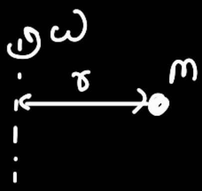
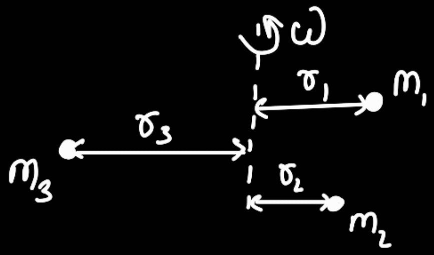
ROTATIONAL MOTION

Moment of Inertia

System of Particles



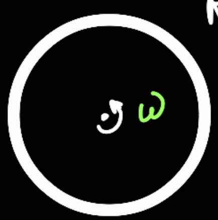
$I = mr^2$



$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

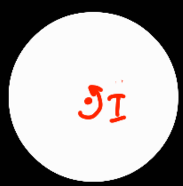
Continuous mass System.

Ring (m, r)



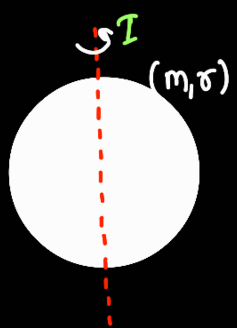
$\boxed{I = mr^2}$

Disc (m, r)

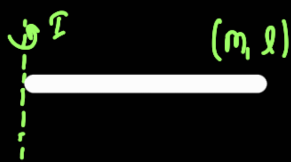


$\boxed{I = \frac{mr^2}{2}}$

Disc



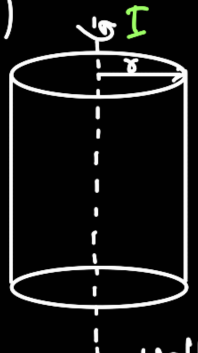
$\boxed{I = \frac{mr^2}{4}}$



$\boxed{I = \frac{ml^2}{3}}$

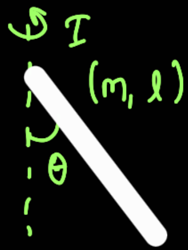
$\boxed{I = \frac{ml^2}{12}}$

(m, r)



$\boxed{I = mr^2}$

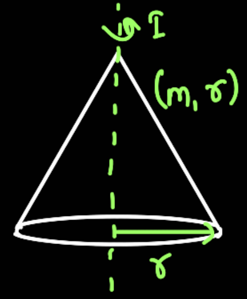
Hollow cylinder



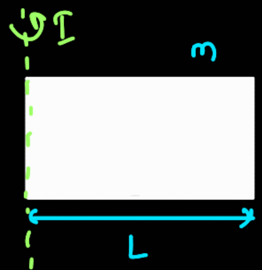
$\boxed{I = \frac{ml^2 \sin^2\theta}{3}}$

Solid cylinder

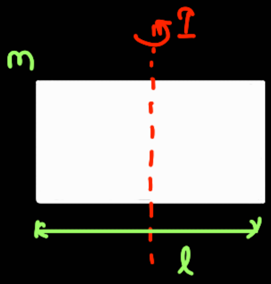
$$I = \frac{mR^2}{2}$$



$$I = \frac{mr^2}{2}$$

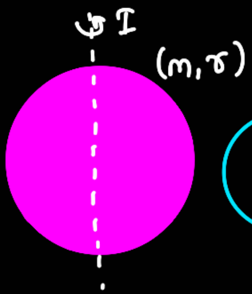


$$I = \frac{mL^2}{3}$$



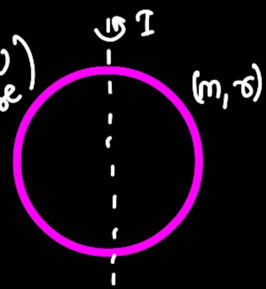
$$I = \frac{ml^2}{12}$$

Solid Sphere



$$I = \frac{2}{5} mR^2$$

(Hollow Sphere)



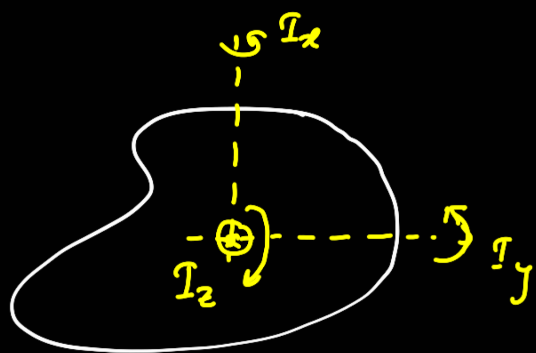
$$\frac{2}{3} mR^2$$

Perpendicular axis theorem

$$I_x + I_y = I_z$$

Same plane & ⊥ar

⊥ to plane of I_x & I_y.

