Optimal Wideband Spectrum Sensing Order Based on Decision-making Tree in Cognitive Radio

Han Han, Jin-long Wang, Qi-hui Wu, Yu-zhen Huang
Institute of Communications Engineering, PLA University of Sci. & Tech
Nanjing, China, Email: hhhd20042008@163.com

***Abstract*—In the cognitive radio networks, the cognitive user must scan the multi-channel to acquire an idle channel before its cognitive transmission. If the sensing capability is limited, the user must sense the multi-channel one by one. So the sensing order is very crucial to the realization of maximum throughput. In this paper, we aim to find the optimal wideband sensing order in polynomial time which is much faster than brute-force search and dynamic programming approach. First, we present two greed solutions, with the trait: fast but not optimal. Also, we analyze and improve the potential function in the optimality principle of sequencing. Then, we propose the decision-making tree method with the proper branching rule and the lopping rule, to search the optimal wideband spectrum sensing order. In the end, through the performance analysis, we validate the low computational complexity and the optimality.**

***Index Terms*—cognitive radio, spectrum sensing, wideband, sensing order, decision-making tree**

I. Introduction

Cognitive radio (CR)[1] is a dynamic spectrum access technology, aimed to solve the crisis of spectrum shortage caused by the fixed spectrum allocation. A cognitive user (also referred to as second user) can change its transmitting or receiving parameters to communicate efficiently while avoiding interference with licensed users (also referred to as primary user). Therefore, the aforehand action is to sense the primary user's activity. When the primary user is depart, the second transmission can be carried on in the idle channel. When the primary user is busy transmitting, the cognitive user could stop to wait or scan the other channels. It will keep scanning the multi-channel until an idle channel is found. Since the transmission cannot be performed whiles the sensing, then a problem is proposed: how to maximize the cognitive transmission in a given time.

To solve the problem, there are two approaches. One is to discover the idle channel as fast as possible, leaving more time to transmit. The other one is to discover the channel with more achievable rates. In this research, the channel is sensed one after another, so the sensing order is a considerable object to optimize. For the former approach, the intuitive order is the descending order of the average idle probabilities, and for the later approach, the descending order of the achievable rates. This issue has recently received much attention. In [2], Hyoil Kim aims at minimizing the delay in finding an idle channel, and an optimal sensing order is proposed, which is the descending order of the idle probabilities. He also researches the order problem in the scenario in which the idle probability, the sensing duration and the channel capacity are all considered in [3]. A referenced parameter is obtained. But its goal is to minimize the average delay while guaranteeing the cumulative capacity, which is different from our research. In [4], the selection of the order is modeled as an assignment problem. Its solution could resort to the classical Hungarian algorithm, but when the number of users is large, the huge complexity forces the user to take up a suboptimal greedy algorithm. Q. Zhao proposes a universal channel selection framework for spectrum sensing and access in [5]. And in [6], the author investigates the channel selection strategy based on the LA (*Learning Automata*). The algorithm adjusts the probability of selecting each available channel and converges to the sub-optimal solution. The research in [7] formulates the problem to a multi-armed bandit problem. In [8], H. Jiang considers the Rayleigh fading channel, and formulates the channel sensing problem as an optimal stopping rule problem. When the adaptive modulation is not presented, the optimal order is the descending order of the channel availabilities, while the adaptive modulation is presented, a dynamic programming approach is proposed. However the complexity is also very huge when the number of the channels is large. In [9], the author gives a sensing order considering the expected maximum throughput, but it is optimal only when the availability of each channel is the same.

In this paper, the optimal sensing order is investigated for a single-user case in multi-channel. We will take the channel capacity (a function of the bandwidth and the SNR) and the idle probability into consideration. Every channel has different capacity and different idle probability which has the generality in the primary channels. And it should be noted that the multi-channel may distribute in a broad spectrum scope called wideband. So the brute force search or the DP (*Dynamic Programming*) ~~cannot attain the optimal order in a given polynomial time [8]. Though some algorithms [2][8][9] have~~

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proposed the order as the descending order of one parameter (the channel capacity or the idle probability), they always assume the other one is the same among different channels. In this paper, our aim is to exploit the optimal sensing order in a limited computational time, so as to achieve the maximal expected throughput. Through the research on the potential function defined in the optimality principle of the sequencing, we find the difference, as well as the method (the decision-making tree method) to solve the problem. In the decision-making tree, we define the proper branching rule and the lopping rule. And then, by searching in the dominance set, we get the optimal wideband sensing order with the computational complexity $O(N^3/2)$, in despite of the large spatial complexity.

