Window Scaling and Resonance

Logarithmic Window Scaling

Since window sizes vary as powers of 2, the appropriate coordinate is:

$$\xi = \log_2(W) \tag{1}$$

James Law in logarithmic coordinates:

$$K(\xi) = \frac{\Lambda_{\text{eff}}}{2^{\xi}} \times \left[1 + A(2^{\xi}) \sin(2\pi f_0 \xi + \varphi)\right] \tag{2}$$

Resonance Detection

Constructive interference maxima occur when:

$$\sin(2\pi f_0 \log_2(W) + \varphi) = 1 \tag{3}$$

Solving for resonance window sizes:

$$2\pi f_0 \log_2(W_{\text{res}}) + \varphi = \frac{\pi}{2} + 2\pi n, \quad n \in \mathbb{Z} \tag{4}$$

$$\log_2(W_{\text{res}}) = \frac{1}{f_0}\left(\frac{1}{4} + n - \frac{\varphi}{2\pi}\right) \tag{5}$$

$$W_{\text{res}} = 2^{\frac{1}{f_0}\left(\frac{1}{4}+n-\frac{\varphi}{2\pi}\right)} \tag{6}$$

For $f_0 = 2/3$, $\varphi = 0$:

$$W_{\text{res}} = 2^{0.375+1.5n} = \{2^{0.375}, 2^{1.875}, 2^{3.375}, 2^{4.875}, \ldots\} \tag{7}$$

$$W_{\text{res}} \approx \{1.3, 3.7, 10.4, 29.4, 83.2, 235, 665, 1880, 5320, 15060, \ldots\} \text{ bytes (raw)} \tag{8}$$

Rounding to practical power-of-2 or nearby values:

$$W_{\text{res}}^{\text{practical}} \approx \{1024, 4096, 6144, 16384, 32768, \ldots\} \tag{9}$$

Anti-Resonance Detection

Destructive interference (zeros) occur when:

$$\sin(2\pi f_0 \log_2(W) + \varphi) = 0 \tag{10}$$

$$W_{\text{anti}} = 2^{\frac{n}{f_0} - \frac{\varphi}{2\pi f_0}} \tag{11}$$

For $f_0 = 2/3$, $\varphi = 0$:

$$W_{\text{anti}} \approx \{512, 2048, 8192, 32768, \ldots\} \tag{12}$$

Golden Ratio Window Penalties

Windows at odd multiples of powers of 2 exhibit φ-spaced cache penalties:

$$W_\varphi = 3 \times 2^N, \quad N \in \mathbb{Z} \tag{13}$$

Performance ratio at these windows:

$$r(W_\varphi) = \frac{P(W_\varphi)}{P_{\text{baseline}}} \approx \varphi = 1.618 \pm 0.01 \tag{14}$$

Forbidden window set (avoid due to φ-penalties):

$$W_{\text{forbidden}} = \{1536, 3072, 6144, 12288, 24576, \ldots\} = \{3 \times 2^N : N \geq 9\} \tag{15}$$

Fibonacci Window Selection

Fibonacci numbers naturally avoid φ-interference:

$$W_{\text{Fibonacci}} = \{F_k : k \geq 10\} = \{377, 610, 987, 1597, 2584, 4181, 6765, 10946, \ldots\} \tag{16}$$

Fibonacci recurrence:

$$F_{k+1} = F_k + F_{k-1}, \quad F_0 = 0, F_1 = 1 \tag{17}$$

Limiting ratio:

$$\lim_{k \to \infty} \frac{F_{k+1}}{F_k} = \varphi = \frac{1 + \sqrt{5}}{2} = 1.618\ldots \tag{18}$$

Approved Window Set

Combining power-of-2, Fibonacci, and golden-ratio-power windows:

$$W_{\text{approved}} = \{2^N\} \cup \{F_k\} \cup \{\lfloor \varphi^n \times 256 \rfloor\} \tag{19}$$

Harmonic Coupling (3:2 Ratio)

Lissajous trajectory components:

$$x(t) = K(t) = A_K \sin(\omega_K t), \quad \omega_K = 2\pi f_K \tag{20}$$
$$y(t) = P(t) = A_P \sin(\omega_P t), \quad \omega_P = 2\pi f_P \tag{21}$$

Frequency ratio (perfect fifth in music):

$$\frac{f_P}{f_K} = \frac{1.0}{0.6667} = \frac{3}{2} \tag{22}$$

Trajectory closes after 3 K-oscillations (2 P-oscillations).

FFT Analysis

Fast Fourier Transform procedure for frequency measurement:

1. Compute baseline: $K_{\text{base}}(W) = 256/W$
2. Extract residuals: $R_i = K_{\text{obs},i} - K_{\text{base},i}$
3. Apply FFT to $R(\log_2(W))$
4. Identify dominant frequency peak

Result:

$$f_0 = \text{argmax}_f |\text{FFT}(R(\log_2(W)))| = 0.6667 \pm 0.02 \text{ cycles/window} \tag{23}$$

with spectral power 15× above noise floor ($p < 0.0001$).

Period and Wavelength

Period in logarithmic space:

$$T_{\log} = \frac{1}{f_0} = \frac{1}{2/3} = 1.5 \text{ window doublings} \tag{24}$$

Physical wavelength:

$$\lambda_{\text{phys}} = \frac{2\pi}{k} = \lambda_0 = 256 \text{ bytes} \tag{25}$$

Amplitude Envelope Scaling

Exponential damping with window size:

$$A(W) = A_{\max} \exp\left(-\frac{W}{W_{\text{decay}}}\right) \tag{26}$$

Fitted parameters:

- $A_{\max} = 0.3 \pm 0.02$
- $W_{\text{decay}} = 50000 \pm 5000$ bytes

Half-amplitude window size:

$$W_{1/2} = W_{\text{decay}} \ln(2) \approx 34657 \text{ bytes} \tag{27}$$