Channel and Power Adaptation for Cognitive Radios in Multiuser OFDM Systems

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***Abstract*— In this paper, we investigate joint power and channel allocation problem for the orthogonal frequency-division multiplexing (OFDM) based multi-user cognitive radio systems with quality-of-service (QoS) in terms of signal-to-interference-plus-noise ratio (SINR) and proportional rate constraints. The QoS requirement of each cognitive radio user imposed by its minimum SINR and interference introduced by unlicensed secondary users to primary users make power and channel allocation problem more complex. We present a formal analysis for a proposed approach and illustrate the approach with numerical results obtained from simulations.**

I. Introduction

Wireless service providers are expecting over 15-fold increase in wireless traffic by 2017 [1] because of the exponential growth of lightweight hand-held devices to access wireless networks. Moreover, because of the static spectrum allocation by licensing bodies to service providers for exclusive use for long time and vast geographic area, there are no more useful frequency bands left for further development of wireless services and applications. As a consequence, if the problem is left unaddressed, many wireless service providers will be facing severe spectrum shortage [2], [3].

Opportunistic spectrum access using cognitive radio technology is an emerging concept for an efficient utilization of RF spectrum by allowing unlicensed secondary users to access the licensed spectrum without causing harmful interference to the licensed primary users [3]–[6]. In opportunistic dynamic spectrum access, secondary users are required to either sense the channels and identify the idle bands or search them on the geolocation database to access licensed bands opportunistically [3], [7]–[11]. Note that the spectrum sensing is an essential step in DSA which has been well studied and readers are referred to [12], [13] and references therein for the details.

In this paper, we consider joint channel and power allocation for dynamic spectrum access in OFDM based cognitive radio network where secondary users are required to satisfy imposed constraints in terms of minimum SINR, transmit power, proportional rate and interference limit constraints. The problem considered in this paper is very similar to the problem in [14], except two constraints; interference constraints and minimum SINR constraints. Note that the channel allocation [14] is done based on maximum SINR to get high achievable rates which creates harmful interference to primary users in OFDM based cognitive radio systems. Thus, we jointly consider SINR of the channel and interference generated by the secondary users as constraint in optimization problem and presented a solution.

The reminder of the paper is organized as flowing: system model and problem statement is presented in Section II. Optimal channel and power allocation with simulation results is presented in Section III. Conclusion and furthermore work is presented in Section IV.

II. System Model and Problem Statement

We consider that a multiple access primary wireless system with primary users $\mathcal{M} = \{1, 2, \ldots, M\}$ operates in a set of sub channels $\mathcal{N} = \{1, 2, \ldots, N\}$ with bandwidth of W where secondary cognitive radio users $\mathcal{K} = \{1, 2, \ldots, K\}$ share those channels dynamically and opportunistically.

Received signal for a user k in a channel n can be expressed as

$$\mathbf{y}_{k,n} = \sum_{k=1}^{K} \sqrt{p_{k,n}} h_{k,n} b_k \mathbf{s}_k + \mathbf{n} \tag{1}$$

where $E[\mathbf{n}\mathbf{n}^{\dagger}] = \sigma^2 \mathbf{I}_N$. The mutual interference power for the kth secondary user in the nth channel from other secondary users is given by

$$I_{n,k} = \text{Trace}\left[\sum_{j=1, j\neq k}^{K} p_{j,n} h_{j,n}^2 \mathbf{s}_j \mathbf{s}_j^{\dagger}\right], \quad \forall n \in \mathcal{N} \tag{2}$$

The mutual interference power generated in the nth channel for the mth primary user for unit power is given by

$$I_{n,m}^{PU} = \text{Trace}\left[\sum_{k=1}^{K} h_{m,n,k}^2 \mathbf{s}_k \mathbf{s}_k^{\dagger}\right], \quad \forall m \in \mathcal{M} \tag{3}$$

where $h_{m,n,k}$ is the channel gain in a channel n from a transmitter of a user k to primary user m.

