35 Years

Previous Solved Papers

GATE 2022

Mechanical Engineering

- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated

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GATE - 2022
Mechanical Engineering

Topicwise Previous GATE Solved Papers (1987-2021)

Editions

1st Edition	:	2008
2nd Edition	:	2009
3rd Edition	:	2010
4th Edition	:	2011
5th Edition	:	2012
6th Edition	:	2013
7th Edition	:	2014
8th Edition	:	2015
9th Edition	:	2016
10th Edition	:	2017
11th Edition	:	2018
12th Edition	:	2019
13th Edition	:	2020
14th Edition	**:**	**2021**

Preface

B. Singh (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2022 Solved Papers : Mechanical Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE-2022

Mechanical Engineering

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Engineering Mathematics

UNIT

I

CONTENTS

Engineering Mathematics

Syllabus

Linear Algebra : Matrix algebra, systems of linear equations, eigenvalues and eigenvectors.

Calculus: Functions of single variable, limit, continuity and differentiability, mean value theorems, indeterminate forms; evaluation of definite and improper integrals; double and triple integrals; partial derivatives, total derivative, Taylor series (in one and two variables), maxima and minima, Fourier series; gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, applications of Gauss, Stokes and Green's theorems.

Differential equations: First order equations (linear and nonlinear); higher order linear differential equations with constant coefficients; Euler-Cauchy equation; initial and boundary value problems; Laplace transforms; solutions of heat, wave and Laplace's equations.

Complex variables: Analytic functions; Cauchy-Riemann equations; Cauchy's integral theorem and integral formula; Taylor and Laurent series.

Probability and Statistics: Definitions of probability, sampling theorems, conditional probability; mean, median, mode and standard deviation; random variables, binomial, Poisson and normal distributions.

Numerical Methods: Numerical solutions of linear and non-linear algebraic equations; integration by trapezoidal and Simpson's rules; single and multi-step methods for differential equations.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	6	**15**
2004	3	5	**13**
2005	5	10	**25**
2006	4	8	**20**
2007	4	8	**20**
2008	6	9	**24**
2009	4	8	**20**
2010	5	3	**11**
2011	5	4	**13**
2012	5	5	**15**
2013	5	5	**15**
2014 Set-1	5	4	**13**
2014 Set-2	5	4	**13**
2014 Set-3	5	4	**13**
2014 Set-4	5	4	**13**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2015 Set-1	4	3	**10**
2015 Set-2	4	4	**12**
2015 Set-3	6	5	**16**
2016 Set-1	5	4	**13**
2016 Set-2	5	4	**13**
2016 Set-3	5	4	**13**
2017 Set-1	5	4	**13**
2017 Set-2	4	4	**12**
2018 Set-1	5	4	**13**
2018 Set-2	5	4	**13**
2019 Set-1	5	4	**13**
2019 Set-2	5	4	**13**
2020 Set-1	6	4	**14**
2020 Set-2	5	4	**13**
2021 Set-1	5	3	**11**
2021 Set-2	5	4	**13**

1 Linear Algebra

1.1 For the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ the eigen values are

(a) 3 and –3 (b) –3 and –5
(c) 3 and 5 (d) 5 and 0

[2003 : 1 Mark]

1.2 Consider the system of simultaneous equations

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

This system has

(a) unique solution
(b) infinite number of solutions
(c) no solution
(d) exactly two solutions

[2003 : 2 Marks]

1.3 The sum of the eigen values of the matrix given below is

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(a) 5 (b) 7
(c) 9 (d) 18

[2004 : 1 Mark]

1.4 For which value of x will the matrix given below become singular?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(a) 4 (b) 6
(c) 8 (d) 12

[2004 : 2 Marks]

1.5 A is a 3 × 4 real matrix and $Ax = B$ is an inconsistent system of equations. The highest possible rank of A is

(a) 1 (b) 2
(c) 3 (d) 4

[2005 : 1 Mark]

1.6 Which one of the following is an eigenvector of the matrix

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$

[2005 : 2 Marks]

1.7 With *a* 1 unit change in *b*, what is the change in x in the solution of the system of equations $x + y = 2$, $1.01\,x + 0.99\,y = b$?

(a) zero (b) 2 units
(c) 50 units (d) 100 units

[2005 : 2 Marks]

1.8 Let x denotes a real number. Find out the INCORRECT statement.

(a) $S = \{x : x > 3\}$ represents the set of all real numbers greater than 3
(b) $S = \{x : x^2 < 0\}$ represents the empty set.
(c) $S = \{x : x \in A \text{ and } x \in B\}$ represents the union of set A and set B.
(d) $S = \{x : a < x < b\}$ represents the set of all real numbers between *a* and *b*, where *a* and *b* are real numbers.

[2006 : 1 Mark]

1.9 Eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1.

What are the eigenvalues of the matrix $S^2 = SS$?

(a) 1 and 25 (b) 6 and 4
(c) 5 and 1 (d) 2 and 10

[2006 : 2 Marks]

1.10 Match the items in columns I and II.

Column I

P. Singular matrix
Q. Non-square matrix
R. Real symmetric
S. Orthogonal matrix

Column II

1. Determinant is not defined
2. Determinant is always one
3. Determinant is zero
4. Eigenvalues are always real
5. Eigenvalues are not defined

(a) P–3, Q–1, R–4, S–2
(b) P–2, Q–3, R–4, S–1
(c) P–3, Q–2, R–5, S–4
(d) P–3, Q–4, R–2, S–1

[2006 : 2 Marks]

1.11 Multiplication of matrices *E* and *F* is *G*. Matrices *E* and *G* are

$$E=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \text{ and } G=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

What is the matrix *F*?

(a) $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}\cos\theta & \cos\theta & 0\\ -\cos\theta & \sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(d) $\begin{bmatrix}\sin\theta & -\cos\theta & 0\\ \cos\theta & \sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

[2006 : 2 Marks]

1.12 If a square matrix *A* is real and symmetric, then the eigenvalues
(a) are always real
(b) are always real and positive
(c) are always real and non-negative
(d) occur in complex conjugate pairs

[2007 : 1 Mark]

1.13 The number of linearly independent eigenvectors of $\begin{bmatrix}2&1\\0&2\end{bmatrix}$ is
(a) 0 (b) 1
(c) 2 (d) infinite

[2007 : 2 Marks]

1.14 The matrix $\begin{bmatrix}1&2&4\\3&0&6\\1&1&p\end{bmatrix}$ has one eigenvalue equal to 3. The sum of the other two eigenvalues is
(a) p (b) $p-1$
(c) $p-2$ (d) $p-3$

[2008 : 1 Mark]

1.15 For what value of a, if any, will the following system of equations in x, y and z have a solution?

$$\begin{aligned}2x+3y&=4\\ x+y+z&=4\\ x+2y-z&=a\end{aligned}$$

(a) Any real number
(b) 0
(c) 1
(d) There is no such value

[2008 : 2 Marks]

1.16 The eigenvectors of the matrix $\begin{bmatrix}1&2\\0&2\end{bmatrix}$ are written in the form $\begin{bmatrix}1\\a\end{bmatrix}$ and $\begin{bmatrix}1\\b\end{bmatrix}$. What is $a+b$?
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

[2008 : 2 Marks]

1.17 For a matrix $[M]=\begin{bmatrix}\frac{3}{5}&\frac{4}{5}\\ x&\frac{3}{5}\end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T=[M]^{-1}$. The value of x is given by
(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

[2009 : 1 Mark]

1.18 One of the eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is

(a) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$

(c) $\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

[2010 : 2 Marks]

1.19 Eigenvalues of a real symmetric matrix are always

(a) positive (b) negative

(c) real (d) complex

[2011 : 1 Mark]

1.20 Consider the following system of equations

$2x_1 + x_2 + x_3 = 0$

$x_2 - x_3 = 0$

$x_1 + x_2 = 0$

This system has

(a) a unique solution

(b) no solution

(c) infinite number of solutions

(d) five solutions

[2011 : 2 Marks]

1.21 For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors is given as

(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

[2012 : 2 Marks]

