{t-1}^{3m}	-	-0.01	-	0.00
	-	-0.77	-	-0.25
$E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$	-0.33	-0.31	-0.11	-0.10
	-6.04	-5.62	-2.22	-2.09
$\Delta E^{\mathbb{P}}[\sigma_{t-1}^{\tau,h}]$	-	-0.07	-	-0.03
	-	-2.17	-	-0.85
Yield Slope$_{t-1}$	-0.12	-0.12	-0.10	-0.10
	-3.98	-3.81	-3.32	-3.31
Vol Slope$_{t-1}$	-0.31	-0.30	-0.06	-0.06
	-6.20	-5.97	-1.34	-1.33
Swap Spread$_{t-1}$	-0.02	-0.02	0.03	0.03
	-0.93	-0.79	1.02	1.01
VIX$_{t-1}$	0.02	0.02	0.05	0.05
	0.55	0.58	1.34	1.33
Credit Spread$_{t-1}$	0.06	0.06	-0.04	-0.04
	1.45	1.25	-0.88	-0.87
EPU$_{t-1}$	-0.02	-0.02	-0.01	-0.01
	-0.99	-0.86	-0.37	-0.34
Ted Spread$_{t-1}$	0.02	0.02	-0.01	-0.01
	0.73	0.67	-0.32	-0.32
Adj R^2	2.23	2.66	0.44	0.53

A2: Figures

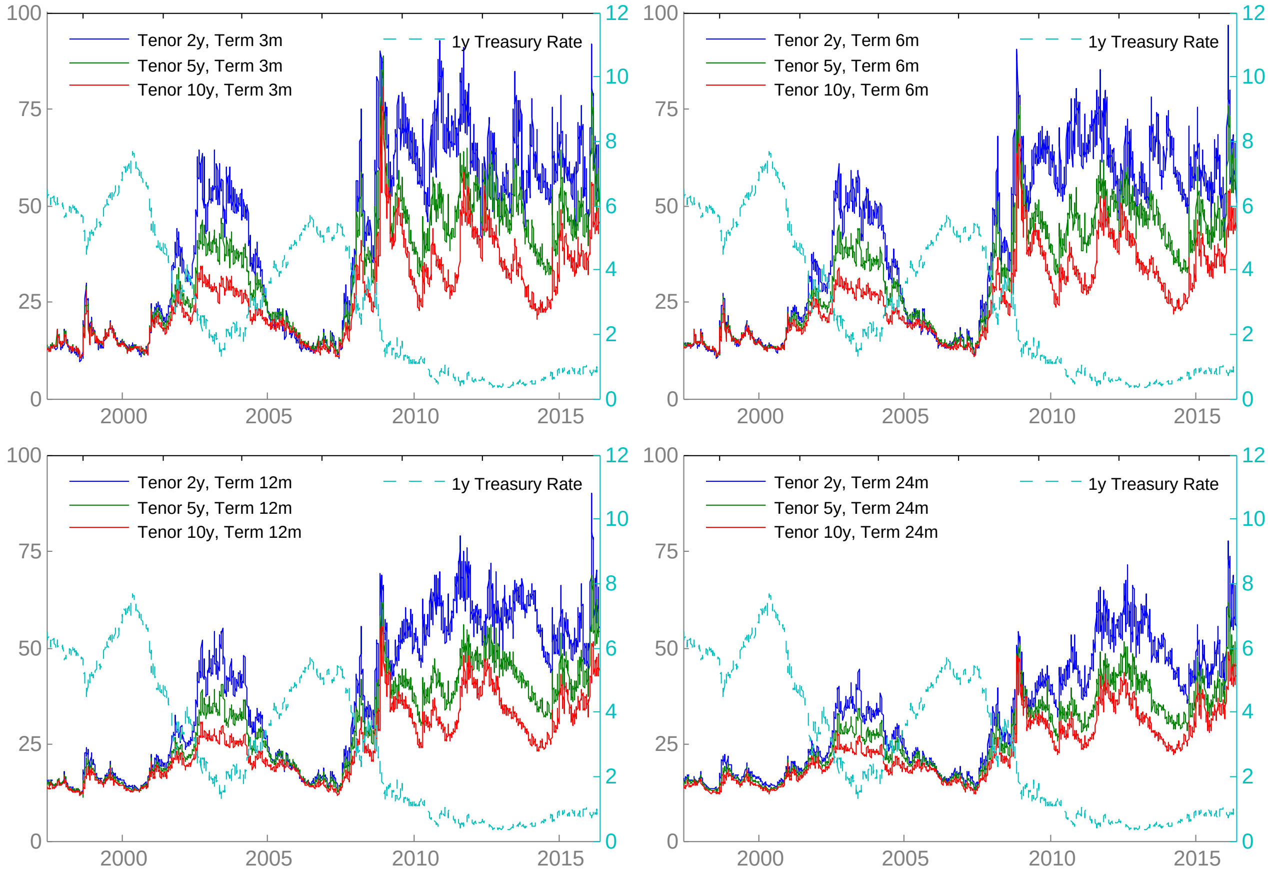

Figure 1. Swaption Implied Volatilities

This figure plots Swaption implied volatilities for tenors of 2, 5 and 10 years and terms of 3, 6, 12 and 24 months along with the one year Treasury rate. The figures are in percent, annualized. The data is from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

