Question 1

1 pts

I hereby state that I will neither provide nor accept unauthorized assistance on this exam. Furthermore, I will not communicate with anybody else about the exam before Monday October 17, 2022.

- True
- False

Question 2

2 pts

Suppose A and B are independent events in some sample space with a probability measure. Then A^c and B^c are independent.

- True
- False

Question 1

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2 pts

Suppose A and B are independent events in some sample space with a probability measure. Then A^c and B^c are independent.

week 1

- True
- False

If A & B are independent, their complements are also independent

Question 3

2 pts

Suppose A and B are events in some sample space with a probability measure with both $0 < \mathbb{P}(A) < 1$ and $0 < \mathbb{P}(B) < 1$. If $\mathbb{P}(A|B) = 1$ then $\mathbb{P}(B^c|A^c) = 1$.

week 1

conditional probability

$P(A|B) = P(B^c|A^c)$

- ☒ True
- ○ False

Question 4

2 pts

Suppose X is a continuous random variable with cumulative probability function $F(x)$. If $a \leq b$ then $F(a) \leq F(b)$.

week 3

- ☒ True
- ○ False

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Question 5

2 pts

Suppose X is a continuous random variable with density function $f(x)$. The function $f(x)$ can never be greater than one.

week 3

- True
- False

total area under $f(x) = 1$

density function CAN be > 1, but area/probability CAN'T

Question 6

2 pts

Suppose X is a random variable and a, b are constants. Then $\mathrm{stdev}(aX + b) = |a| \cdot \mathrm{stdev}(X)$.

Here stdev(X) means standard deviation of X.

week 4

$var(Y) = var(ax+b) = a^2 \cdot var(x)$

- True
- False

Question 7 — 2 pts

Suppose $X \sim B(m,p)$ and $Y \sim B(n,p)$ where $p \in (0,1)$ and $m < n$. Then $\mathrm{var}\left(\frac{X}{m}\right) > \mathrm{var}\left(\frac{Y}{n}\right)$.

$m<n$

$\frac{1}{m} > \frac{1}{n}$

$\frac{p(1-p)}{m} > \frac{p(1-p)}{n}$

- [x] True
- [ ] False

week 3/4

Question 8 — 2 pts

If X and Y are independent random variables then $\mathbb{E}(X \cdot Y) = \mathbb{E}(X) \cdot \mathbb{E}(Y)$.

- [x] True
- [ ] False

week 4

slide 9

Question 9 — 2 pts

If X and Y are independent random variables then $\mathrm{var}(X - Y) = \mathrm{var}(X) - \mathrm{var}(Y)$.

week 4

slide 17

- [ ] True
- [x] False

$\mathrm{var}(X-Y) = \mathrm{var}(X) + \mathrm{var}(Y)$

Question 10 — 2 pts

The correlation of two random variables is always between -1 and 1.

week 6

slide 6

- [x] True
- [ ] False

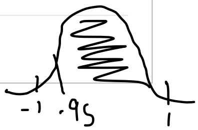
Question 11

2 pts

Suppose Z is a standard normal variable. Then for all $a \neq 0$ we have $\mathbb{P}(a-1 \leq Z \leq a+1) < \mathbb{P}(-1 \leq Z \leq 1)$.

use $a = .05$ to test $\longrightarrow$ week 3 slide 26

- ~~True~~
- False



Question 12

2 pts

If the random variables $X_1, \ldots, X_n$ are iid standard normal then $\frac{X_1 + \cdots + X_n}{\sqrt{n}}$ is standard normal.

- ~~True~~
- False

week 4 slide

Standardization

$\mu = 0 \quad \sigma = 1$

in standard normal

$EU(x_1, \ldots, x_n)$

$= \frac{1}{\sqrt{n}} \to$ constant

$E(x_1, \ldots, x_n)$

$\downarrow$

0

$$\mathrm{var}\left(\frac{x_1 + \ldots + x_n}{\sqrt{n}}\right)$$

$$\frac{1}{n}\left(\frac{1 + \ldots + n}{n}\right) = 1$$

Question 13

2 pts

$p < .05$

If a hypothesis test rejects the null hypothesis at the 0.05 significance level, then it would also reject the null hypothesis at the 0.1 significance level.