The rest of the paper is organized as follows: Section Ⅱ presents the system model. Section Ⅲ gives one simple greed algorithm. Section Ⅳ introduces the optimality principle of sequencing before the analysis of the second greed algorithm. The decision-making tree method is proposed in Section Ⅴ with its complexity analysis and optimality analysis in Section Ⅵ. In the last section are the conclusions.

II. SYSTEM MODEL

Considering a single cognitive user case, there are N independent channels in the system. For channel i, the bandwidth is B_i, $i=1,2,...N$ and the signal-to-noise (SNR) is $SNR_i, i=1,2,...N$. Every primary user in each band is idle with the probability θ_i which is also referred to as the availability probability or the idle probability. Of course, θ_i is variable in a long time. Here, the cognitive user performs the sensing in a slotted fashion. In order to simplify the order problem, we assume the B_i, SNR_i, θ_i of channel i are constants in a given time slot, which is known to the cognitive user through some detecting methods.

At the beginning of each time slot, the cognitive user senses the multi-channel in a determinable order $s=(s_1,s_2,...,s_N)$ where s_i is one channel from the multi-channel in the sequence i. Once an idle channel is found, the sensing process is terminated, and the transmission is performed in the remnant time of the slot as depicted in Fig.1. In fact, it's possible that no idle channel is found after the sensing process, in which circumstance the transmission have to be postponed to the next slot. The sensing time for every channel is the same, which is $\tau<\frac{1}{N}$ (the slot time is 1). We assume the sensing time is enough to the channel detection and the SNR estimation, where the detection error is omitted. And we also assume the adaptive modulation is adopted in order to maintain the Shannon Capacity of the idle channel. Our aim is to determine the sensing order in a time slot, so as to achieve the maximal expected throughput which is given by

$$Q=\mathrm{E}[C_{s_k}(1-k\tau)]=\sum_{k=1}^{N}\left(\prod_{i=1}^{k-1}(1-\theta_{s_i})\right)\theta_{s_k}C_{s_k}(1-k\tau) \quad (1)$$

where the C_i is the Shannon Capacity of the channel i, defined as $C_i=B_i\log(1+SNR_i)$. In conclusion, the mathematic model of the problem is as follows.

$$s^o=\arg\max_s \quad Q \quad (2)$$

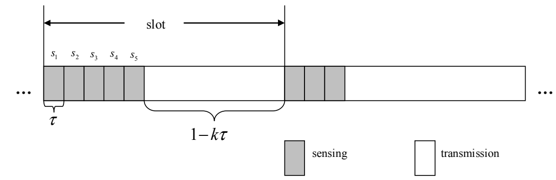

Fig.1. the slotted fashion (N=10). 5 channels are sensed in the former slot while 3 in the later slot. And k=5 in the former slot while k=3 in the later slot.

III. THE FIRST GREED ALGORITHM

From the above equation, we easily think the brute force search could discover the optimal sensing order. But the computational complexity is $O(N\cdot N!)$ if the complexity in calculating the expected throughput is $O(1)$. Fortunately, we can rewrite the equation (1) as follows.

$$Q_k=\mathrm{E}[C_{s_k}(1-k\tau)]=\theta_{s_k}C_{s_k}(1-k\tau)+(1-\theta_{s_k})Q_{k+1} \quad (3)$$

where Q_k is the expected throughput in the k-th sensing channel. And Q_1 is our target. Specially, when $k=N$, $Q_N=\theta_{s_N}C_{s_N}(1-N\tau)$. Therefore, a simple and greed solution to the NP hard problem can be concluded following the steps.

Step 1: the last channel to sense: $s_N=\arg\min_{i\in\{1,2,...,N\}} \theta_iC_i(1-N\tau)$. And the chosen channel set $\Lambda=\{s_N\}$, $k=N-1$.

