Assignment 1

G.Rajanarsavva

Download all python codes from

https://github.com/grajanarsavva/Matrix−theory/codes

and latex-tikz codes from

https://github.com/grajanarsavva/Matrix−theory

1 Question No. 2.9

Draw a $\triangle ABC$ in which $\angle C$ =90°,$\angle B$ =30° and $a+b+c=11$

2 Explanation

Given,

(2.0.1)

and we know $a+b+c=11$
By using Sin Rule:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \tag{2.0.2}$$
$$\implies b\sin C=c\sin B \tag{2.0.3}$$
$$b\sin 90=csin30 \tag{2.0.4}$$
$$\implies c=2b \tag{2.0.5}$$

$$a\sin B=b\sin A \tag{2.0.6}$$
$$a\sin 30=b\sin 60 \tag{2.0.7}$$
$$\implies a=\sqrt{3}b \tag{2.0.8}$$

Then, AX=B
which can be expressed as the matrix equation

$$\begin{pmatrix}0 & -2 & 1\\ 1 & -\sqrt{3} & 0\\ 1 & 1 & 1\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 11\end{pmatrix} \tag{2.0.9}$$

$$-2b+c=0 \tag{2.0.10}$$
$$a-\sqrt{3}b=0 \tag{2.0.11}$$
$$a+b+c=11 \tag{2.0.12}$$
$$\implies c=2b \tag{2.0.13}$$
$$\implies a=\sqrt{3}b \tag{2.0.14}$$

substitute a,b c values in $a+b+c=11$ will get

$$where, a=\left(\frac{11\sqrt{3}}{3+\sqrt{3}}\right); b=\left(\frac{11}{3+\sqrt{3}}\right); c=\left(\frac{22}{3+\sqrt{3}}\right) \tag{2.0.15}$$

$$\implies a=4.02; \tag{2.0.16}$$
$$\implies b=2.32; \tag{2.0.17}$$
$$\implies c=4.64 \tag{2.0.18}$$

The Vertices of $\triangle ABC$ are

$$\mathbf{A}=\begin{pmatrix}0\\ c\end{pmatrix}=\begin{pmatrix}0\\ 4.64\end{pmatrix} \tag{2.0.19}$$
$$\mathbf{B}=\begin{pmatrix}0\\ 0\end{pmatrix} \tag{2.0.20}$$
$$\mathbf{C}=\begin{pmatrix}a\\ 0\end{pmatrix}=\begin{pmatrix}4.02\\ 0\end{pmatrix} \tag{2.0.21}$$
$$\mathbf{D}=\begin{pmatrix}p\\ 0\end{pmatrix}=\begin{pmatrix}4.02\\ 0\end{pmatrix} \tag{2.0.22}$$

Now,

$$AB=\|\mathbf{A}-\mathbf{B}\|^2=\|\mathbf{A}\|^2=c^2=21.61 \tag{2.0.23}$$
$$BC=\|\mathbf{C}-\mathbf{B}\|^2=\|\mathbf{C}\|^2=a^2=16.21 \tag{2.0.24}$$
$$AC=\|\mathbf{A}-\mathbf{C}\|^2=b^2=5.4 \tag{2.0.25}$$

from AC

$$b^2=\|\mathbf{A}-\mathbf{C}\|^2=(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) \tag{2.0.26}$$
$$=\mathbf{A}^T\mathbf{A}+\mathbf{C}^T\mathbf{C}-\mathbf{A}^T\mathbf{C}-\mathbf{C}^T\mathbf{A} \tag{2.0.27}$$
$$=\|\mathbf{A}\|^2+\|\mathbf{C}\|^2-2\mathbf{A}^T\mathbf{C} \tag{2.0.28}$$
$$=a^2+b^2-2ap \tag{2.0.29}$$

yielding

$$p=\left(\frac{a^2+c^2-b^2}{2a}\right)=4.026; \tag{2.0.30}$$
$$\|\mathbf{A}\|^2=c^2=p^2+q^2 \tag{2.0.31}$$
$$\implies q=\pm\sqrt{c^2-p^2}=\pm 2.324 \tag{2.0.32}$$

Plot the $\triangle ABC$ is as follows:

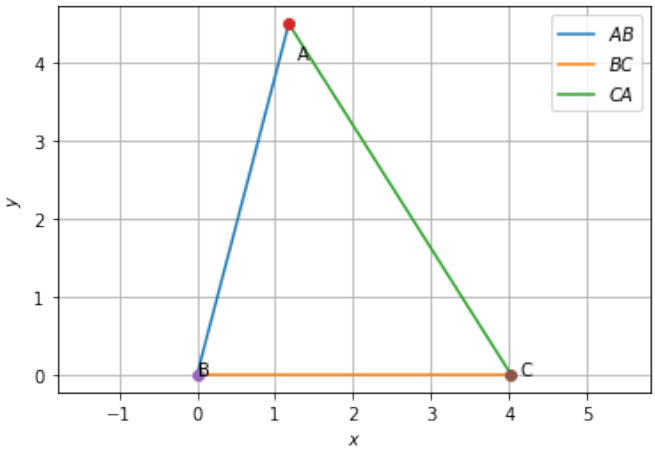

Fig. 2.1: $\triangle ABC$