Week 5
slide 34

- ✗ True
- ○ False

if $p < .05$,
$p < .10$,
so yes, reject

Question 14

2 pts

Suppose the probability that an airline passenger is a no-show" (that is, he/she does not show up for a flight) is 0.1. To take advantage of this opportunity, an airline decides to sell 10 seats for a 9-seat plane, 20 seats for a 18-seat plane, and 30 seats for a 27-seat plane. Which of the three planes is the least likely to be overbooked?

(A plane is overbooked if it does not have enough seats for the passengers who show up.)

week 2

- ✗ The 9-seat plane
- ○ The 18-seat plane
- ○ The 27-seat plane
- ○ All equally likely
- ○ Cannot be determined

$p = .1 \rightarrow$ no show $\quad p = .9 \rightarrow$ show

1 − BINOM.DIST(x, n, p, 1)

1 − BINOM.DIST(9, 10, .9, 1) = .3487

1 − BINOM.DIST(18, 20, .9, 1) = .3917

1 − BINOM.DIST(27, 30, .9, 1) = .4114

least % is least likely to be overbooked

Question 15
2 pts

Suppose var(X+Y) = 10 and var(X-Y) = 6.

Find cov(X,Y). If the information is insufficient, write 99999.

1

week 4

slide 6-9

var (x+y) = var (x) +var (y) + 2 cov (x,y)

var (x-y) = var(x)+ var (y) - 2 cov (x,y)

4 cov (x,y) = 4

cov (x,y) = 1

Question 16
2 pts

Suppose you take a Probability exam that consists of 100 multiple-choice questions. Your level of mastery of the subject is so that you can answer each question correctly with probability 0.9. What is the probability that you answer at least 85 of the 100 questions correctly?

week 2

- 0.85
- 0.90
- X 0.9601
- 0.99
- None of the above.

p correct = .9

p ≥ 85 ⟶

1 - BINOM (84, 100, .9, 1) = .9601

17) 1 - poiss.dist (0, 2, 0)

atleast one call

≥ 1

1 - poiss. dist (0, 2, 1)

expon. dist (1, 2, 1)

week 3

slide 13/14

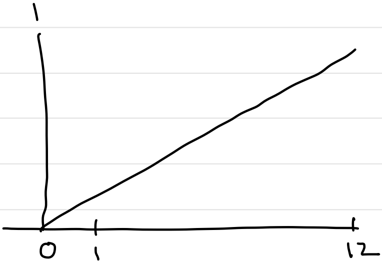
expon. dist (5, .4, 1)

Question 18

2 pts

A continuous random variable X has cumulative distribution function F(x) and the range of values that it takes is from $-\infty$ to ∞. Let A be the event {1<= X <= 7 } and let B be the event {3 <= X <= 12}. What is $\mathbb{P}(A \cup B)$?

week



- F(12) - F(1)
- F(7)-F(1)+F(12)-F(3)
- (F(7)-F(1))*(F(12)-F(3))
- (F(7)-F(1))/(F(12)-F(3))
- None of the above

Question 19

3 pts

Suppose you want to estimate the population proportion p via the sample proportion $\hat{p}$ Find the minimum number of samples that will guarantee a margin of error of 0.1 or less at the 0.95 confidence level?

97

week 5

slide 25 - 27

norm.s.inv(1 - .05/2) = 1.96

$$.1 = 1.96 * \frac{\sqrt{.5(1-.5)}}{\sqrt{n}}$$

$$.1 = 1.96 * \frac{.5}{\sqrt{n}}$$

$.1\sqrt{n} = .98$

$n = 96.04$

Question 20

2 pts

Consider the following experiment: You flip a fair coin repeatedly until the outcome is tails. What is the probability that you have to flip the coin at least four times?

