UNIT- 1
NUMBER SYSTEMS & CODES

TOPIC 1.2
ARITHMETIC IN NUMBER SYSTEM

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BINARY ARITHMETIC

1. BINARY ADDITION
2. BINARY SUBTRACTION
3. BINARY MULTIPLICATION
4. BINARY DIVISION

BINARY ADDITION

0 + 0 = 0
1 + 0 = 1
0 + 1 = 1
1 + 1 = 0   1   (Carry bit)

1 1 0 1	(13 decimal)
+0 0 0 1	(+1 decimal)
1 1 1 0	(14 decimal)

BINARY ADDITION

10 +7 convert into binary n then add

```
   - - - -   (10 decimal)
 + - - - -   (+7 decimal)
 ---------
   - - - -   (-- decimal)
 ---------
```

21 +30 convert into binary n then add

```
   - - - -   (21 decimal)
 + - - - -   (+30 decimal)
 ---------
   - - - -   (-- decimal)
 ---------
```

BINARY SUBTRACTION

0 - 0 = 0
1 - 0 = 1
0 - 1 = 1   1 (Carry bit)
1 - 1 = 0

Borrow

1 1 0 1	(13 decimal)
- 0 0 1 1	(-3 decimal)
1 0 1 0	(10 decimal)

BINARY SUBTRACTION

10 – 4 convert into binary and then subtract

- - - -	(10 decimal)
- - - -	(- 4 decimal)
- - - -	(-- decimal)

14 – 6 convert into binary and then subtract

- - - -	(14 decimal)
- - - -	(- 6 decimal)
- - - -	(-- decimal)

BINARY MULTIPLICATION

$$
\begin{array}{rr}
1\,0\,0\,0 & =8_{10} \\
\times\ \underline{0\,1\,1\,0} & =6_{10} \\
0\,0\,0\,0 & \\
+\,1\,0\,0\,0 & \\
+\,1\,0\,0\,0 & \\
+\,\underline{0\,0\,0\,0} & \\
0\,1\,1\,0\,0\,0\,0 & =48_{10}
\end{array}
$$

BINARY MULTIPLICATION

- - - - $=2_{10}$

X - - - - $=7_{10}$

- -- - - - - - - - -

$=???_{10}$

- - - - $=6_{10}$

X - - - - $=5_{10}$

- -- - - - - - - - -

$=???_{10}$

BINARY DIVISION

```
011 ) 0 1 1 0 0 1 0    ( 1
      0 1 1
        0 0 0          (0
        0 0 0
          0 0 0        (0
          0 0 0
            0 0 1      (0
            0 0 0
              0 1 0    (0
```

Q=10000=16_{10}

R=10= 2_{10}

BINARY DIVISION

111) 0100110

Solution is

Q=----2=---10

R=---2= ---10

010)110110

Q=----2=---10

R=---2= ---10

BINARY DIVISION

Solution:
Your problem → binary division 0100110/111

Solution is

```
          1 0 1
     ___________
111 | 1 0 0 1 1 0
    - 1 1 1          = 111 x 1
    -----------
        1 0 1
          - 0        = 111 x 0
        -------
        1 0 1 0
      - 1 1 1        = 111 x 1
      ---------
            1 1
```

111 table
111 × 1 = 111
111 × 10 = 1110

∴ 100110 ÷ 111 = 101 Remainder 11

BINARY DIVISION

Solution is

```
      1 1 0 1 1
   ┌───────────
10 │ 1 1 0 1 1 0
   − 1 0            = 10 × 1
   ───────────
       1 0
     − 1 0          = 10 × 1
   ───────────
           1
         − 0        = 10 × 0
       ───────
           1 1
         − 1 0      = 10 × 1
       ───────
             1 0
           − 1 0    = 10 × 1
           ─────
               0
```

10 table
10 × 1 = 10
10 × 10 = 100

∴ 110110 ÷ 10 = 11011

REPRESENTATION OF NEGATIVE NUMBER

Sign-Magnitude representation-

1. " + " sign before a number indicates it as a positive number
2. "-" sign before a number indicates it as a negative number

- Not very convenient on computers
- Replace " + "sign by " 0 "and " - "sign by " 1 "

$(+1100101)_2 = (01100101)_2$

$(+101.001)_2 = (0101.001)_2$

$(-10010)_2 = (110010)_2$

$(-110.101)_2 = (1110.101)_2$

Representing signed numbers

- **1's complement** binary numbers are very useful in Signed number representation. Positive numbers are simply represented as Binary number... But in case of negative binary number representation, **we** represent in **1's complement**.
- If the number is negative then it is represented using **1's complement**.