1.22 $x + 2y + z = 4$

$2x + y + 2z = 5$

$x - y + z = 1$

The system of algebraic given below has

(a) A unique solution of $x = 1$, $y = 1$ and $z = 1$

(b) only the two solutions of ($x = 1$, $y = 1$, $z = 1$) and ($x = 2$, $y = 1$, $z = 0$)

(c) infinite number of solutions

(d) no feasible solution [2012 : 2 Marks]

1.23 Choose the CORRECT set of functions, which are linearly dependent.

(a) $\sin x$, $\sin^2 x$ and $\cos^2 x$

(b) $\cos x$, $\sin x$ and $\tan x$

(c) $\cos 2x$, $\sin^2 x$ and $\cos^2 x$

(d) $\cos 2x$, $\sin x$ and $\cos x$ [2013 : 1 Mark]

1.24 The eigen values of a symmetric matrix are all

(a) complex with non-zero positive imaginary part

(b) complex with non-zero negative imaginary part

(c) real

(d) pure imaginary [2013 : 1 Mark]

1.25 Given that the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is – 12, the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is

(a) – 96 (b) – 24

(c) 24 (d) 96

[2014 : 1 Mark, Set-1]

1.26 The matrix form of the linear system $\frac{dx}{dt} = 3x - 5y$ and $\frac{dy}{dt} = 4x + 8y$ is

(a) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(b) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & 8 \\ 4 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(c) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(d) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & 8 \\ 3 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

[2014 : 1 Mark, Set-1]

1.27 One of the eigenvectors of matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$ (b) $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$

(c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

[2014 : 1 Mark, Set-2]

1.28 Consider a 3 × 3 real symmetric matrix S such that two of its eigenvalues are $a \neq 0$, $b \neq 0$ with respective eigenvectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$. If $a \neq b$ then $x_1y_1 + x_2y_2 + x_3y_3$ equals

(a) a (b) b
(c) ab (d) 0

[2014 : 1 Mark, Set-3]

1.29 Which one of the following equations is a correct identity for arbitrary 3 × 3 real matrices P, Q and R?

(a) $P(Q + R) = PQ + RP$
(b) $(P - Q)^2 = P^2 - 2PQ + Q^2$
(c) $\det(P + Q) = \det P + \det Q$
(d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$

[2014 : 1 Mark, Set-4]

1.30 If any two columns of a determinant $P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$ are interchanged, which one of the following statements regarding the value of the determinant is CORRECT?

(a) Absolute value remains unchanged but sign will change
(b) Both absolute value and sign will change
(c) Absolute value will change but sign will not change
(d) Both absolute value and sign will remain unchanged **[2015 : 1 Mark, Set-1]**

1.31 At least one eigenvalue of a singular matrix is

(a) positive (b) zero
(c) negative (d) imaginary

[2015 : 1 Mark, Set-3]

1.32 The lowest eigenvalue of the 2 × 2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is ______.

[2015 : 1 Mark, Set-3]

1.33 For given matrix $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ where $i = \sqrt{-1}$, the inverse of matrix P is

(a) $\frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$ (b) $\frac{1}{25}\begin{bmatrix} i & 4-i \\ 4+3i & -i \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ (d) $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

[2015 : 2 Marks, Set-3]

1.34 The solution to the system of equations is $\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$

(a) 6, 2 (b) –6, 2
(c) –6, –2 (d) 6, – 2

[2016 : 1 Mark, Set-1]

1.35 The condition for which the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive, is

(a) $k > \frac{1}{2}$ (b) $k > -2$

(c) $k > 0$ (d) $k < -\frac{1}{2}$

[2016 : 1 Mark, Set-2]

1.36 A real square matrix A is called skew-symmetric if

(a) $A^T = A$ (b) $A^T = A^{-1}$
(c) $A^T = -A$ (d) $A^T = A + A^{-1}$

[2016 : 1 Mark, Set-3]

1.37 The number of linearly independent eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is________

[2016 : 2 Marks, Set-3]

1.38 The product of eigenvalues of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

(a) – 6 (b) 2
(c) 6 (d) – 2

[2017 : 1 Mark, Set-1]

1.39 Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

Which one of the following statements about P is INCORRECT?

(a) Determinant of P is equal to 1.
(b) P is orthogonal.
(c) Inverse of P is equal to its transpose.
(d) All eigenvalues of P are real numbers.

[2017 : 2 Marks, Set-1]

1.40 The determinant of a 2 × 2 matrix is 50. If one eigenvalue of the matrix is 10, the other eigen value is ___________.

[2017 : 1 Mark, Set-2]

1.41 Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigenvectors corresponding to eigenvalues λ_1 and λ_2 are $x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix}$ and $x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$, respectively. The value of $x_1^T x_2$ is ___________.

[2017 : 2 Marks, Set-2]

1.42 The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is

(a) 1 (b) 2
(c) 3 (d) 4

[2018 : 1 Marks, Set-1]

1.43 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is ________ (correct to two decimal places).

[2018 : 1 Marks, Set-2]

1.44 Consider the matrix

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The number of distinct eigen values of P is

(a) 2 (b) 0
(c) 1 (d) 3

[2019 : 1 Mark, Set-1]

1.45 The set of equations

$x + y + z = 1$
$ax - ay + 3z = 5$
$5x - 3y + az = 6$

has infinite solution if a =

(a) – 4 (b) – 3
(c) 3 (d) 4

[2019 : 2 Mark, Set-1]

1.46 In matrix equation $[A]\{X\} = \{R\}$

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{X\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}$$

One of the eigen values of Matrix $[A]$ is

(a) 4 (b) 15
(c) 8 (d) 16

[2019 : 1 Mark, Set-2]

1.47 Multiplication of real valued square matrices of same dimension is

(a) not always possible to compute
(b) associative
(c) always positive definite
(d) commutative

[2020 : 1 Mark, Set-1]

1.48 A matrix P is decomposed into its symmetric part S and skew symmetric part V. If

$$S = \begin{pmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{pmatrix}, \quad V = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{pmatrix}$$

then matrix P is

(a) $\begin{pmatrix} -2 & 9/2 & -1 \\ -1 & 81/4 & 11 \\ -2 & 45/2 & 73/4 \end{pmatrix}$

(b) $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$

[2020 : 1 Mark, Set-2]

1.49 Let $\boldsymbol{I}$ be a 100 dimensional identity matrix and E be the set of its distinct (no value appears more than once in $\boldsymbol{E}$) real eigenvalues. The number of elements in $\boldsymbol{E}$ is ______.

[2020 : 1 Mark, Set-2]

1.50 Consider an $n \times n$ matrix A and a non-zero $n \times 1$ vector p. Their product $Ap = \alpha^2 p$, where $\alpha \in \Re$ and $\alpha \notin \{-1, 0, 1\}$. Based on the given information, the eigen value of A^2 is:

(a) $\sqrt{\alpha}$ (b) α^2

(c) α (d) α^4

[2021 : 1 Mark, Set-2]

1.51 Let the superscript T represent the transpose operation. Consider the function $f(x) = \frac{1}{2}x^T Qx - r^T x$, where x and r are $n \times 1$ vectors and Q is a symmetric $n \times n$ matrix. The stationary point of $f(x)$ is

(a) $\frac{r}{r^T r}$ (b) $Q^T r$

(c) $Q^{-1} r$ (d) r

[2021 : 2 Marks, Set-2]

1.52 Consider a vector p in 2-dimensional space. Let its direction (counter-clockwise angle with the positive x-axis) be θ. Let p be an eigen vector of a 2×2 matrix A with corresponding eigen value λ, $\lambda > 0$. If we denote the magnitude of a vector v by $\|v\|$, identify the VALID statement regarding p', where $p' = Ap$.