.125

$p = .5$

$X \geq 4$

$X \leq 3$

$(1-p)^x$

$(1-.5)^3 = .125$

week 2
slide 8-9

Question 21

2 pts

week 3

The next two questions concern the following setup.

Suppose X is a normal random variable, the 0.1-quantile of X is 0.1, and the 0.9-quantile of X is 0.9.

Find the standard deviation of X.

$$\frac{.8\ (.9 - .1)}{\text{norm.s.inv}(.9) - \text{norm.s.inv}(.1)}$$

$= .3121 = \sigma$

Question 22

2 pts

Find the 0.95-quantile of X.

1.013

$\mu + \sigma * \text{norm.s.inv}(.1) = .1$

$\mu + .3121 * \text{norm.s.inv}(.1) = .1$

$\mu + -.39997 = .1$

$\mu = .4997$

$.4997 + .3121 * \text{norm.s.inv}(.95) = 1.013$

Question 23

3 pts

Suppose $X_1, \ldots, X_n$ are iid standard normal and consider their sample mean $\overline{X} := \frac{X_1 + \cdots + X_n}{n}$. Find the smallest n such that $|\overline{X}| \leq 0.2$ with probability at least 0.95. In other words, find the smallest n such that $\mathbb{P}(|\overline{X}| \leq 0.2) \geq 0.95$.

$\mathbb{P}(|\overline{X}| \leq 0.2) \geq .95$

$\mathbb{P}(-.2 \leq \overline{x} \leq .2) \geq .95$

acceptance interval: $\mu \pm z_{a/2} \cdot \frac{\sigma}{\sqrt{n}}$

standard normal: $\mu = 0$ $\sigma = 1$ $a = .05$

$0 + 1.96 \cdot \frac{1}{\sqrt{n}} \leq x \leq 0 - 1.96 \cdot \frac{1}{\sqrt{n}}$

$1.96 \cdot \frac{1}{\sqrt{n}} = .2$

$n = 96.04 = \boxed{97}$

Question 24

2 pts

Week 3
slide 19-20

The next three questions, including this one, refer to the following setup.

Suppose X is a discrete random variable that takes values $1, 2, \ldots, 10$ with probability $1/10$ each.

Find $\mathbb{E}(X)$.

$E(x) = \text{avg} = \dfrac{1+2+3+4+5+6+7+8+9+10}{10}$

$= 5.5$

5.5

Question 25

2 pts

Find $\operatorname{var}(X)$. $= 8.25$

$\operatorname{var}(x) = \dfrac{(10-5.5)^2 + (9-5.5)^2 + (8-5.5)^2 + (\ldots) \text{ etc}}{10}$

Question 26

3 pts

Suppose $X_1, \ldots, X_{100}$ are iid with the same distribution as X. Use the central limit theorem to compute the following probability approximately: $\mathbb{P}(530 \le X_1 + \cdots + X_{100} \le 570)$.

Week 4
slide 28-30

$Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\mu = 5.5 \times 100 = 550$

$\text{var} = 8.25 \times 100 = 825$

$\sigma = \sqrt{\text{var}} = \sqrt{825} = 28.7228$

$n = 100$

$= \text{norm.dist}(570, 550, 28.7228, 1) - \text{norm.dist}(530, 550, 28.7228, 1)$

$= \boxed{.51377}$

Question 27

2 pts

Week 4
slide 16

The next four questions, including this one, concern the following setup.

Suppose stocks A,B,C,D have annual expected returns and standard deviations as indicated in the following table:

	stock A	stock B	stock C	stock D
expected return	10%	14%	18%	22%
standard deviation	10%	15%	20%	25%

ER-σ = 0 = -1 = -2 = -3

var = 100 var = 15^2 = 225 var = 400 var = 625

Furthermore, suppose the annual returns are independent and have normal distribution.

Which one of the stocks is least likely to attain a negative return over the next year?

