2022 Intro to Prob & Stats Final Exam

Due Oct 13 at 11:59am **Points** 100 **Questions** 50
Available Oct 5 at 12am - Oct 13 at 11:59am **Time Limit** 180 Minutes
Allowed Attempts Unlimited

Instructions

NOTE: THESE WERE THE INSTRUCTIONS IN 2022

Please read these instructions carefully before you start.

1) The exam is three hours long.

2) The exam is open book, open notes, and open laptop.

3) The exam is an individual assignment. You must not communicate with anyone else except the instructor about the contents of the exam before Monday October 17.

4) The exam contains 50 questions total. For questions that require a numerical calculation, use at least four digits of accuracy after the decimal point (e.g., 5.7456, 0.1243)

Javier

Take the Quiz Again

Attempt History

	Attempt	Time	Score
KEPT	Attempt 19	180 minutes	57 out of 100
LATEST	Attempt 19	180 minutes	57 out of 100
	Attempt 18	180 minutes	0 out of 100
	Attempt 17	less than 1 minute	6 out of 100
	Attempt 16	2 minutes	0 out of 100
	Attempt 15	less than 1 minute	2 out of 100
	Attempt 14	less than 1 minute	6 out of 100
	Attempt 13	less than 1 minute	4 out of 100
	Attempt 12	less than 1 minute	0 out of 100
	Attempt 11	less than 1 minute	2 out of 100

Attempt	Time	Score
Attempt 10	less than 1 minute	6 out of 100
Attempt 9	less than 1 minute	6 out of 100
Attempt 8	180 minutes	2 out of 100
Attempt 7	less than 1 minute	2 out of 100
Attempt 6	36 minutes	4 out of 100
Attempt 5	180 minutes	0 out of 100
Attempt 4	1 minute	2 out of 100
Attempt 3	4 minutes	0 out of 100
Attempt 2	42 minutes	29 out of 100
Attempt 1	330 minutes	23 out of 100

Submitted Oct 12 at 3:09pm

Download the Excel file **Final2022.xlsx (https://canvas.cmu.edu/courses/35101/files/10116733?wrap=1)** **(https://canvas.cmu.edu/courses/35101/files/10116733/download?download_frd=1)**

This file contains the datasets "SAT" and "rates" that you will need to answer some questions below. It also has a "stocks" sheet that may facilitate some of your calculations.

Question 1

1 / 1 pts

I hereby state that I will neither provide nor accept unauthorized assistance on this exam. Furthermore, I will not communicate with anybody else about the exam before Monday October 17, 2022.

Correct!

- (●) True
- ( ) False

Question 2

2 / 2 pts

Suppose A and B are independent events in some sample space with a probability measure. Then A^c and B^c are independent.

Correct!

- (selected) True
- False

Question 3

2 / 2 pts

Suppose A and B are events in some sample space with a probability measure with both $0 < \mathbb{P}(A) < 1$ and $0 < \mathbb{P}(B) < 1$. If $\mathbb{P}(A|B) = 1$ then $\mathbb{P}(B^c|A^c) = 1$.

Correct!

- (selected) True
- False

Question 4

2 / 2 pts

Suppose X is a continuous random variable with cumulative probability function $F(x)$. If $a \leq b$ then $F(a) \leq F(b)$.

Correct!

- (selected) True
- False

Question 5

2 / 2 pts

Suppose X is a continuous random variable with density function $f(x)$. The function $f(x)$ can never be greater than one.

- ○ True
- ◉ False

Correct!

Question 6

2 / 2 pts

Suppose $X \sim B(m,p)$ and $Y \sim B(n,p)$ where $p \in (0,1)$ and $m < n$. Then $\mathbf{var}\left(\frac{X}{m}\right) > \mathbf{var}\left(\frac{Y}{n}\right)$.

- ◉ True
- ○ False

Correct!

Question 7

2 / 2 pts

If X and Y are independent random variables then $\mathbb{E}(X \cdot Y) = \mathbb{E}(X) \cdot \mathbb{E}(Y)$.

- ◉ True
- ○ False

Correct!

Question 8

2 / 2 pts

If X and Y are independent random variables then $\mathbf{var}(X - Y) = \mathbf{var}(X) - \mathbf{var}(Y)$.

- ○ True

Correct!

- ◉ False

Question 9

2 / 2 pts

The correlation of two random variables is always between -1 and 1.

Correct!

- ◉ True
- ○ False

Question 10

2 / 2 pts

Suppose Z is a standard normal variable. Then for all $a \neq 0$ we have $\mathbb{P}(a-1 \leq Z \leq a+1) < \mathbb{P}(-1 \leq Z \leq 1)$.

Correct!

- ◉ True
- ○ False

Unanswered

Question 11

0 / 2 pts

If the random variables $X_1, \ldots, X_n$ are iid standard normal then $\frac{X_1+\cdots+X_n}{\sqrt{n}}$ is standard normal.

