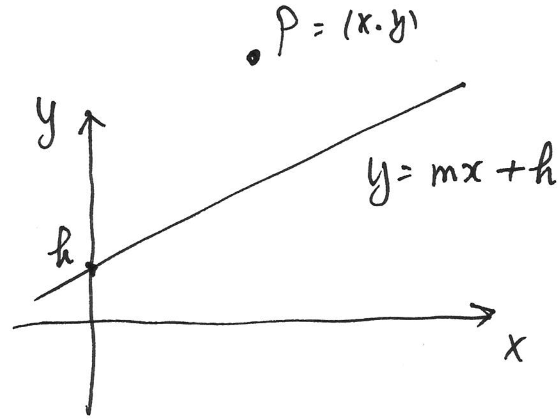
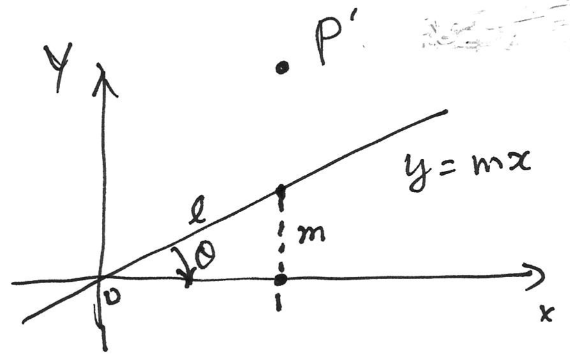
Q2.



Step 1. Translate the line down by $(dx, dy) = (0, h)$ such that the line will pass the origin

$$p' = T(0, -h) \cdot P$$



Step 2. Rotate around the origin for $-\theta$ such that the line with on x-axis.

$$R(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

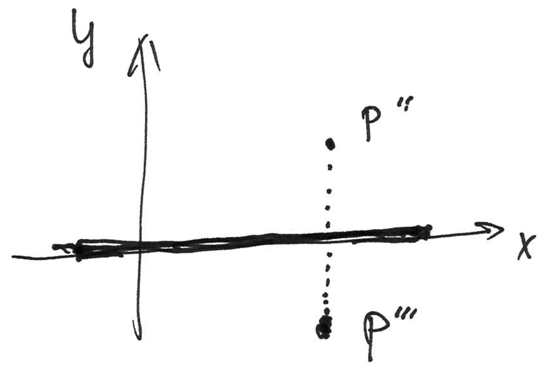
$$= \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}.$$

The $\sin(\theta)$ and $\cos(\theta)$ values can be directly derived from the triangle shown above.

$\sin(\theta) = \frac{m}{l}$ where $l = \sqrt{1^2 + m^2}$

$\cos(\theta = \frac{1}{l}$

$$P'' = R(-\theta) \cdot P'$$



Step 3. Flip P'' about x-axis

$$P''' = M_{Flip\text{-}x} \cdot P''$$

where $M_{Flip\text{-}x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

Step 4: Inverse of step 2:

$$R^{-1}(-\theta) = R(\theta)$$

Step 5: Inverse of Step 1:

$$T^{-1}(0,-h) = T(0,h).$$

~~So~~ concatanate the five steps together:

$$M = T(0,h) \cdot R(\theta) \cdot M_{Flip\text{-}x} \cdot R(-\theta) \cdot T(0,-h)$$

The matrix M will flip a point P about an arbitrary line $y = mx + h$.

Q3

(a) $P(\alpha) = P_1 + \alpha(P_2 - P_1)$

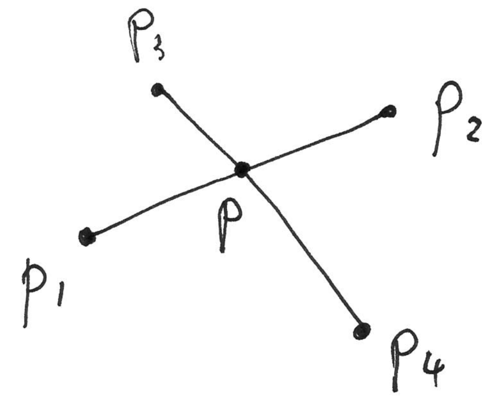
or

$P(\alpha) = (1-\alpha)P_1 + \alpha P_2$

(b). $x(\alpha) = x_1 + \alpha(x_2 - x_1)$

$y(\alpha) = y_1 + \alpha(y_2 - y_1)$

(c)



The intersection point P is on line P_1 to P_2.

so $\quad P = P_1 + \alpha_1(P_2 - P_1) \qquad (1)$

Similarly P is also on line P_3 to P_4.

so $\quad P = P_3 + \alpha_2(P_4 - P_3) \qquad (2)$

Therefore the problem becomes solving for α_1 and α_2.

(1) and (2) are equal:

$$P_1 + \alpha_1 (P_2 - P_1) = P_3 + \alpha_2 (P_4 - P_3)$$

It appears that we have only one equation for solving two unknowns. That is generally not possible. But, the above equation is the vector form. We actually have two equations:

$$\begin{cases} x_1 + \alpha_1 (x_2 - x_1) = x_3 + \alpha_2 (x_4 - x_3) \\ y_1 + \alpha_1 (y_2 - y_1) = y_3 + \alpha_2 (y_4 - y_3) \end{cases}$$

Now you can solve α_1 and α_2.

[Hint: if $x_1, y_1, \ldots$ have numerical values, the equations will look simpler and more easily be solved],

Your answer should solve α_1 ~~and~~ α_2 in the symbolic form]

(d). If $0 \le \alpha_1 \le 1$, P_1 is between P_1 and P_2 otherwise, not.

Similarly for $0 \le \alpha_2 \le 1$.

Q4.

Consider the Vector from V_1 to V_2.

$$V_{1-2} = V_2 - V_1$$
$$= (V_{2x} - V_{1x},\ V_{2y} - V_{1y},\ V_{2z} - V_{1z})$$

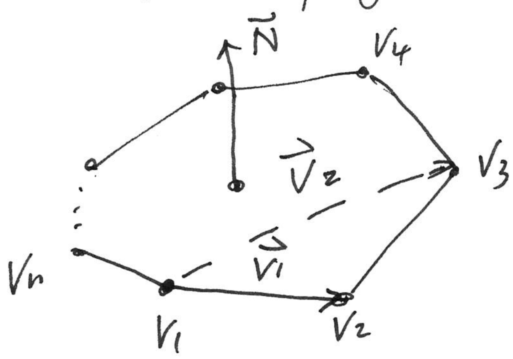
and the Vector from V_2 to V_3

$$V_{2-3} = V_3 - V_1$$

If V_{1-2} and V_{2-3} are parallel, then dot-product between the two:

$$V_{1-2} \bullet V_{2-3} = |V_{1-2}| \cdot |V_{2-3}| \cdot \cos(0^\circ)$$
$$= |V_{1-2}| \cdot |V_{2-3}|.$$

Q5. Assume $V_1, V_2, \dots V_n$ $(n \geq 3)$ be the list of vertices of the polygon.



Let $\vec{V_1} = V_2 - V_1$

$\vec{V_2} = V_3 - V_1$

the cross-product

$$\vec{V_1} \times \vec{V_2} = \vec{N}$$

give the normal vector.

For remaining vertices $V_4, \ldots V_n$, the vector from V_1 to V_i, where $4 \leq i \leq n$ must be perpendicular to $\vec{N}$ if V_i is on the same plane. i.e. the dot-product

$$\vec{N} \cdot (V_i - V_1) = \emptyset.$$