(a) Direction of $p' = \theta, \|p'\| = \lambda\|p\|$

(b) Direction of $p' = \lambda\theta, \|p'\| = \|p\|$

(c) Direction of $p' = \theta, \|p'\| = \|p\|/\lambda$

(d) Direction of $p' = \lambda\theta, \|p'\| = \lambda\|p\|$

[2021 : 2 Marks, Set-1]

■■■■

Answers Linear Algebra

1.1	(c)	1.2	(c)	1.3	(b)	1.4	(a)	1.5	(b)	1.6	(a)	1.7	(c)	1.8	(c)
1.9	(a)	1.10	(a)	1.11	(c)	1.12	(a)	1.13	(c)	1.14	(c)	1.15	(b)	1.16	(b)
1.17	(a)	1.18	(a)	1.19	(c)	1.20	(c)	1.21	(b)	1.22	(c)	1.23	(c)	1.24	(c)
1.25	(a)	1.26	(a)	1.27	(d)	1.28	(d)	1.29	(d)	1.30	(a)	1.31	(b)	1.32	(2)
1.33	(a)	1.34	(d)	1.35	(a)	1.36	(c)	1.38	(b)	1.39	(d)	1.40	(5)	1.41	(0)
1.42	(b)	1.44	(c)	1.45	(d)	1.46	(d)	1.47	(b)	1.48	(c)	1.49	(1)	1.50	(d)
1.51	(c)	1.52	(a)												

Explanations Linear Algebra

1.1 (c)

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

Now $|A - \lambda I| = 0$

where λ = eigen value

$$\therefore \quad \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)^2 - 1 = 0$$

or, $(4-\lambda)^2 - (1)^2 = 0$

or, $(4 - \lambda + 1)(4 - \lambda - 1) = 0$

or, $(5-\lambda)(3-\lambda) = 0$

$\therefore \quad \lambda = 3, 5$

1.2 (c)

Given equation are

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

Given system can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Applying row operation

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ -1 \end{bmatrix}$$

and applying $R_3 \to 3R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 3 \end{bmatrix}$$

Since the rank of coefficient matrix is 2 and rank of argument matrix is 3, which are not equal. Hence system has **no solution**.

1.3 (b)

Sum of eigen values of given matrix

= sum of diagonal element of given matrix

$= 1 + 5 + 1 = \mathbf{7}$

1.4 (a)

For singularity of matrix

$$= \begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0$$

$\Rightarrow 8\,(0 - 12) - x\,(0 - 2 \times 12) = 0$

$\therefore \quad x = \mathbf{4}$

1.5 (b)

$$C = [A : B]_{3 \times 5}$$

$\therefore \quad \rho[C_{3\times 5}] \le \min\{3, 5\}$

& $\because$ The system is inconsistent

$\rho\,(A) < \rho\,(C)$

$\therefore \quad \rho\,(A) < 3$

Hence maximum possible rank of

$A = \mathbf{2}$

1.6 (a)

Eigen vector is an independent vector. Here in option *a* last two terms are zero. Hence only this option is correct.

1.7 (c)

Given $\quad x + y = 2 \quad$ ...(i)

$1.01\,x + 0.99\,y = b \quad$ ...(ii)

Multiply 0.99 is equation (i), and substract from equation (ii); we get

$(1.01 - 0.99)\,x = b - 2 \times 0.99$

$0.02x = b - 1.98$

$\therefore \quad 0.02\,\Delta x = \Delta b$

$\therefore \quad \Delta x = \dfrac{1}{0.02} = \mathbf{50\ unit}$

1.9 (a)

If $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_4$ are the eigen values of A. Then the eigen values of

A^m are $\lambda_1^m, \lambda_2^m, \lambda_3^m \ldots.$

$\Rightarrow \quad S^2$ are $S_1^1, S_2^2, S_3^2 \ldots.$

S matrix has eigen values 1 and S.

$\Rightarrow \quad S^2$ matrix has eigen values 1^2 and S^2

i.e. **1 and 25**

1.11 (c)

$$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The according to problem

$E \times F = G$

or $$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence we see that product of $(E \times F)$ is unit matrix so F will be the inverse of E.

$$F = E^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.12 (a)

The eigen values of any real and symmetric matrix **are always real.**

1.13 (c)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix} = 0$$

$\Rightarrow \quad (2-\lambda)^2 = 0$

$\Rightarrow \quad \lambda = \mathbf{2}$

No. of linearly independent eigen vectors

= No. of distinct eigen values

1.14 (c)

Sum of the eigen values of matrix is

= sum of diagonal values present in the matrix

$\therefore\ 1 + 0 + p = 3 + \lambda_2 + \lambda_3$

$\Rightarrow p + 1 = 3 + \lambda_2 + \lambda_3$

$\Rightarrow \lambda_2 + \lambda_3 = p + 1 - 3 = \boldsymbol{p - 2}$

1.15 (b)

Augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & -1 & a \end{array}\right] \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & -2 & -4 \\ 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & a-4 \end{array}\right] R_1 \to R_1 - R_3$$

$$\sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & -a \\ 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & a-4 \end{array}\right]$$

will have solution if $\boldsymbol{a = 0}$

as Rank A = Rank (aug. A)

1.16 (b)

$$\begin{vmatrix} (1-\lambda) & 2 \\ 0 & (2-\lambda) \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)(2-\lambda) = 0$

$\therefore \quad \lambda = 1, 2$

Putting the value of $\lambda = 1$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix} = 0$$

$\Rightarrow \quad a = 0$

putting the value of $\lambda = 2$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = 0$$

$$\Rightarrow \quad b = \frac{1}{2}$$

$$\therefore \quad a + b = \frac{1}{2}$$

1.17 (a)

If $\quad A^T = A^{-1}$

then A is orthogonal matrix.

Therefore $A.A^{-1} = A^{-1} A = I$

and $A^T A = A A^T = I$

Since M is orthogonal matrix

$$M^T M = I$$

$$\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left(\frac{3}{5}\right)^2 + x^2 & \left(\frac{3}{5}\cdot\frac{4}{5}\right) + \frac{3}{5}x \\ \left(\frac{4}{5}\cdot\frac{3}{5}\right) + \frac{3}{5}\cdot x & \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ Compare both sides a_{12}

$$a_{12} = \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{5}\right)x = 0$$

$$\Rightarrow \quad \frac{3}{5}x = -\frac{3}{5}\cdot\frac{4}{5}$$

$$\Rightarrow \quad x = -\frac{4}{5}$$

1.18 (a)

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Characteristic equation of A is

$$\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda = 1, 4$$

The eigen value problem is $[A - \lambda I]\hat{x} = 0$

$$\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Putting $\lambda = 1$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0 \quad \text{... (i)}$$

$$x_1 + 2x_2 = 0 \quad \text{... (ii)}$$

Solution is $x_2 = k, x_1 = -2k$

$$\hat{X}_1 = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

i.e. $\quad x_1 : x_2 = -2 : 1$

Since, choice (A) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is in same ratio of x_1 to x_2.

$\therefore$ Choice (A) is an eigen vector.

Engineering Mechanics

UNIT

II

CONTENTS

Engineering Mechanics

Syllabus

Free-body diagrams and equilibrium; friction and its applications including rolling friction, belt-pulley, brakes, clutches, screw jack, wedge, vehicles, etc.; trusses and frames; virtual work; kinematics and dynamics of rigid bodies in plane motion; impulse and momentum (linear and angular) and energy formulations; Lagrange's equation.

