Assignment 1

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1 QUESTION No. 2.9

Draw a $\triangle ABC$ in which $\angle C = 90°, \angle B = 30°$ and $a + b + c = 11$

2 EXPLANATION

Given,

$$(2.0.1)$$

and we know $a + b + c = 11$
By using Sin Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad (2.0.2)$$

$$\implies b \sin C = c \sin B \qquad (2.0.3)$$

$$b \sin 90 = c \sin 30 \qquad (2.0.4)$$

$$\implies c = 2b \qquad (2.0.5)$$

$$a \sin B = b \sin A \qquad (2.0.6)$$

$$a \sin 30 = b \sin 60 \qquad (2.0.7)$$

$$\implies a = \sqrt{3}b \qquad (2.0.8)$$

Then, AX=B
which can be expressed as the matrix equation

$$\begin{pmatrix} 0 & -2 & 1 \\ 1 & -\sqrt{3} & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \qquad (2.0.9)$$

$$-2b + c = 0 \qquad (2.0.10)$$

$$a - \sqrt{3}b = 0 \qquad (2.0.11)$$

$$a + b + c = 11 \qquad (2.0.12)$$

$$\implies c = 2b \qquad (2.0.13)$$

$$\implies a = \sqrt{3} \qquad (2.0.14)$$

substitute a,b c values in $a + b + c = 11$ will get

$$where, a = \left(\frac{11\sqrt{3}}{3 + \sqrt{3}}\right); b = \left(\frac{11}{3 + \sqrt{3}}\right); c = \left(\frac{22}{3 + \sqrt{3}}\right) \qquad (2.0.15)$$

$$\implies a = 4.02; \qquad (2.0.16)$$

$$\implies b = 2.32; \qquad (2.0.17)$$

$$\implies c = 4.64 \qquad (2.0.18)$$

The Vertices of $\triangle ABC$ are

$$A = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 4.64 \end{pmatrix} \qquad (2.0.19)$$

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (2.0.20)$$

$$C = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4.02 \\ 0 \end{pmatrix} \qquad (2.0.21)$$

$$D = \begin{pmatrix} p \\ 0 \end{pmatrix} = \begin{pmatrix} 4.02 \\ 0 \end{pmatrix} \qquad (2.0.22)$$

Now,

$$AB = \|A - B\|^2 = \|A\|^2 = c^2 = 21.61 \qquad (2.0.23)$$

$$BC = \|C - B\|^2 = \|C\|^2 = a^2 = 16.21 \qquad (2.0.24)$$

$$AC = \|A - C\|^2 = b^2 = 5.4 \qquad (2.0.25)$$

from AC

$$b^2 = \|A - C\|^2 = (A - C)^T(A - C) \qquad (2.0.26)$$

$$= A^T A + C^T C - A^T C - C^T A \qquad (2.0.27)$$

$$= \|A\|^2 + \|C\|^2 - 2A^T C \qquad (2.0.28)$$

$$= a^2 + b^2 - 2ap \qquad (2.0.29)$$

yielding

$$p = \left(\frac{a^2 + c^2 - b^2}{2a}\right) = 4.026; \qquad (2.0.30)$$

$$\|A\|^2 = c^2 = p^2 + q^2 \qquad (2.0.31)$$
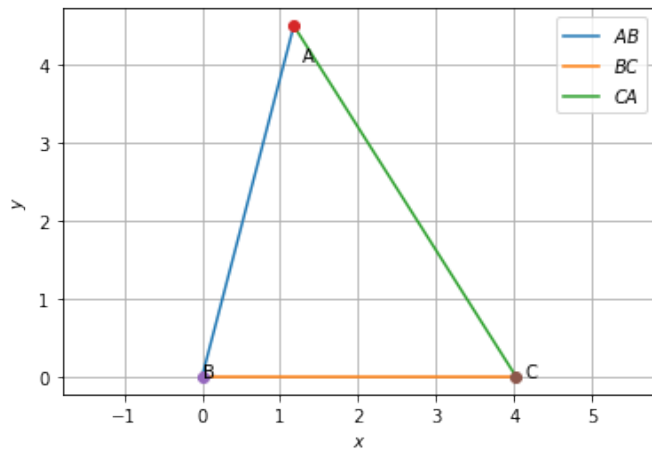
$$\implies q = \pm\sqrt{c^2 - p^2} = \pm 2.324 \qquad (2.0.32)$$

Plot the $\triangle ABC$ is as follows:

Fig. 2.1: $\triangle ABC$