Similarly the mutual interference power cast by the mth primary users with power $p_{m,n}$ into the nth channel is given by

$$I_{n,m,k}^{SU} = \text{Trace}\left[\sum_{m=1}^{M} p_{m,n} h_{m,n}^2 \mathbf{s}_m \mathbf{s}_m^\dagger\right], \quad \forall k \in \mathcal{K} \quad (4)$$

The SINR for a user k in the nth channel for match filtering is expressed as

$$\gamma_{k,n} = \frac{p_{k,n}.h_{k,n}^2}{I_{n,k} + I_{n,m,k}^{SU} + \sigma_k^2} = \frac{p_{k,n}}{z_{k,n}} \quad (5)$$

where

$$z_{k,n} = \frac{I_{n,k} + I_{n,m,k}^{SU} + \sigma_k^2}{h_{k,n}^2}.$$

Hence the attainable data rate for the nth used by the kth secondary user is calculated using Shannon capacity as

$$\begin{aligned} r_{k,n} &= W \log_2(1 + \theta_k.\gamma_{k,n}) \\ &= W \log_2(1 + \theta_k.\tfrac{p_{k,n}.h_{k,n}^2}{I_{n,k}+I_{n,m,k}^{SU}+\sigma_k^2}) \end{aligned} \quad (6)$$

where

$$\theta_k = -\frac{1.5}{\ln(5.BER_k)}$$

is the SNR gap for a specified Bit-Error-Rate (BER) [15] Then, the sum capacity of the kth secondary user is

$$R_k = \sum_{n=1}^{N} \rho_{k,n} r_{k,n} = \sum_{n=1}^{N} \rho_{k,n} W \log_2(1 + \theta_k.\gamma_{k,n}) \quad (7)$$

where $\rho_{k,n} \in \{0, 1\}$, that is, if kth secondary user uses nth channel, $\rho_{k,n} = 1$, otherwise $\rho_{k,n} = 0$.

Our goal in this paper is to maximize the achievable rates of secondary users by satisfying QoS and other imposed constraints. The optimization problem focused in this paper is expressed formally as

$$\begin{aligned} \underset{\rho_{k,n},p_{k,n}}{\text{maximize}} \quad & R_k = \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n} W \log_2(1 + \theta_k.\gamma_{k,n}) \\ \text{subject to} \quad & Co1 : p_{k,n} \geq 0, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\ & Co2 : \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \leq P_T^{max} \\ & Co3 : \rho_{k,n} = \{0, 1\}, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\ & Co4 : \sum_{k=1}^{K} \rho_{k,n} = 1, \quad \forall n \in \mathcal{N} \\ & Co5 : p_{k,n} I_{n,m}^{PU} \leq I_m^P \\ & Co6 : R_1 : R_2 : \ldots : R_K = \alpha_1 : \alpha_2 : \ldots : \alpha_K \\ & Co7 : \gamma_{k,n}^* \leq \gamma_{k,n} \end{aligned} \quad (8)$$

where P_T^{max} is the maximum allowed transmission power and I_m^P is predefined interference power limit of the mth primary user. $Co1$ and $Co2$ are the transmission power constraints. Constraints $Co3$ and $Co4$ show that each channel can only be allocated to only one secondary user. The constraint $Co5$ is for interference limit for all primary users generated by secondary users. $Co6$ are the proportional rate constraints of secondary users with predefined values that are used to ensure fairness among secondary users. Constraint $Co7$ is the quality of service requirement imposed by minimum required SINR $\gamma_{k,n}$ for a given user k in a given channel n. Note that the values of minimum required SINRs are chosen based on admissibility condition [16] as $\frac{\overline{\gamma}_{n,k}}{1+\overline{\gamma}_{n,k}} <$ processing gain of a given network or using rate requirement of secondary users that could be admitted to the system in OFDM based network. From predefined values, to find the fairness factor [14], we can calculate the radio of the rate of kth to the sum rate of all secondary users as

$$\mu_k = \frac{\alpha_k}{\sum_{k=1}^{K} \alpha_k} \quad (9)$$

Note that when $\mu_k = \frac{1}{K}, \forall k$, all secondary users can obtain the same data rates which is fair sharing of spectrum among them.

III. Channel and Power Allocation

Channel selection in OFDM based cognitive radio network is performed by considering instantaneous SINRs of secondary users and interference introduced by secondary users to primary users.

When we have that $\mu_k P_T^{max} I_{n,m}^{PU} \leq I_m^P$, $\forall m \in \mathcal{M}$, the given channel n is power limited since the interference constraint of the users is satisfied. Otherwise, the channel is interference limited as the role of interference will be there in the system.

