

Question 2 2 pts

Suppose A and B are independent events in some sample space with a probability measure. Then A^c and B^c are independent.

True
 False

Question 3 2 pts

Suppose A and B are events in some sample space with a probability measure with both $0 < P(A) < 1$ and $0 < P(B) < 1$. If $P(A|B) = 1$ then $P(B^c|A^c) = 1$.

True
 False

Question 4 2 pts

Suppose X is a continuous random variable with cumulative probability function $F(x)$. If $a \leq b$ then $F(a) \leq F(b)$.

True
 False

Question 5 2 pts

Suppose X is a continuous random variable with density function $f(x)$. The function $f(x)$ can never be greater than one.

True
 False

Question 6 2 pts

Suppose X is a random variable and a, b are constants. Then $\text{stdev}(aX + b) = |a| \cdot \text{stdev}(X)$. Here $\text{stdev}(X)$ means standard deviation of X .

True
 False

Question 7 2 pts

Suppose $X \sim B(m, p)$ and $Y \sim B(n, p)$ where $p \in (0, 1)$ and $m < n$. Then $\text{var}\left(\frac{X}{m}\right) > \text{var}\left(\frac{Y}{n}\right)$.

True
 False

Question 8 2 pts

If X and Y are independent random variables then $E(X \cdot Y) = E(X) \cdot E(Y)$.

True
 False

Question 9 2 pts

If X and Y are independent random variables then $\text{var}(X - Y) = \text{var}(X) - \text{var}(Y)$.

True
 False

Question 10 2 pts

The correlation of two random variables is always between -1 and 1 .

True
 False

Question 11 2 pts

Suppose Z is a standard normal variable. Then for all $a \neq 0$ we have $P(a-1 \leq Z \leq a+1) < P(-1 \leq Z \leq 1)$.

True
 False

Question 12 2 pts

If the random variables X_1, \dots, X_n are iid standard normal then $\frac{X_1 + \dots + X_n}{\sqrt{n}}$ is standard normal.

Q3) $P(A|B)=1 \Rightarrow P(B^c|A^c)=1$
true

$P(A|B)=1 \Leftrightarrow$ "if B happens, then A happens"
 \Rightarrow "if A does not happen, then B does not happen"
 $\Leftrightarrow P(B^c|A^c)=1$

P Table

	B	B^c	
A	$P(A B)$?	$P(A)$
A^c	0	$P(A^c)$	$P(A^c)$
	$P(B)$	$P(B^c)$	1

$$P(A^c) = P(A^c \cap B^c)$$

$$\therefore P(A|B)=1 \Leftrightarrow 1 = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow 1 = \frac{P(A^c \cap B^c)}{P(A^c)} = P(B^c|A^c)$$

$$\begin{aligned} \text{Var}(x+y) &= \text{Var}(Ix + -Iy) = I^2 \cdot \text{Var}(x) + (-I)^2 \cdot \text{Var}(y) \\ &\quad + 2(I)(-I) \text{cov}(x, y) \\ &\text{(relevant to Q15)} \end{aligned}$$

Q4) as you move to b from a, area will be larger by definition

standard normal $P(a-1 \leq z \leq a+1)$

Q1D
1/2

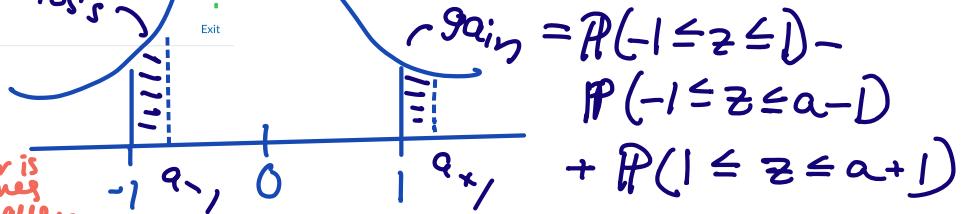
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Question 12 2 pts

If the random variables X_1, \dots, X_n are iid standard normal then $\frac{X_1 + \dots + X_n}{\sqrt{n}}$ is standard normal.