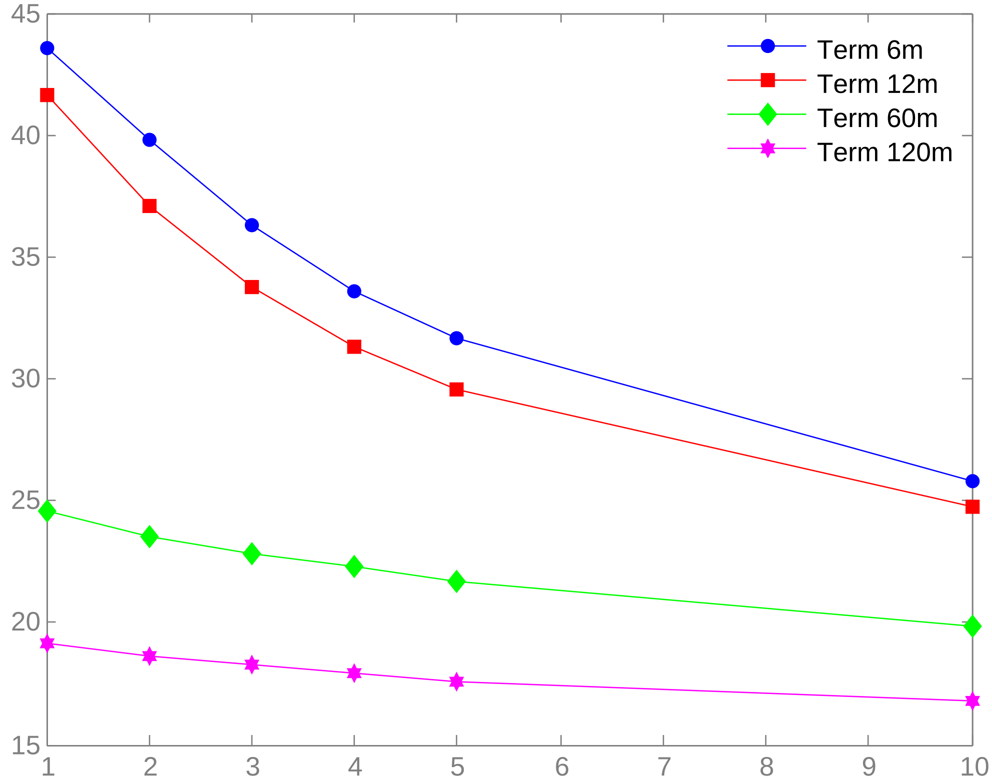

Figure 2. Cross Section of Swaption Implied Volatilities

This figure plots the cross-section of Swaption implied volatilities for tenors of 1, 2, 3, 4, 5 and 10 years and terms of 6, 12, 60 and 120 months. The figures are in percent, annualized. The data is from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

