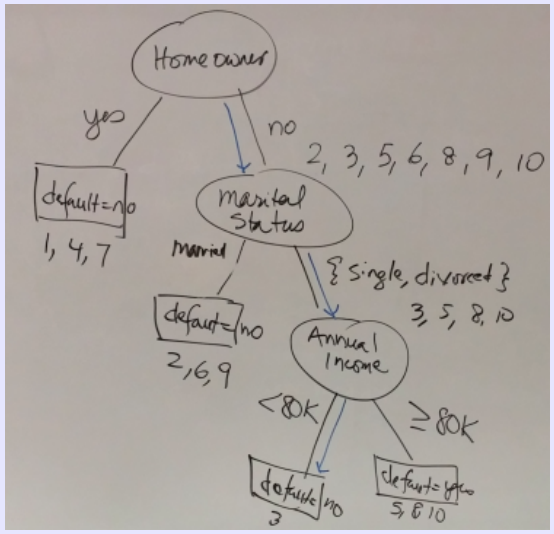
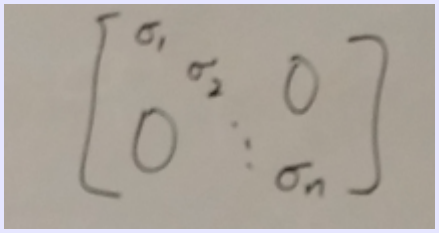
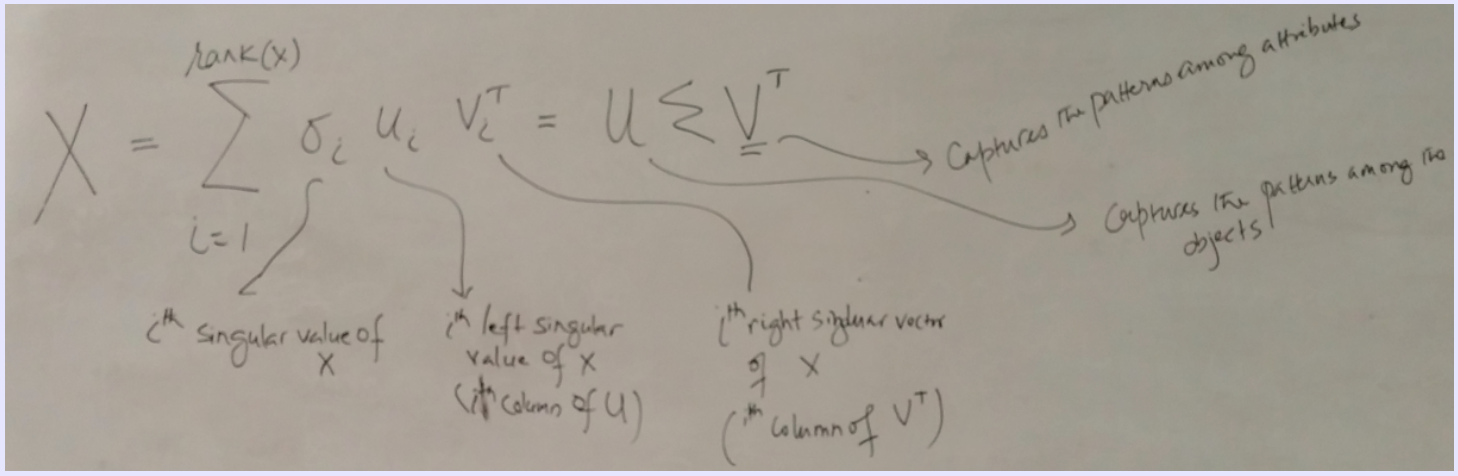
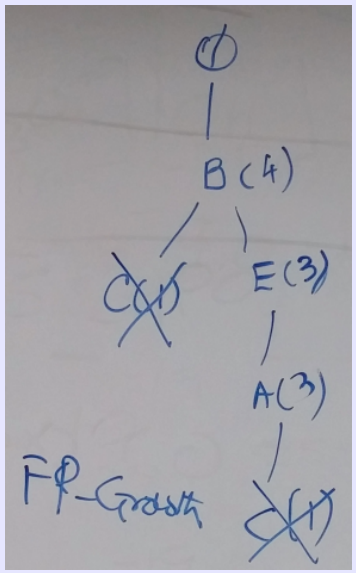