True
 False

$\bar{X} = \frac{X_1 + \dots + X_n}{n} \Rightarrow N(0, \frac{1}{n})$ var is n-times smaller



Question 13 2 pts

If a hypothesis test rejects the null hypothesis at the 0.05 significance level, then it would also reject the null hypothesis at the 0.1 significance level.

True
 False

[Redacted]

Question 14 2 pts

Suppose the probability that an airline passenger is a "no-show" (that is, he/she does not show up for a flight) is 0.1. To take advantage of this opportunity, an airline decides to sell 10 seats for a 9-seat plane, 20 seats for a 18-seat plane, and 30 seats for a 27-seat plane. Which of the three planes is the least likely to be overbooked?

(A plane is overbooked if it does not have enough seats for the passengers who show up.)

The 9-seat plane
 The 18-seat plane
 The 27-seat plane
 All equally likely
 Cannot be determined
 None of the above

Q14) let $X = \text{shows}$ (for 9-seat)
 $X \sim B(10, 0.9)$

want $P(\text{overbooking})$

$$P(X > 9) = 1 - P(X \leq 9)$$

$$= 1 - \text{binom.dist}(9, 10, 0.9, 1)$$

$$\text{Var}(x-y) = \text{Var}(x) + \text{Var}(y) + 2ab \cdot \text{cov}(x,y)$$

Q15) Suppose $\text{var}(X+Y) = 10$ and $\text{var}(X-Y) = 6$. Find $\text{cov}(XY)$. If the information is insufficient, write 999999.

$\begin{aligned} 10 &= \text{var}(x) + \text{var}(y) + 2ab \cdot \text{cov}(x,y) \\ 6 &= \text{var}(x) + \text{var}(y) - 2ab \cdot \text{cov}(x,y) \end{aligned}$

1

subtract
systems of
equations

$$\begin{aligned} 16 &= 4ab \cdot \text{cov}(x,y) \\ \Rightarrow ab \cdot \text{cov}(x,y) &= 4 \end{aligned}$$

Question 16 2 pts

Suppose you take a Probability exam that consists of 100 multiple-choice questions. Your level of mastery of the subject is so that you can answer each question correctly with probability 0.9. What is the probability that you answer at least 85 of the 100 questions correctly?

0.85
 0.90
 0.9001
 0.99
 None of the above.

Q16) let $X = \text{score on exam}$

$$X \sim B(100, 0.9)$$

want $P(X \geq 85) = 1 - P(X \leq 84)$

$$= 1 - \text{binom.dist}(84, 100, 0.9, 1)$$

Q17) $A = \{1 \leq x \leq 7\}$

$B = \{3 \leq x \leq 12\}$

Question 17 2 pts

Suppose calls arrive at a 1-800 phone line following a Poisson process with average arrival rate of 24 calls per hour.

Which of the following is NOT a correct way of computing the probability that at least one call arrives within the next 5 minutes?

1-poisson.dist(0,2.0) = 1 - dpois(0,2)
 1-poisson.dist(0,2.1) = 1 - ppois(0,2)
 expov.distr(1,2.1) = pexp(1,2)
 expov.distr(5,12.1) = pexp(5,12)
 None of the above. In other words, all of the above are correct.

= 0 (reason)

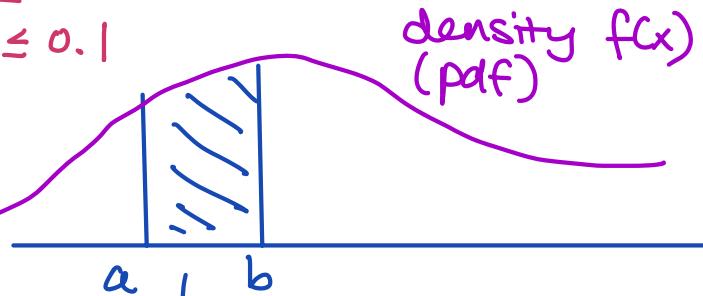
Question 18 2 pts

A continuous random variable X has cumulative distribution function $F(x)$ and the range of values that it takes is from $-\infty$ to ∞ . Let A be the event $\{1 \leq X \leq 7\}$ and let B be the event $\{3 \leq X \leq 12\}$. What is $P(A \cup B)$?