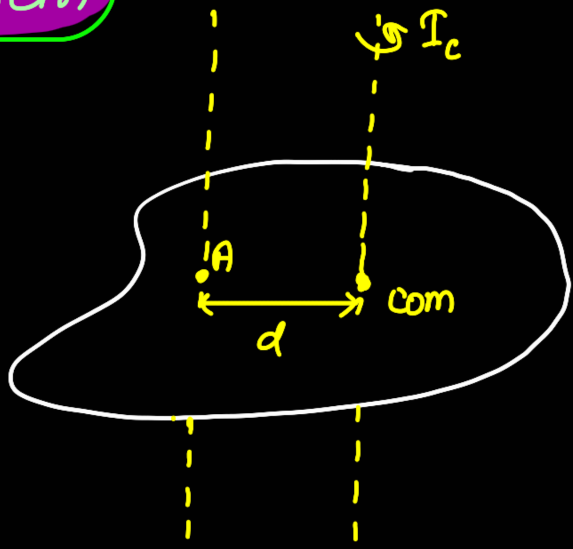


* I_x, I_y, I_z same point se pass hona chaya.
* com se pass ho zaroori nhi.

* Applicable for 2-D bodies

  3-D Bodies me apply nhi hoga.

Parallel Axis Theorem



$$I_A = I_c + md^2$$

* I_c com se pass hona zaroori hai

* Here, md^2 me mass = total mass of system hai.

* Radius of Gyration

Torque

$$\vec{\tau} = \vec{r} \times \vec{F} = r_\perp F = r F_\perp$$

Force agar kisi point se pass kar raha hai to us force ka torque us point ke about zero hoga

$\tau_O = 0$

Agar net Force 0 hai to har point ke about torque same hoga.

Equilibrium

→ Translational Eq. $\vec{F}_{net} = 0$

+

→ Rotational Eq. $\vec{\tau}_{net} = 0$ (about any point)

Fix-axis Rotation Motation

$$\boxed{K.E. = \frac{1}{2} I \omega^2}$$ (rotational K.E.)

About axis

$$\boxed{\vec{\tau} = I\alpha}$$

net torque about fix Axis

abt. fix axis

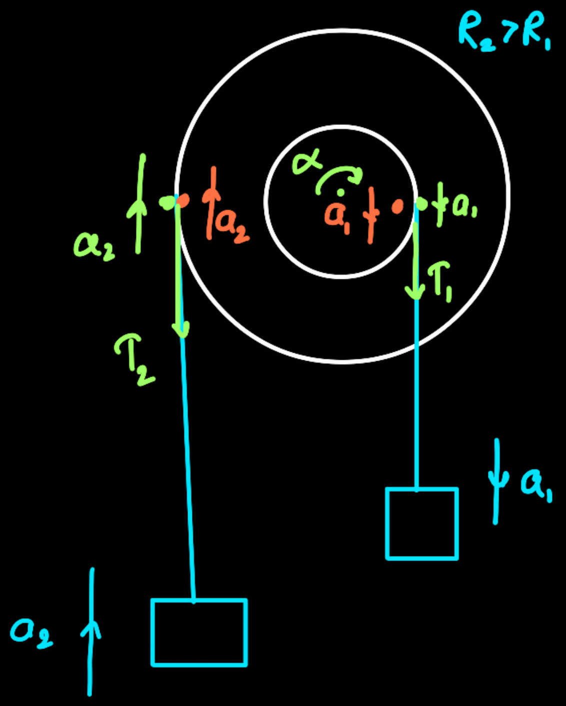
$$\boxed{(WD)_{ext\,\tau} = (\Delta K.E.)_{rotational}}$$

Jab bhi kabhi vel./ω puche
work energy theorem ko yaad karo

Jab bhi kabhi α puche to
$\vec{\tau}_{net} = I\alpha$ ko yaad karo

Jab bhi hinge force ke bare me
puche balma $(F = ma)$ ko yaad karo

Pulley System.