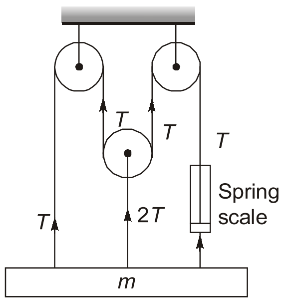
Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	1	–	–	–	**1**
1992	1	–	–	–	**1**
1993	1	–	–	–	**1**
1994	1	–	–	–	**1**
1995	–	1	–	–	**2**
1996	2	3	–	–	**8**
1997	–	1	–	–	**2**
1998	1	–	–	–	**1**
1999	–	1	–	–	**2**
2000	2	–	–	–	**2**
2003	5	–	–	–	**5**
2004	–	3	–	–	**6**
2005	2	4	–	–	**10**
2006	–	2	–	–	**4**
2007	1	1	–	–	**3**
2008	1	2	–	–	**5**
2009	1	1	–	–	**3**
2011	1	2	–	–	**5**
2012	2	3	–	–	**8**
2013	–	1	–	–	**2**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2014 Set-1	1	2	**5**
2014 Set-2	1	2	**5**
2014 Set-3	1	3	**7**
2014 Set-4	–	4	**8**
2015 Set-1	3	4	**11**
2015 Set-2	1	2	**5**
2015 Set-3	2	3	**8**
2016 Set-1	2	2	**6**
2016 Set-2	1	3	**7**
2016 Set-3	2	3	**8**
2017 Set-1	2	1	**4**
2017 Set-2	–	1	**2**
2018 Set-1	1	2	**5**
2018 Set-2	1	1	**3**
2019 Set-1	1	2	**5**
2019 Set-2	2	1	**4**
2020 Set-1	1	1	**3**
2020 Set-2	2	1	**4**
2021 Set-1	1	2	**5**
2021 Set-2	–	2	**4**

1 FBD, Equilibrium, Plane Trusses and Virtual Work

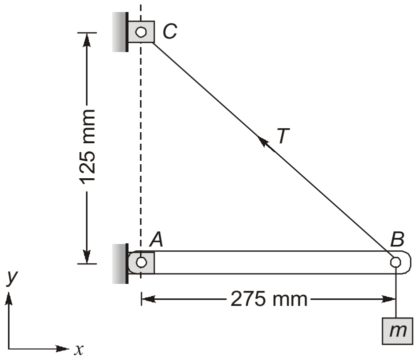
1.1 A spring scale indicates a tension T in the right hand cable of the pulley system shown in figure. Neglecting the mass of the pulleys and ignoring friction between the cable and pulley the mass m is



(a) $2T/g$ (b) $T(1 + e^{4\pi})/g$
(c) $4T/g$ (d) None of these

[1995 : 2 Marks]

1.2 A mass 35 kg is suspended from a weightless bar AB which is supported by a cable CB and a pin at A as shown in figure. The pin reactions at A on the bar AB are



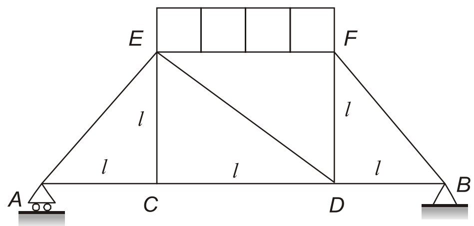
(a) $R_x = 343.4$ N, $R_y = 755.4$ N
(b) $R_x = 343.4$ N, $R_y = 0$
(c) $R_x = 755.4$ N, $R_y = 343.4$ N
(d) $R_x = 755.4$ N, $R_y = 0$

[1997 : 2 Marks]

1.3 A truss consists of horizontal members (AC, CD, DB and EF) and vertical members (CE and DF) having length l each. The members AE, DE and BF are inclined at 45° to the horizontal. For the uniformly distributed load p per unit length on the members EF of the truss shown in figure given below, the force in the member CD is

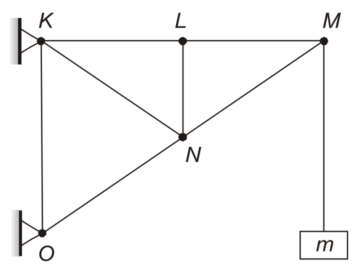
(a) $\dfrac{pl}{2}$ (b) pl

(c) 0 (d) $\dfrac{2pl}{3}$



[2003 : 1 Mark]

1.4 The figure shows a pin-jointed plane truss loaded at the point M by hanging a mass of 100 kg. The member LN of the truss is subjected to a load of



(a) 0 Newton
(b) 490 Newtons in compression
(c) 981 Newtons in compression
(d) 981 Newtons in tension

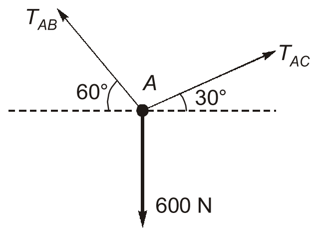
[2003 : 1 Mark]

1.5 If a system is in equilibrium and the position of the system depends upon many independent variables, the principle of virtual work states that the partial derivatives of its total potential energy with respect to each of the independent variable must be

(a) –1.0 (b) 0
(c) 1.0 (d) ∞

[2006 : 2 Marks]

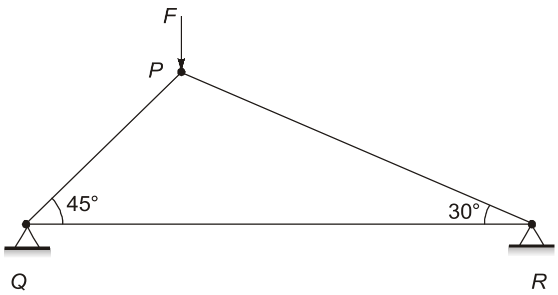
1.6 If point A is in equilibrium under the action of the applied forces, the values of tensions T_{AB} and T_{AC} are respectively.



(a) 520 N and 300 N (b) 300 N and 520 N
(c) 450 N and 150 N (d) 150 N and 450 N

[2006 : 2 Marks]

1.7 Consider a truss *PQR* loaded at *P* with a force *F* as shown in the figure.

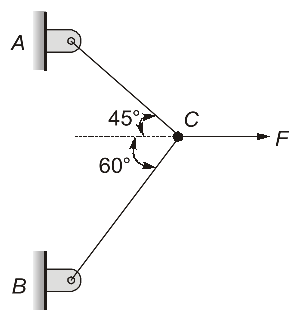


The tension in the member *QR* is
(a) 0.5 F (b) 0.63 F
(c) 0.73 F (d) 0.87 F

[2008 : 2 Marks]

Common Data Questions (1.8 and 1.9):

Two steel truss members, *AC* and *BC*, each having cross sectional area of 100 mm², are subjected to a horizontal force *F* as shown in figure. All the joints are hinged.



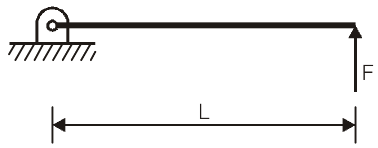
1.8 If *F* = 1 kN, the magnitude of the vertical reaction force developed at the point *B* in kN is
(a) 0.63 (b) 0.32
(c) 1.26 (d) 1.46

[2012 : 2 Marks]

1.9 The maximum force *F* in kN that can be applied at *C* such that the axial stress in any of the truss members DOES NOT exceed 100 MPa is
(a) 8.17 (b) 11.15
(c) 14.14 (d) 22.30

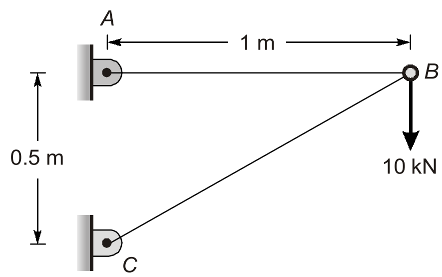
[2012 : 2 Marks]

1.10 A pin jointed uniform rigid rod of weight *W* and length *L* is supported horizontally by an external force *F* as shown in the figure below. The force *F* is suddenly removed. At the instant of force removal, the magnitude of vertical reaction developed at the support is



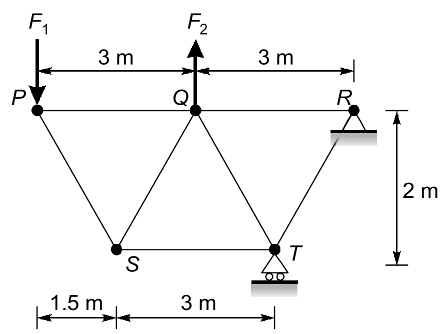
(a) zero (b) *W*/4
(c) *W*/2 (d) *W* [2013 : 2 Marks]

1.11 A two member truss *ABC* is shown in the figure. The force (in kN) transmitted in member *AB* is _____.



[2014 : 1 Mark, Set-2]

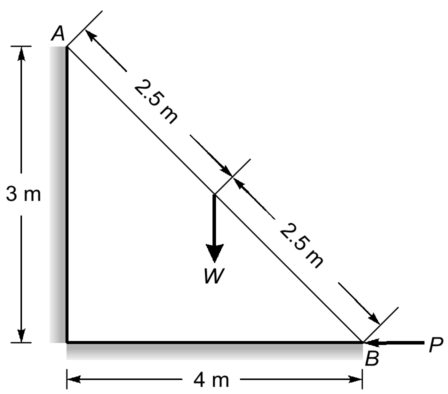
1.12 For the truss shown in the figure, the forces F_1 and F_2 are 9 kN and 3 kN, respectively. The force (in kN) in the member *QS* is (All dimensions are in m)



(a) 11.25 tension
(b) 11.25 compression
(c) 13.5 tension
(d) 13.5 compression

[2014 : 2 Marks, Set-4]

1.13 A ladder *AB* of length 5 m and weight (W) 600 N is resting against a wall. Assuming frictionless contact at the floor (B) and the wall (A), the magnitude of the force *P* (in newton) required to maintain equilibrium of the ladder is _______.