$F(12) - F(1)$

CONTINUOUS random variables:

$$\text{Q19) } \text{MOE} = \frac{Z_{0.05} \sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \leq 0.1$$



$$P(a \leq X \leq b) = F(b) - F(a)$$

cumulative distribution f(x)

* largest possible value of $\hat{p}(1-\hat{p})$ = 0.25 when $p=0.5$

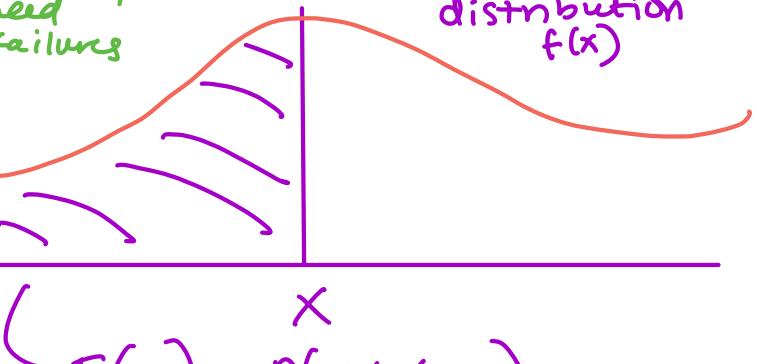
$$\Rightarrow n \geq 96.04$$

Q20) heads first 3

$$P(X=4) = (1-p)^{x-1}$$

$$= (0.5)^3 (0.5)$$

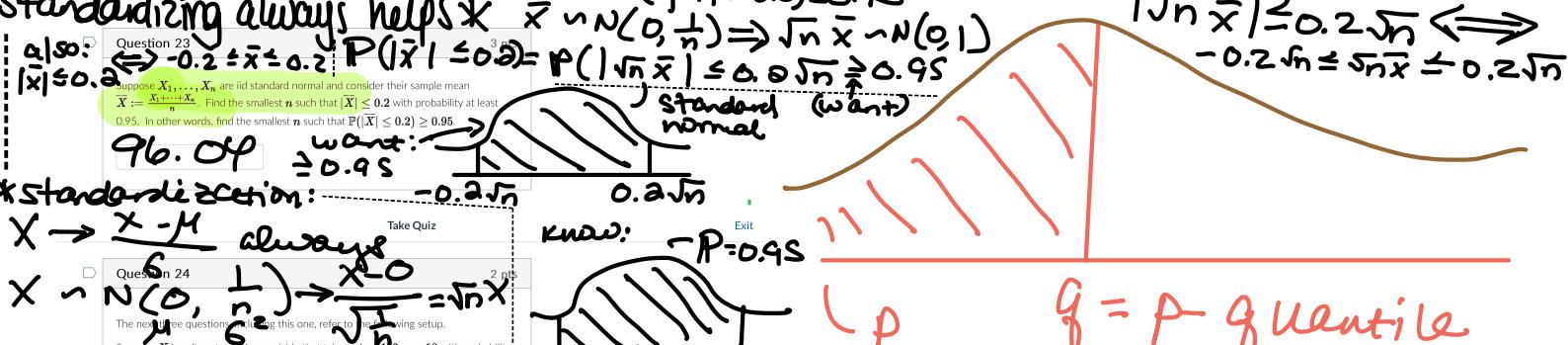
eliminate successes
b/c I only need failures



$$F(x) = P(X \leq x)$$

Q21) 1. D134

$$= \text{norm.inv}(0.95, 0.5, 6)$$



$$\text{also: } 1\sqrt{n}\bar{x} \leq 0.2\sqrt{n} \iff -0.2\sqrt{n} \leq \bar{x} \leq 0.2\sqrt{n}$$

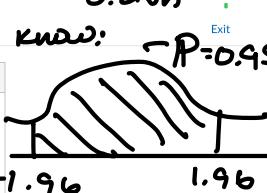
* standardization: $X \rightarrow \frac{X-\mu}{\sigma}$ always

$$X \sim N(0, \frac{1}{n}) \rightarrow \frac{X-0}{\sqrt{n}} = \sqrt{n}X$$

The next three questions share this one, refer to the following setup.