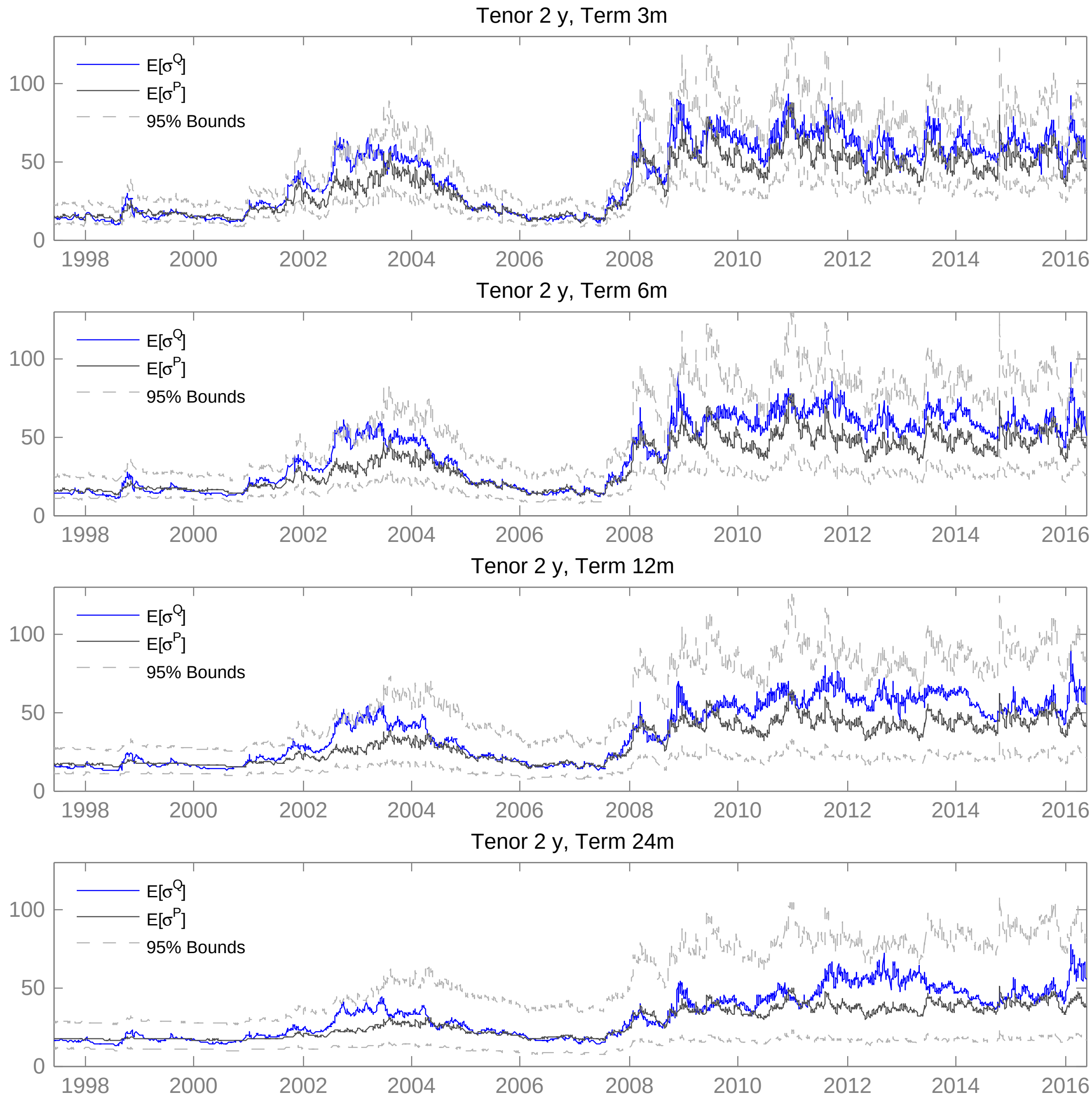

Figure 3. Swaption Impield Volatilities and Forecasts of Realized Volatility

This figure plots Swaption implied volatilities and expected realized volatility forecasts $E^{\mathbb{P}}[\sigma_t^{\tau,h}]$ along with their 95% confidence bounds, for the 2 years tenor, and terms going from 3m to 24 months. The expected realized volatility forecasts and their confidence bounds are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

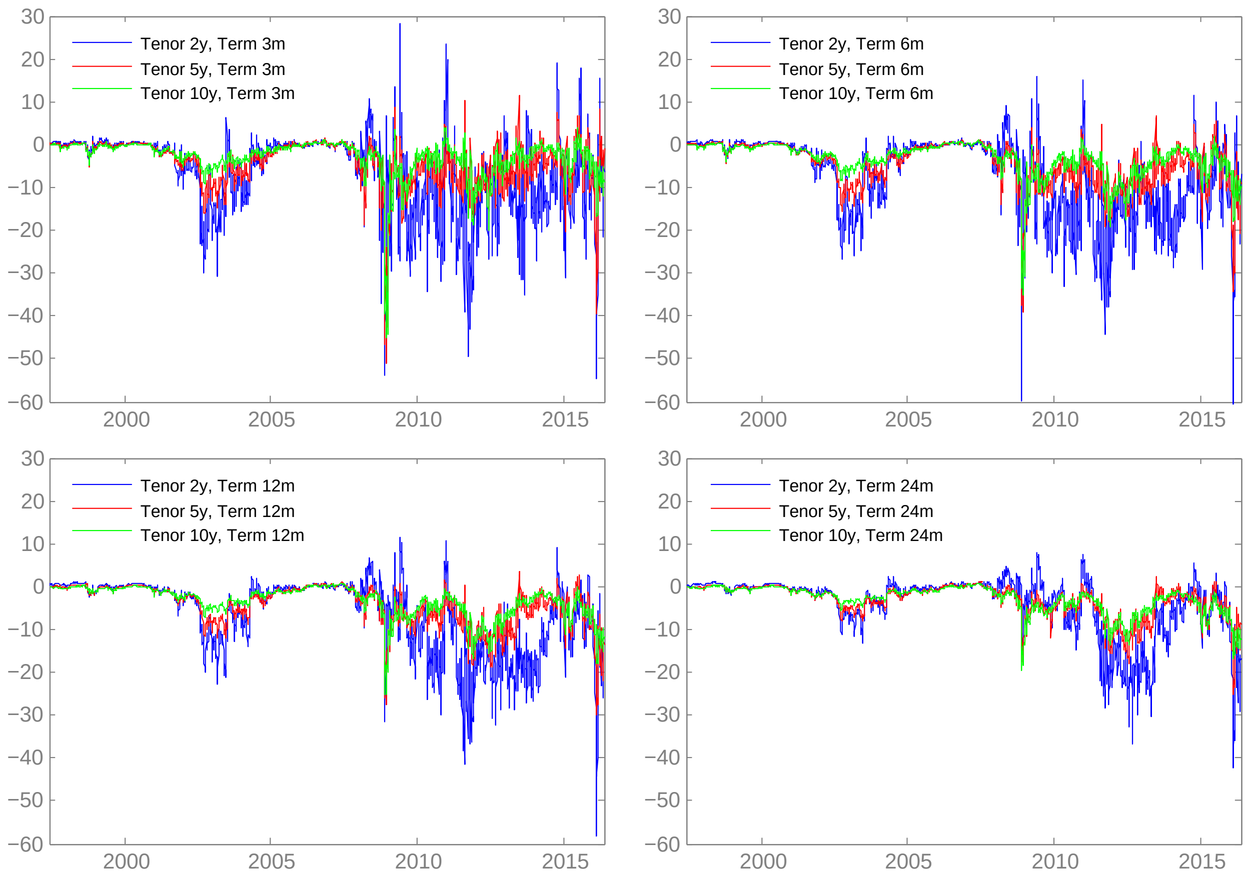

Figure 4. Variance Risk Premia

The figure plots variance risk premia, computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[\sigma_t^{\tau,h}]^2 - E^{\mathbb{Q}}[\sigma_t^{\tau,h}]^2$, with tenor of 2 years and terms of 3 months. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

