

## Artificial Intelligence (BIS515B )

Textbooks:

1. Stuart J. Russell and Peter Norvig, Artificial Intelligence, 3<sup>rd</sup> Edition, Pearson, 2015

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## **Module-5**

# **Quantifying Uncertainty**

# Uncertainty

**Uncertainty** in Artificial Intelligence (AI) refers to situations --where there is insufficient information or ambiguity in data or decision-making processes.

- In AI, uncertainty arises because **-real-world data** is often **noisy, incomplete, or subject to change.**
- This uncertainty poses **challenges** for AI systems that aim to **make accurate and reliable decisions.**

# Baye's Rule and its use

## Bayes' Rule and Its Use

### Two forms of the product rule

$$\checkmark P(a \wedge b) = P(a | b)P(b) \quad \text{and} \quad P(a \wedge b) = P(b | a)P(a).$$

Equating the two right-hand sides and dividing by  $P(a)$ , we get

$$P(b | a) = \frac{P(a | b)P(b)}{P(a)}. \quad (13.12)$$

This equation is known as **Bayes' rule** (also Bayes' law or Bayes' theorem). This simple equation underlies most modern AI systems for probabilistic inference.

When there r two events, this product rule can be modified.

These r 2 forms of product rule

1. Prob. Of a and b = prob. Of a given b into prob. Of b
2. As AND is cumutative, we can replace a with b.

As LHS r equal, we will get this eqation, this is a Bayes rule.

# Unification

1. In a task such as medical diagnosis, the doctor knows  $P(\text{symptoms} \mid \text{disease})$ . Here a doctor knows that the disease meningitis causes the patient to have a stiff neck, say, 70% of the time. The doctor also knows some unconditional facts: the prior probability that a patient has meningitis is 1/100, and the prior probability that any patient has a stiff neck is 1%. Derive a diagnosis,  $P(\text{disease} \mid \text{symptoms})$ .

Assume ' $s$ ' be the proposition that the patient has a stiff neck and ' $m$ ' be the proposition that the patient has meningitis.

$$P(s \mid m) = 0.7$$

$$P(m) = 1/100$$

$$P(s) = 0.01$$

$$P(m \mid s) = \frac{P(s \mid m)P(m)}{P(s)} = \frac{0.7 \times 1/100}{0.01} = 0.7 = 0.7 \times 10/10 = 7/10$$

7 In 10 patient with stiff neck has probability of having meningitis

# Baye's Rule and its use

Derive probability that a patient has the flu (F) given they have a fever (V)?

**Given Data:**

- The flu causes a fever in 75% of cases.
- The probability that a patient has the flu is 1/5,000.
- The probability that a patient has a fever is 4%.

Let  $a$  be the proposition that a patient has a fever and  $b$  be the proposition that a patient has the flu.

**Solution:**

Given:

- $P(a|b) = 0.75$
- $P(b) = \frac{1}{5000}$
- $P(a) = 0.04$

Using Bayes' theorem:

$$P(b|a) = \frac{P(a|b) \cdot P(b)}{P(a)} = \frac{0.75 \cdot \frac{1}{5000}}{0.04}$$

$$P(b|a) = \frac{0.75}{200} = 0.00375$$

So, the probability that a patient has the flu given they have a fever is 0.375%, or 1 in 267 patients.

# Baye's Rule and its use

Bayes Rule

1. In a task such as medical diagnosis, the doctor knows  $P(\text{symptoms} \mid \text{disease})$ . Here a doctor knows that the disease meningitis causes the patient to have a stiff neck, say, 70% of the time. The doctor also knows some unconditional facts: the prior probability that a patient has meningitis is 1/50,000, and the prior probability that any patient has a stiff neck is 1%. Derive a diagnosis,  $P(\text{disease} \mid \text{symptoms})$ .

Assume ‘ $s$ ’ be the proposition that the patient has a stiff neck and ‘ $m$ ’ be the proposition that the patient has meningitis.

$$P(s \mid m) = 0.7$$

$$P(m) = 1/50000$$

$$P(s) = 0.01$$

$$P(m \mid s) = \frac{P(s \mid m)P(m)}{P(s)} = \frac{0.7 \times 1/50000}{0.01} = 0.0014 .$$

we expect less than 1 in 700 patients with a stiff neck to have meningitis.

# Baye's Rule and its use

What is the probability that a patient has strep throat (S) given they have a sore throat (T)?

**Given Data:**

- Strep throat causes a sore throat in 85% of cases.
- The probability that a patient has strep throat is 1/1,000.
- The probability that a patient has a sore throat is 10%.

**Solution:**

Let  $a$  be the proposition that a patient has a sore throat and  $b$  be the proposition that a patient has strep throat.

**Solution:**

Given:

- $P(a|b) = 0.85$
- $P(b) = \frac{1}{1000}$
- $P(a) = 0.1$

Using Bayes' theorem:

$$P(b|a) = \frac{P(a|b) \cdot P(b)}{P(a)} = \frac{0.85 \cdot \frac{1}{1000}}{0.1}$$

$$P(b|a) = \frac{0.85}{100} = 0.0085$$

So, the probability that a patient has strep throat given they have a sore throat is 0.85%, or 1 in 118 patients.

# Baye's Rule and its use

## Example,

diagnosis, we often have conditional probabilities on causal relationships (that is, the doctor knows  $P(\text{symptoms} \mid \text{disease})$ ) and want to derive a diagnosis,  $P(\text{disease} \mid \text{symptoms})$ . For example, a doctor knows that the disease meningitis causes the patient to have a stiff neck, say, 70% of the time. The doctor also knows some unconditional facts: the prior probability that a patient has meningitis is 1/50,000, and the prior probability that any patient has a stiff neck is 1%. Letting  $s$  be the proposition that the patient has a stiff neck and  $m$  be the proposition that the patient has meningitis, we have

$$P(s \mid m) = 0.7$$

$$P(m) = 1/50000$$

$$P(s) = 0.01$$

$$P(m \mid s) = \frac{P(s \mid m)P(m)}{P(s)} = \frac{0.7 \times 1/50000}{0.01} = 0.0014. \quad (13.14)$$

That is, we expect less than 1 in 700 patients with a stiff neck to have meningitis. Notice that even though a stiff neck is quite strongly indicated by meningitis (with probability 0.7), the probability of meningitis in the patient remains small. This is because the prior probability of stiff necks is much higher than that of meningitis.

$$\frac{\frac{7}{10}}{\frac{1}{5000}} \approx \frac{\frac{7}{10}}{\frac{1}{5000}} = \frac{1}{700}$$