Rotation wali pulley me dhyan dene wali batte:-

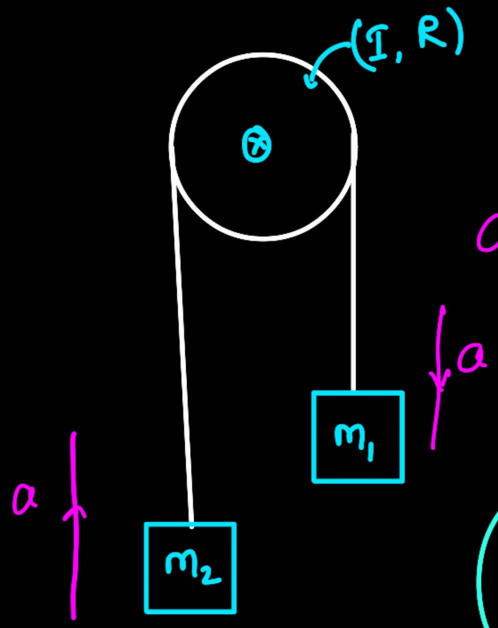
(i) Dono side tension same nhi hoga ab to T_1, T_2 manna hai

(ii) Pulley ke lia $\tau = I\alpha$ & constrained motion me $a = r\alpha$ lagana hai

$\boxed{a_1 = R_1\alpha}$ $\boxed{a_2 = R_2\alpha}$

& NLM Equations.

For simple/ single pulley system.



In NLM

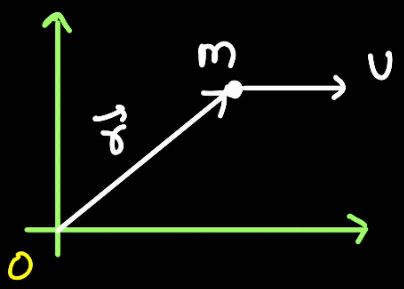
$$a = \frac{(\text{Kichne wala Force}) - (\text{Rokne wala Force})}{(\text{total mass})}$$

In Rotation

$$\boxed{a = \frac{(\text{Khichne walla Force}) - (\text{Rokne walla force})}{m_1 + m_2 + I/R^2}}$$

Angular Momentum

(i) for particle



$$\boxed{\vec{L} = \vec{r}\times m\vec{v} = m\vec{v}\,\vec{r}_\perp = \vec{r}\,m\vec{v}_\perp}$$

$\vec{r}$ wo vector hai jiski puch us point par hai jiske about torque pucha hai

mass se multiply kerna nhi bhulna galti ho chuki hai ek bar

(ii) Angular Momentum of Rigid Body.

$$\boxed{(\vec{L}_o)_{axis} = I_{axis}\,\omega}$$

Conservation of angular momentum:

$$(\tau_{net})_{ext.} = 0 \Rightarrow \boxed{\vec{L}_i = \vec{L}_f}$$

τ

Jis point ke about torque 0 hai usi ke about angular momentum conserve hoga

$$\boxed{\vec{P}_{sys} = m_{total}\,\vec{V}_{com}}$$

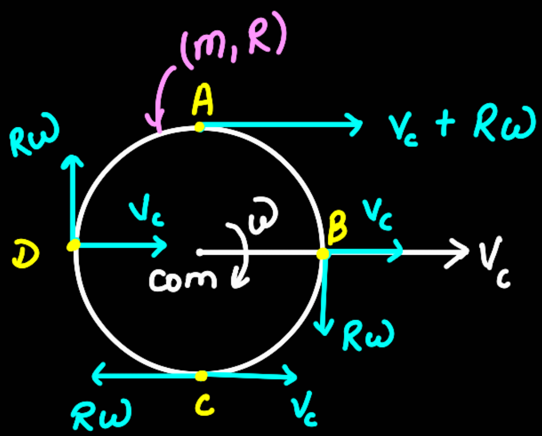
Angular Impulse (H)

Impulse = change in momentum

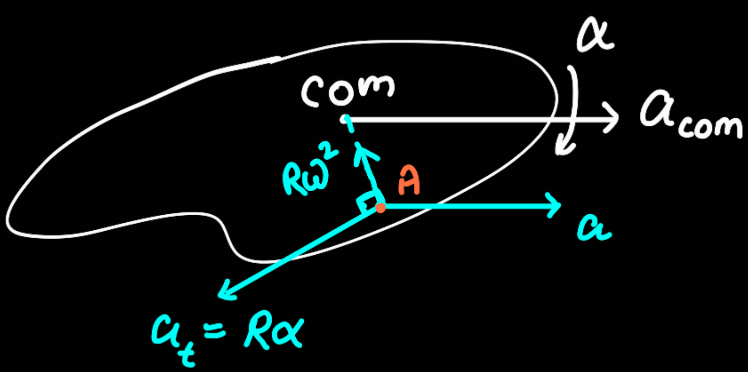
Angular Impulse = change in angular momentum

$$\boxed{H = \vec{r} \times \vec{J} = \Delta \vec{L} = \int \vec{\tau}\, dt}$$

Combined Rotational and Translational Motion (CRTM)