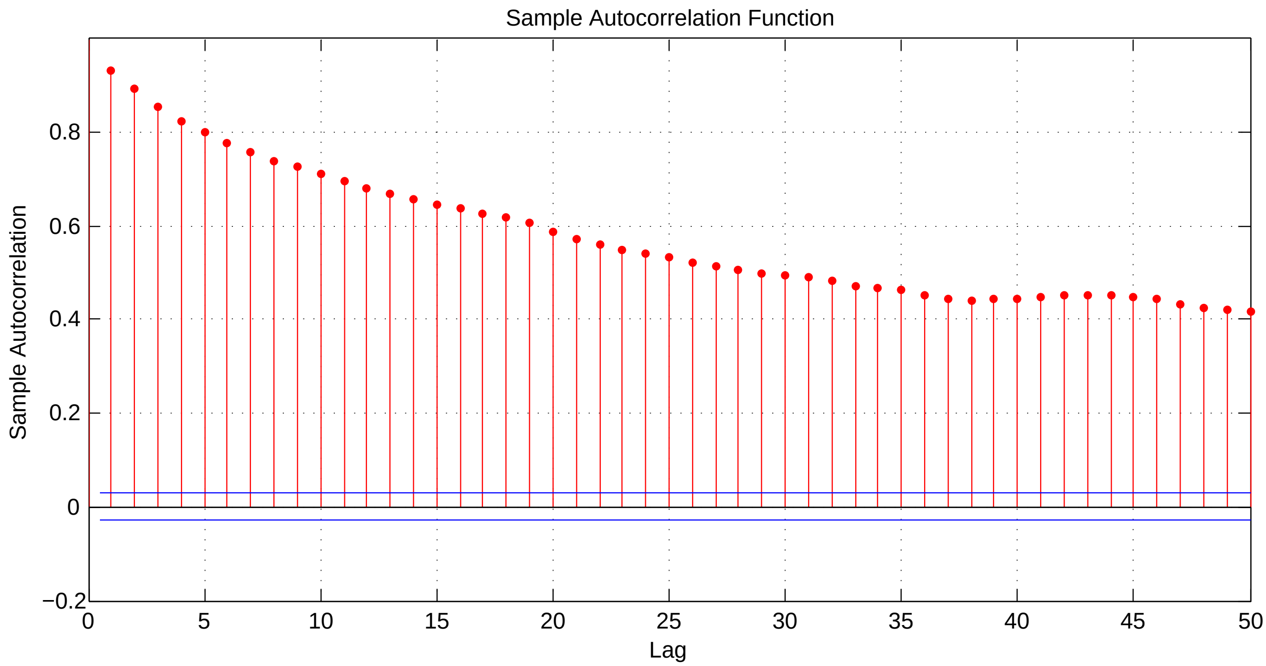

Figure 5. Autocorrelation Function for Variance Risk Premia

The figure plots the autocorrelation function of variance risk premia with tenor of 2 years and terms of 3 months, computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[\sigma_t^{\tau,h}]^2 - E^{\mathbb{Q}}[\sigma_t^{\tau,h}]^2$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

