

Ques-2 Given

$$y = mx + h \quad \& \quad P(x, y)$$

- 1) perform translation & Rotation to move the given line on  $x$ -axis

$$\text{Translation } T = T(0, -h)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -h \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rotation } R = R(-\theta)$$

$$= \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, we've got the given line  $y = mx + h$  on the  $x$ -axis.  
Now, we apply reflection(S)

y Reflecting.

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) inverse of step(1)

$$T^{-1} = T(0, h)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & h \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^{-1}(\theta) = (R(\theta))^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4) Concatenating step (1), (2) & (3)

$$(T^{-1}(R^{-1}(S(R(T)))))) = M = \begin{bmatrix} \cos 2\theta & \sin 2\theta & -b \sin \theta \\ \sin 2\theta & -\cos 2\theta & b \cos \theta \\ 0 & 0 & 1 \end{bmatrix}$$

Ques-3 Given two 2D-Points.

$$P_1 = (x_1, y_1) \quad P_2 = (x_2, y_2)$$

Let  $v_1$  be the vector

$$v_1 = P_2 - P_1$$

a) the parametric line expression in vector form:

$$\begin{aligned} P(\alpha) &= P_1 + \alpha v_1 \\ &= P_1 + \alpha (P_2 - P_1) \\ &= \alpha P_2 + P_1(1 - \alpha) \end{aligned}$$

b) the parametric line expression in individual coordinates:

$$x(\alpha) = \alpha x_2 + x_1(1 - \alpha)$$

$$y(\alpha) = \alpha y_2 + y_1(1 - \alpha)$$

Ques-4

Let be three vertex

$$V_1 = (x_1, y_1, z_1)$$

$$V_2 = (x_2, y_2, z_2)$$

$$V_3 = (x_3, y_3, z_3)$$

the value

if area of triangle is equal to "zero", then  
given three vertices are collinear.

$$\text{Area of triangle} = \frac{1}{2} \|\vec{A} \times \vec{B}\|$$

$$A = V_2 - V_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$B = V_3 - V_1 = (x_3 - x_1, y_3 - y_1, z_3 - z_1)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x_2 - x_1) & (y_2 - y_1) & (z_2 - z_1) \\ (x_3 - x_1) & (y_3 - y_1) & (z_3 - z_1) \end{vmatrix}$$

$$(A \times B) = \underbrace{[(y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1)]}_{(A \times B)_x} - \underbrace{[(x_2 - x_1)(z_3 - z_1) - (x_3 - x_1)(z_2 - z_1)]}_{(A \times B)_y} + \underbrace{[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)]}_{(A \times B)_z}$$

$$\text{Area of triangle} = \frac{1}{2} \sqrt{(A \times B)_x^2 + (A \times B)_y^2 + (A \times B)_z^2} = 0$$