Range of n-bit numbers

One's complement numbers:

0111111	-->	+	63
0000110	-->	+	6
0000000	-->	+	0
1111001	-->	-	6
1000000	-->	-	63

- 0 is represented by 000.....0
- 7- bit number covers the range from +63 to -63.
- n-bit number has a range from $+(2^{n-1} - 1)$ to $-(2^{n-1} - 1)$

Example: One's complement

1111001= (1)(111001)

- First (sign) bit is 1: The number is negative
- One’s Complement of
- 111001 is = 000110

$=(6)_{10}$

One's complement of a number

Complement all the digits-

- If A is an integer in one's complement form, then one's complement of A = -A

This applies to fractions as well.

A = 0.101 , $(+0.625)_{10}$

- One's complement of A = 1.010, $(-0.625)_{10}$ Mixed number

B = 010011.0101 , $(+19.3125)_{10}$

One's complement of B = 101100.1010 , $(-19.3125)_{10}$

Two's Complement Representation

If MSD is a 0 -The number is positive

- Remaining (n-1) bits directly indicate the

magnitude If the MSD is 1-The number is negative

- Magnitude is obtained by complementing all the remaining (n-1) bits and adding a 1

Two's complement allows negative and positive numbers to be added together without any special logic. ... This means that subtraction and addition of both positive and negative numbers can all be done by the same circuit in the CPU.

Range of n-bit numbers

Two's complement numbers:

0111111	+ 63
0000110	+ 6
0000000	+ 0
1111010	- 6
1000001	- 63
1000000	- 64

- 0 is represented by 000.....0
- 7- bit number covers the range from +63 to -64.
- n-bit number has a range from $+(2^{n-1} - 1)$ to $-(2^{n-1})$

Example: Two's complement

1111010 = (1)(111010)

First (sign) bit is 1:The number is negative

Complement 111010 and add 1= 000101 + 1

= 000110 = $(6)_{10}$

Two's complement of a number

Complement all the digits

If A is an integer in one's complement form, then

- Two's complement of A= -A
- This applies to fractions as well
  - A = 0.101 , (+0.625)10

- Two's complement of A = 1.011$(-0.625)_{10}$ Mixed number
  - B = 010011.0101, $(+19.3125)_{10}$
  - Two's complement of B = 101100.1011, $(- 19.3125)_{10}$

The brackets around the msb (the sign bit) are included here for clarity but brackets are not normally used. Because only 7 bits are used for the actual number, the range of values the system can represent is from (−127)10 or (11111111)2, to (+127)10.

A comparison between signed binary, pure binary and decimal numbers is shown in Table in next ppt. Notice that in the signed binary representation of positive numbers between +010 and +12710, all the positive values are just the same as in pure binary.

Table 1.4.1			
Binary	**Decimal**	**Signed Binary**	
11111111	255	−127	
11111110	254	−126	
11111101	253	−125	
11111100	252	−124	
			−
10000011	131	−3	
10000010	130	−2	
10000001	129	−1	
10000000	128	−0	
01111111	127	+127	
01111110	126	+126	
01111101	125	+125	
01111100	124	+124	
			+
00000011	3	+3	
00000010	2	+2	
00000001	1	+1	
00000000	0	+0	

Convert decimal number into 2’s complement form,assuming an 8 bit binary representation for all

1) 1
2) 72
3) -127

Convert 2’s complement form into decimal

1) 1111
2) 001101

BINARY SUBTRACTION using 1'S COMPLIMENT

The steps to be followed in subtraction by 1's complement are:

Suppose (A-B)

i) To write down 1's complement of the subtrahend(B).

ii) To add this with the minuend(A).

iii) If the result of addition has a carry over then it is dropped and an 1 is added in the **first bit**.

iv) If there is no carry over, then 1's complement of the result of addition is obtained to get the final result and it is negative.

