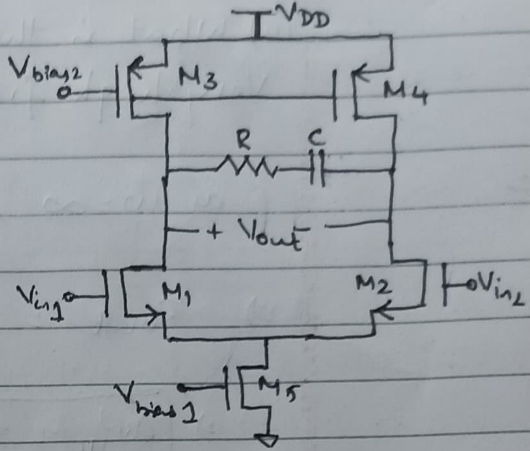
Assignment 3.

1. In the following circuit assume transistors M_1 and M_2, and transistors M_3 and M_4 are identical and $\gamma = 0$, $\lambda \neq 0$.

i) Find the expression for the small signal differential voltage gain $(V_{out}/V_{in1} - V_{in2})$ of the circuit.



let impedance Z

$= R + \frac{1}{j\omega C}$.

as Z is parallel to M_3 and to M_1 when AC ground is applied,

Rout can be considered as $(r_{o1} \| r_{o3} \| \frac{Z}{2})$ on one side.

$\Rightarrow R_{out} = (r_{o1} \| r_{o3} \| Z/2)$.

$\Rightarrow \boxed{A_V = -g_{m1}(r_{o1} \| r_{o3} \| Z/2)}$

ii) What is the gain of the circuit in very low frequencies?

at low frequencies $\omega = 0$.

$\Rightarrow Z = R + \frac{1}{j\omega C} = R + \frac{1}{0} = \infty$.

at $z = \infty$.

$$\boxed{A_{V_{LF}} = -g_{m_1}(r_{o_1} \| r_{o_3})}$$

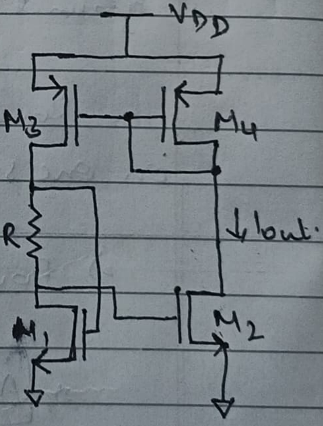
iii) What is the gain of the circuit at very high frequencies?

at $\omega = \infty$, $Z = R + \frac{1}{j\omega C} = R + \frac{1}{\infty} = R$.

$\Rightarrow$ $R_{out} = r_{o_1} \| r_{o_3} \| R/_2$.

$$\Rightarrow \boxed{A_{V_{HF}} = -g_{m_1}(r_{o_1} \| r_{o_3} \| R/_2)}$$

2. Assuming all transistors in saturation and ignoring CLM and body effect, Also, $(W/L)_3 = (W/L)_4$.



i) Find an expression for Iout.

$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 \Rightarrow I_1 = I_3 = I_{out}$

$I_{out} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS_2} - V_{th})^2$.

$\Rightarrow V_{GS_2} = V_{thn} + \sqrt{\frac{2 I_{out}}{\mu_n C_{ox}(W/L)_2}}$.

$\text{lly } V_{GS_1} = V_{thn} + \sqrt{\frac{2 I_{out}}{\mu_n C_{ox}(W/L)_1}}$.

Also,

$V_{GS_1} - V_{GS_2} = I_{out} \times R$.

$$\Rightarrow V_{GS_1} - V_{GS_2} = \left[V_{thn} + \sqrt{\frac{2 I_{out}}{\mu_n C_{ox}(W/L)_1}}\right] - \left[V_{thn} - \sqrt{\frac{2 I_{out}}{\mu_n C_{ox}(W/L)_2}}\right]$$

$$\Rightarrow R I_{out} = \sqrt{\frac{2 I_{out}}{\mu_n C_{ox}}}\left(\sqrt{\left(\frac{L}{W}\right)_1} - \sqrt{\left(\frac{L}{W}\right)_2}\right)$$

$$\Rightarrow R \sqrt{I_{out}} = \sqrt{\frac{2}{\mu_n C_{ox}}}\left(\sqrt{\frac{L_1}{W_1}} - \sqrt{\frac{L_2}{W_2}}\right)$$

$$\Rightarrow \boxed{I_{out} = \frac{2}{\mu_n C_{ox} R^2}\left(\sqrt{\frac{L_1}{W_1}} - \sqrt{\frac{L_2}{W_2}}\right)^2}$$

