

Question: Fit a linear regression model onto the following data.

Initial parameter values are given, perform 2 iterations of Gradient Descent and find the final parameters of model. Take  $\eta = [0.1, 0.0]$  for 2 iterations.

Soln:

$f_1$	$f_2$	Price
0.2	0.6	2
0.3	0.7	7
0.1	0.4	6
0.7	0.9	10

Initial parameters

$$\begin{cases} \theta_0 = 0.06 \\ \theta_1 = 0.07 \\ \theta_2 = -0.10 \end{cases}$$

$$h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x_1^{(i)} + \theta_2 x_2^{(i)}$$

$$\text{Given: } \begin{cases} \theta_0 = 0.06 \\ \theta_1 = 0.07 \\ \theta_2 = -0.10 \end{cases}$$

$f_1$ ( $x_1$ )	$f_2$ ( $x_2$ )	$y$ (actual)	$y$ (predicted) $h_{\theta}(x)$
0.2	0.6	0.02	$0.06 + 0.07 \times 0.2 + (-0.1) \times 0.6 = 0.014$
0.3	0.7	0.07	$0.06 + 0.07 \times 0.3 + (-0.1) \times 0.7 = 0.011$
0.1	0.4	0.06	$0.06 + 0.07 \times 0.1 + (-0.1) \times 0.4 = 0.027$
0.7	0.9	0.010	$0.06 + 0.07 \times 0.7 + (-0.1) \times 0.9 = 0.019$

$$\begin{aligned} \text{MSE}(y_a, y_p) &= \frac{1}{m} \sum_{i=1}^m (y^{(i)} - h_{\theta}(x^{(i)}))^2 \\ &= \frac{1}{2 \times 4} \left( (0.02 - 0.014)^2 + (0.07 - 0.011)^2 + (0.06 - 0.027)^2 + (0.01 - 0.019)^2 \right) \end{aligned}$$

$$0.000585$$

=

from gradient descent update rule:

$$\theta_j := \theta_j - \eta \nabla_{\theta_j} J(\theta)$$



$$= -0.098$$

Old Thetas      New Thetas (Iteration 1)

Given:  $\begin{cases} \theta_0 = 0.6225 \\ \theta_1 = 0.0703 \\ \theta_2 = -0.098 \end{cases}$        $\begin{cases} 0.0689 \\ 0.07159 \\ -0.005 \end{cases}$

Now Computation for iteration 2

$f_1(x_1)$	$f_2(x_2)$	$y(\text{actual})$	$y_p(\text{using new } \theta's)$ $\theta_0 + \theta_1 x_1 + \theta_2 x_2$
0.2	0.6	0.02	$0.06225 + 0.0703 \times 0.2 + -0.098 \times 0.6 = 0.0175$
0.3	0.7	0.07	$0.06225 + 0.0703 \times 0.3 + -0.098 \times 0.7 = 0.1474$
0.1	0.4	0.06	$0.06225 + 0.0703 \times 0.1 + -0.098 \times 0.4 = 0.0300$
0.7	0.9	0.010	$0.06225 + 0.0703 \times 0.7 + -0.098 \times 0.9 = 0.0232$

$$\begin{aligned} \text{MSE}(y_a, y_p) &= \frac{1}{m} \sum_{i=1}^m (y^{(i)} - h_{\theta}(x^{(i)}))^2 \\ &= \frac{1}{8} \left[ (0.02 - 0.0175)^2 + (0.07 - 0.1474)^2 + (0.06 - 0.0300)^2 + (0.010 - 0.0232)^2 \right. \\ &\quad \left. + (-0.0132)^2 \right] \end{aligned}$$

$$= \frac{1}{8} \left[ 0.00707125 \right]$$

$$= 0.000883 \text{ (new error)} \quad \left( \begin{array}{l} \text{Yay!! error is} \\ \text{reducing!} \end{array} \right)$$