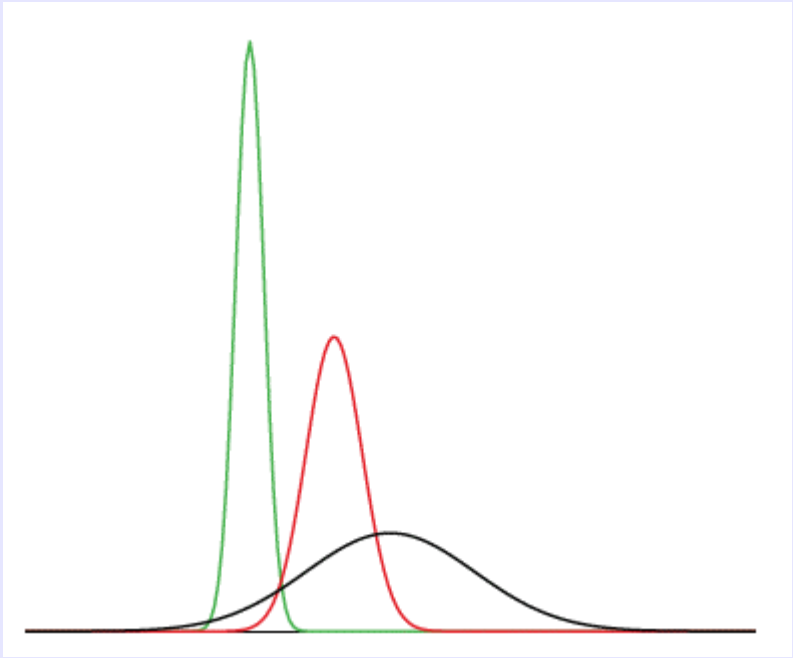


CS 422: Data Mining
Vijay K. Gurbani, Ph.D.,
Illinois Institute of Technology

Lecture 2: **Random variables, measures of central tendency and distributions**

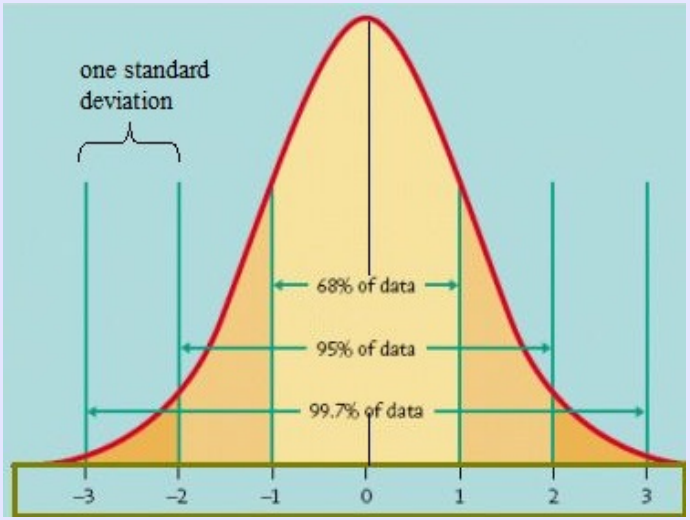
Distributions

- Normal / Gaussian distribution



Green: μ=-3.0, σ=0.5
Red: μ=0.0, σ=1.0
Black: μ=2.0, σ=3.0

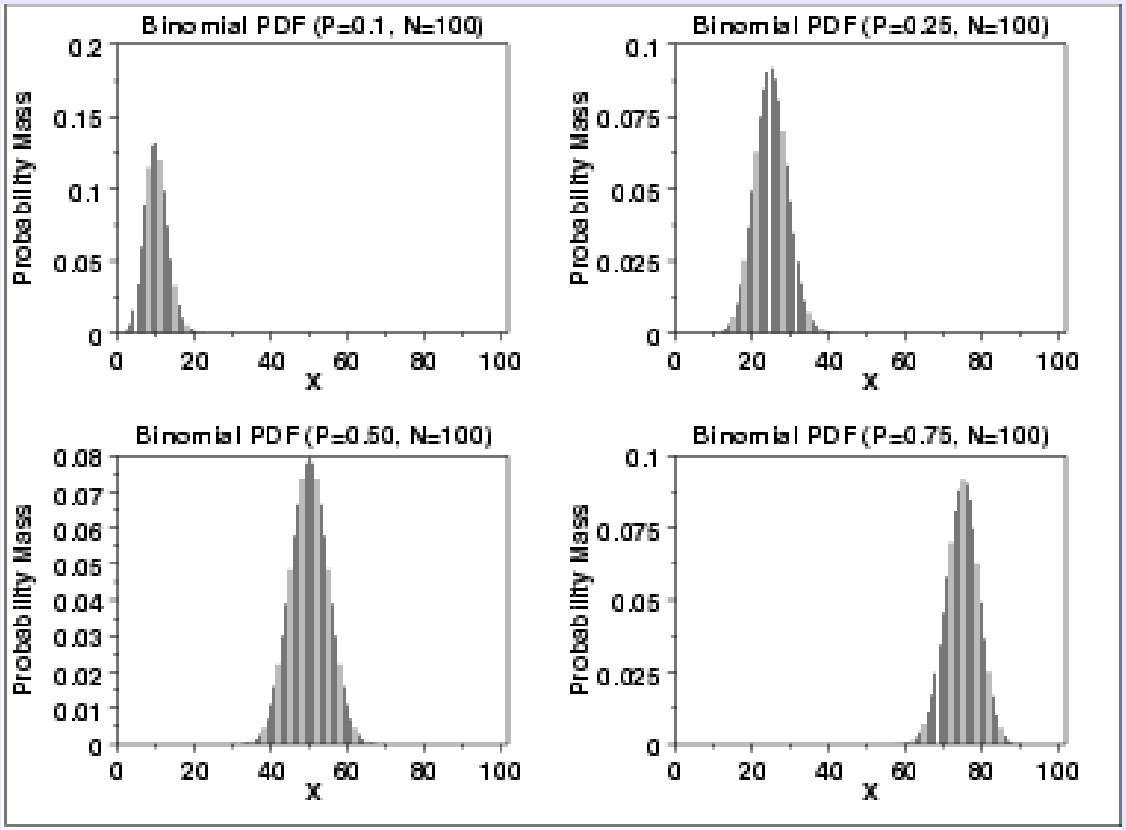
- Parameterized by mean (μ) and standard deviation (σ)
