**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) mean = 45min

Std = 8 min

But transmission will begin after 10 min after the car dropped off.

Now, the mean shifted to (45+10) = 55 min

As mentioned servicing transmission is normally distributed.

p(x>60) = 1-stats.norm.cdf(60, loc = mew, scale = std)

= 1- stats.norm.cdf(60, loc = 55, scale = 8)

= 0.2676.

**Option B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans) no.of employees older than 44 = (1-stats.norm.cdf(44,loc=38,scale=6))\*400 = 60.

No.of employes in between 38 and 44

= stats.norm.cdf(44,loc=38,scale=6)-stats.norm.cdf(38,loc=38,scale=6)

= 136.

**False, no.of employees in between 38 and 44 is greater than no.of employees older than 44 at center.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) no.of employees under the age of 30 = stats.norm.cdf(30,loc=38,scale=6)\*400

= 36.

**True, we can except 36 employees under the age of 30 for training.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean, and the variance.

From the properties of normal random variables,

if X*1* ~ N(μ*1*, σ*1* 2) and X*2* ~ N(μ*2*, σ*2*2) are two independent identically distributed random variables then

* the sum of normal random variables is given by

*X1 +X*2 ~  N(μ*1+* μ*2,* σ*1* 2+ σ*2*2)

and the difference of normal random variables is given by

*X1 -X*2 ~  N(μ*1-* μ*2,* σ*1* 2+ σ*2*2)

       When Z = aX the product of X is given by

Z ~ N(aμ*1,* a^2 σ*1* 2)

Given to find, 2X1

Thus, following the property of multiplication, we get

2X1 ~ N(2μ*1,* 4σ*1* 2)

and following the property of addition,

*X1 +X*2 ~  N(2μ*,* 2σ2)

The mean of 2x1 and x1 +x2is same but the var(https://tex.z-dn.net/?f=%5Csigma%5E2) of  2x1 is 2 times more than the variance of x1+x2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) D.

stats.norm.interval(0.99,loc=100,scale=20)= (48.48,151.51)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans)

Company ~ N(5+7,(3^2+4^2)) ~N(12,5^2)

Converting into rupees = N(54,00,00,000, 22,50,00,000)

Mean = 540 million rupees

Std = 225 million

stats.norm.interval(0.95,loc=540,scale=225)

95%probability annual profit range of the company = (99.081, 980.991) in millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans)

To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table,

5 percentile = -1.645

5th percentile of profit (in Rupees) for the company = np.round(540+(-1.645)\*(225))

= 170

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans)

Divison 1 Profit1 ~ N(5, 32)

Divison 2 Profit2 ~ N(7,4^2)

P(div1<0) = stats.norm.cdf(0,loc=5,scale=3)=0.047

P(div2<0) = stats.norm.cdf(0, loc=7,scale=4) = 0.040.

Divison1 has slightly high probability than division2. So, divison1 have larger probability of making a loss in given year.