By considering fairness factor in (9), initial transmit power for kth secondary user is allocated as

$$p_{k,n} = \mu_k P_T^{max} \quad (10)$$

which is proportional to the required rate of secondary user. Similarly, when a channel is interference limited, the optimal transmit power should be

$$p_{k,n} = \min_{m \in \mathcal{M}} \left(\frac{I_m^P}{I_{n,m}^{PU}}\right). \quad (11)$$

First, channels are allocated to secondary users who can get the maximum rates and satisfy the constraint $Co7$. Secondary user k who does not meet its minimum required SINR in a given channel n (i.e., $\gamma_{k,n}^* > \gamma_{k,n}$) should not use the channel n since it does not satisfy constraint $Co7$. If the secondary user k uses the channel n without $\gamma_{k,n}^* > \gamma_{k,n}$, it just creates interference to other active users and consumes its power.

In addition, when secondary user has minimum desired SINR in each channel the power allocation to meet the constraint $Co7$ using (5) can be computed as

$$p_{k,n} = \gamma_{k,n}^* z_{n,k} \quad (12)$$

Thus, the optimal power allocation for kth secondary user in a channel n is represented as

$$p_{n,k}^{opt} = \min\{\mu_k P_T^{max}, \min_{m\in\mathcal{M}} \left(\frac{I_m^P}{I_{n,m}^{PU}}\right), \gamma_{k,n}^* z_{k,n}\} \quad (13)$$

Substituting (13) into (6), we get optimal rate for a given user k in a channel n as

$$r_{k,n}^{opt} = W \log_2(1 + \theta_k \cdot \frac{p_{n,k}^{opt} . h_{k,n}^2}{I_{n,k} + I_{n,m,k}^{SU} + \sigma_k^2}) \quad (14)$$

In the following section, we present the joint channel and power allocation algorithm, and numerical results obtained from simulation.

IV. The Algorithm and Numerical Results

Based on the discussion presented above, the resource allocation (joint channel and power allocation) algorithm is formally presented as **Algorithm 1** A set of channels allocated to kth secondary user is denoted by $\mathcal{C}_k \subseteq \mathcal{N}$, $B \setminus b$ implies that element b is removed from the set B, and $B \cup A$ represents set union operation between sets A and B.

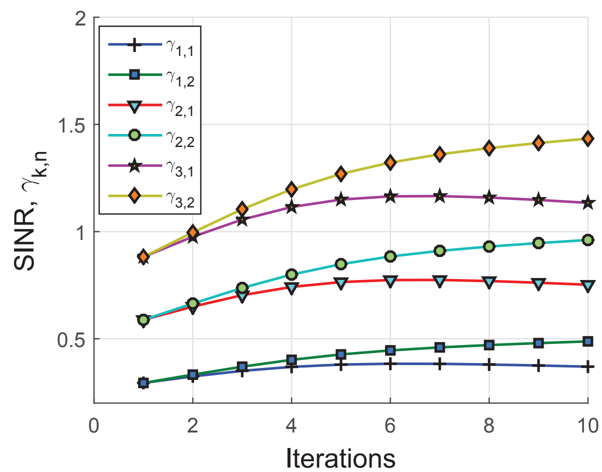
Algorithm 1 Joint channel and power allocation.

1: **Input**: I_m^P, P_T^{max}, $\{R_k\}_{k=1}^K = 0$, $\{\mathcal{C}_k\}_{k=1}^K = 0$, $\gamma_{k,n}^*$ values
2: **Output**: Optimal achievable rate $R_k, \forall k in \mathcal{K}$
3: Calculate $p_{n,k}^{opt}$ using (13), $\gamma_{k,n}$ using (5) and $r_{k,n}^{opt}$ using (14).
4: **for** $k = 1, 2, 3, \ldots, K$ **do**
5: Find k' and n' that satisfy both $r_{k',n'}^{opt} \geq r_{k,n}^{opt}$ AND $\gamma_{k,n} \geq \gamma_{k,n}^*$, $\forall n \in \mathcal{N}$ and $\forall k \in \mathcal{K}$
6: $\mathcal{C}_k := \mathcal{C}_k \cup n'$, $\mathcal{N} := \mathcal{N} \setminus n'$, $\mathcal{K} := \mathcal{K} \setminus k'$
7: $R_k = r_{n',k'}^{opt}$
8: **end for**
9: Set $\mathcal{K} = \{1, 2, 3, \ldots, K\}$
10: **while** $\mathcal{N}$ is not NULL **do**
11: Find k' for $\min_{k\in\mathcal{K}} \left(\frac{R_k}{\alpha_k}\right)$
12: For k', find n' that satisfy both $r_{k',n'}^{opt} \geq r_{k',n}^{opt}$ AND $\gamma_{k,n} \geq \gamma_{k,n}^*$, $\forall n \in \mathcal{N}$
13: $\mathcal{C}_{k'} := \mathcal{C}_{k'} \cup n'$, $\mathcal{N} := \mathcal{N} \setminus n'$
14: $R_k = R_k + r_{k',n'}^{opt}$
15: **end while**