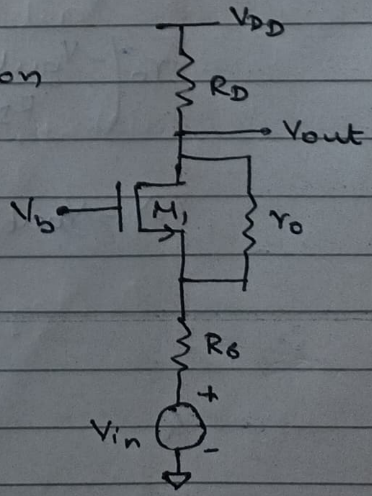
ii) What would be the percentage change in I_{out} if V_{DD} is increased by 10%.

As all transistors are in saturation, the I_{out} is independent of V_{DD}. Hence, there will be no change in I_{out}.

iii) How would the expression for I_{out} derived in part (i) change if $\gamma \neq 0$ and why?

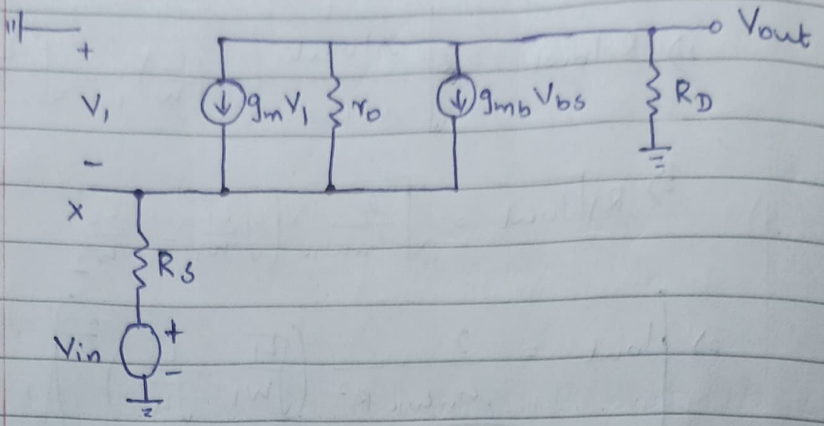
As $V_{SB_1} = V_{SB_2} = 0$, the resultant body effect would not change the I_{out}.

3. Calculate the gain of the common gate amplifier as shown below considering both CLM and Body effect.



The current through R_S is same as $-V_{out}/R_D$.

Small signal:



KVL at ip:

$$V_1 - \frac{V_{out}}{R_D} \cdot R_S + V_{in} = 0. \quad \text{—(1)}$$

KVL at op:

$$r_o\left(-\frac{V_{out}}{R_D} - g_m V_1 - g_{mb} V_1\right) - \frac{V_{out}}{R_D} \cdot R_S + V_{in} = V_{out} \quad \text{—(2)}$$

from (1)

$$V_1 = V_{out} \cdot \frac{R_S}{R_D} - V_{in}.$$

Substituting value of V_1 in (2).

$$r_o\left(-\frac{V_{out}}{R_D} - (g_m + g_{mb})\left(V_{out} \cdot \frac{R_S}{R_D} - V_{in}\right)\right) - V_{out}\frac{R_S}{R_D} + V_{in} = V_{out}$$

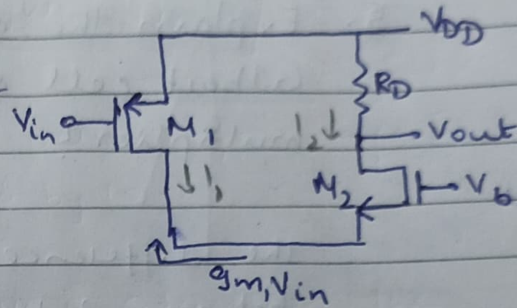
$$\Rightarrow -V_{out} \cdot \frac{r_o}{R_D} - r_o(g_m + g_{mb}) \cdot \frac{R_S}{R_D} \cdot V_{out} - V_{out}\frac{R_S}{R_D} - V_{out}$$

$$= -V_{in} - (g_m + g_{mb}) r_o V_{in}.$$

$\Rightarrow V_{out}\left(\frac{r_o + (g_m+g_{mb})r_o R_S + R_S + R_D}{R_D}\right) = V_{in}(1 + r_o(g_m+g_{mb}))$

$$\Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{[(g_m+g_{mb})r_o + 1]\,R_D}{r_o + (g_m+g_{mb})r_o R_S + R_S + R_D}}$$

4. Explain the working of folded cascode circuit using large-signal analysis.



If V_{in} becomes more +ve, $|I_{D1}|$ decreases, forcing I_{D2} to increase, hence decreasing V_{out} to drop.