Correct Answer

- ○ True
- ○ False

Unanswered

Question 12

0 / 2 pts

If the random variables $X_1, \ldots, X_n$ are iid binomial then $X_1 + \cdots + X_n$ is binomial.

orrect Answer

- True
- False

Question 13

2 / 2 pts

If a hypothesis test rejects the null hypothesis at the 0.05 significance level, then it would also reject the null hypothesis at the 0.1 significance level.

Correct!

- True
- False

Question 14

2 / 2 pts

If two random variables X, Y are such that $\mathbf{cov}(X, Y) = 0$ then X and Y are independent.

- True

Correct!

- False

Question 15

2 / 2 pts

Suppose the probability that an airline passenger is a no-show" (that is, he/she does not show up for a flight) is 0.1. To take advantage of this opportunity, an airline decides to sell 10 seats for a 9-seat plane, 20 seats

for a 18-seat plane, and 30 seats for a 27-seat plane. Which of the three planes is the least likely to be overbooked?

(A plane is overbooked if it does not have enough seats for the passengers who show up.)

Correct!

- (●) The 9-seat plane
- ( ) The 18-seat plane
- ( ) The 27-seat plane
- ( ) All equally likely
- ( ) Cannot be determined
- ( ) None of the above

Question 16

2 / 2 pts

Suppose you take a Probability exam that consists of 100 multiple-choice questions. Your level of mastery of the subject is so that you can answer each question correctly with probability 0.9. What is the probability that you answer at least 85 of the 100 questions correctly?

- ( ) 0.85
- ( ) 0.90

Correct!

- (●) 0.9601
- ( ) 0.99
- ( ) None of the above.

Question 17

2 / 2 pts

Suppose calls arrive at a 1-800 phone line following a Poisson process with average arrival rate of 24 calls per hour.

Which of the following is NOT a correct way of computing the probability that at least one call arrives within the next 5 minutes?

- 1-poisson.dist(0,2,0) = 1 - dpois(0,2)
- 1-poisson.dist(0,2,1) = 1 - ppois(0,2)
- expon.dist(1,2,1) = pexp(1,2)

Correct!

- (●) expon.dist(5,12,1) = pexp(5,12)
- None of the above. In other words, all of the above are correct.

Question 18

2 / 2 pts

A continuous random variable X has cumulative distribution function F(x) and the range of values that it takes is from $-\infty$ to ∞. Let A be the event {1<= X <= 7 } and let B be the event {3 <= X <= 12}. What is $\mathbb{P}(A \cup B)$?

Correct!

- (●) F(12) - F(1)
- F(7)-F(1)+F(12)-F(3)
- (F(7)-F(1))*(F(12)-F(3))
- (F(7)-F(1))/(F(12)-F(3))
- None of the above

Question 19

2 / 2 pts

Suppose you want to estimate the population proportion p via the sample proportion $\hat{p}$ What is the minimum number of samples that will guarantee a margin of error of 0.05 or less at the 0.95 confidence level?

- 95
- 196
- Correct! 385
- 1960
- None of the above.

Unanswered

Question 20

0 / 2 pts

Consider the following experiment: You flip a fair coin repeatedly until the outcome is tails. What is the probability that you have to flip the coin at least four times?

You Answered

Correct Answers 0.125 (with margin: 0.001)

Question 21

0 / 2 pts

Suppose X is a normal random variable, the 0.1-quantile of X is 0.1, and the 0.9-quantile of X is 0.9.

Find the standard deviation of X.

You Answered 0.2465

Correct Answers 0.312 (with margin: 0.01)

Unanswered

Question 22

0 / 2 pts

Suppose $X_1, \ldots, X_n$ are iid standard normal and consider their sample mean $\overline{X} := \frac{X_1 + \cdots + X_n}{n}$. Find the smallest n such that $|\overline{X}| \leq 0.2$ with probability at least 0.95. In other words, find the smallest n such that $\mathbb{P}(|\overline{X}| \leq 0.2) \geq 0.95$.

You Answered

Correct Answers

97 (with margin: 1)

Unanswered

Question 23

0 / 2 pts

The next four questions, including this one, refer to the following setup.

Suppose X is a discrete random variable that takes values $1, 2, \ldots, m$ with probability $1/m$ each for some integer $m \geq 1$.