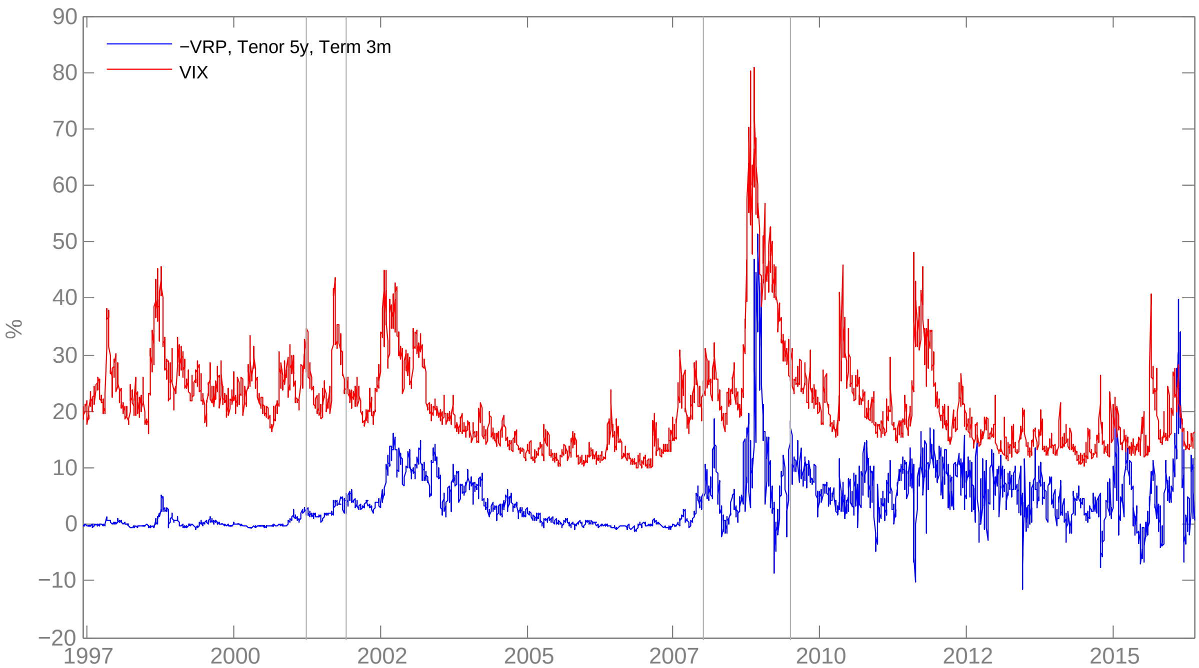

Figure 6. Swaption Variance Risk Premia and the VIX

The figure plots minus the variance risk premia, computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$, with a tenor of 5 years and terms of 3 months along with the VIX. The figures are in percent, annualized. The data has been taken from FRED (the Federal Reserve Economic Data of the St. Louis Fed) and Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

Figure 7. Structural Breaks in the Data

The figure plots the Swaption implied volatility and variance risk premia, with tenor 1 year and term 6 months, along with structural break points (in grey dashed vertical lines), determined by the methods developed in (Bai and Perron, 1998). In red dashed lines, are reported the means of the data by the regimes corresponding to the break points. The green vertical lines denote two events associated with the structural breaks. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

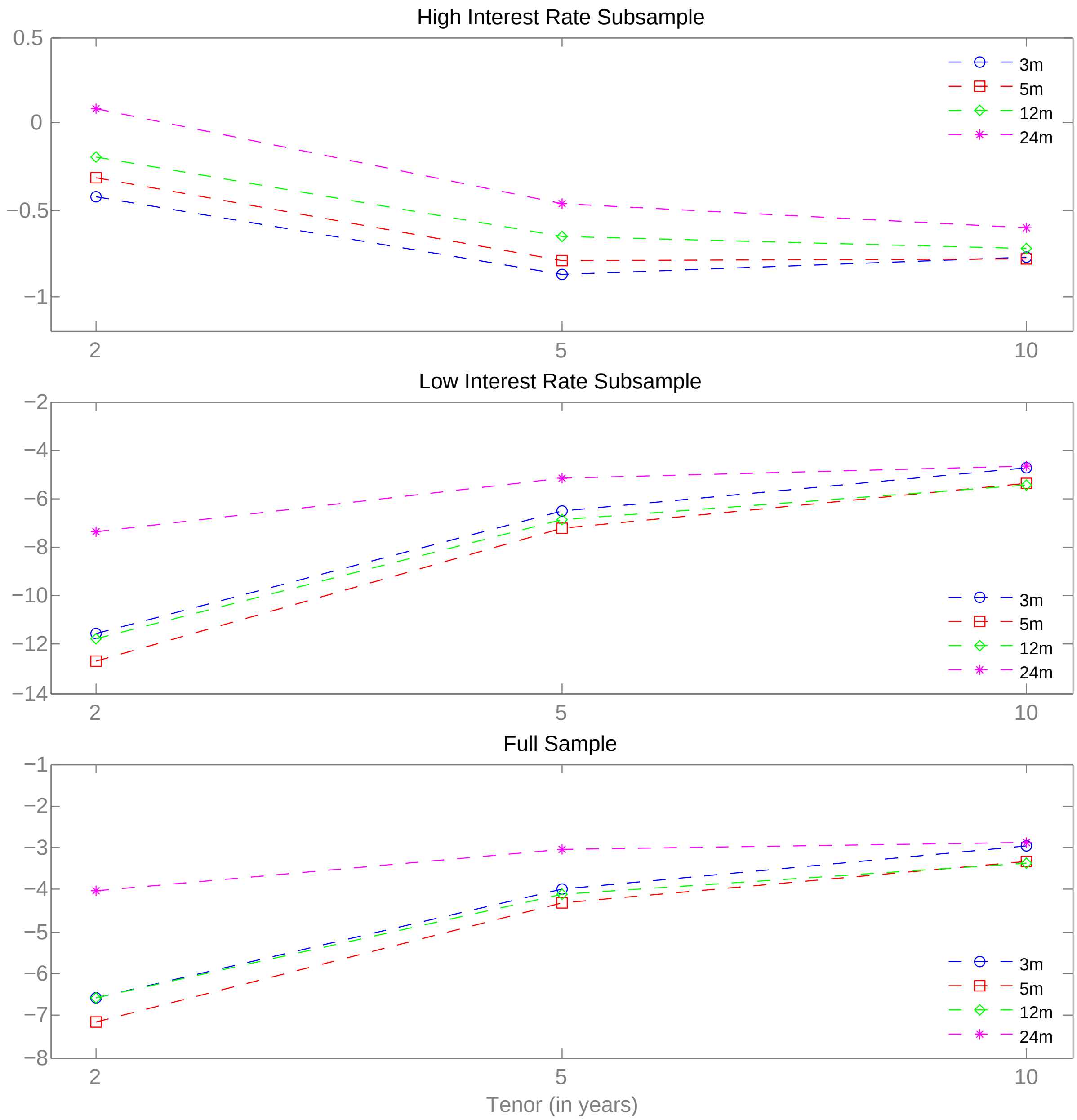

Figure 8. Cross-Section of Variance Risk Premia by Subsamples

The figure plots the cross-section of variance risk premia in the tenor dimension by the subsamples corresponding to the break points. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