$V_{\text{any point on object}}$ = Vel. of com + (Angular Vel. of that point)



$$\vec{a}_{A/com} = \vec{a}_A - \vec{a}_{com}$$

$$\vec{a}_A = \vec{a}_{com} + \vec{a}_{A/com}$$

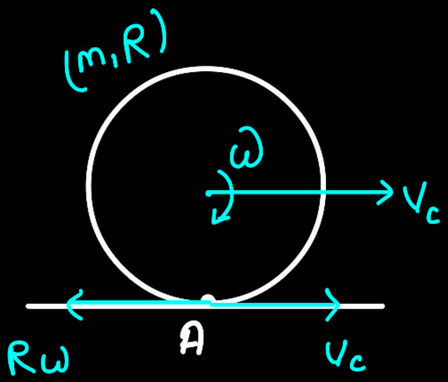
$$\boxed{\vec{a}_A = \vec{a}_{com} + R\vec{\alpha} + R\vec{\omega}}$$

CRTM me accl^n likhna hai to 3 terms aienge $\Bigg\}$
1. a_{com}
2. $a_t = R\alpha$
3. $a_c = R\omega^2$

Kinetic Energy in CRTM

$$K.E. = \frac{1}{2} m V_{com}^2 + \frac{1}{2} I_{com} \omega^2$$

ROLLING



For pure rolling, $V_A = 0$

i.e no relative motion b/w point A & ground.

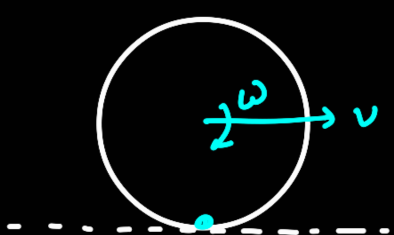
$$V_{com} = R\omega$$

Pure rolling kab start hoga puche to :-

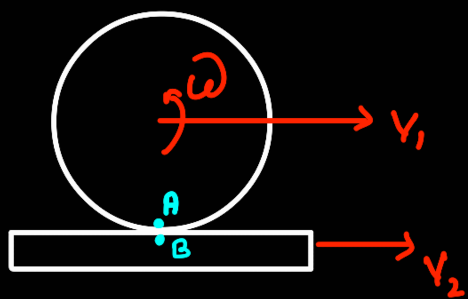
$$v = u + at$$
$$\omega_0 = \omega_i + \alpha t$$
$$\tau = \vec{r} \times \vec{F} = I\vec{\alpha}$$

← In 3 eq^n ka use karo

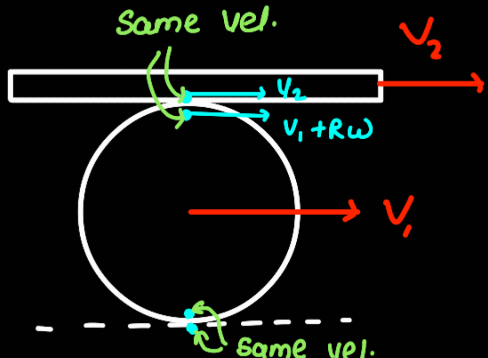
Condition for pure rolling.



$$v = R\omega$$



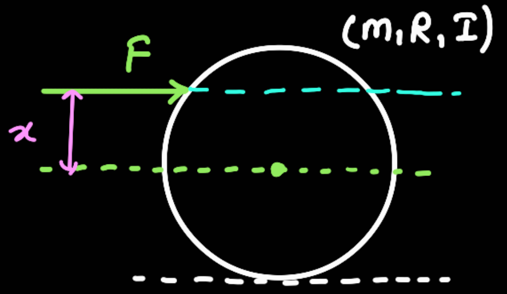
$$V_A = V_B$$
$$V_1 - R\omega = V_2$$



$$V_1 = R\omega$$
$$V_1 + R\omega = V_2$$

- For pure rolling ⇒ $v = R\omega$
  To maintain pure rolling ⇒ $a = R\alpha$

- Pure rolling me agar friction aia to wo hamesha static hoga.