Suppose X is a discrete random variable that takes values 1, 2, ..., 10 with probability 1/10 each. Find $E(X)$.

$$0.55$$



$g = p$ quantile

$$P(X \leq g) = p$$

④ $P(S.3 \leq \frac{x_1 + \dots + x_{100}}{100} \leq S.7) \approx N$

$$\text{⑤ approx. } N + \text{use CLT} \quad \text{⑥ } \sqrt{n} \geq 9.8 \Rightarrow n \geq (9.8)^2$$

$$\text{⑦ norm.dist } P(S.3 \leq \frac{x_1 + \dots + x_n}{100} \leq S.7) = \text{norm.dist}(S.7, S.5, \sqrt{0.085}, 1) - \text{norm.dist}(S.3, \dots)$$

CLT: $E(\frac{x_1 + \dots + x_{100}}{100}) = S.5$

Question 26) 0.5138 vs $(\frac{x_1 + \dots + x_{100}}{100})^{3.00} = 8.25$

Question 27) $\frac{100}{100} = N(550, 8.25)$

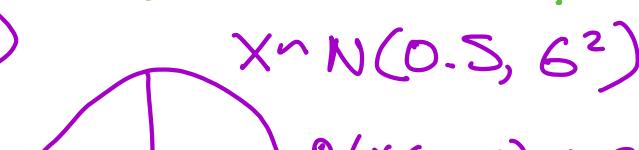
Q28) which has smallest tail? when Q is furthest from μ in C^2

stock A	stock B	stock C	stock D
expected return	10%	15%	20%
standard deviation	10%	15%	20%

continuous distributions:

t-tails are wider,
w/ more narrow normal

Q29)



$$\begin{aligned} & \therefore 1 - P(X \leq 24) \\ & = 1 - \frac{4}{5} = \boxed{\frac{1}{5}} \end{aligned}$$

Take Quiz

Question 33 2 pts

Suppose the Fed does not increase the federal funds rate before the end of the day on November 10, 2022. Given that information and assuming Powell's original announcement will hold, find the probability that the Fed does not increase the federal funds rate before the end of the day on November 24, 2022.

0.3333

Q33) the new $\Rightarrow F(24) \int \frac{24-10}{30-10} = \frac{14}{20} = \frac{7}{10}$

$H_0 = P(X \leq 24) \sim U(10, 30) = 1 - \frac{7}{10} = \boxed{\frac{3}{10}}$

Take Quiz

Question 34 2 pts

The next four questions, including this one, concern the following setup.

The Environmental Protection Agency (EPA) collects data, such as horsepower and highway mileage on 30 different models of small trucks, with horsepower between 280

$X = \text{hp}$
 $Y = \text{mpg}$

Take Quiz Q34

Exit

Regression Statistics	
Multiple R square	0.5492
R-square	0.3016
Adjusted R square	0.2766
Standard error	3.5532
Observations	30
Coefficients	Std Error
Intercept	31.1658
Horse Power	-0.0286
	0.0082
	3.4772
	0.0017

if statistically significant, reject H_0

t tests --

P-value

What regression coefficients are statistically significant at the 5% significance level?

$\alpha = 0.05$ meaning strong dependence

- Intercept only
- Slope only
- Both intercept and slope
- Neither intercept nor slope

-stat: ① reject if $|t_{n-1, \alpha/2}| > t_{n-1, \alpha/2}$ * Java suggests using p-value

test 1: $|t_{n-1, \alpha/2}| = 3.4772$ given

$$P(|t_{n-1}| > |t_{n-1, \alpha/2}|)$$

if p-value $< \alpha$

test 2: $p\text{-Value} = 0.0017$

$\alpha = 0.05$

$0.0017 < 0.05$

$\therefore p\text{-value} < \alpha$

$\therefore \text{reject } H_0$

Take Quiz

Question 35 2 pts

Compute the margin of error of a 95% confidence interval for the slope coefficient.

0.0168

$$n-1, \alpha/2 = -2.9 \xrightarrow{\text{approx}} +1.96$$

$$\Rightarrow |t_{n-1, \alpha/2}| = 1.96$$

$$\therefore 1.96 \times 0.0286 = 0.0452$$

$\therefore \text{reject } H_0$

in (simple) linear regression:

$$R\text{-square} = r_{xy}^2 = 0.3016$$

$$\therefore \sqrt{r_{xy}^2} = r_{xy} = \text{sample correlation} = \sqrt{0.3016}$$

* it is negative b/c the $x = \text{hp}$ coefficient is negative!

