MCL261 Nonlinear Optimization Assignment 2: due November 3, 2021 at 10 PM
Total marks: 15

Submission Instructions
Submit the code on Moodle. A submission link will be created shortly. Submit code as a zip file (no other file types will be accepted). The zip files must contain .py files for each question (if there are n questions, there must be n .py files). Your script must provide the answer to all parts of the question upon execution; therefore, ensure that output is presented in a readable format.

The Python scripts **must** be easy to read: all variables and key quantities of interest must be clearly named, all computations must be commented, all assumptions must be very clearly stated, and finally, you may even add a short commented explanation of your approach to the problem at the beginning of the .py file. File naming conventions: **EntryNum_Name_AssignmentNum.zip** (with each file inside named as **EntryNum_Name_AssignmentNum_QuestionNum.py**). Submissions that do not follow the above convention will not under any circumstances be graded.

1. (3 marks) Find a root for the function below using Newton's method (via Python).

$$f(x) = \sin x - 3xe^{-x^3}$$

2. (4 marks) Use Newton's method for minimization to solve the optimization problem below. Use an initial solution of $(1, 1)^T$. Verify that you have found a minimum.

$$min\ f(x_1, x_2) = 5x_1^4 + 6x_2^4 - 6x_1^2 + 2x_1x_2 + 5x_2^2 + 15x_1 - 7x_2 - 13$$

3. (4 marks) Solve the unconstrained optimization problem below using the steepest descent method (via Python). Use the backtracking line search method to find a suitable step length at every iteration. Set $\mu = 0.2$ as a default value. Ensure and show that you find a minimum.

$$min\ 8x_1^4 + 3x_2^2 - 6x_1x_2 + 2x_2$$

4. (2 marks) Research on time required to solve three questions of an exam with respect to overall performance is conducted. It shows that the time devoted to each of the question affects the overall score as $P(x_1, x_2, x_3) = -(x_1^2 - 2x_1 + 4x_2^2 - 0.03x_3^2 + 0.04x_3)$. Here x_1, x_2 and x_3 represent the hours invested for each of the questions in the exam. What would be the best strategy for the student to maximize the score in exam, assuming every student takes exactly 2 hours to

finish the exam. Solve this in Python using the functions available for solving constrained optimization problems in SciPy.

5. (2 marks) Solve the constrained problem below using the constrained optimization solvers in SciPy.

$$\begin{aligned} min\ & x_1^2 + e^{x_2^2+2x_3^2} + 4x_3 \\ subject\ to:\ & x_1 - x_2 + 10x_3 \leq -2 \\ & -x_2 + x_3 = 1 \end{aligned}$$