- PDF: $\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
- mean = median = mode



Distributions

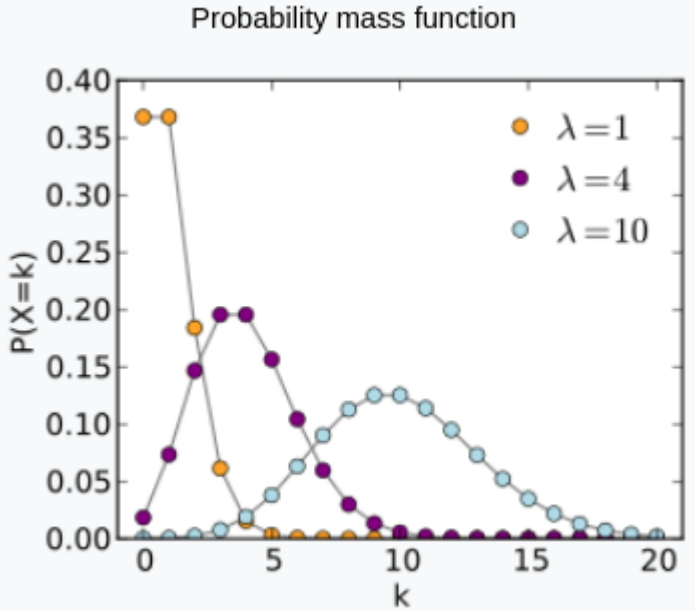
- Binomial distribution: parameterized by *n* (number of trials) and *p* (probability of success in each trial)
- PMF: $\binom{n}{x}(p)^x(1-p)^{(n-x)} \quad \text{for } x = 0, 1, 2, \cdots, n$ for observing *x* successes in *n* trials.
- Mean: np
- Median: ⌊np⌋ or ⌈np⌉
- Variance: np(1-p)