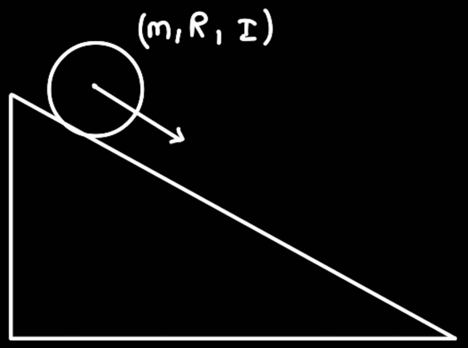
- 

$$x = \frac{I}{mR}$$

Ye vo point hai jis point par Force lagane par pure rolling ho jiega without frxⁿ

Agar blue wali line ke upar force laga to frxⁿ age lagega
aur agar us line ke niche force laga to frxⁿ piche lagega.

Rolling on Inclined Plane



$\boxed{a = g\sin\theta}$ ← In case of slipping

$$\boxed{a = \frac{g\sin\theta}{1 + \left(\frac{I}{mR^2}\right)}}$$ ← In case of Pure rolling

$\boxed{I\uparrow \rightarrow a\downarrow \rightarrow t\uparrow \rightarrow v\downarrow \rightarrow \omega\downarrow}$

(to reach lowest point)

translational KE ↓

$\because \boxed{W_g = (KE)_{trans.} + (KE)_{rot.}}$

Rotational KE ↑

↑(WET)↑

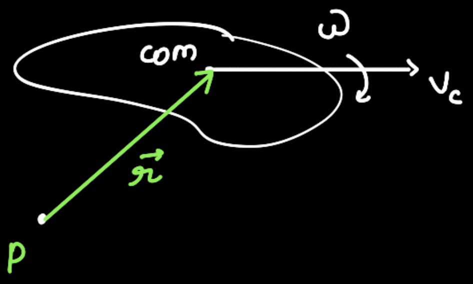
Angular momentum in CRTM

$$\boxed{\vec{L}_{\substack{body\\abt\ P}} = I_c\vec{\omega} + \vec{r}\times m\vec{v}_c}$$

moment of Inertia about com.

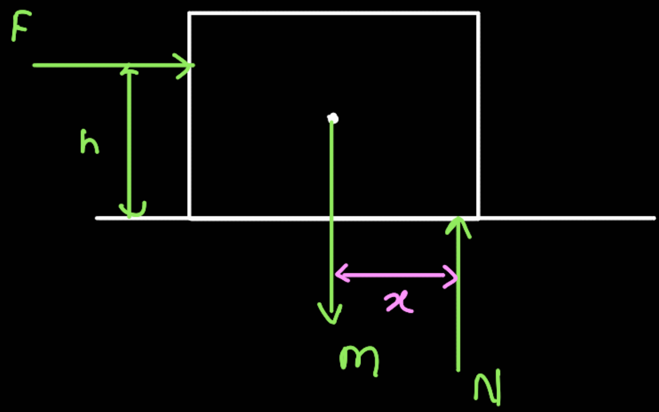
Total mass

vel. of com.



Jise impulse hone par momentum badalta hai usi tarah angular impulse hone par angular momentum badalta hai

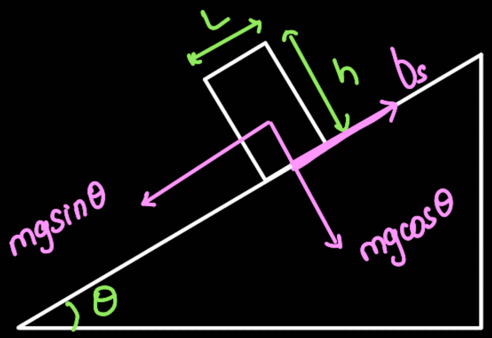
TOPPLING



Normal shifts in case of Toppling.

$F\uparrow \rightarrow x\uparrow$ (i.e. normal shifts)

For finding F_{min} for toppling, Normal ko shift kar ke kone par le jao



$$\left(\theta_{min}\right)_{toppling} = \tan^{-1}\left(\frac{L}{h}\right)$$

$$\left(\theta_{min}\right)_{sliding} = \tan^{-1}(\mu_s)$$

If θ increasing gradually,

$(\theta_{min})_T < (\theta_{min})_S \rightarrow$ First topple then slides.

$(\theta_{min})_T > (\theta_{min})_S \rightarrow$ Slides first then topple