BINARY SUBTRACTION

1'S COMPLIMENT

Step1-

0 0 0 0 0 1 0 1 (+5 decimal) 1
1 0 0 0 0 0 1 0 (-2 decimal)
0 1 1 1 1 1 0 1 (1'comp of -2) 2

1 0 0 0 0 0 1 0
\+ 1

0 0 0 0 0 1 1 (+3 decimal)

BINARY SUBTRACTION using 1'S COMPLIMENT

Evaluate: 110101 – 100101

Solution:
1's complement of 100101 is 011010. Hence

Minued -		1 1 0 1 0 1
1's complement of subtrahend -		0 1 1 0 1 0
Carry over -	1	0 0 1 1 1 1
		1
		0 1 0 0 0 0

BINARY SUBTRACTION using 1'S COMPLIMENT

Evaluate: 1011001 – 1101010

Solution:

1's complement of 1101010 is 0010101. Hence

Minued -		1 0 1 1 0 0 1
1's complement of subtrahend -	+	0 0 1 0 1 0 1
no Carry(ans) -		1 1 0 1 1 1 0
Convert ans. into 1's complement again		0 0 1 0 0 0 1

1011001 – 1101010= 0010001

BINARY SUBTRACTION using 1'S COMPLIMENT

1011.001 – 110.10
Solution:
1's complement of 0110.100 is 1001.011 Hence

Minued -		+	1 0 1 1 . 0 0 1
1's complement of subtrahend -			<u>1 0 0 1 . 0 1 1</u>
Carry over -	1		0 1 0 0 . 1 0 0
			<u>1</u>
			0 1 0 0 . 1 0 1

Hence the required difference is 100.101

BINARY SUBTRACTION using 1'S COMPLIMENT

1) Evaluate: 1111 – 0110

Solution: ??

2) 101011 – 111001

Solution: ??

3) 10110.01 – 11010.10

1s complement of (B)=00101.01

Add(A+B)   1 0 1 1 0 .0 1
0 0 1 0 1 .0 1

1 1 0 1 1 .1 0

No carry ,convert ans. into 1's complement=(00100.01)

BINARY SUBTRACTION using 2'S COMPLIMENT

The operation is carried out by means of the following steps:
(A-B)
(i) At first, 2's complement of the subtrahend(B) is found.
(ii) Then it is added to the minuend(A).
(iii) If the final carry over of the sum is 1, it is dropped and the result is positive.
(iv) If there is no carry over, the two's complement of the sum will be the result and it is negative.

BINARY SUBTRACTION using 2'S COMPLIMENT

(i) 110110 - 10110

Solution:

The numbers of bits in the subtrahend is 5 while that of minuend is 6. We make the number of bits in the subtrahend equal to that of minuend by taking a `0' in the sixth place of the subtrahend.

Now, 2's complement of 010110 is (101101 + 1) i.e.101010. Adding this with the minuend.

	1 1 0 1 1 0	Minuend
	1 0 1 0 1 0	2's complement of subtrahend
Carry over 1	1 0 0 0 0 0	Result of addition

After dropping the carry over we get the result of subtraction to be 100000.

BINARY SUBTRACTION using 2'S COMPLIMENT

(ii) **10110 – 11010**

Solution:

2's complement of 11010 is (00101 + 1) i.e. 00110. Hence

Minued -	1 0 1 1 0
2's complement of subtrahend -	0 0 1 1 0
Result of addition -	1 1 1 0 0

As there is no carry over, the result of subtraction is negative and is obtained by writing the 2's complement of 11100 i.e.(00011 + 1) or 00100.

Hence the difference is – 100.

BINARY SUBTRACTION using 2'S COMPLIMENT

(ii) **10110 – 11010**

Solution:

2's complement of 11010 is (00101 + 1) i.e. 00110. Hence

Minued -	1 0 1 1 0
2's complement of subtrahend -	0 0 1 1 0
Result of addition -	1 1 1 0 0

As there is no carry over, the result of subtraction is negative and is obtained by writing the 2's complement of 11100 i.e.(00011 + 1) or 00100.