NEGATIVE

Q37) horse power coefficient = -0.0286

$$y = \text{mpg} = (320) - 0.0286 = \boxed{-9.152}$$

(x) (coefficient of x)

Q38) confidence interval: h. +

~~\$S_{b_1}~~ is "Standard Err" & MOE: $t_{n-2, \frac{\alpha}{2}} \cdot S_{b_1}$

Compute the sample variance of SAT. $n=24$ (Q38)
 $1134.6015 = \text{var. s}(SAT)$

Q39) two options:

Question 39: Compute the margin of error of the 95% confidence interval for SAT.
 ± 35.7570

Question 40: Compute the sample correlations between each of the three pairs of the variables SAT, Income, and GPA. Then determine which of the following statements about hypothesis tests hold at the 5% significance level:
 We reject the null hypothesis that the correlation between SAT and Income is zero.
 We reject the null hypothesis that the correlation between SAT and GPA is zero.
 We reject the null hypothesis that the correlation between GPA and Income is zero.
 reject BOTH the null hypothesis that the correlation between SAT and Income is zero AND the null hypothesis that the correlation between SAT and GPA is zero.
 None of the above

$t_{n-2, \frac{\alpha}{2}} = t_{28, 0.025} \quad \& \quad S_{b_1} = 0.0082$

$= 0.0082 \cdot t.\text{inv}(0.975, 28)$

MOE: $\pm t_{n-2, \frac{\alpha}{2}} \cdot \frac{6}{\sqrt{n}}$ (we have variance & n, so we have St. dev.)
 * in previous weeks, CI used $n-1$ d.f. but for this, use $n-2$ *

$= \frac{\sqrt{\text{Sample variance from Q38}}}{\sqrt{24}} \cdot t.\text{inv}(0.975, 22)$

(Q41) V. similar to Q40
 Question 41: $t\text{-stat} = 0.5584 \quad p\text{-value} = 0.00002$ -- So REJECT

The worksheet interest "rates" in the Excel file Final2022.xlsx shows daily data from the beginning of the current year (2022) through October 12 on the following variables: yield rates for the 1-month treasury bill, 2-year treasury note, 10-year treasury note, 30-year treasury bond.

1mo x 2yr: REJECT
 1mo x 10yr: REJECT
 1mo x 30yr: REJECT

Compute the sample correlation between 1-month yield rates and the other three rates: 2-year, 10-year, and 30-year rates. Then determine which of the following statements about hypothesis tests hold at the 5% significance level:
 We reject the null hypothesis that the correlation between the 1-month rate and the 2-year rate is zero.
 We reject the null hypothesis that the correlation between the 1-month rate and the 10-year rate is zero.
 We reject the null hypothesis that the correlation between the 1-month rate and the 30-year rate is zero.
 All of the above
 None of the above

② Data → Data Analysis → Descriptive Statistics: ✓ Summary Statistics;
 - USE Standard Error * $t.\text{inv}(0.975, 22)$

(Q40) reject H₀ if p-value < α OR $|t\text{-stat}| > t_{n-2, \frac{\alpha}{2}}$
 $t\text{-stat} = r_{xy} \cdot \sqrt{\frac{n-2}{1-r_{xy}^2}}$ BUT if we run regression analysis, t-stat & p-values are given
 SO... run Data → Data Analysis → Regression:
 X-values: SAT; Y-values: Income, GPA,
 then X-values: GPA w/ Y-values: Income

$H_0: r_{(SAT, \text{Income})} = 0$; $H_0: r_{(SAT, \text{GPA})} = 0$; $H_0: r_{(\text{GPA}, \text{Income})} = 0$	$p\text{-value} = 0.00002$; $p\text{-value} = 0.00002$; $p\text{-value} = 0.00002$
$0.00002 < 0.05$	$0.00002 < 0.05$
Reject	Reject
$ t\text{-stat} = 2.4990 $	$ t\text{-stat} = 5.4014 $
$2.4990 > 2.0739$	$5.4014 > 2.0739$
Reject	Reject