- X Stock A
- ○ Stock B
- ○ Stock C
- ○ Stock D
- ○ None of the above

Question 28

3 pts

Week 4
slide 10

Consider the following three possible allocations of capital to the above four stocks

Allocation 1: 50% stock A, 50% stock D

Allocation 2: 50% stock B, 50% stock C

Allocation 3: 25% stock A, 25% stock B, 25% stock C, 25% stock D

smallest

Compute the standard deviations of each of the above three allocations and write down the value of the smallest of them.

All 1: $(.5^2 * .01) + (.5^2 * .0625) = .018125$

All 2: $(.5^2 * .0225) + (.5^2 * .0400) = .015625$

All 3: $(.25^2 * .01) + (.25^2 * .0225) + (.25^2 * .04) + (.25^2 * .0625) = .00843$

All 1 = $\sqrt{.018125}$ = σ = .134629

All 2 = .125

$\boxed{\text{All 3} = .09182}$

Question 29

3 pts

ER All 1: $(.5 * .1) + (.5 * .22) = .16$

All 2: .16

All 3: .16

For each of the above three allocations compute the "Sharpe ratio":

(expected return)/(standard deviation). $\boxed{1.74}$

Write down the value of the largest of them.

ratio = .16 / .134629 = 1.188

.16 / .125 = 1.28

.16 / .09182 = 1.74

Question 30

2 pts

Which of the above four stocks or three allocations is most likely to attain or exceed a target return of 15%?

- ○ Stock A
- ○ Stock B
- ○ Stock C
- X Stock D
- ○ Allocation 1
- ○ Allocation 2
- ○ Allocation 3
- ○ They all have the same chance or attaining or exceeding 15%

Question 31

2 pts

Week 3 slide 6

The next three questions, including this one, concern the following setup.

Suppose the Federal Reserve Chairman Jerome Powell announces that the Fed will increase the federal funds rate sometime in November, 2022 and the increase could occur at any moment. This means that the amount of time X (in days) between the beginning of the day on November 1, 2022 and the moment when the federal funds rate increase occurs is a uniform random variable with range of values [0,30]. In other words, X ~ U(0,30).

Find the probability that the Fed increases the federal funds rate before the end of the day on November 10, 2022.

$1/30 \cdot (10-0) = 10/30 = \boxed{.3333}$

$f(x) = 1/30$

Question 32

2 pts

Find the probability that the Fed does not increase the federal funds rate before the end of the day on November 24, 2022.

.2

$1 - 1/30 \cdot (24-0) = .2$

Week 3 slide 7

Question 33

2 pts

Suppose the Fed does not increase the federal funds rate before the end of the day on November 10, 2022. Given that information and assuming Powell's original announcement will hold, find the probability that the Fed does not increase the federal funds rate before the end of the day on November 24, 2022.

$1 - 1/20 \cdot (24-10) = \boxed{.3}$

30-10

Q 34 : Table given

Question 35

2 pts

Compute the margin of error of a 95% confidence interval for the slope coefficient.

.016072

standard error

$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ norm.s.inv

$1.96 \cdot .0082 = .016072$

Week 5 slide 17

Question 36

2 pts

Multiple R in regression

Compute the sample correlation between mileage and horsepower. If the information available is insufficient, enter the value 99999.

-.54918

Week 6 slide 20

$R^2 = r^2_{xy}$

34) BOTH (p value < .05 in table)

$R^2 = .3016$

$R = \sqrt{.3016}$

$= .54918$

$= -.54918$

Question 37

2 pts

What mileage would you predict for a car with horsepower of 320? (Give a point estimate only.)

22.0138

$Y_{n+1} = b_0 + b_1 X_{n+1}$

$y = mx+b$

$y = -.0286(320) + 31.1658$

Week 6 slide 29

Question 38

$n < 30$ = t-dist

$n > 30$ = z-score (CLT)

2 pts

Week 5 slide 5

It is generally believed that SAT scores are related to income. Another commonly used predictor of SAT scores is the grade point average (GPA). The worksheet "SAT" in the Excel file Final2022.xlsx contains data on 24 randomly chosen students.