Determine which of the following is the moment generating function $\psi_X(t)$ of X for $t \neq 0$:

- $\frac{e^{mt}-1}{e^t-1}$
- $\frac{e^{mt}-1}{m(e^t-1)}$
- $\frac{e^t(e^{mt}-1)}{e^t-1}$

Correct Answer

- $\frac{e^t(e^{mt}-1)}{m(e^t-1)}$
- None of the above

Unanswered

Question 24

0 / 2 pts

Suppose $m = 10$. Find $\mathbb{E}(X)$.

ou Answered

orrect Answers 5.5 (with margin: 0.1)

Unanswered

Question 25

0 / 2 pts

Suppose $m = 10$. Find $\mathbf{var}(X)$.

ou Answered

orrect Answers 8.25 (with margin: 0.01)

Unanswered

Question 26

0 / 3 pts

Suppose $m = 10$ and $X_1, \ldots, X_{100}$ are iid with the same distribution as X. Use the central limit theorem to compute the following probability approximately: $\mathbb{P}(530 \leq X_1 + \cdots + X_{100} \leq 570)$.

ou Answered

orrect Answers 0.5138 (with margin: 0.02)

Question 27

2 / 2 pts

The next two questions, including this one, refer to the following setup. Suppose $X_1, \ldots, X_n$ is a random sample from the uniform distribution

$U(0, \theta)$ for some unknown parameter $\theta > 0$. In other words, $X_1, \ldots, X_n$ are iid and $X_1, \ldots, X_n \sim U(0, \theta)$.

Consider the following estimator $\hat{\theta}_n$ of θ:

$$\hat{\theta}_n := \max\{X_1, \ldots, X_n\}.$$

The estimator $\hat{\theta}_n$ is unbiased.

- True
- False

Correct!

Question 28

2 / 2 pts

The estimator $\hat{\theta}_n$ is consistent.

- True
- False

Correct!

Question 29

2 / 2 pts

The next four questions, including this one, concern the following setup.

Suppose stocks A,B,C,D have annual expected returns and standard deviations as indicated in the following table:

	stock A	stock B	stock C	stock D
expected return	10%	14%	18%	22%
standard deviation	10%	15%	20%	25%

Which one of the stocks is least likely to attain a negative return over the next year?

Correct!

- (●) Stock A
- ( ) Stock B
- ( ) Stock C
- ( ) Stock D
- ( ) None of the above

Question 30

2 / 2 pts

Consider the following three possible allocations of capital to the above four stocks

Allocation 1: 50% stock A, 50% stock D

Allocation 2: 50% stock B, 50% stock C

Allocation 3: 25% stock A, 25% stock B, 25% stock C, 25% stock D

Which of the above allocations has the lowest risk (standard deviation)?

- ( ) Allocation 1
- ( ) Allocation 2

Correct!

- (●) Allocation 3
- ( ) All three allocations have the same risk

Question 31

2 / 2 pts

Which of the above three allocations has the highest ratio (expected return)/(standard deviation).

In investments, this ratio is called the "Sharpe ratio".

- Allocation 1
- Allocation 2
- Allocation 3 — Correct!
- All three allocations have the same Sharpe ratio

Question 32

0 / 2 pts

Which of the above four stocks or three allocations is most likely to attain or exceed a target return of 15%?

- Stock A
- Stock B
- Stock C
- Stock D — orrect Answer
- Allocation 1
- Allocation 2
- Allocation 3 — ou Answered
- They all have the same chance or attaining or exceeding 15%

Question 33

2 / 2 pts

The next three questions, including this one, concern the following setup.

Suppose the Federal Reserve Chairman Jerome Powell announces that the Fed will increase the federal funds rate sometime in November, 2022

and the increase could occur at any moment. This means that the amount of time X (in days) between the beginning of the day on November 1, 2022 and the moment when the federal funds rate increase occurs is a uniform random variable with range of values [0,30]. In other words, X ~ U(0,30).

Find the probability that the Fed increases the federal funds rate before the end of the day on November 10, 2022.

Correct!

0.334

orrect Answers

0.3333 (with margin: 0.001)

Question 34

2 / 2 pts

Find the probability that the Fed does not increase the federal funds rate before the end of the day on November 24, 2022.

Correct!

0.2

orrect Answers

0.2 (with margin: 0.01)

Question 35

0 / 2 pts

Suppose the Fed does not increase the federal funds rate before the end of the day on November 10, 2022. Given that information and assuming Powell's original announcement will hold, find the probability that the Fed does not increase the federal funds rate before the end of the day on November 24, 2022.

ou Answered

0.7

orrect Answers

0.3 (with margin: 0.01)

Question 36

2 / 2 pts

The next four questions, including this one, concern the following setup.

The Environmental Protection Agency (EPA) collects data, such as horsepower and highway mileage, on 30 different models of small trucks with horsepower between 280 and 420. Analysts at a major automobile company would like to use this data to better understand the relationship between horsepower (the predictor variable X) and highway mileage (the response variable Y). The results of the regression analysis are listed below.