Step 2: the k-th channel to sense: $s_k=\arg\min_{i\in\{1,2,...,N\}\setminus\Lambda} \theta_iC_i(1-k\tau)+(1-\theta_i)Q_{k+1}$, since the $(k+1)$-th sensing channel has been estimated.

Step 3: $\Lambda=\Lambda\cup\{s_k\}$. $k=k-1$, go to step 2.

IV. THE SECOND GREED ALGORITHM

A. The Optimality Principle of Sequencing

Firstly, we will briefly introduce an important idea in the sequencing principles [10] to introduce the following parts. Considering the objects to sort is denoted as $\{1,2,...,N\}$, a basic element of the sequence set $\boldsymbol{S}$ is $s=(s_1,s_2,...,s_N)$, $s_i\in\{1,2,...,N\}$. Obviously the cardinality of $\boldsymbol{S}$ is $N!$. And optimality criterion is to maximum the target function $f(s)$. The aim is to find the corresponding sequence. Then the optimality principle of sequencing is here.

The sequence $s=(s_1,s_2,...,s_N)$ could also be expressed as $s=(\pi_{k-1}ij\pi^*_{k+2})$ where π_{k-1} is the former part, i.e. $(s_1,s_2,...,s_{k-1})$, π^*_{k+2} is the later part, i.e. $(s_{k+2},s_{k+3},...,s_N)$, $i=s_k$ and $j=s_{k+1}$.

Definition 1: If there exists $f(\pi_{k-1}ij\pi^*_{k+2}) \geq f(\pi_{k-1}ji\pi^*_{k+2})$ for k, then $f(\pi_{m-1}ij\pi^*_{m+2}) \geq f(\pi_{m-1}ji\pi^*_{m+2})$ for all $m \in \{1,2,...,N\} \setminus k$. If the above relationship comes into existence, then i, j satisfy *the Agreeable Neighbor Relation*.

Definition 2: If there exists a function $g:\{1,2,...,N\} \to \Re$ and it satisfies:

$$\begin{aligned} g(i) \geq g(j) &\Rightarrow f(\pi_{k-1}ij\pi^*_{k+2}) \geq f(\pi_{k-1}ji\pi^*_{k+2}) \\ g(i) = g(j) &\Rightarrow f(\pi_{k-1}ij\pi^*_{k+2}) = f(\pi_{k-1}ji\pi^*_{k+2}) \end{aligned}, \quad (4)$$

for all $k \in \{1,2,...,N\}$, then g is called *the Potential Function*. Here, $\Re$ means the real number set.

Definition 3: If there exists a sequence $s = (s_1, s_2, ..., s_N)$ which satisfies the equation:

$$g(s_1) \geq g(s_2) \geq ... \geq g(s_N), \quad (5)$$

then the sequence is called *the Protruding Sequence*. And all the Protruding Sequences compose *the Dominance Set* Θ.

Lemma: If the sequencing problem with the agreeable neighbor relation has a potential function g, then the corresponding protruding sequence is the optimal one:

$$s = s^o. \quad (6)$$

Proof: If there is another optimal sequence s^* whose two neighbor elements s_k, s_{k+1} satisfy $g(s_k) \leq g(s_{k+1})$, i.e. the potential function don't stand up, exchange the element s_k and s_{k+1} to get a new sequence $s^{*\bullet} = (..., s_{k+1}, s_k, ...)$. According to the definition 2, we have $f(s^{*\bullet}) \geq f(s^*)$ which is contradictive to the optimality. In addition, if there are several sequences whose elements all meet the condition of the lemma, they must be equivalent because of the equality in definition 2.