Hence the difference is – 100.

BINARY SUBTRACTION using 2'S COMPLIMENT

.

(iii)1010.11 – 1001.01
Solution:
2's complement of 1001.01 is 0110.11. Hence

Minued -		1 0 1 0 . 1 1
2's complement of subtrahend -		0 1 1 0 . 1 1
Carry over	1	0 0 0 1 . 1 0

After dropping the carry over we get the result of subtraction as 1.10.

EXAMPLES

- - - -   - - - -   (+6 decimal) → 1
- - - -   - - - -   (-4 decimal)
- - - -   - - - -   (1'comp of -4)
- - - -   - - - -   (2's comp of -4) → 2

\- - - - -   - - - -   (-- decimal)

EXAMPLES

- - - -   - - - -   (+7 decimal) → 1
- - - -   - - - -   (-3 decimal)
- - - -   - - - -   (1’comp of -3)
- - - -   - - - -   <u>(2’s comp of -3)</u> → 2

1 − 2 ⇒ -   - - - -   - - - -   (-- decimal)

OCTAL ARITHMETIC

1
- OCTAL ADDITION

2
- OCTAL SUBTRACTION

3
- OCTAL MULTIPLICATION

4
- OCTAL DIVISION

OCTAL ARITHMETIC

1

- OCTAL ADDITION

Note-Perform addition but if addition is grater than or equal to 8 then subtract from 8 and also generate carry bit.

1. OCTAL ADDITION

(i) $(162)_8 + (537)_8$

Solution:

$$
\begin{array}{cccl}
1 & 1 & & \text{-------- Carry bit} \\
1 & 6 & 2 & \\
5 & 3 & 7 & \\
\hline
7 & 10 & 9 & \\
- & 8 & 8 & \\
\hline
7 & 2 & 1 &
\end{array}
$$

Therefore, sum = 721_8

OCTAL ADDITION

$(136)_8 + (636)_8$

Solution:

$$
\begin{array}{rl}
 & \;\;1 \quad\quad\quad \text{<---- carry} \\
 & 1\ 3\ 6 \\
+ & \\
 & \underline{6\ 3\ 6} \\
 & 7\ 7\ 4
\end{array}
$$

Therefore, sum = 774_8

OCTAL ADDITION

$(25.27)_8 + (13.2)_8$

Solution:

$$
\begin{array}{r l}
1 & \quad \text{<---- carry} \\
2\,5\,.\,2\,7 & \\
\underline{1\,3\,.\,2} & \\
4\,0\,.\,4\,7 &
\end{array}
$$

Therefore, sum = $(40.47)_8$

OCTAL ADDITION

$(167)_8 + (425)_8$

Solution: ??

OCTAL ARITHMETIC

2

- OCTAL SUBTRACTION

OCTAL SUBTRACTION

The Following methods can be used for octal subtraction:-

1) Direct Subtraction.
2) Convert the no. to binary, perform the subtraction and convert the result back to Octal.
3) Use the 7's complement method.
4) Use the 8's complement method.

OCTAL SUBTRACTION

1) Direct Subtraction-

i)456-173=

$$
\begin{array}{rl}
8 & \text{borrow} \\
{}^{3}456 & = 302_{10} \\
-173 & = 123_{10} \\
\hline
263 & = 179_{10}
\end{array}
$$

ii)653-177= ??

OCTAL SUBTRACTION using 7's complement

3)Use 7's complement method-

i) Find 7's complement of (512)

$$\begin{array}{r} 7\ 7\ 7 \\ -\quad 5\ 1\ 2 \\ \hline 2\ 6\ 5 \end{array}$$

Hence 7's Complement of(512) is (265)

OCTAL SUBTRACTION using 7’s complement

3)Use 7’s complement method-

Procedure of subtraction using 7’s complement ,Eg.(A-B)

1.At first, find 7's complement of the B (subtrahend).

2. Then add it to the A (minuend).

3. If the final carry over of the sum is 1, then it is dropped and 1 is added into the result.

4. If there is no carry over, then 7's complement of the sum is the final result and it is negative.

Find Subtraction of 342 and 614 using 7's complement method

Solution:

Here A = 342, B = 614.