If V_{in} decreases from V_{DD} to zero, for $V_{in} > V_{DD} - |V_{thn1}|$, M_1 is off and M_2 carries all of I_2, yielding $V_{out} = V_{DD} - I_1 R_D$. For $V_{in} < V_{DD} - |V_{thn1}|$, M_1 turns on in saturation,

$$I_{D2} = I_1 - \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{in} - |V_{thn1}|)^2.$$

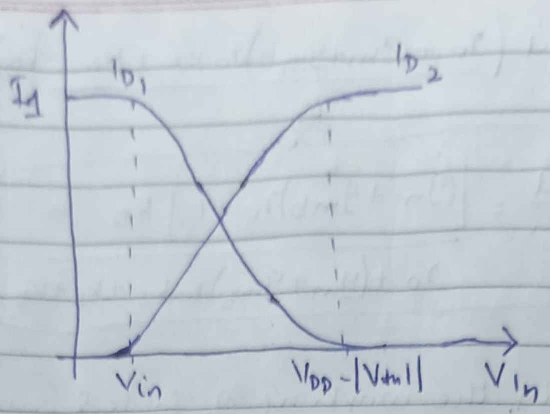
As V_{in} drops, I_{D2} decreases further, falling to zero, if $I_{D1} = I_1$.

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{in1} - |V_{thn1}|)^2 = I_1.$$

Thus,

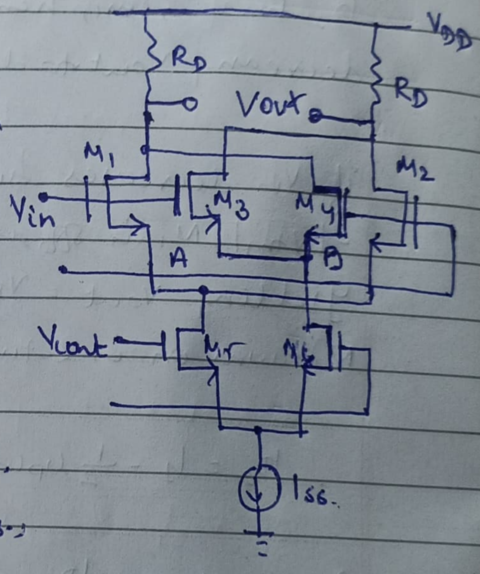
$$V_{in1} = V_{DD} - \sqrt{\frac{2I_1}{\mu_p C_{ox}(W/L)_1}} - |V_{thn1}|.$$

If V_{in} falls below this level, I_{D1} tends to be greater than I_1 and M_1 enters the triode region.



5. Explain the operation of Gilbert cell. Can Gilbert cell operation as an analog voltage multiplier?

The difference between 2 terminals of V_{cont} steer a lot part of I_{SS} from V_{in} to V_{out}.



Since,

$V_{out_1} = R_D I_{D_1} - R_D I_{D_2}$.

$V_{out_2} = R_D I_{D_4} - R_D I_{D_3}$.

$$V_{out} = R_D (I_{D_1} + I_{D_4}) - R_D (I_{D_2} + I_{D_3}).$$

where,

$$\frac{V_{out_1}}{V_{in}} = -g_m R_D, \quad \frac{V_{out_2}}{V_{in}} = +g_m R_D.$$