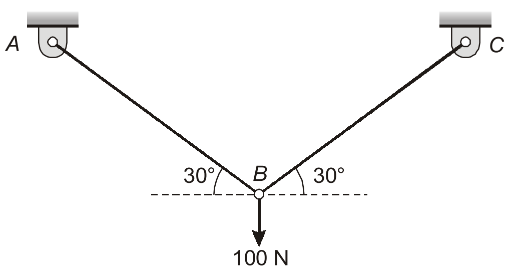
[2014 : 2 Marks, Set-4]

1.14 In a statically determinate plane truss, the number of joints (j) and the number of members (m) are related by

(a) $j = 2m - 3$
(b) $m = 2j + 1$
(c) $m = 2j - 3$
(d) $m = 2j - 1$

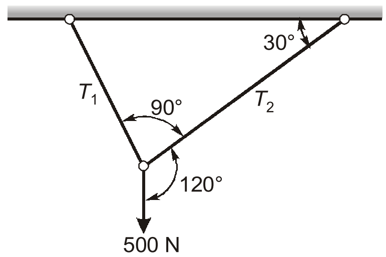
[2014 : 1 Mark, Set-4]

1.15 Two identical trusses support a load of 100 N as shown in the figure. The length of each truss is 1.0 m, cross-sectional area is 200 mm^2; Young's modulus E = 200 GPa. The force in the truss *AB* (in N) is ______



[2015 : 1 Mark, Set-1]

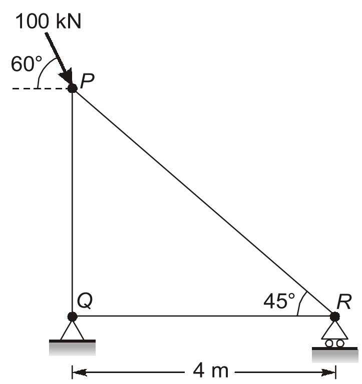
1.16 A weight of 500 N is supported by two metallic ropes as shown in the figure. The values of tensions T_1 and T_2 are respectively



(a) 433 N and 250 N
(b) 250 N and 433 N
(c) 353.5 N and 250 N
(d) 250 N and 353.5 N

[2015 : 1 Mark, Set-3]

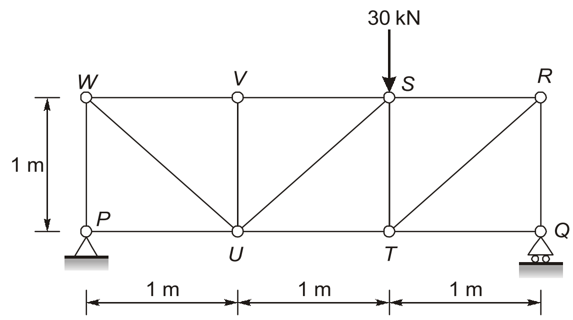
1.17 For the truss shown in figure, the magnitude of the force in member *PR* and the support reaction at *R* are respectively



(a) 122.47 kN and 50 kN
(b) 70.71 kN and 100 kN
(c) 70.71 kN and 50 kN
(d) 81.65 kN and 100 kN

[2015 : 2 Marks, Set-1]

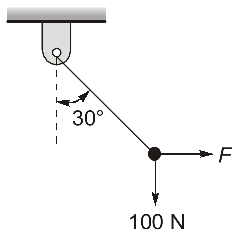
1.18 For the truss shown in the figure, the magnitude of the force (in kN) in the member *SR* is



(a) 10
(b) 14.14
(c) 20
(d) 28.28

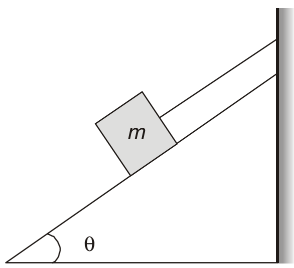
[2015 : 2 Marks, Set-2]

1.19 A rigid ball of weight 100 N is suspended with the help of a string. The ball is pulled by a horizontal force F such that the string makes an angle of 30° with the vertical. The magnitude of force F (in N) is_______.



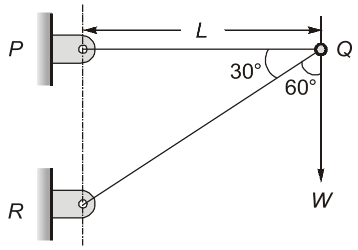
[2016 : 1 Mark, Set-1]

1.20 A block of mass m rests on an inclined plane and is attached by a string to the wall as shown in the figure. The coefficient of static friction between the plane and the block is 0.25. The string can withstand a maximum force of 20 N. The maximum value of the mass (m) for which the string will not break and the block will be in static equilibrium is _______ kg.
Take $\cos\theta = 0.8$ and $\sin\theta = 0.6$.
Acceleration due to gravity $g = 10$ m/s^2



[2016 : 2 Marks, Set-1]

1.21 A two member truss PQR is supporting a load W. The axial forces in members PQ and QR are respectively



(a) $2W$ tensile and $\sqrt{3}W$ compressive

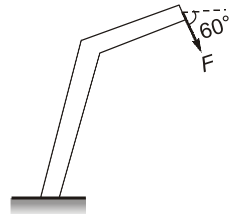
(b) $\sqrt{3}W$ tensile and $2W$ compressive

(c) $\sqrt{3}W$ compressive and $2W$ tensile

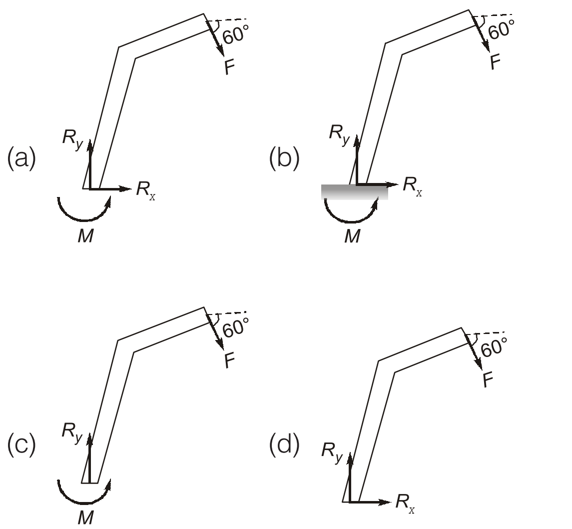
(d) 2W compressive and $\sqrt{3}W$ tensile

[2016 : 2 Marks, Set-1]

1.22 *A* force F is acting on a bent bar which is clamped at one end as shown in the figure.

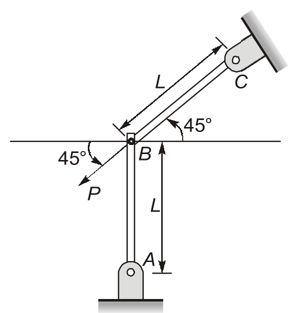


The CORRECT free body diagram is



[2016 : 1 Mark, Set-3]

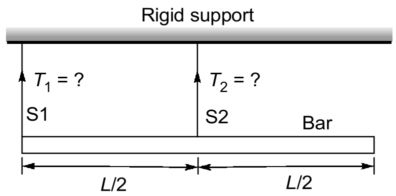
1.23 In the figure, the load P = 1 N, length L = 1 m, Young's modulus E = 70 GPa, and the cross-section of the links is a square with dimension 10 mm × 10 mm. All joints are pin joints.