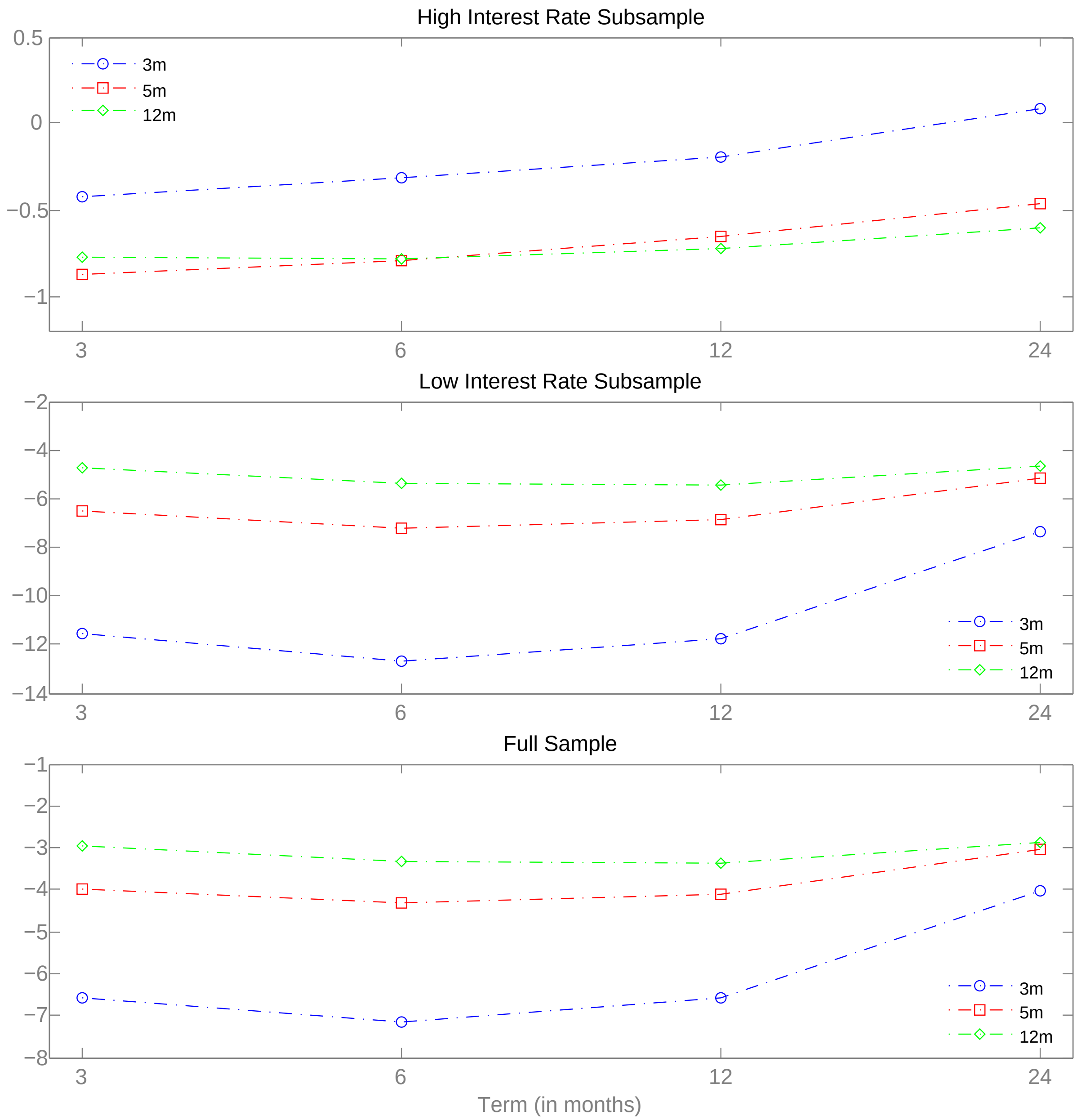

Figure 9. Term Structure of Variance Risk Premia by Subsamples

The figure plots the term structure of variance risk premia (i.e. in the term dimension) by the subsamples corresponding to the break points. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