B. The Potential Function in This Paper

In this paper, the target function is $f(s) = Q$. We assume the optimal sensing order is s^o. Similarly, we exchange s_k and s_{k+1}, getting the counterpart sensing order $s^{o\bullet} = (..., s_{k+1}, s_k, ...)$. If $f(s^o) \geq f(s^{o\bullet})$, we have:

$$f(s^{o\bullet}) - f(s^o) = \prod_{i=1}^{k-1}(1-\theta_{s_i})\{[(1-k\tau)-(1-\theta_{s_k})(1-(k+1)\tau)]\theta_{s_{k+1}}C_{s_{k+1}}$$
$$+[(1-k\tau)-(1-\theta_{s_{k+1}})(1-(k+1)\tau)]\theta_{s_k}C_{s_k}\} < 0$$

$$\Rightarrow [\frac{\tau}{\theta_{s_{k+1}}} + 1-(k+1)\tau]C_{s_k} \geq [\frac{\tau}{\theta_{s_k}} + 1-(k+1)\tau]C_{s_{k+1}}, \text{ i.e.}$$

$$\frac{C_{s_k}}{\frac{\tau}{\theta_{s_k}} + 1-(k+1)\tau} \geq \frac{C_{s_{k+1}}}{\frac{\tau}{\theta_{s_{k+1}}} + 1-(k+1)\tau}. \quad (7)$$

Let the Potential Function be $g(s_k) = \dfrac{C_{s_k}}{\frac{\tau}{\theta_{s_k}} + 1-(k+1)\tau}$. This means:

$$\begin{aligned} g(s_k) \geq g(s_{k+1}) &\Rightarrow f(\pi_{k-1}s_ks_{k+1}\pi^*_{k+2}) \geq f(\pi_{k-1}s_{k+1}s_k\pi^*_{k+2}) \\ g(s_k) = g(s_{k+1}) &\Rightarrow f(\pi_{k-1}s_ks_{k+1}\pi^*_{k+2}) = f(\pi_{k-1}s_{k+1}s_k\pi^*_{k+2}) \end{aligned} \quad (8)$$

only in the k-th sensing stage. Actually, the potential function is related to the sensing stage, which does not follow the agreeable neighbor relation. So the protruding order in the dominance set is not always the optimal sensing order. Here, we extend the potential function of channel i in sensing stage k to $g(i,k)$, which will be mentioned later. There exists a protruding order we can easily find out as the second greed solution. Follow the steps:

Step 1: initialization: $k=1$, the chosen channel set $\Lambda = \phi$ (ϕ is the empty set).

Step 2: check if $k > N$. If yes, terminate; else, go to step 3.

Step 3: find the channel to sense in k-th sensing stage: $s_k = \arg\max_{n \in \{1,2,...,N\} \setminus \Lambda} g(n,k)$. And $\Lambda = \Lambda \cup \{s_k\}$.

Step 4: $k = k+1$, go to step 2.

Because $g(s_k, k) \geq g(s_{k+1}, k)$ for all $k < N$, it's a protruding order. But it's not always the optimal one. In the next section, we will give an optimal solution based on the conclusions in this section.

V. The Optimal Sensing Order Based on the Decision-Making Tree

A. The Decision-making Tree

How to select the optimal wideband sensing order using the potential function $g(i,k)$? Since the potential function is also related to the sensing stage, directly applying the conclusion in section Ⅳ suggests an ineffective solution. Here, the essential motive is to exhume all the protruding orders. Decision-making Tree will help us.

The root node is the start point, and the leaf node is the channel we try to select at every stage (the 'stage' here is shown in Fig.2). From the father node to the son node means that the branch is available. We define the target channel set as the strategy set, i.e. $\Gamma = \{1,2,...,N\}$. In every node, we select one strategy based on certain rule (details in the next section) to produce corresponding son node. At the end leaves of the tree, we get the sensing order by tracing back from the root.

B. The Branching Rule and The Lopping Rule

No two channels in a given order are the same, so we define *the active strategy set* $\mathrm{A} = \Gamma \setminus \mathrm{B}$ for every leaf node, where B is the set containing all the ancestral nodes. The son node must come from the active strategy set of its father node. In addition, according to the potential function,

$$g(s_k, k) \geq g(s_{k+1}, k) \Rightarrow f(\pi_{k-1}s_ks_{k+1}\pi^*_{k+2}) \geq f(\pi_{k-1}s_{k+1}s_k\pi^*_{k+2})$$

So the father node s_f in the stage k and the son node s_s in the stage $k+1$ must satisfy:

$$g(s_f, k) \geq g(s_s, k). \quad (9)$$

Therefore, we define *the available strategy set* $\mathrm{I} = \{m \mid g(m,k) \leq g(n,k)\}$ for the node n in stage k. The son node must come from the available strategy set of its father node.