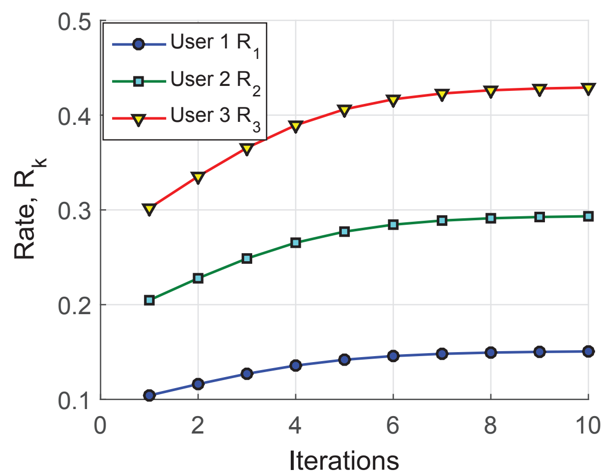
In steps $3 - 7$, **Algorithm 1** allocates the channels to secondary users who satisfy both conditions $r_{k',n'}^{opt} \geq r_{k,n}^{opt}$ AND $\gamma_{k,n} \geq \gamma_{k,n}^*$, $\forall n \in \mathcal{N}$ and $\forall k \in \mathcal{K}$. In steps $9 - 13$, **Algorithm 1** allocates remaining channels to the users who have minimum rates but satisfy the condition $\gamma_{k,n} \geq \gamma_{k,n}^*$, $\forall n \in \mathcal{N}$ and $\forall k \in \mathcal{K}$. **Algorithm 1** ensures the fairness by assigning channels to all secondary users and remaining channels (if any) are allocated to the secondary users who have lower achievable rates.

To evaluate the proposed scenario, we simulated the scenario with $N = 6$ channels and $K = 3$ secondary users. For fair spectrum sharing among secondary users, we assume that each user has equal number of channels (i.e., 2 channels per user) and the minimum required SINR of secondary users were chosen as $\{\gamma_{1,1}^*, \gamma_{1,2}^*\} = \{0.25, 0.35\}$, $\{\gamma_{2,1}^*, \gamma_{2,2}^*\} = \{0.65, 0.70\}$, and $\{\gamma_{3,1}^*, \gamma_{3,2}^*\} = \{0, 90, 0.95\}$. We assume that $\alpha_1 : \alpha_2 : \alpha_3 = 1 : 2 : 3$, $I_m^P = 0.1$ mW and $P_T^{max} = 1$ mW.

We have simulated the scenario with given configuration and have plotted the numerical results for both SINR and rate as shown in Fig. 1. We observed that each secondary user is able to satisfy the $C07$ (i.e., $\gamma_{k,n}^* \leq \gamma_{k,n}$) in each channel as shown in Fig. 1 (a) and maximize achievable rates of secondary users as shown in Fig. 1 (b).



(a) SINR ($\gamma_{k,n}$) variation



(b) Rate (R_k) variation

Fig. 1. Variation of SINR and Rate of each secondary user.

V. Conclusion and Further Work

We have presented the algorithm for joint power and channel allocation for OFDM based cognitive radio networks where each secondary user satisfies the quality-of-service constraint, interference constraint and proportional rate constraints. We have evaluated the proposed algorithm with the help of numerical results obtained from simulations. The numerical results also supported the convergence of the proposed algorithm to an equilibrium point. As the further work, we plan to include rigorous proof of an equilibrium and more numerical results.

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