Find A - B = ? using 7's complement

7's complement of a number is obtained by subtracting all bits from 777.

7's complement of 614 is

	7	7	7
-	6	1	4
	1	6	3

Now Add this 7's complement of B to A

	1		
	3	4	2
+	1	6	3
	5	2	5

Here there is no carry, answer is - (7's complement of the sum obtained)

7's complement of a number is obtained by subtracting all bits from 777.

7's complement of 525 is

	7	7	7
-	5	2	5
	2	5	2

So answer is -252

OCTAL SUBTRACTION using 7's complement

Use 7's complement method- find $(161)_8 - (243)_8$

Sloution =??

OCTAL SUBTRACTION using 8's complement

4)Use 8's complement method-

i)Find 8's complement of (512)

$$
\begin{array}{r l}
 & 7\ 7\ 7 \\
- & 5\ 1\ 2 \\
\hline
 & 2\ 6\ 5 \\
 & 1 \quad \text{(Add 1 to 7's complement)} \\
\hline
 & 2\ 6\ 6
\end{array}
$$

Hence 8's Complement of(512) is (266)

OCTAL SUBTRACTION using 8's complement

4)**<u>Use 8's complement method-</u>**

1. At first, find 8's complement of the B(subtrahend).
2. Then add it to the A(minuend).
3. If the final carry over of the sum is 1, then it is dropped and the result is positive.
4. If there is no carry over, then 8's complement of the sum is the final result and it is negative.

Here A = 342, B = 614.

Find A - B = ? using 8's complement

8's complement of a number is 1 added to it's 7's complement number.

7's complement of 614 is

	7	7	7
-	6	1	4
	1	6	3

Now add 1 : 163 + 1 = 164

Now Add this 8's complement of B to A

	1		
	3	4	2
+	1	6	4
	5	2	6

Here there is no carry, answer is - (8's complement of the sum obtained)

8's complement of a number is 1 added to it's 7's complement number.

7's complement of 526 is

	7	7	7
-	5	2	6
	2	5	1

Now add 1 : 251 + 1 = 252

So answer is -252

OCTAL SUBTRACTION using 8's complement

Use 8's complement method- find $(536)_8 - (345)_8$

Solution =??

OCTAL ARITHMETIC

3

- OCTAL MULTIPLICATION

OCTAL MULTIPLICATION

Step1 – Convert both octal no. to binary

Step 2 – perform simple binary multiplication

Step 3 - After performing multiplication, whatever answer you get in binary ,convert it to equivalent octal number.

Eg . (12)*(7)=

Step1-Convert into binary- (12)= 001010

(7) = 111

Step 2- 1010

* 1 1 1

1000110

Step 3 - 1 0 6 ans. is $=(106)_8$

OCTAL ARITHMETIC

4

- OCTAL DIVISION

OCTAL DIVISION

Step1 – Convert both octal no. to binary
Step 2 – perform simple binary Division.
Step 3 - convert given binary quotient and reminder to octal number.

Eg. (24)/(4)=

Step1-Convert into binary- (24)= 010100
(4) = 100

Step 2- 100)10100(101 quotient
000 reminder

Step 3- Convert into octal $(101)_2 = (5)_8$

Octal Multiplication
Solve 14* 5= ??

Octal Division
Solve 5)30 = ??

HEXADECIMAL ARITHMETIC

1. HEXA. ADDITION
2. HEXA. SUBTRACTION
3. HEXA. MULTIPLICATION
4. HEXA. DIVISION

HEXADECIMAL ARITHMETIC

1

- HEXA. ADDITION

HEXADECIMAL ADDITION

Add 9 +8

1 Carry bit

```
  9
+ 8
-----
  17
- 16
-----
   1
-----
1  1
```

HEXADECIMAL ADDITION

$4A6_{16} + 1B3_{16} = 659_{16}$

$$
\begin{array}{rl}
1 & \text{carry} \\
4\,A\,6 & = 1190_{10} \\
+\,1\,B\,3 & = 435_{10} \\
\hline
6\,5\,9 & = 1625_{10}
\end{array}
$$

HEXADECIMAL ADDITION

Add A1 +23 =

Solution = ??

