

Homework 2

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Problem 1: Consider the bus ridership example. Intuitively, L_1 and L_2 are not independent, show that $\text{var}(L_1 - L_2)$ does not equal $\text{var}(L_1) + \text{var}(L_2)$. Determine the difference. (Find the 3 variances using simulation)

L_i = # of passengers on the bus as it leaves stop i

B_i = # of passengers boarding at stop i

$P(\text{leave}) = 0.2 \mid B(0) = 0.5 \mid B(1) = 0.4 \mid B(2) = 0.1$

```
set.seed(123)
nreps <- 10000000
nstops <- 2

# Simulate the number of riders at each stop
riders_at_stop <- matrix(0, nrow = nreps, ncol = nstops)

for (i in 1:nreps) {
  riders <- 0
  for (j in 1:nstops) {
    if (riders > 0) {
      for (k in 1:riders) {
        if (runif(1) < 0.2) {
          riders <- riders - 1
        }
      }
    }
    new_riders <- sample(0:2, 1, prob = c(0.5, 0.4, 0.1))
    riders <- riders + new_riders
    riders_at_stop[i, j] <- riders
  }
}

# Calculate the variances
var_L1 <- var(riders_at_stop[, 1])
var_L2 <- var(riders_at_stop[, 2])
var_L1_minus_L2 <- var(riders_at_stop[, 1] - riders_at_stop[, 2])
cov_L1_L2 <- cov(riders_at_stop[, 1], riders_at_stop[, 2])

# Print the results
cat("var(L1) = ", var_L1, "\n")
```

```
## var(L1) = 0.4401128
```

```
cat("var(L2) = ", var_L2, "\n")
```

```
## var(L2) = 0.8172579
```

```
cat("var(L1 - L2) = ", var_L1_minus_L2, "\n")
```

```
## var(L1 - L2) = 0.5536452
```

```
cat("var(L1) + var(L2) = ", var_L1 + var_L2, "\n")
```

```
## var(L1) + var(L2) = 1.257371
```

```
cat("cov(L1, L2) = ", cov_L1_L2, "\n")
```

```
## cov(L1, L2) = 0.3518627
```

```
cat("Difference = ", var_L1_minus_L2 - (var_L1 + var_L2), "which is twice the negative of the covariance")
```

```
## Difference = -0.7037255 which is twice the negative of the covariance
```