Slide source: http://www.itl.nist.gov/div898/handbook/eda/section3/eda366i.htm



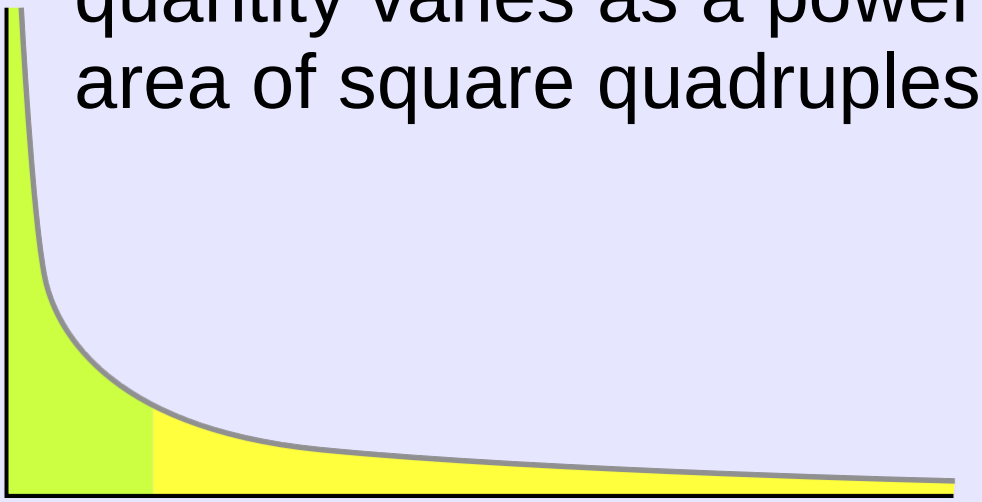
Distributions

- Poisson distribution: expresses the probability of a given number of events (k) occurring in a fixed interval of time if these events occur with a known constant mean rate (λ) and independently of the time since the last event.
- PMF: $\frac{\lambda^k e^{-\lambda}}{k!}$
- Mean: λ
- Median: $\lambda - \ln 2 \leq \nu < \lambda + \frac{1}{3}.$
- Variance: λ



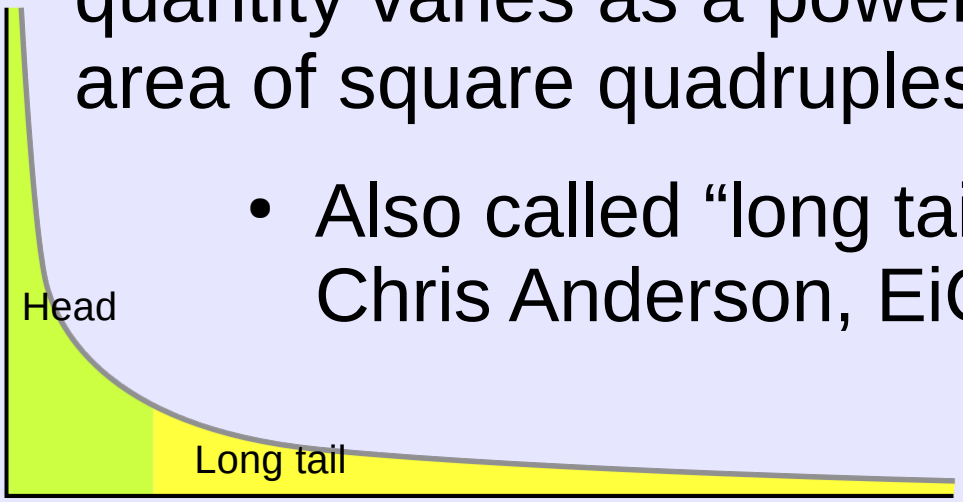
Distributions

- Power-law distributions: relationships where one quantity varies as a power of another. (Example: area of square quadruples when length is doubled.)



Distributions

- Power-law distributions: relationships where one quantity varies as a power of another. (Example: area of square quadruples when length is doubled.)
  - Also called “long tailed” distributions (coined by Chris Anderson, EiC of *Wired*), or the 80-20 rule.



  - Unlike other distributions, the moments of Power Law are hard to define: under certain conditions the first moment is defined, but rest are infinite!