HEXADECIMAL ARITHMETIC

2

- HEXA. SUBTRACTION

HEXADECIMAL SUBTRACTION

The Following methods can be used for octal subtraction:-

1) Direct Subtraction.
2) Convert the no. to binary, perform the subtraction and convert the result back to Hexadecimal.
3) Use the 15’s complement method.
4) Use the 16’s complement method.

HEXADECIMAL SUBTRACTION

1) Direct Subtraction-

Eg.73 -1C=

$$
\begin{array}{rcc}
 & 6 & 16 \\
 & \cancel{7} & 3 \\
- & 1 & 12 \\
\hline
 & 5 & 7
\end{array}
$$

16 → (3 is less than 12 so we add 16)

HEXADECIMAL SUBTRACTION

1) Direct Subtraction-

$4A6_{16} - 1B3_{16} = 2F3_{16}$

$$
\begin{array}{rcl}
16 & & \text{borrow} \\
{}^{3}4\,A\,6 & = & 1190_{10} \\
-\,1\,B\,3 & = & 435_{10} \\
\hline
2\,F\,3 & = & 755_{10}
\end{array}
$$

REPRESENTATION OF NEGATIVE NUMBER

15’S COMPLIMENT

1.At first, find 15's complement of the B(subtrahend).
2. Then add it to the A(minuend).
3. If the final carry over of the sum is 1, then it is dropped and 1 is added to the result.
4. If there is no carry over, then 15's complement of the sum is the final result and it is negative.

Here A = B06, B = C7C.
Find A - B = ? using 15's complement

15's complement of a number is obtained by subtracting all bits from FFF.
15's complement of C7C is

F	F	F	
-	C	7	C
3	8	3	

Now Add this 15's complement of B to A

B	0	6	
+	3	8	3
E	8	9	

Here there is no carry, answer is - (15's complement of the sum obtained)

15's complement of a number is obtained by subtracting all bits from FFF.
15's complement of E89 is

F	F	F	
-	E	8	9
1	7	6	

So answer is -176

REPRESENTATION OF NEGATIVE NUMBER

16’S COMPLIMENT

1.At first, find 16's complement of the B(subtrahend).
2. Then add it to the A(minuend).
3. If the final carry over of the sum is 1, then it is dropped and the result is positive.
4. If there is no carry over, then 16's complement of the sum is the final result and it is negative.

Here A = B06, B = C7C.
Find A - B = ? using 16's complement

16's complement of a number is 1 added to it's 15's complement number.
15's complement of C7C is

F	F	F	
-	C	7	C
3	8	3	

Now add 1 : 383 + 1 = 384
Now Add this 16's complement of B to A

B	0	6	
+	3	8	4
E	8	A	

Here there is no carry, answer is - (16's complement of the sum obtained)
16's complement of a number is 1 added to it's 15's complement number.
15's complement of E8A is

F	F	F	
-	E	8	A
1	7	5	

Now add 1 : 175 + 1 = 176
So answer is -176

HEXADECIMAL ARITHMETIC

3

- HEXA. MULTIPLICATION

HEXADECIMAL MULTIPLICATION

Step1 – Convert both hexadecimal number to binary

Step 2 – perform simple binary multiplication

Step 3 - After performing multiplication, whatever answer you get in binary ,convert it to equivalent hexadecimal number.

Eg . (12)*(7)=

Step1-Convert into binary- (8)= 1000

(7) = 0111

Step 2- 1 0 0 0

* 0 1 1 1

0111000

Step 3 - 3 8 ans. is $=(38)_{16}$

HEXADECIMAL ARITHMETIC

4

- HEXA. DIVISION

HEXADECIMAL DIVISION

Step1 – Convert both hexadecimal number to binary

Step 2 – perform simple binary Division.

Step 3 - convert given binary quotient and reminder to hexadecimal number.

Eg. (24)/(8)=

Step1-Convert into binary- (24)= 00100100

(8) = 1000

Step 2- 1000)100100(100 quotient

100 reminder

Step 3- Convert into hexadecimal

Quotient $(0100)_2 = (4)_{16}$, Reminder $(0100)_2 = (4)_{16}$