The stress (in Pa) in the AB is_______
(Indicate compressive stress by a negative sign and tensile stress by a positive sign.)

[2016 : 2 Marks, Set-2]

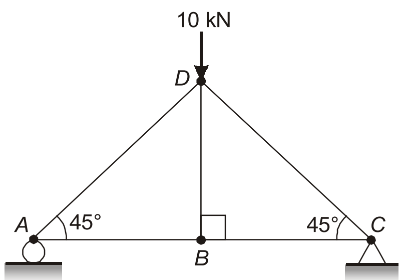
1.24 A bar of uniform cross section and weighing 100 N is held horizontally using two massless and inextensible strings S_1 and S_2 as shown in the figure.



(a) T_1 = 100 N and T_2 = 0 N
(b) T_1 = 0 N and T_2 = 100 N
(c) T_1 = 75 N and T_2 = 25 N
(d) T_1 = 25 N and T_2 = 75 N

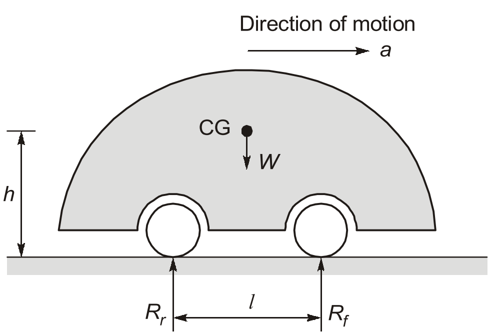
[2018 : 1 Marks, Set-1]

1.25 A truss is composed of members *AB*, *BC*, *CD*, *AD* and *BD*, as shown in the figure. A vertical load of 10 kN is applied at point *D*. The magnitude of force (in kN) in the member *BC* is _______.



[2019 : 2 Mark, Set-1]

1.26 A car is having weight *W* is moving in the direction as shown in the figure. The centre of gravity (CG) of the car is located at height *h* from the ground, midway between the front and rear wheels. The distance between the front and rear wheels, is *l*. The acceleration of the car is *a*, and acceleration due to gravity is *g*. The reactions on the front wheels (R_f) and rear wheels (R_r) are given by



(a) $R_f = \frac{W}{2} + \frac{W}{g}\left(\frac{h}{l}\right)a;\ R_r = \frac{W}{2} - \frac{W}{g}\left(\frac{h}{l}\right)a$

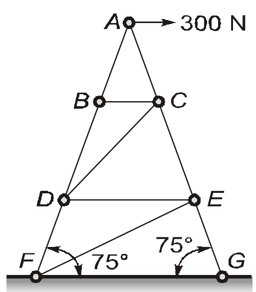
(b) $R_f = R_r = \frac{W}{2} + \frac{W}{g}\left(\frac{h}{l}\right)a$

(c) $R_f = R_r = \frac{W}{2} - \frac{W}{g}\left(\frac{h}{l}\right)a$

(d) $R_f = \frac{W}{2} - \frac{W}{g}\left(\frac{h}{l}\right)a;\ R_r = \frac{W}{2} + \frac{W}{g}\left(\frac{h}{l}\right)a$

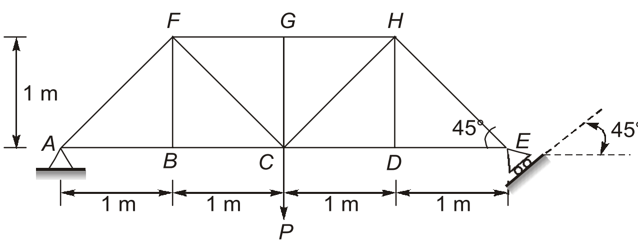
[2019 : 2 Mark, Set-1]

1.27 The figure shows an idealized plane truss. If a horizontal force of 300 N is applied at point *A*, then the magnitude of the force produced in member *CD* is ________ N.



[2019 : 1 Mark, Set-2]

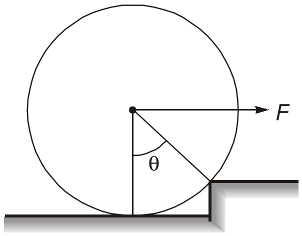
1.28 The members carrying zero force (i.e. zero-force members) in the truss shown in the figure, for any load $P > 0$ with no appreciable deformation of the truss (i.e. with no appreciable change in angles between the members), are



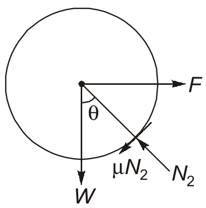
(a) BF, DH and GC only
(b) BF, DH, GC, CD and DE only
(c) BF and DH only
(d) BF, DH, GC, FG and GH only

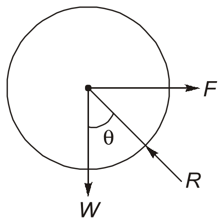
[2020 : 1 Mark, Set-1]

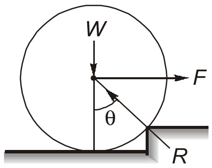
1.29 An attempt is made to pull a roller of weight W over a curb (step) by applying a horizontal force F as shown in the figure.

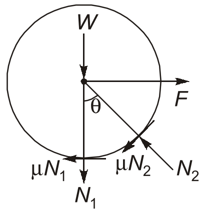


The coefficient of static friction between the roller and the ground (including the edge of the step) is μ. Identify the correct free body diagram (FBD) of the roller when the roller is just about to climb over the step.

(a)


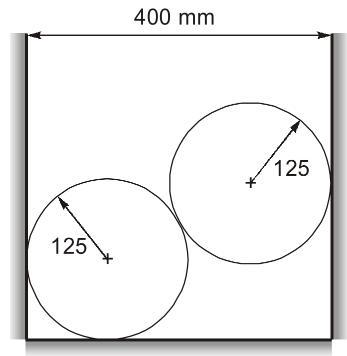
(b)


(c)


(d)


[2020 : 1 Mark, Set-2]

1.30 Two smooth identical spheres each of radius 125 mm and weight 100 N rest in a horizontal channel having vertical walls. The distance between vertical walls of the channel is 400 mm.

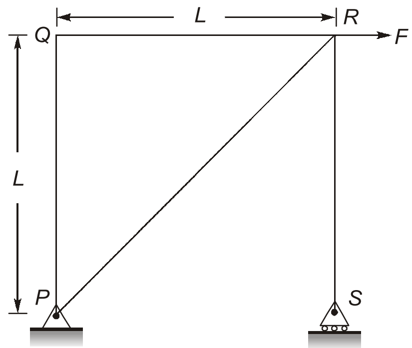


All dimensions are in mm

The reaction at the point of contact between two spheres is ________ N. [Round off to end one decimal place]

[2021 : 2 Marks, Set-1]

1.31 A plane truss $PQRS$ ($PQ = RS$, and $\angle PQR = 90°$) is shown in the figure.



The forces in the members PR and RS, respectively, are _______.

(a) $F\sqrt{2}$ (tensile) and F(tensile)

(b) $F\sqrt{2}$ (tensile) and F (compressive)

(c) F(compressive) and $F\sqrt{2}$ (compressive)

(d) F(tensile) and $F\sqrt{2}$ (tensile)

[2021 : 1 Mark, Set-2]

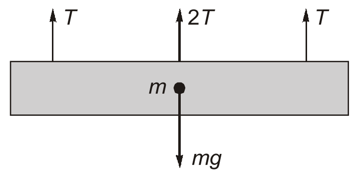
Answers **FBD, Equilibrium, Plane Trusses and Virtual Work**

1.1 (c)	1.2 (d)	1.3 (a)	1.4 (a)	1.5 (b)	1.6 (a)	1.7 (b)	1.8 (a)	1.9 (b)
1.10 (b)	1.11 (20)	1.12 (a)	1.13 (400)	1.14 (c)	1.15 (100)	1.16 (a)	1.17 (c)	1.18 (c)
1.19 (57.74)		1.20 (5)	1.21 (b)	1.22 (a)	1.23 (0)	1.24 (b)	1.25 (5)	1.26 (d)
1.27 (0)	1.28 (b)	1.29 (b)	1.30 (1.25)		1.31 (b)			

Explanations FBD, Equilibrium, Plane Trusses and Virtual Work

1.1 (c)

The free body diagram of mass m,



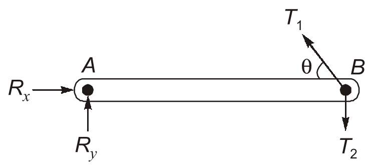
In equilibrium condition,

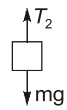
$$4T = mg$$

or $$m = \frac{4T}{g}$$

1.2 (d)

Since point A is hinge support, so there will be horizontal and vertical reactions at point A.