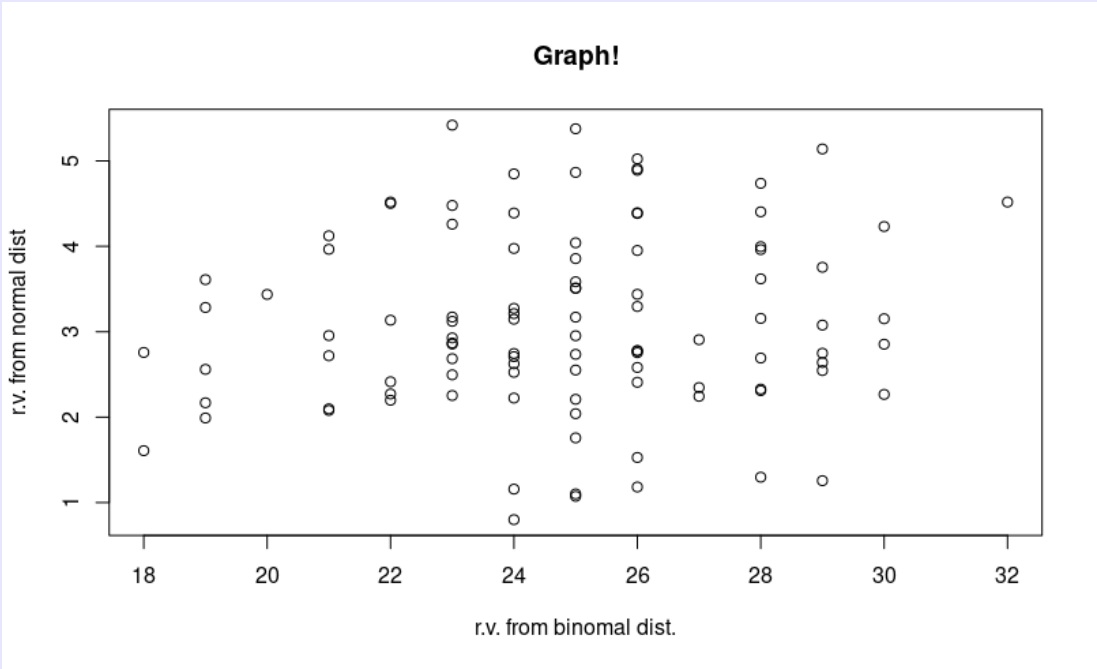
Graphs

- Visualization is important!
- Excellent resource: Claus O. Wilke, *Fundamentals of Data Visualization*, O'Reilly Publishing. (Uses R/RStudio/R Markdown.)
- No time to look at details into graphs, but:
  - Boxplots
  - CDFs
  - Normal x-y plots

Graphs

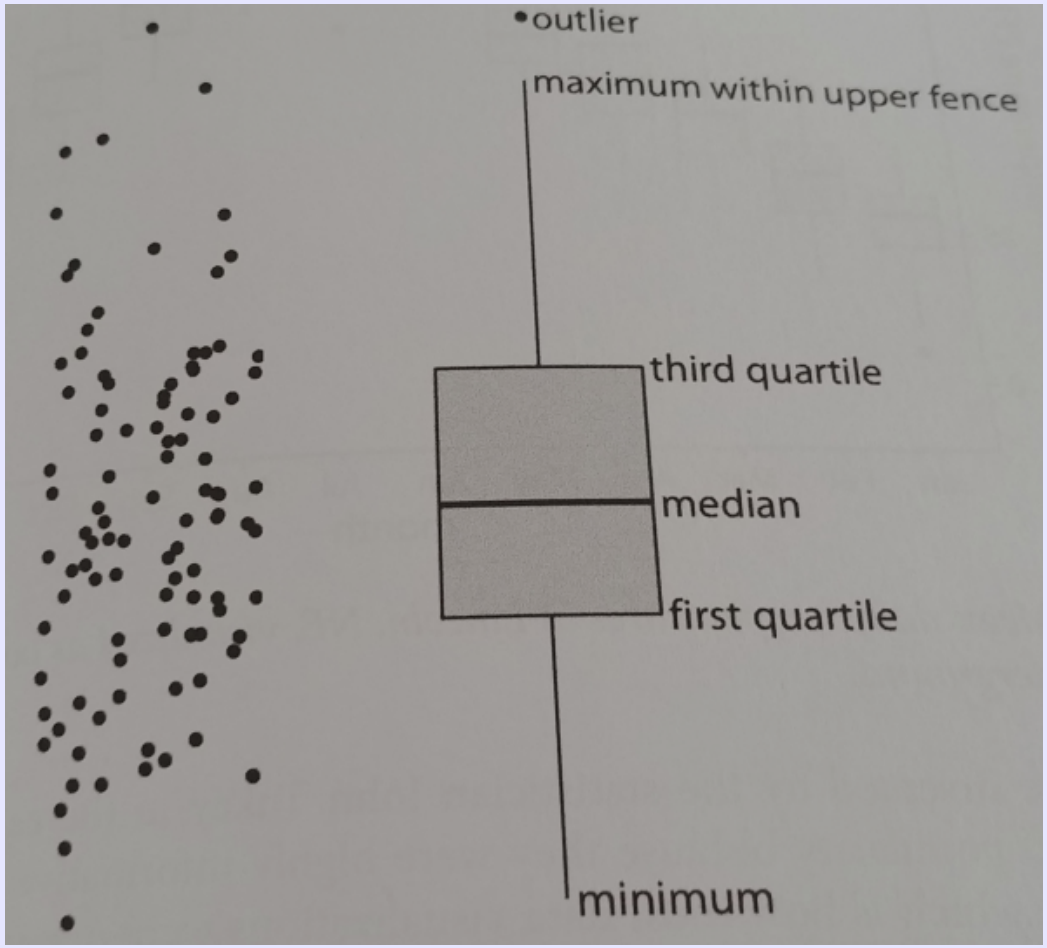
- x-y plots

```
Console ~/
> a <- rbinom(100, 50, 0.5)
> head(a)
[1] 25 23 26 26 28 21
> a
  [1] 25 23 26 26 28 21 23 22 26 25 29 24 28 25 26 24 26 27 19 27 29 22 26 28 28 30 25 28 23 19 25 26 32 21 26 24 29 25 25 27 24 24
 [43] 23 24 30 21 28 24 26 29 23 26 22 19 29 23 18 28 21 22 30 28 23 23 29 22 26 24 25 25 22 24 26 21 26 20 25 26 24 18 26 23 23 25
 [85] 24 30 25 25 24 24 25 28 23 25 19 25 19 29 28 21
> b <- rnorm(100, 3.0, 1.0)
> head(b)
[1] 1.072663 4.259295 4.890823 4.908722 3.619042 2.078458
> plot(a, b, xlab="r.v. from binomal dist.", ylab="r.v. from normal dist", main="Graph!")
```