The Branching Rule: A father node branches to its son nodes according to the every strategy from $\mathrm{Z} = A \cap I$.

The Lopping Rule: A father node has no son nodes if $A \cap I = \phi$, where ϕ is the empty set.

C. The Optimal Sensing Order Based on Decision-making Tree

Step 1: initialization. $k = 1$. Construct the root node which is actually the non channel. Its active strategy set is $\mathrm{A} = \Gamma$, and the available strategy set is $\mathrm{I} = \Gamma$. Its sons are all the possible channels first sensed and then the son nodes are produced.

Step 2: set $\mathrm{A} = \mathrm{A}_f \setminus s_f$ for all the son nodes, where the subscript f denotes its father node. Calculate the potential function $g(i,k),\ i \le N$. Then the available strategy set $\mathrm{I} = \{m \mid g(m,k) \le g(n,k)\}$ for the node n in the current stage k is achieved.

Step 3: branch or lop according to the Branching Rule and the Lopping Rule.

Step 4: $k = k+1$. If $k < N$, go to step 2; else, a protruding order is obtained in every leaf node in the stage N (trace back to the root node and reverse order), i.e. the Dominance Set Θ is obtained.

Step 5: calculate the target function of all the protruding orders, and select the optimal sensing order: $s^o = \arg\max_{s \in \Theta} \ f(s)$.

Give an example after we define the potential matrix $G = (g(i,k))$, where $g(i,k)$ is the potential function of channel i in stage k. For $N = 4$, we have detected the channel capacity C and the idle probability θ for all the channels, $C = [5.407, 6.051, 7.768, 6.615]$, $\theta = [0.9192, 0.4826, 0.0736, 0.3428]$. Then

$$G = \begin{bmatrix} 5.950 & 6.685 & 7.629 & 8.882 \\ 6.008 & 6.670 & 7.496 & 8.556 \\ 3.598 & 3.773 & 3.965 & 4.179 \\ 6.059 & 6.670 & 7.418 & 8.355 \end{bmatrix}.$$

Following the above steps, we can get the decision-making tree in Fig.2.

Tracing back, then we have $\Theta = \{(2,1,4,3),(4,1,2,3)\}$, with the corresponding target functions (the expected throughput) are 4.752 and 4.763. So the optimal sensing order is $(4,1,2,3)$.

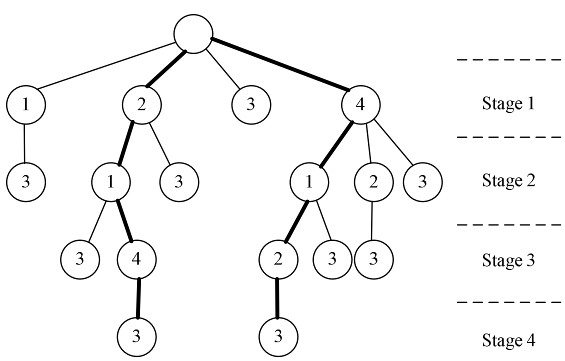

Fig.2. the decision-making tree in the example

VI. PERFORMANCE ANALYSIS

A. Complexity Analysis

From the procedure of constructing the decision-making tree, we can conclude that the main computational complexity is from two parts: one is the calculating of the potential function, the other is the calculating of the target function of the protruding orders. The computational complexity of the former one is $O(N^2)$ if the complexity of calculating one potential function is $O(1)$. The computational complexity of the later one may be large or small, depending on the cardinality of the dominance set. It is $O(N * cardinality)$ if the complexity of calculating one expected throughput is $O(1)$. From the analysis of the cardinality (detailed in the APPENDIX), we know it's $O(C_N^2)$. So the total computational complexity is $O(N^2 + N * \frac{N(N-1)}{2}) = O(N^3/2)$, which is much less than the brute force search $O(N \cdot N!)$ and the dynamic programming approach $O(N \cdot 2^{N-1})$ [8].

The weakness of the proposed method is that the spatial complexity is much larger than the brute force search and the dynamic programming approach.