Compute the sample variance of SAT. Excel: var.S : 7134.601

Question 39

2 pts

Compute the margin of error of the 95% confidence interval for SAT.

35.66

$\sigma = \sqrt{7134.601} = 84.4666$

t-stat = 2.068

t.inv(1-α/2, n-1)

$moe = t_{stat} \cdot \frac{\sigma}{\sqrt{n}}$

$2.068 \cdot \frac{84.4666}{\sqrt{24}} = 33.79$

Question 40

2 pts

Compute the sample correlations between each of the three pairs of the variables SAT, Income, and GPA. Then determine which of the following statements about hypothesis tests hold at the 5% significance level:

compare x-variable p-values

- ○ We reject the null hypothesis that the correlation between SAT and Income is zero.
- ○ We reject the null hypothesis that the correlation between SAT and GPA is zero.
- ○ We reject the null hypothesis that the correlation between GPA and Income is zero.
- X We reject BOTH the null hypothesis that the correlation between SAT and Income is zero AND the null hypothesis that the correlation between SAT and GPA is zero.
- ○ None of the above

Question 41

2 pts

The next four questions, including this one, concern the following setup.

The worksheet interest "rates" in the Excel file Final2022.xlsx shows daily data from the beginning of the current year (2022) through October 12 on the following variables: yield rates for the 1-month treasury bill, 2-year treasury note, 10-year treasury note, 30-year treasury bond.

Compute the sample correlation between 1-month yield rates and the other three rates: 2-year, 10-year, and 30-year rates. Then determine which of the following statements about hypothesis tests hold at the 5% significance level:

- ○ We reject the null hypothesis that the correlation between the 1-month rate and the 2-year rate is zero.
- ○ We reject the null hypothesis that the correlation between the 1-month rate and the 10-year rate is zero.
- ○ We reject the null hypothesis that the correlation between the 1-month rate and the 30-year rate is zero.
- ○ All of the above
- ○ None of the above

Question 42

2 pts

Run a regression of 30-year yield rates on 2-year yield rates. That is, the response (Y) variable is the 30-year yield rate and the predictor (X) variable is the 2-year yield rate.

What is the R-square of this regression model?

.943165

r square on regression table

Question 43

2 pts

Write down the estimate of the slope.

x-variable on regression table

.53049

Question 44

2 pts

What regression coefficients are statistically significant at the 5% significance level?

p-values < .05 reject significant

- ○ Intercept only
- ○ Slope only
- X Both intercept and slope
- ○ Neither intercept nor slope

week 5 slide 25

Question 45

2 pts

=norm.s.inv(.005)

The next four questions, including this one, concern the following setup.

z= 2.5758

According to the most recent Gallup poll of 812 randomly selected adults in the US, 42% of them approve of the job Joe Biden is doing as president.

Compute the margin of error of a 0.99 confidence interval for Joe Biden's approval rating.

$$\text{moe} = 2.5758 \cdot \frac{\sqrt{.42(1-.42)}}{\sqrt{812}} = 0.0446$$

Question 46

2 pts

Test the null hypothesis that Joe Biden's approval rating is 45% against the two-sided alternative that it is different from 45% at the 0.01 significance level.

To that end, compute first the relevant statistic (Z-stat or t-stat, whatever applies).

Week 5 Slide 45

-1.718

$$z = \frac{.45-.42}{\sqrt{.42(1-.42)}/\sqrt{812}} = 1.73$$

$\hat{p} = .42$

$p_0 = .45$

Question 47

2 pts

Compute the p-value of the above test.

2* (1-norm.s.dist (1.73, 1)) = $\boxed{.0832}$

↓

two-sided

Question 48

2 pts

As a result of the above test, we reject the null hypothesis that Joe Biden's approval rating is 45% against the two-sided alternative that it is different from 45% at the 0.01 significance level.

- True
- False

p-val > .01 (α)

DO NOT REJECT