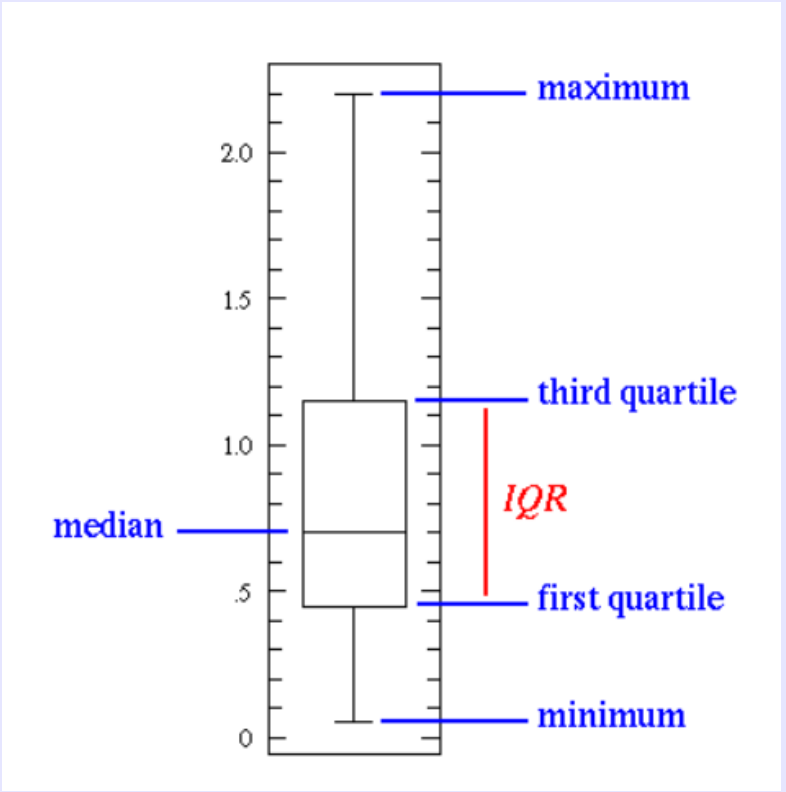


Graphs

- Boxplots



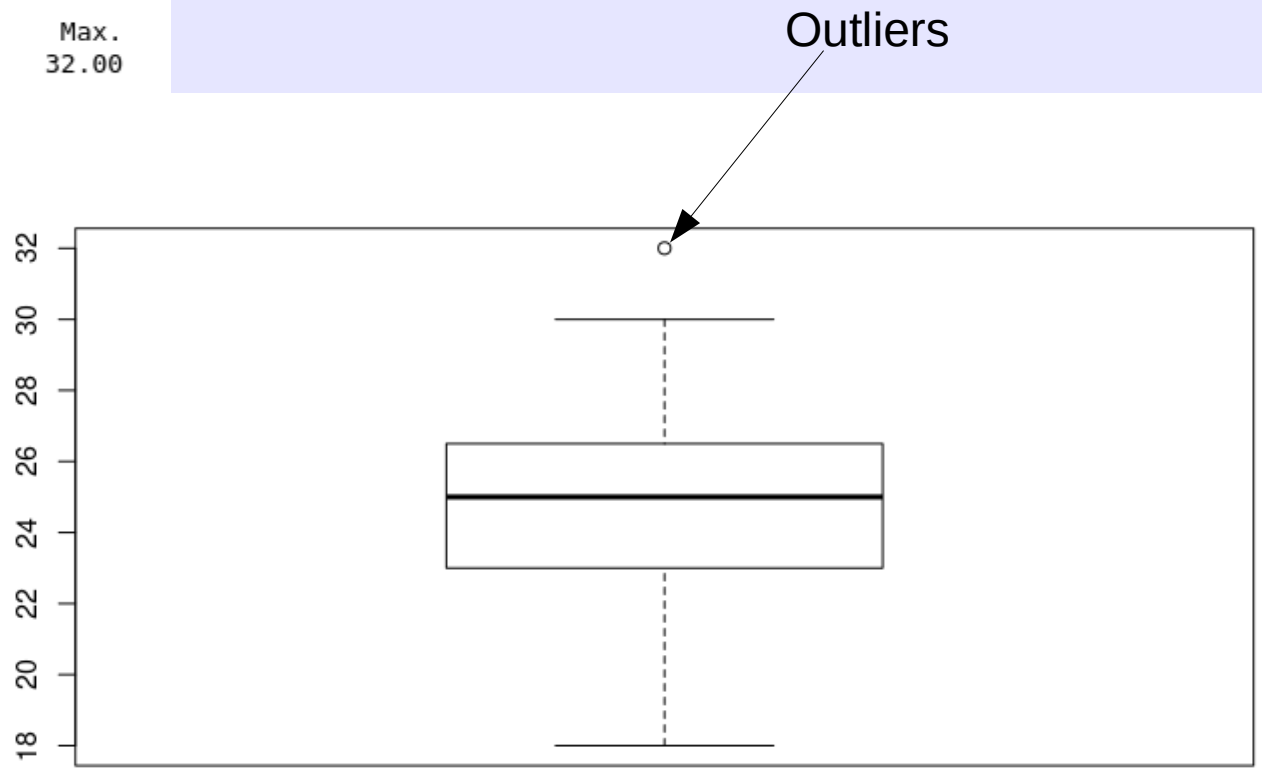
Graphic source: Fundamentals of Data Visualization, Claus O. Wilke, O'Reilly Publishing, 2019.



Graphs

- Boxplots

```
Console ~/
> summary(a)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  18.00   23.00   25.00   24.80   26.25   32.00
> boxplot(a)
>
```



Variability outside the upper and lower quartiles are the ”whiskers”

Graphs

- Empirical cumulative distribution function
  - CDF(*y*) of a dataset *X* at a value *y* is the ratio of samples that are **lower** than the value *y*:

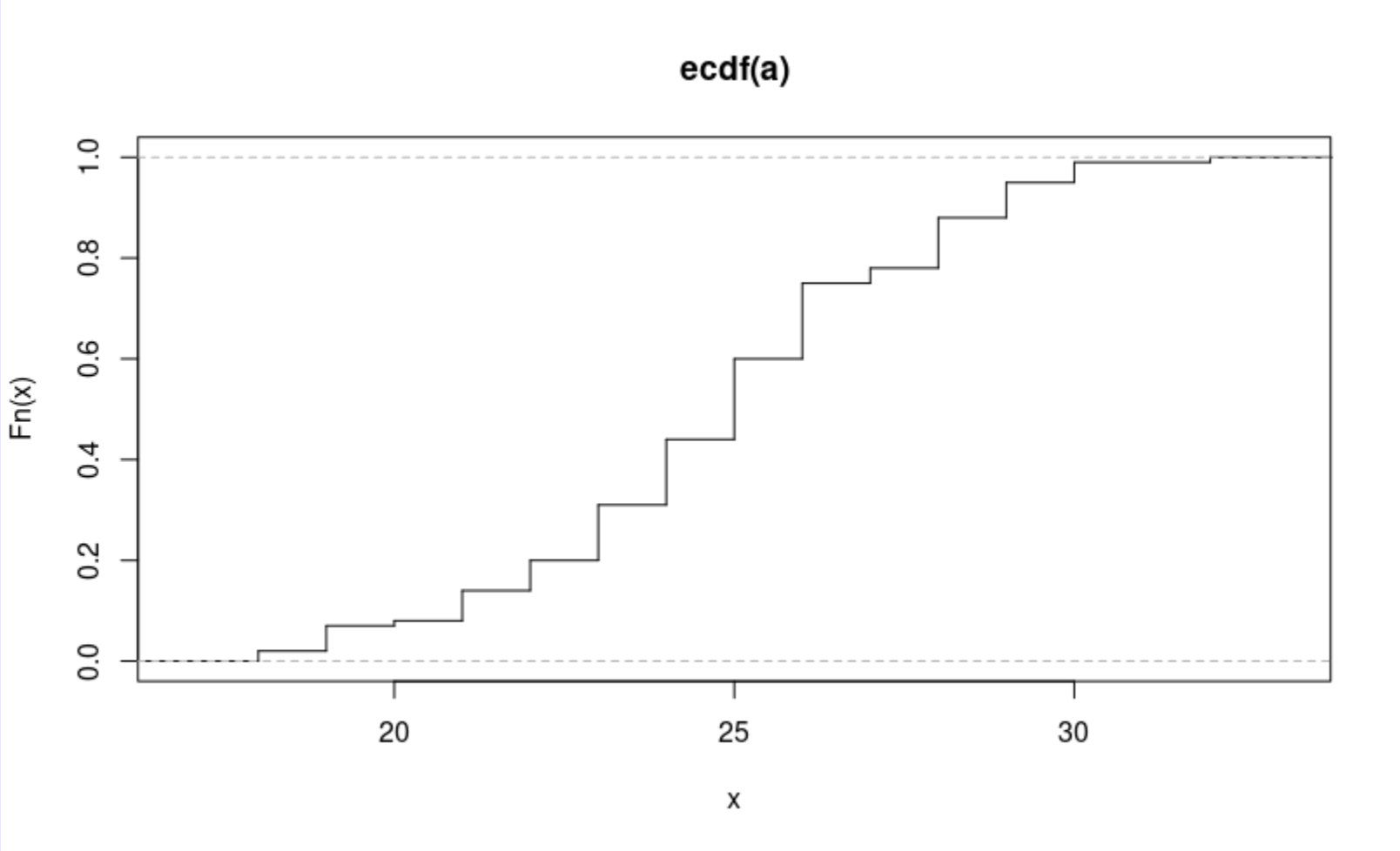
$$\mathrm{CDF(X,y)} = \frac{|i| : i \leq y}{|X|}$$

  - The derivative of the CDF is the PDF.
  - Example: *X* = [2, 7, 8, 9, 10, 15, 16, 20]
    - CDF(*X, 15*) = 6/8 = 0.75

Graphs

- ECDF

```
Console ~/
> plot(ecdf(a), verticals = T, do.points=F)
>
```



A note on debugging

- When you run into problems, learn how to debug your code.
  - See
    - https://support.rstudio.com/hc/en-us/articles/205612627-Debugging-with-RStudio
    - https://adv-r.hadley.nz/debugging.html
- Sometimes, even after debugging, things are not clear. In such cases:
  - Isolate the problem as best as you can.
  - Reproduce the problem on a small dataset.
  - And get in touch with the TA or me.
- Remember: The better you can describe your problem to me or the TA, the quicker it is for us to help you.
- Debugging is an art! Become proficient at it.

```
> df <- read.csv(...)
> index <- sample(1:nrow(df), 0.20*nrow(df))
> small.df <- df[index, ]
> rm(index)
> dim(df)
[1] 11034    31
> dim(small.df)
[1] 2206   31
```