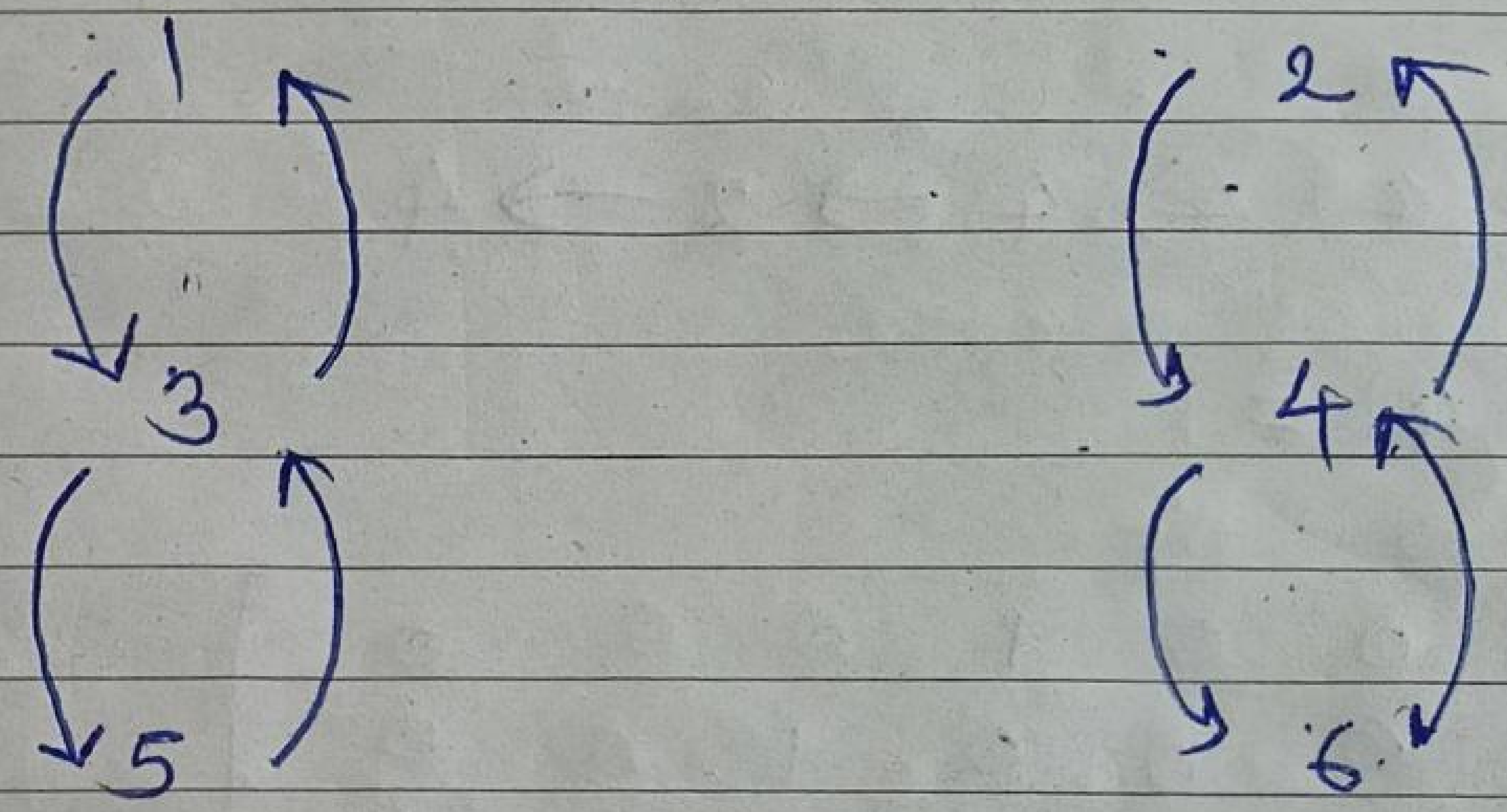
Q2 Find the transitive closure of R using Warshall's algorithm.
$A = \{1, 2, 3, 4, 5, 6\}$.
$R = \{(x, y) \mid |x - y| = 2\}$.

→ Given : $A = \{1, 2, 3, 4, 5, 6\}$.
R is a relation on the set A

$R = \{(x, y) \mid |x - y| = 2\}$.

$R = \{(1,3)\ (3,1),\ (3,5)\ (5,3)$
$(2,4)\ (4,2),\ (4,6)\ (6,4)$.



Here, $v_1 = 1$, $v_2 = 2$, $v_3 = 3$,
$v_4 = 4$, $v_5 = 5$, $v_6 = 6$

$$W_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

W_0 has entry 1 when $v_1 = 1$ & or $v_2 = 2$ as its interior vertex.

Now, $(3,3) \Rightarrow 3 \rightarrow 1 \rightarrow 3$

$(3,3) = 1$

$$W_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

W_2 has ~~many~~ entry 1, when $v_1 = 1$ & or $v_2 = 2$ as its interior vertex

Now, $(4,4) \Rightarrow 4 \rightarrow 2 \rightarrow 4$

$(4,4) = 1$.

$$W_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

W_3 has entry 1, when $v_1 = 1$, $v_2 = 2$, & or $v_3 = 3$, as its interior vertex

Now, $(1,1) \Rightarrow 1 \rightarrow 3 \rightarrow 1$.

$(1,1) = 1$

$(5,5) \Rightarrow 5 \rightarrow 3 \rightarrow 5$

$(1,5) \Rightarrow 1 \rightarrow 3 \rightarrow 5$

$(1,5) = 1.$

$(5,1) \Rightarrow 5 \rightarrow 3 \rightarrow 1.$

$(5,1) = 1.$

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

We have entry 1, when $v_1 = 1$, $v_2 = 2$, $v_3 = 3$, 1 or $v_4 = 4$ as interior vertex.

Now,

$(2,2) \Rightarrow 2 \rightarrow 4 \rightarrow 2$

$(2,2) = 1.$

$(6,6) \Rightarrow 6 \rightarrow 4 \rightarrow 6.$

$(6,6) = 1.$

$(2,6) \Rightarrow 2 \rightarrow 4 \rightarrow 6$

$(2,6) = 1.$

$(6,2) \Rightarrow 6 \rightarrow 4 \rightarrow 2.$

$(6,2) = 1$

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$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

W_5, has entry 1, when $v_1 = 1$, $v_2 = 2$, $v_3 = 3$, $v_4 = 4$ or $v_5 = 5$ as its interior vertex'

$\therefore W_5 = W_4$.

W_6 has entry 1, when $v_1 = 1$, $v_2 = 2$, $v_3 = 3$, $v_4 = 4$, $v_5 = 5$ or $v_6 = 6$ as its interior vertex

$\therefore W_6 = W_5$.

$\therefore$ The transitive closure of R is

$R' = \{ (1,1), (1,3), (1,5),$
$(2,2), (2,4), (2,6)$
$(3,1), (3,3), (3,5)$
$(4,2), (4,4), (4,6)$
$(5,1), (5,3), (5,5)$
$(6,2), (6,4), (6,6) \}$.

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