SUMMARY OUTPUT					
Regression Statistics					
Multiple R square	0.5492				
R-square	0.3016				
Adjusted R square	0.2766				
Standard error	3.5532				
Observations	30				
	Coefficients	Std Error	t-stat	p-value	
Intercept	31.1658	1.9332	16.1212	0.0000	
Horse Power	-0.0286	0.0082	-3.4772	0.0017	

What regression coefficients are statistically significant at the 5% significance level?

- Intercept only

Slope only

Correct! Both intercept and slope

Neither intercept nor slope

Question 37

2 / 2 pts

Compute the margin of error of a 95% confidence interval for the slope coefficient.

Correct! 0.0161

orrect Answers 0.0168 (with margin: 0.001)

Question 38

0 / 2 pts

Compute the sample correlation between mileage and horsepower. If the information available is insufficient, enter the value 99999.

ou Answered 0.5491

orrect Answers -0.549 (with margin: 0.01)
-0.549 (with margin: 0.01)

Question 39

2 / 2 pts

What mileage would you predict for a car with horsepower of 320? (Give a point estimate only.)

Correct! 22.0138

orrect Answers 22.0138 (with margin: 0.1)

Question 40

2 / 2 pts

It is generally believed that SAT scores are related to income. Another commonly used predictor of SAT scores is the grade point average (GPA). The worksheet "SAT" in the Excel file Final2022.xlsx contains data on 24 randomly chosen students.

Compute the sample variance of SAT.

Correct!

7,134.6015

orrect Answers

7,134.6 (with margin: 0.1)

Question 41

0 / 2 pts

Compute the margin of error of the 95% confidence interval for SAT.

ou Answered

3.677

orrect Answers

35.75 (with margin: 0.1)

Question 42

2 / 2 pts

Compute the sample correlations between each of the three pairs of the variables SAT, Income, and GPA. Then determine which of the following statements about hypothesis tests hold at the 5% significance level:

We reject the null hypothesis that the correlation between SAT and Income is zero.

We reject the null hypothesis that the correlation between SAT and GPA is zero.

We reject the null hypothesis that the correlation between GPA and Income is zero.

Correct!

We reject BOTH the null hypothesis that the correlation between SAT and Income is zero AND the null hypothesis that the correlation between SAT and GPA is zero.

None of the above

Unanswered

Question 43

0 / 2 pts

The next four questions, including this one, concern the following setup.

The worksheet interest "rates" in the Excel file Final2022.xlsx shows daily data from the beginning of the current year (2022) through October 12 on the following variables: yield rates for the 1-month treasury bill, 2-year treasury note, 10-year treasury note, 30-year treasury bond.

Compute the sample correlation between 1-month yield rates and the other three rates: 2-year, 10-year, and 30-year rates. Then determine which of the following statements about hypothesis tests hold at the 5% significance level:

We reject the null hypothesis that the correlation between the 1-month rate and the 2-year rate is zero.

We reject the null hypothesis that the correlation between the 1-month rate and the 10-year rate is zero.

We reject the null hypothesis that the correlation between the 1-month rate and the 30-year rate is zero.

orrect Answer

All of the above

None of the above

Unanswered

Question 44

0 / 2 pts

Run a regression of 30-year yield rates on 2-year yield rates. That is, the response (Y) variable is the 30-year yield rate and the predictor (X) variable is the 2-year yield rate.

What is the R-square of this regression model?

ou Answered

orrect Answers

0.9432 (with margin: 0.001)

Unanswered

Question 45

0 / 2 pts

Write down the estimate of the slope.

ou Answered

orrect Answers

0.53 (with margin: 0.001)

Unanswered

Question 46

0 / 2 pts

What regression coefficients are statistically significant at the 5% significance level?

- Intercept only
- Slope only

orrect Answer

- Both intercept and slope
- Neither intercept nor slope

Unanswered

Question 47

0 / 2 pts

The next four questions, including this one, concern the following setup.

According to the most recent Gallup poll of 812 randomly selected adults in the US, 42% of them approve of the job Joe Biden is doing as president.

Compute the margin of error of a 0.99 confidence interval for Joe Biden's approval rating.

ou Answered

orrect Answers

0.044 (with margin: 0.001)
4.4 (with margin: 0.1)

Unanswered

Question 48

0 / 2 pts

Test the null hypothesis that Joe Biden's approval rating is 45% against the two-sided alternative that it is different from 45% at the 0.01 significance level.

To that end, compute first the relevant statistic (Z-stat or t-stat, whatever applies).

ou Answered

orrect Answers

1.7183 (with margin: 0.1)
-1.7183 (with margin: 0.1)

Unanswered

Question 49

0 / 2 pts

Compute the p-value of the above test.

ou Answered

orrect Answers

0.08 (with margin: 0.01)

Unanswered

Question 50

0 / 2 pts

As a result of the above test, we reject the null hypothesis that Joe Biden's approval rating is 45% against the two-sided alternative that it is different from 45% at the 0.01 significance level.

- True
- False

orrect Answer