1 $\hat{y} = ax_i + b$

$\sum(y_i - \hat{y}_i)^2$

$\sum(y_i - (ax_i + b))^2$

Taking the partial derivitive,

$\frac{\partial}{\partial a} \sum(y_i - (ax_i + b))^2 = \sum 2(y_i - (ax_i + b))(-x_i)$

$\frac{\partial}{\partial a} = -2\sum x_i(y_i - (ax_i + b)$

$\frac{\partial}{\partial b} \sum(y_i - (ax_i + b))^2 = -2\sum(y_i - (ax_i + b))$

Setting the partial derivitives to 0

$-\cancel{2}\sum x_i(y_i - (ax_i + b)) = 0$

$-2\sum(y_i - (ax_i + b)) = 0$

solving the equations,

$\sum y_i - \sum ax_i - \underbrace{\sum b}_{NB} = 0$

so

$b = \frac{\sum y_i - a\sum x_i}{N}$ $\quad a = \frac{\sum x_i y_i - \cancel{b\sum x_i}}{\sum x_i^2}$

$B_1 = a$
$B_0 = b$

Thus,

$b = \bar{y} - a\bar{x}$

substituting into a equation,

$\sum x_i (x_i - (\bar{y} - a\bar{x} + ax_i))) = 0$

$\sum x_i (x_i - \bar{y} - a(x_i - \bar{x})) = 0$

$\sum x_i (x_i - \bar{y}) - \sum a x_i (x_i - \bar{x}) = 0$

$$a = \frac{\sum x_i (x_i - \bar{y})}{\sum x_i (x_i - \bar{x})} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

reducing the following expression,

$\sum x_i (x_i - \bar{y}) - \sum \bar{x}(x_i - \bar{y})$

$= \sum x_i (y_i - \bar{y}) - \bar{x} \sum (y_i - \bar{y})$

$= \sum x_i (y_i - \bar{y}) - \bar{x} \sum (y_i - n\bar{y}) = 0$

y goes to 0 so

$= \sum x_i (y_i - \bar{y})$

and reducing the denominator,

$\sum (x_i - \bar{x})^2 = \sum (x_i - \bar{x})(x_i - \bar{x})$

$= \sum x_i (x_i - \bar{x}) - \sum \bar{x}(x_i - \bar{x})$

$= \sum x_i (x_i - \bar{x}) - \bar{x} \sum (x_i - \bar{x})$

$= \sum x_i (x_i - \bar{x})$

so $$\boxed{\hat{y} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} x + \bar{y} - a\bar{x}}$$