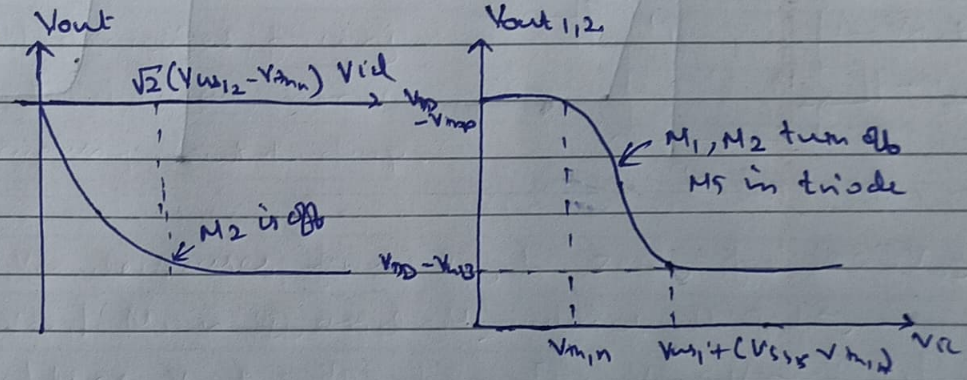
Analog Voltage multiplier:

Since the gain of the circuit is a function of $V_{cont} = V_{cont1} - V_{cont2}$, we have

$V_{out} = V_{in} \cdot f(V_{cont})$. Expanding $f(V_{cont})$ in a Taylor series and retaining only the first-order term, αV_{cont}, we have $V_{out} = \alpha V_{in} V_{cont}$. Thus, the circuit can multiply voltages. This property accompanies any voltage-controlled variable-gain amplifier.

6. Assuming all the circuit to be symmetric, plot V_{out} as V_{in1} and V_{in2} vary differentially from 0 to V_{DD}

a)



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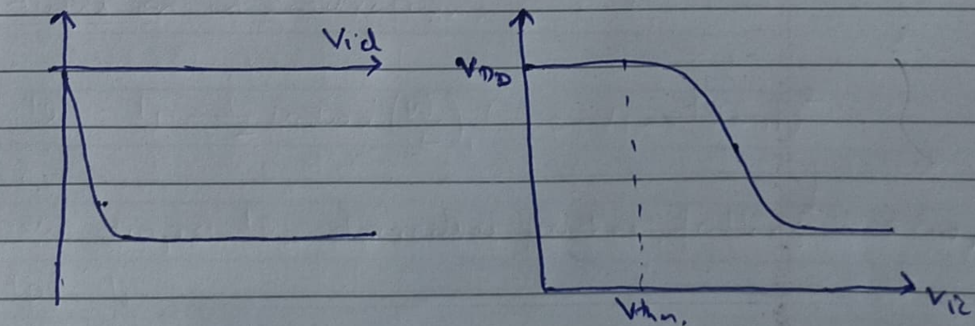
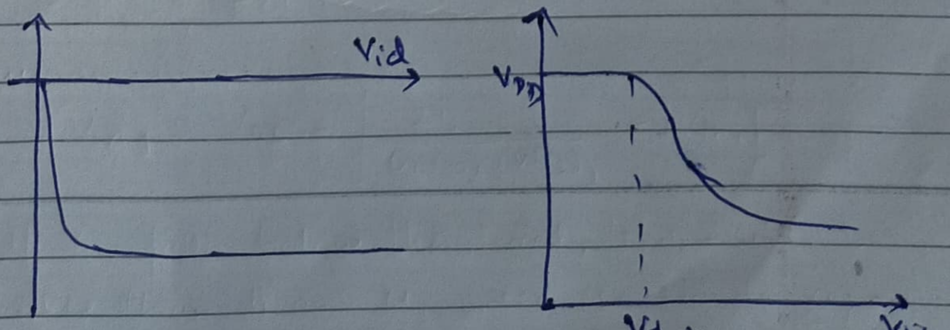
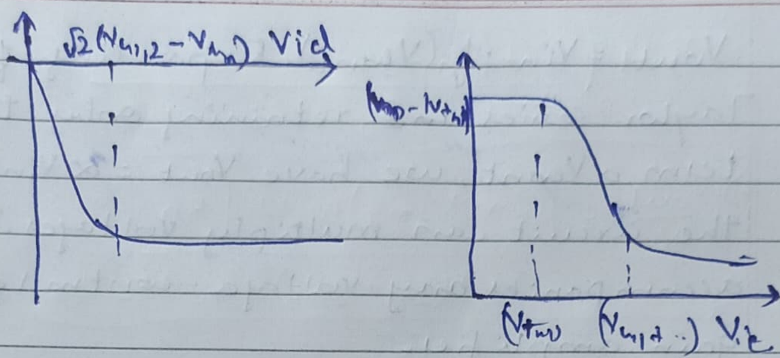


c)



d)



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