For block $\quad T_2 = mg = 343.35 \text{ N} \quad$ ...(i)

$$\tan\theta = \frac{125}{275} = 0.4545$$

$$\theta = \tan^{-1}(0.4545) = 24.44°$$

For bar $\quad \Sigma M_A = 0$

$$T_1 \sin\theta \times l = T_2 \times l$$

$$\Rightarrow \quad T_1 = \frac{T_2}{\sin\theta} = \frac{343.35}{\sin 24.44°}$$

$$= 829.74 \text{ N}$$

$$\Sigma F_y = 0$$

$$+R_y - T_2 + T_1 \sin\theta = 0$$

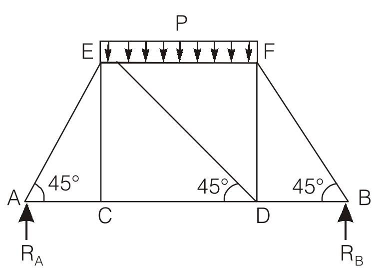
$$\therefore \quad R_y = 0$$

$$\Sigma F_x = 0$$

$$R_x - T_1\cos\theta = 0$$

$$R_x = \mathbf{755.39\ N}$$

1.3 (a)



$$R_A + R_B = Pl$$

Taking moment about A

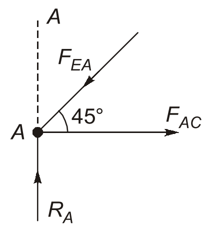
$$Pl(l + l/2) = R_B \times 3l$$

$$\therefore \quad R_B = \frac{Pl}{2}$$

$$\therefore \quad R_A = \frac{Pl}{2}$$

Joint A

Now at joint A



$$\Sigma F_V = 0$$

$$F_{EA} \sin 45° = \frac{Pl}{2}$$

$$\therefore \quad F_{EA} = \frac{Pl}{\sqrt{2}}$$

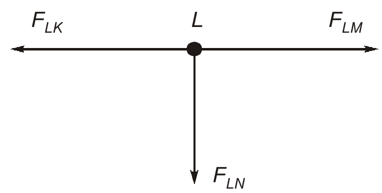
$$F_{AC} = F_{EA}\cos 45° = \frac{Pl}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$F_{AC} = \frac{Pl}{2}$$

At joint C

$$F_{CA} = F_{CD} = \frac{Pl}{2} \qquad [\because F_{CE} = 0]$$

1.4 (a)



$\Sigma F_H = 0 \quad \& \quad \Sigma F_V = 0$

At joint "L"

$$\therefore \quad F_{LK} - F_{LM} = 0 \quad (\Sigma F_H = 0)$$

$$F_{LN} = 0 \quad (\Sigma F_V = 0)$$

Hence no force is acting on the truss number LN.

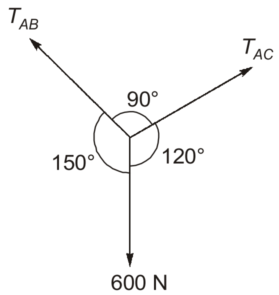
1.5 (b)

In equilibrium, potential energy is minimum.

If any system is in equilibrium and subjected to many independent variables, partial derivatives of its total potential energy with respect to each of the independent variable must be zero.

1.6 (a)

<u>Method I:</u>



By Lami's theorem

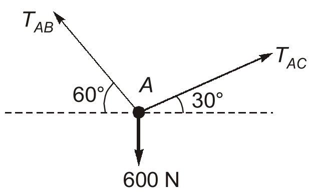
$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 150°} = \frac{600}{\sin 90°}$$

$\therefore \quad T_{AB} = 600 \sin 120° = 519.61 \approx 520$ N

and $\quad T_{AC} = 600 \sin 150° = \mathbf{300\ N}$

<u>Method II:</u>

In equilibrium,



Horizontal forces,

$$\Sigma F_x = 0,$$

$$T_{AC} \cos 30° - T_{AB} \cos 60° = 0$$

$$T_{AC} = T_{AB} \frac{\cos 60°}{\cos 30°} = \frac{T_{AB}}{\sqrt{3}} \quad \text{...(i)}$$

Vertical forces,

$$\Sigma F_y = 0,$$

$$T_{AC} \sin 30° + T_{AB} \sin 60° - 600 = 0$$

$$T_{AC} \sin 30° + T_{AB} \sin 60° = 600$$

$$T_{AC} + \sqrt{3}\ T_{AB} = 1200 \quad \text{...(ii)}$$

From equation (i) and (ii)

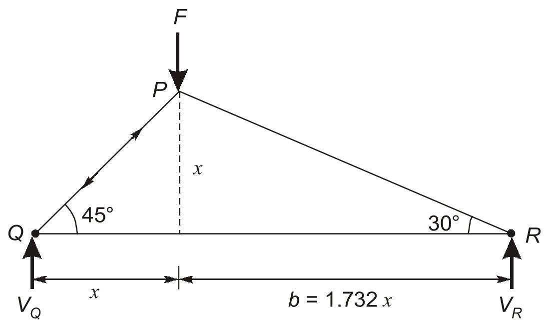
$$\frac{T_{AB}}{\sqrt{3}} + \sqrt{3} T_{AB} = 1200$$

$$4T_{AB} = 1200\sqrt{3}$$

$$T_{AB} = 300\sqrt{3} = 519.61 \text{ N} = 520 \text{ N}$$

and $$T_{AC} = \frac{T_{AB}}{\sqrt{3}} = \frac{300\sqrt{3}}{\sqrt{3}} = 300 \text{ N}$$

1.7 (b)



$$\tan 30° = \frac{x}{b}$$

$$b = \frac{x}{\tan 30°} = 1.732\, x$$

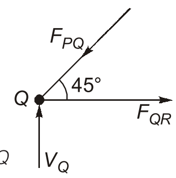
Taking moment about Q

$$F \times x = V_R \times 2.732\, x$$

$$V_R = 0.366\, F$$

$$V_Q = F - 0.366\, F$$

$$= 0.634\, F$$



FBD of joint Q

Let force in the member PQ is F_{PQ}

$\therefore F_{PQ} \sin 45° = V_Q$

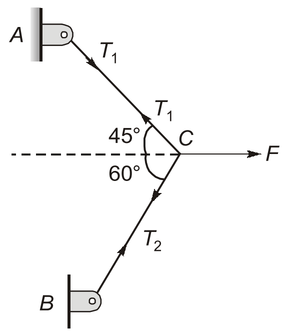
$\Rightarrow F_{PQ} \sin 45° = 0.634$ F

Force in member QR

$$F_{QR} = F_{PQ} \cos 45° = \mathbf{0.634}\boldsymbol{F}$$

1.8 (a)

<u>Method I:</u>



Using Lami's theorem

$$\frac{T_1}{\sin 120°} = \frac{T_2}{\sin 135°} = \frac{F}{\sin 105°}$$

$T_1 = 0.8965F$

$T_2 = 0.732F$

Vertical reaction at B,

$$R_B = T_2 \cos 30° = 0.732 \cos 30°$$

R_B = 0.634 kN

Method II:

$$\Sigma F_x = 0,$$

$$(F_{AC})_x + (F_{BC})_x = F \qquad \text{...(i)}$$

$$\Rightarrow F_{AC} \cos 45° + F_{BC} \cos 60° = F$$

$$\Sigma F_y = 0,$$

$$F_{AC} \sin 45° = F_{BC} \sin 60°$$

$$F_{AC} = \frac{F_{BC} \sin 60°}{\sin 45°} = 1.224\, F_{BC}$$

$$\Rightarrow \quad 1 = 0.865\, F_{BC} + 0.5\, F_{BC}$$

$$\therefore \quad F_{BC} = \frac{1}{1.365} = 0.732 \text{ kN}$$

Vertical force at B, $(R_B)_V = F_{BC} \sin 60°$

$$= 0.732 \sin 60° = 0.634 \text{ kN}$$

1.9 (b)