B. Optimality Analysis

Obviously, the proposed method will find the optimal sensing order to maximum the expected throughput. In the simulation, we will show it comparing with the other four methods: one is the simple channel sensing order denoted by 'zhuang' proposed in [9], one is the intuitive sensing order denoted by 'intuitive' proposed in [8], one is the first greed algorithm denoted by 'greed1', the last one is the second greed algorithm denoted by 'greed2'. And the decision-making tree method in this paper is denoted by 'DMT'. In this paper, Monte Carlo simulation method is adopted. There are 50 channels with their average idle probability increasing from 0.1 to 0.9. The unit sensing time is 0.01 second.

In Fig.3, the decision-making tree method has the optimal performance, better than the others. When the average idle probability is low, the intuitive method in the order of the idle probability gains more throughput than Zhuang's method, because the channel with large throughputs may be in use by the primary user. When the average idle probability is high, Zhuang's method achieves almost the optimal performance with the explanation that the first channel to sense may be idle in all likelihood. And the performance of the 'greed1' solution is between the above two methods when the idle probability is higher than 0.4. The second greed algorithm is nearly optimal because the protruding orders in the dominance set almost satisfy the agreeable neighbor relation. So in reality, we could use the second greed algorithm to obtain the nearly optimal performance.

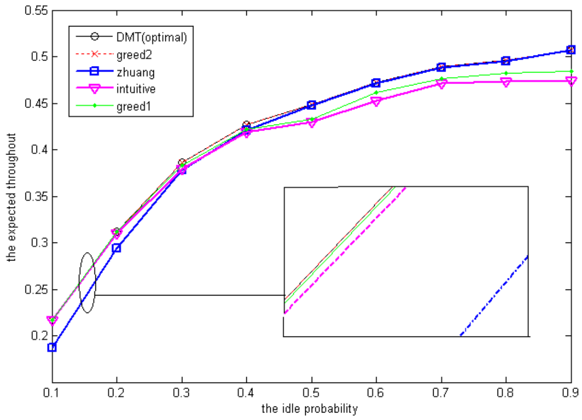

Fig.3. the performance comparison

VII. Conclusions

In this paper, we investigate the wideband sensing order in the slotted fashion. Every channel has different capacity and different idle probability. Through the optimality principle of sequencing, we conclude that directly applying the principle isn't feasible. But we aim to find all the limited protruding orders using the decision-making tree, so as to search the optimal sensing order. By defining the proper branching rule and the lopping rule, we get it with low complexity.

APPENDIX

Analysis of the cardinality of the dominance set

Before we analyze the cardinality, we present the following lemma.

Lemma: the curves of every two channels' potential function would not cross each other twice.

Proof: For every channel i, the potential function $g(i,k)=\dfrac{C_{s_k}}{\dfrac{\tau}{\theta_{s_k}}+1-(k+1)\tau}$ is a bald increasing function. If channel i and j cross each other twice, we assume the stages of the two crossovers are d_1 and d_2 ($0\le d_1\le d_2\le N$)without the loss of the generality. In the stage d_1-1,

$$g(i,d_1-1)>g(j,d_1-1)$$

$$\Rightarrow C_i\frac{\tau}{\theta_j}+(1-d_1\tau)C_i>C_j\frac{\tau}{\theta_i}+(1-d_1\tau)C_j \qquad (*)$$

In the stage d_1,

$$g(i,d_1)<g(j,d_1)$$

$$\Rightarrow C_j\frac{\tau}{\theta_i}+(1-(d_1+1)\tau)C_j>C_i\frac{\tau}{\theta_j}+(1-(d_1+1)\tau)C_i \qquad (\sim)$$

(*)+(~), we have

$$C_i>C_j.$$

Using the same method in the stage d_2, we have $C_j>C_i$ which is contradictive. This completes the proof.

If no crossover exists among the curves of the N channels' potential function, the sensing order would be an only one. If the crossover isn't in the strategy set $Z=A\cap I$ in their stage, the sensing order also would not change. So the worst case is that all the crossovers among the curves of the N channels' potential function are just in everyone's strategy set $Z=A\cap I$ in their stage. We know one more crossover is present, one more variety may appear. We consider the worst case, so that the number of the varied sensing orders is equal to the number of the crossovers. Therefore the cardinality of the dominance set is up bounded by C_N^2.

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