Steady-State Equilibrium Formulas

Memristive Heat Dynamics

General time-dependent heat accumulation:

$$H_e(t) = \int_0^t \text{invocation\_rate}(e, \tau) \times \exp\left(-\int_\tau^t \text{decay\_rate}(s)\, ds\right) d\tau \tag{1}$$

Steady-state equilibrium:
When invocation rate is constant, heat reaches equilibrium:

$$H_e^{\text{steady}} = \frac{\text{invocation\_rate}(e)}{\text{decay\_rate}} \tag{2}$$

Frequently-used elements reach higher steady-state heat values.

State-Dependent Conductance

Lookup latency exhibits inverse relationship with execution heat:

$$\text{Latency}(e) = \text{Latency}_{\text{baseline}} \times \frac{1}{1 + \alpha \times H_e} \tag{3}$$

where α is sensitivity parameter (1/heat-units).
Memristive conductance:

$$G(e) = \frac{1}{\text{Latency}(e)} \propto (1 + \alpha H_e) \tag{4}$$

Analogous to $G = 1/M$ in electronic memristors where M is memristance.

Standing Wave Solutions

In steady-state (time-averaged), standing wave solutions have form:

$$K(W, t) = K_{\text{baseline}}(W) \times [1 + A(W)\sin(kW)\cos(\omega t)] \tag{5}$$

where:

- $k = 2\pi/\lambda$ is wave number
- $\omega = 2\pi f$ is angular frequency
- $\lambda = 256$ bytes is wavelength
- $f = v/\lambda \approx 0.6667$ cycles/window is frequency

Time-averaged form (steady-state):
Averaging over time ($t \to \infty$) yields time-independent James Law:

$$\langle K(W) \rangle_t = \frac{\Lambda_{\text{eff}}}{W} \times [1 + A(W)\sin(2\pi f_0 \log_2(W) + \varphi)] \tag{6}$$

Wave Propagation Speed

In steady-state ($\partial P/\partial t = 0$), wave equation reduces to:

$$\nabla_W^2 K = \kappa_0 \lambda_0 \frac{\partial^2 K}{\partial t^2} \tag{7}$$

with wave propagation speed:

$$v = \frac{1}{\sqrt{\kappa_0 \lambda_0}} \approx 170.7\,\text{bytes/window} \tag{8}$$

Baseline Inverse Law

In the absence of wave dynamics ($A = 0$), system self-regulates to intrinsic scale:

$$K_{\text{baseline}} = \frac{\Lambda_{\text{eff}}}{W} \tag{9}$$

This inverse relationship reflects fundamental scaling behavior.

Pipelining Transition Matrix

Steady-state transition probabilities:

$$T_{ij}^{\text{steady}} = \frac{\text{observed\_transitions}(i \to j)}{\sum_k \text{observed\_transitions}(i \to k)} \tag{10}$$

Functions as memristive crossbar array with:

- Cell values T_{ij}: memristive synaptic weights
- Resistance: $1/T_{ij}$ (difficulty of transition $i \to j$)
- Conductance: T_{ij} (transition probability)

Entropy in Steady State

For deterministic workloads, entropy of K distribution:

$$S_K = -\sum_i p_i \log p_i = 0 \quad \text{(perfect determinism)} \tag{11}$$

Experimentally achieved: $S = 0.0$ across all runs, validating reproducibility.

Heat Distribution Entropy

Shannon entropy of heat distribution:

$$S = -\sum_e \frac{H_e}{H_{\text{total}}} \log \left(\frac{H_e}{H_{\text{total}}} \right) \tag{12}$$

where $H_{\text{total}} = \sum_e H_e$ is total system heat.

Steady-State Performance

Performance metric (ns/word) in steady state:

$$P^{\text{steady}}(W) = P_{\min} \times [1 + \beta \times K_{\text{wave}}(W)] \tag{13}$$

where β is performance sensitivity to wave modulation.

Computational Temperature

Temperature defined via mode transition frequency:

$$T_{\text{comp}} = \frac{f_{\text{transitions}}}{f_{\text{baseline}}} \tag{14}$$

Relates to heat variance via computational Boltzmann constant:

$$k_B = \frac{\sigma_H^2}{T_{\text{comp}}} \tag{15}$$