Method I:

Maximum force = 0.8965F

$$\therefore \text{ Max stress } \frac{0.8965F}{100} \le 100 \text{ MPa}$$

$$\therefore\ 100 \ge \frac{0.8965 \times F}{100}$$

$$\therefore\ F \le 11.154 \text{ kN}$$

Method II:

As, F_{AC} = 0.895 kN

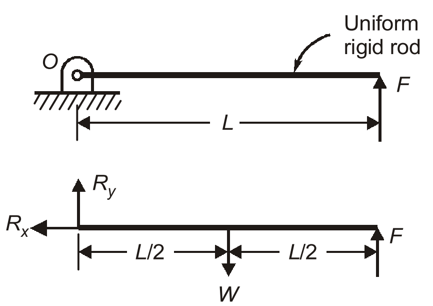
then

$$\frac{F \times F_{AC}}{A} = 100$$

$$\frac{F \times 0.895}{100} = 100$$

$$F = 11173.18 \text{ N} = 11.173 \text{ kN}$$

1.10 (b)



$$\Sigma R_x = 0 \Rightarrow R_x = 0$$

$$\Sigma R_y = 0 \Rightarrow R_y + F = W$$

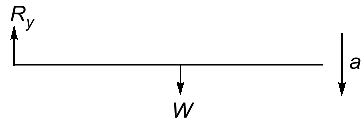
$$R_y = (W - F) \qquad \text{...(i)}$$

If force F removes, then rod will rotate about O, then

$$T_{(\text{torque})} = I\alpha$$

$$W \times \frac{L}{2} = \left(\frac{W}{g}\right)\frac{L^3}{3} \times \alpha \qquad \text{...(ii)}$$

$$W - R_y = \left(\frac{W}{g}\right) a_{cg} \qquad \text{...(iii)}$$



$$a_{cg} = \left(\frac{L}{2}\right)\alpha \qquad \text{...(iv)}$$

Equations (ii), (iii), (iv)

$$W \times \frac{L}{2} = \left(\frac{W}{g}\right)\frac{L^2}{3} \times \frac{a_{c.m}}{\left(\frac{L}{2}\right)}$$

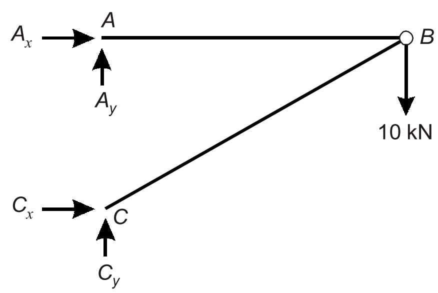
$$W \times \frac{L}{2} = \left(\frac{W}{g}\right) \times \frac{L^2}{3} \times \left(\frac{2}{L}\right) \times \frac{(W - R_y)}{\left(\frac{W}{g}\right)}$$

$$\frac{3W}{4} = W - R_y$$

$$R_y = \frac{W}{4}$$

1.11 Sol.

Method I:



$$AB = 1 \text{ m}$$

$$AC = 0.5 \text{ m}$$

$$BC = \sqrt{1^2 + 0.5^2} = \sqrt{1.25} = 1.118 \text{ m}$$

$$A_x + C_x = 0$$

$$A_y + C_y = 10$$

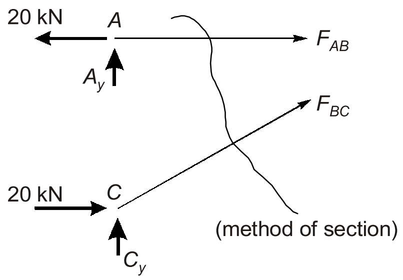
(from force equilibrium)

$$\Sigma M_A = 0$$

$$C_x \times 0.5 = 10 \times 1$$

or $\quad C_x = 20 \text{ kN}$

and $\quad A_x = -20 \text{ kN}$

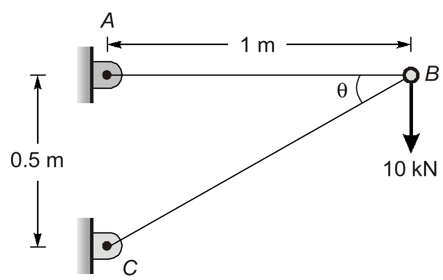


$$\Sigma M_c = 0$$

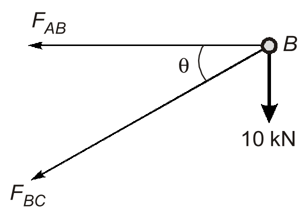
$\Rightarrow \quad F_{AB} \times 0.5 = 20 \times 0.5$

$\therefore \quad F_{AB} = \mathbf{20 \text{ kN}}$

<u>Method II:</u>



Free body diagram of point *B*,



Horizontal Reaction:

$$F_{BC} \cos\theta = -F_{AB} \qquad \text{...(i)}$$

Vertical Reaction:

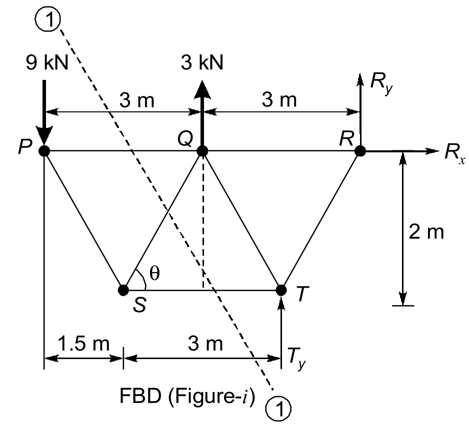
$$F_{BC} \sin\theta = -10 \qquad \text{...(ii)}$$

Putting the value of F_{BC} from equation (ii) to equation (i)

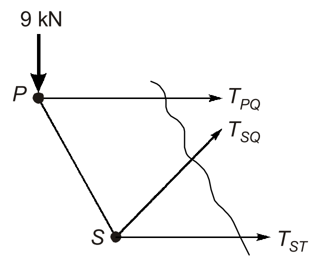
$$\frac{-10}{\sin\theta} \times \cos\theta = -F_{AB}$$

$$\therefore \quad F_{AB} = 10 \times \cot\theta = 10 \times \frac{1}{0.5} = 20 \text{ kN}$$

1.12 (a)



FBD (Figure-*i*)



Section through *PQ*,*QS*, & ST (Figure-*ii*)

$$\tan\theta = \frac{2}{1.5}$$

$$\theta = \mathbf{53.13°}$$

Considering L.H.S. of section (1)......(1)

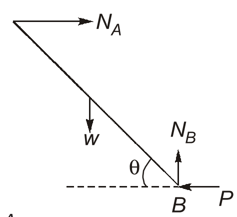
$$\Sigma F_v = 0$$

$$-9 + T_{SQ} \times \sin\theta = 0$$

$$T_{SQ} = \frac{9}{\sin 53.13°} = 11.25 \text{ kN (Tensile)}$$

1.13 Sol.

Drawing FBD of ladder AB



$$\tan\theta = \frac{3}{4}$$

$$(\Sigma F)_y = 0$$

$$\Rightarrow \quad 600 \text{ N} = N_B$$

Taking moment about *A*

$$-600 \times 2.5 \times \cos\theta + N_B \times 5 \times \cos\theta - P \times 5 \times \sin\theta = 0$$

$$\Rightarrow \quad -600 \times 2.5 \times \frac{4}{5} + 600 \times 5 \times \frac{4}{5} - P \times 5 \times \frac{3}{5} = 0$$

$\Rightarrow P = 400$ N