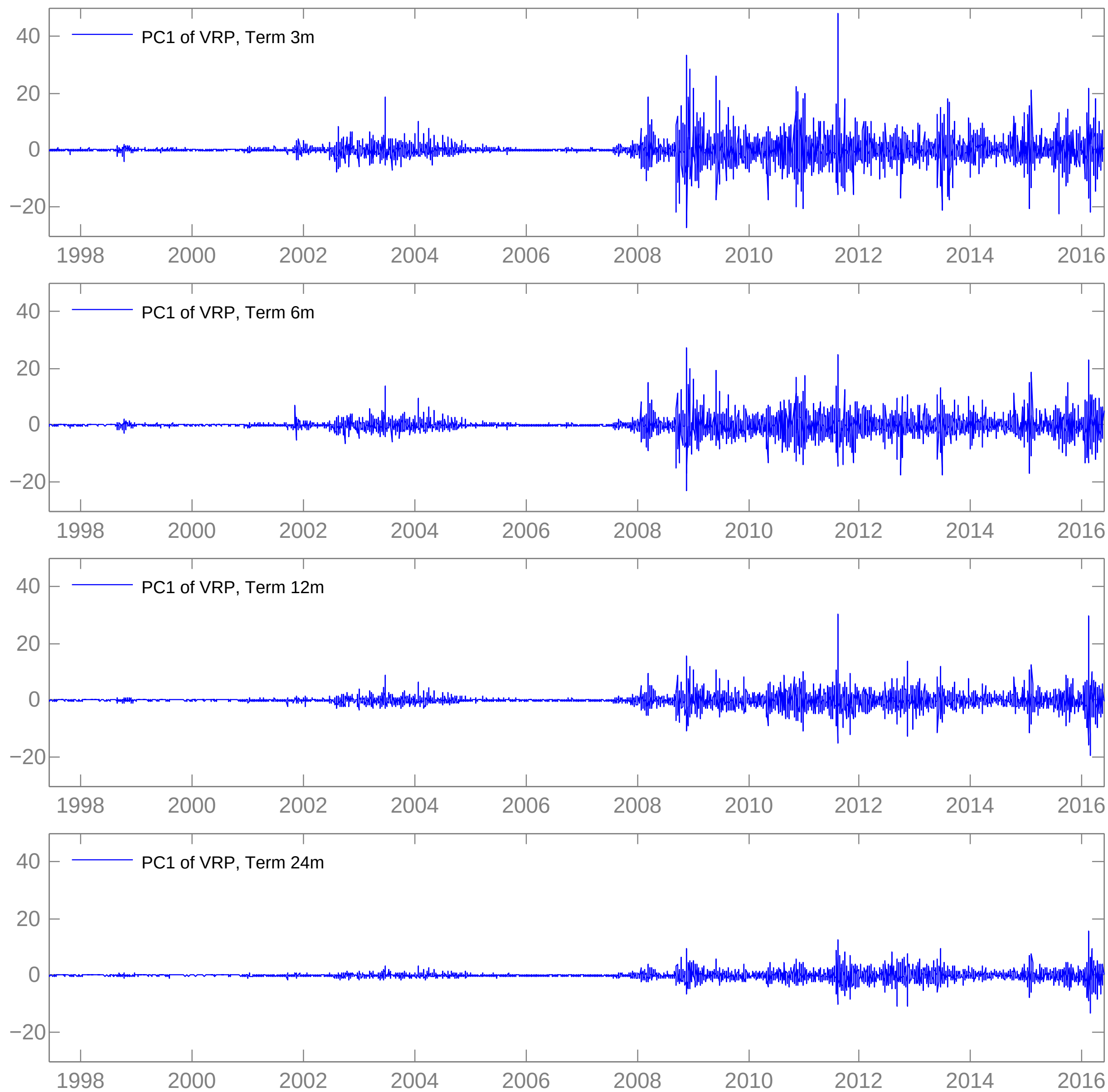

Figure 10. Principal Components of Changes in Variance Risk Premia

The figure plots the principal components of changes in variance risk premia, for terms of 3, 6, 12 and 24 months. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

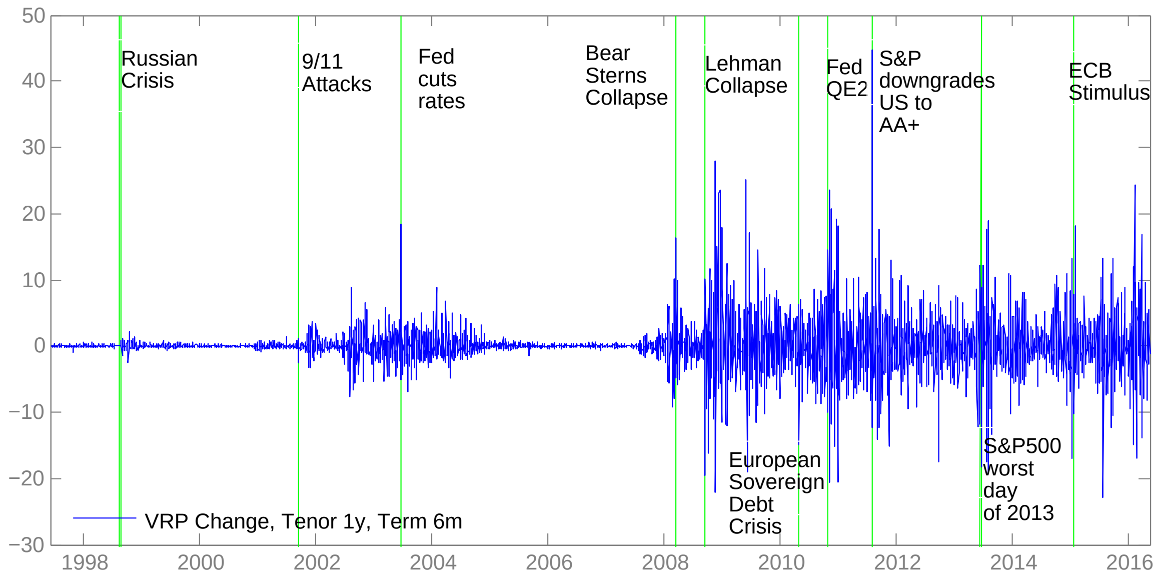

Figure 11. Changes in Variance Risk Premia and Major Financial/Economic Evens

The figure plots changes in variance risk premia (for the 1 year tenor, 6 months term), along with the major financial and economic events (green vertical lines) associated with the the largest changes. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The figures are in percent, annualized. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.

