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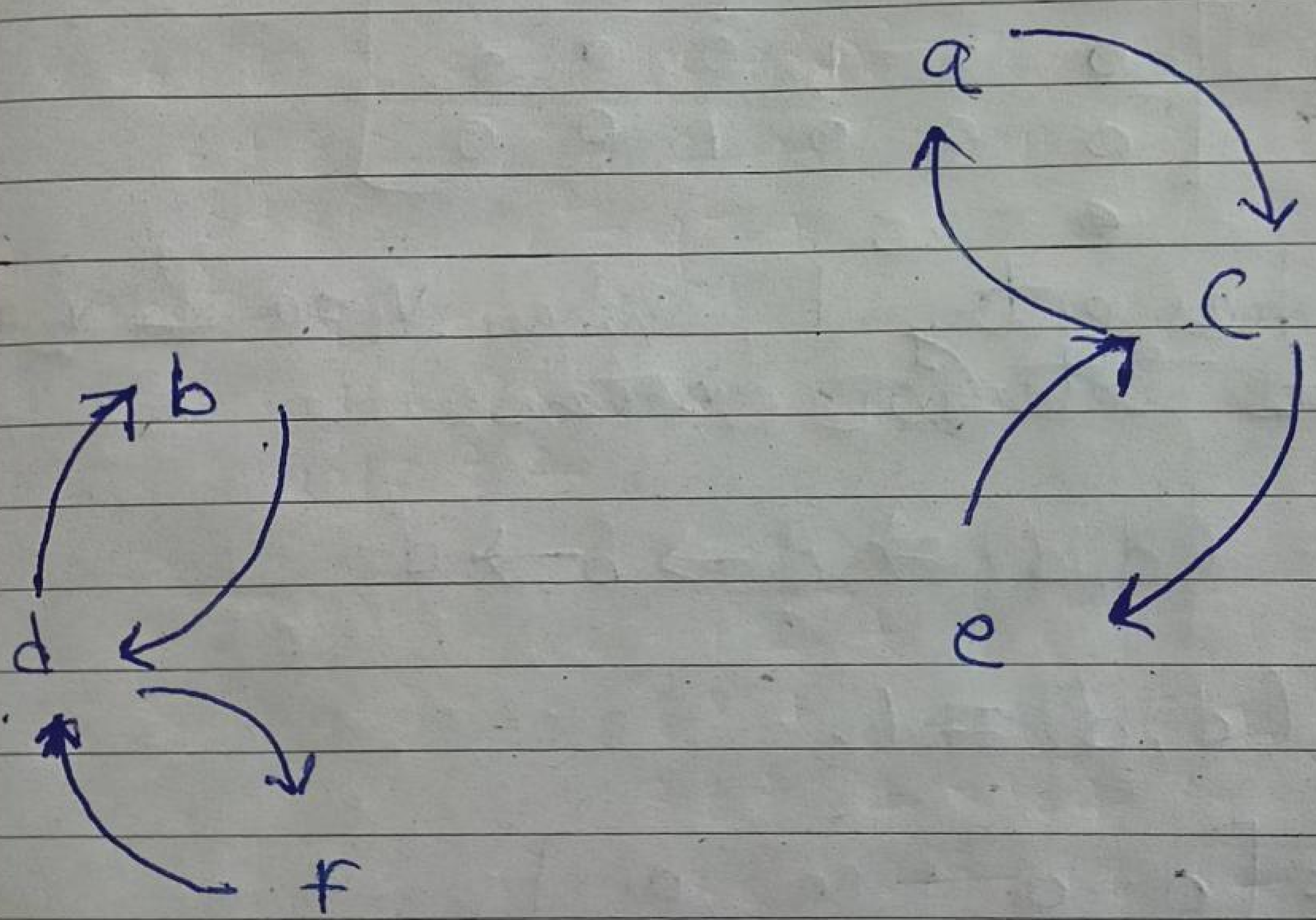
Q.1. Let R be the relation on the set $A = \{a, b, c, d, e, f\}$ &
$R = \{(a, c), (b, d), (c, a), (c, e), (d, b), (d, f), (e, c), (f, d)\}$.
Find the transitive closure of R using Warshall's algorithm.

→ Given: $A = \{a, b, c, d, e, f\}$.
R is relation on the set A.

$R = \{(a, c)\ (b, d)\ (c, a)\ (c, e)\ (d, b)\ (d, f),\ (e, c);\ (f, d)\}$.



Here $v_1 = a$, $v_2 = b$, $v_3 = c$, $v_4 = d$
$v_5 = e$, $v_6 = f$.

$$W_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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W_1 has entry 1, when $v_1 = a$, is its interior vertex'

$v_1 = a$, is a interior vertex

~~$v_1 = a$~~ Now,

$(c, c) \Rightarrow c, a, c$

$\therefore (c, c) = 1$

$$W_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

W_2 has entry 1, when $v_1 = a$ & $v_2 = b$ as its interior vertex.

Now, $(d, d) \Rightarrow d \rightarrow b \rightarrow d$.

$(d, d) = 1$.

$$W_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

W_3 has entry 1, when $v_1 = a$, $v_2 = b$, & $v_3 = c$, are interior vertex, Now, $(a,e) \Rightarrow a \to c \to e$.

$(a,e) = 1$

$(e,e) \Rightarrow e \to c \to e$

$(e,e) \to 1$

$(a,a) \Rightarrow a \to c \to a$

$(a,a) = 1.$

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

W_4 has entry 1, when $v_1 = a$, $v_2 = b$, $v_3 = c$ & $v_4 = d$ as interior vertex.

Now, $(b,f) \Rightarrow b \to d \to f$

$(b,f) = 1.$

$(f,b) \Rightarrow f \to d \to b$

$(b,b) = 1.$

$(f,f) \Rightarrow f \to d \to f$

$(f,f) = 1.$

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$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

W_5 has entry 1, when $v_1 = a$, $v_2 = b$, $v_3 = c$, $v_4 = d$ & $v_5 = e$ as its interior vertex.

$\therefore W_5 = W_4$.

W_5 has entry 1, when $v_1 = a$, $v_2 = b$, $v_3 = c$, $v_4 = d$ & $v_5 = e$ as its interior vertex.

$\therefore W_6 = W_5$.

$\therefore$ The transitive closure of R is

$R' = \{ (a,a), (a,c), (a,e), (b,b), (b,d), (b,f), (c,a), (c,c), (c,e), (d,b), (d,d), (d,f), (e,a), (e,c), (e,e), (f,b), (f,d), (f,f) \}$