$0.6649 > 0.05$

Accept

$|t\text{-stat}| = |-0.4391|$

$0.4391 < 2.0739$

Accept

Q43) Slope estimate: coefficient of x-value

Question 42: Run a regression of 30-year yield rates on 2-year yield rates. That is, the response (Y) variable is the 30-year yield rate and the predictor (X) variable is the 2-year yield rate.
 What is the R-square of this regression model?
 0.9432

Question 43: Write down the estimate of the slope.
 0.5305

Question 44: What regression coefficients are statistically significant at the 5% significance level?
 Intercept only

Take Quiz

Q44) Slope is β -value testing
 Intercept is intercept testing

Do P-value or t-stat tests
 Statistically significant means they are concluded/reject H_0

Q45) $n=812 \hat{p}=0.42$ 0.99 CI for p

$$\hat{p} \pm MOE \quad MOE = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= z_{0.005} \cdot \sqrt{\frac{0.42(0.58)}{812}}$$

$$= \text{norm.s.dist}(0.995) * \text{sqrt}\left(\frac{0.42 * 0.58}{812}\right)$$

Q46) test $H_0: p=0.45$

$$Z\text{-Stat} = \frac{\hat{p} - 0.45}{\sqrt{\frac{0.45(0.55)}{812}}} = \frac{0.42 - 0.45}{\sqrt{\frac{0.45(0.55)}{812}}}$$

$$\Rightarrow Z\text{-stat} = -1.7184$$

P-value: $P(\text{standard normal } |z| > |z\text{-stat}|)$

$$= 2(1 - \text{norm.s.dist}(|z\text{-stat}|, 1))$$

= 0.0857 ∴ do not reject/accept b/c

P-value > 0.05

*accidentally skipped
 Q 25! *

list X & $P(X)$
 in excel,
 $=\text{sumproduct}(X\text{ values}, \text{probabilities})$

= Var.p(array of X-values)

= Var.p(1:10)

= $\lceil (1-\bar{x})^2 + (2-\bar{x})^2 + (3-\bar{x})^2 + \dots + (10-\bar{x})^2 \rceil$

Question 44

2 pts

What regression coefficients are statistically significant at the 5% significance level?

- Intercept only
 - Slope only
 - Both intercept and slope
 - Neither intercept nor slope
- x both t -stars are huge & p-values are very small *

Question 45

2 pts

The next four questions, including this one, concern the following setup.

According to the most recent Gallup poll of 812 randomly selected adults in the US, 42% of them approve of the job Joe Biden is doing as president.

Compute the margin of error of a 0.99 confidence interval for Joe Biden's approval rating.

± 0.0446

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Exit

Question 46

2 pts

Test the null hypothesis that Joe Biden's approval rating is 45% against the two-sided alternative that it is different from 45% at the 0.01 significance level.

To that end, compute first the relevant statistic (Z-stat or t-stat, whatever applies).

-1.7184

Question 47

2 pts

Compute the p-value of the above test.

0.0857

Take Quiz

Exit

Question 47

2 pts

Compute the p-value of the above test.

0.0857

Question 48

2 pts

As a result of the above test, we reject the null hypothesis that Joe Biden's approval rating is 45% against the two-sided alternative that it is different from 45% at the 0.01 significance level.

 True False

Quiz saved at 4:23pm

Submit Quiz

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Exit

Question 24

2 pts

The next three questions, including this one, refer to the following setup.

Suppose X is a discrete random variable that takes values 1, 2, ..., 10 with probability $1/10$ each.Find $E(X)$.

0.55

Question 25

2 pts

Find $\text{var}(X)$.

8.25

= Var.p(array of X-values)

= Var.p(1:10)

= $\lceil (1-\bar{x})^2 + (2-\bar{x})^2 + (3-\bar{x})^2 + \dots + (10-\bar{x})^2 \rceil$

$$\left[(1 - 5.5)^2 + (2 - 5.5)^2 + (3 - 5.5)^2 + (4 - 5.5)^2 + (5 - 5.5)^2 + (6 - 5.5)^2 + (7 - 5.5)^2 + (8 - 5.5)^2 + (9 - 5.5)^2 + (10 - 5.5)^2 \right] / 10$$