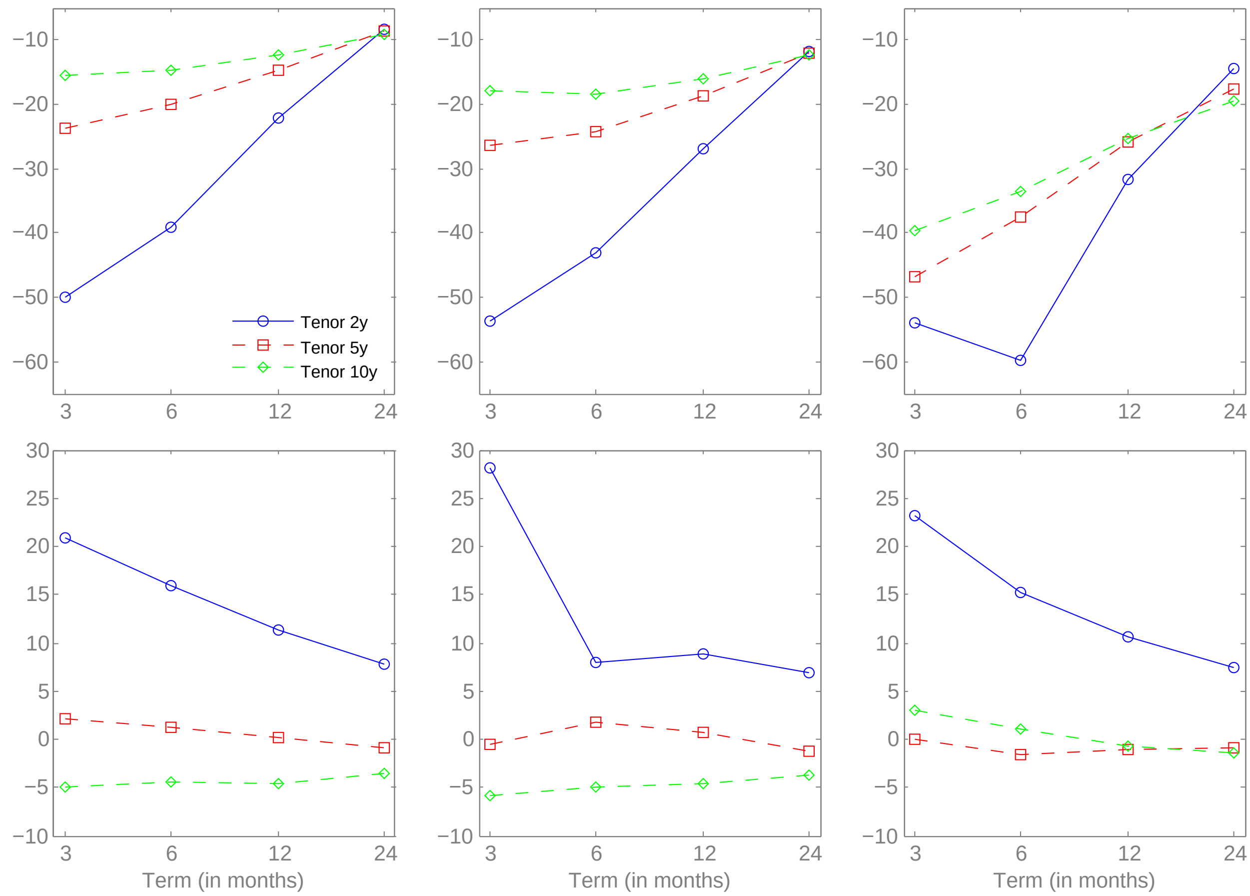

Figure 12. Term Structure of Variance Risk Premia on Particular Dates

The figure plots the term structure of variance risk premia (i.e. in the term dimension) in the dates where the variance risk premium for the 2 year tenor and 3 month term, $VRP_t^{2y,3m}$, takes its highest negative values (the three plots on the top) and its highest positive values (the three plots on the bottom). The figures are in percent, annualized. The variance risk premia are computed as $VRP_t^{\tau,h} = E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2] - E^{\mathbb{Q}}[(\sigma_t^{\tau,h})^2]$. The expected realized volatility forecasts $E^{\mathbb{P}}[(\sigma_t^{\tau,h})^2]$ are computed at each point in time from simulations based on parameter estimates from a GARCH(1,1) model and conditioning at the information available at each point tin time. The data has been taken from Bloomberg, it is sampled at a daily frequency and covers the period June 2, 1997 to May 19, 2016.