1

Assignment 1

G.Rajanarsavva

Download all python codes from

https://github.com/grajanarsavva/Matrix-theory/codes

and latex-tikz codes from

https://github.com/grajanarsavva/Matrix-theory

1 Question No. 2.9

Draw a $\triangle ABC$ in which $\angle C = 90^{\circ}$, $\angle B = 30^{\circ} \angle A = 60^{\circ} and a + b + c = 11$

2 EXPLANATION

By using Sin Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \tag{2.0.1}$$

$$\implies b \sin C = c \sin B$$
 (2.0.2)

$$b \sin 90 = c \sin 30$$
 (2.0.3)

$$\implies c = 2b$$
 (2.0.4)

$$a\sin B = b\sin A \tag{2.0.5}$$

$$a\sin 30 = b\sin 60 \tag{2.0.6}$$

$$\implies a = \sqrt{3}b \tag{2.0.7}$$

Then, AX=B which can be expressed as the matrix equation

$$\begin{pmatrix} 0 & -2 & 1 \\ 1 & -\sqrt{3} & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix}$$
 (2.0.8)

By solving (2.0.8), we get values:

$$\implies a = 4.02627944;$$
 (2.0.9)

$$\implies b = 2.32457352;$$
 (2.0.10)

$$\implies c = 4.64914704$$
 (2.0.11)

The Vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 4.64 \end{pmatrix} \tag{2.0.12}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4.02 \\ 0 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{D} = \begin{pmatrix} p \\ 0 \end{pmatrix} = \begin{pmatrix} 4.02 \\ 0 \end{pmatrix} \tag{2.0.15}$$

Now,

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 = 21.61$$
 (2.0.16)

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 = 16.21$$
 (2.0.17)

$$AC = ||\mathbf{A} - \mathbf{C}||^2 = b^2 = 5.4$$
 (2.0.18)

from AC

$$b^{2} = \|\mathbf{A} - \mathbf{C}\|^{2} = (\mathbf{A} - \mathbf{C})^{T} (\mathbf{A} - \mathbf{C}) \qquad (2.0.19)$$
$$= \mathbf{A}^{T} \mathbf{A} + \mathbf{C}^{T} \mathbf{C} - \mathbf{A}^{T} \mathbf{C} - \mathbf{C}^{T} \mathbf{A} \qquad (2.0.20)$$

=
$$\|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C}$$
 (2.0.21)

$$= a^2 + b^2 - 2ap \tag{2.0.22}$$

yielding

$$p = \left(\frac{a^2 + c^2 - b^2}{2a}\right) = 4.026; \tag{2.0.23}$$

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2$$
 (2.0.24)

$$\implies q = \pm \sqrt{c^2 - p^2} = \pm 2.324$$
 (2.0.25)

Plot the $\triangle ABC$ is as follows:

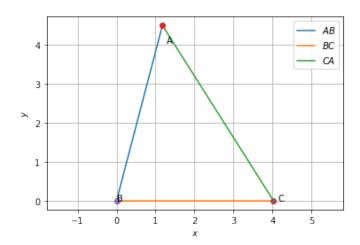


Fig. 2.1: △*ABC*