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Term Premium Estimates for Brazil in a Model with Survey Expectations

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This paper estimates the term premium and equilibrium rates implied in the Brazilian yield curve, using a term structure model that incorporates data from survey expectations. Nominal long-term yields in Brazil are explained mostly by fluctuations in the equilibrium real rate.

Keywords: yield curve, macro-finance, inflation trend, equilibrium real interest rate, shifting endpoints, bond risk premia. JEL E43; E44; E47

1. Introduction

This paper obtains the term premium and equilibrium rates implied in the Brazilian yield curve through the model of Bauer and Rudebusch (2020). The key innovation is to take into account macro-financial trends about inflation and the equilibrium real interest rate in the estimation of the term premium. Since Brazil has been through a process of stabilisation over the last 30 years - since the launch of the Real plan - a natural question that arises is how changes in the macroeconomic environment have impacted the yield curve, or more specifically, long-term interest rates.

This paper is related to the recent strand of research that uses models that merge macroeconomic trends and financial information in estimates of bond risk premia.

Davis et al. (2024) compare estimates of the natural rate, the inflation trend and bond risk premia from several sources and show that they are not compatible with each other, labelling this a natural rate puzzle. They then use a model where bond yields are determined by long-run trends in the natural rate and inflation, finding mostly flat, instead of declining risk premia over the last decades. Cieslak and Povala (2015) find that trend inflation determines the level of interest rates in the long run and across maturities, with their measure accounting for 85% of the unconditional variance of yields, determining the overall level of the yield curve. Similar to Davis et al. (2024), in their model, nominal yields are driven by trend inflation, a real short interest-rate factor and a price-of-risk

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factor.

Brand, Goy and Lemke (2021) incorporate the natural real rate as shifting endpoint in a term structure model, and estimate r-star jointly, using information from yields and macroeconomic trends for the United States and the euro area. Their model nests the semi-structural modelling approach of the r-star literature and term structure models. Fenou and Fontaine (2023) study the joint dynamics of the nominal short rate, inflation and output. They find that nominal shocks increase output gap, inflation and nominal yields in periods of unanchored inflation expectations, while in periods of anchored inflation expectations the impact of these shocks become short-lived.

The rest of the paper is organised as follows. In addition to this introduction, Section 2 introduces the affine term structure model (ATSM) of Bauer and Rudebush (2020). Section 3 describes the data used to estimate the model. Section 4 shows the results. Section 5 concludes.

2. Model

This paper uses the model of Bauer and Rudebusch (2020) to estimate the term premium and equilibrium interest rates implied in the Brazilian yield curve. The model is a Dynamic Term Structure Model (DTSM) that incorporates a shifting endpoint for interest rates i_t^* as a common stochastic trend for yields. The main innovation of the model is that the equilibrium nominal interest rate i_t^* is allowed to be time-varying, implying that bond yields have a common stochastic trend and are $I(1)$.

As explained by Bauer and Rudebusch (2020), this is a convenient way to account for a very persistent process in a dynamic model, taking into account slow-moving interest rate trends. Bauer and Rudebusch (2020) denominate this model as the shifting endpoint (SE) model.

Early references in the literature includes Kozicki and Tinsley (2001), who links long-horizon forecasts of nominal short-term interest rates to market perceptions of the lung-run policy target for inflation. The key insight is that changes in market perceptions of the long-run target for inflation may imply shifts to the long-run trends of nominal interest rate expectations embedded in the term structure[2], i.e., agents try to learn the true inflation target, and this reflects in long-term interest rates. In the terminology of Brand, Goy and Lemke (2019), the natural nominal short rate i_t^* plays the role of a time-varying attractor for short rate expectations over long horizons.

[2] This can be viewed in the Fisher equation: $i_\infty = r_\infty + \pi_\infty$. Shifting agents' perceptions of the long-run inflation target π_∞ will be reflected in shifts in the perceived endpoints of the nominal interest rate i_∞.

For emerging markets like Brazil, this channel can be relevant in periods of unanchored inflation expectations, in which agents try to understand the implicit inflation target pursued by the central bank. One can also think about how changes in the economic outlook reflect in the price action of long-term interest rates.

Following Bauer and Rudebusch (2020), any continuously compounded yield on an n-period discount bond, $y_t^{(n)}$ can be decomposed in the expected average path of short rates over n-periods (given by $\frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}$) and a term premium $TP_t^{(n)}$:

$$y_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)} = i_t^* + \frac{1}{n}\sum_{j=0}^{n-1} E_t i^c_{t+j_{t+j}} + TP_t^{(n)} \quad (1)$$

where i_t is the risk-free nominal short-term rate at time t, i_t^* is the trend in the short-term interest rate and i_t^c is the cyclical component of the short rate, both defined below. The term premium is a compensation for bond investors for the duration risk in longer-term bonds.

The expectations theory of interest rates assumes that the term premium is zero in its strong form, or constant in its weak form. The second equality in equation (1) introduces the trend or equilibrium short rate, i_t^*, and the short rate cycle $i_t^c = i_t - i_t^*$, where i_t^* is the trend in the short-term interest rate:

$$i_t^* = lim_{j\to\infty} E_t i_{t+j} \quad (2)$$

It is assumed that the trend is stochastic, rather than deterministic. Nominal yields are affected by changes in i_t^*, and if this trend is time-varying, yields are nonstationary with a common trend.

By the Fisher equation:

$$i_t = r_t + E_t \pi_{t+1} \Longrightarrow i_t^* = r_t^* + \pi_t^* \quad (3)$$

where r_t is the real short rate, π_t is inflation, and the trends in r_t and π_t are r_t^* and π_t^*, respectively. If inflation or the real interest rate contains long-run trend components, then nominal interest rates will also be driven by the same macroeconomic trends.

The state variables of the model include N linear combination of yields in the vector P_t and *N* = 3 of such yield factors are used. The key feature of the model is the presence of a single stochastic trend τ_t common to the three factors. The common trends representation of the system is:

$$P_t = \bar{P} + \gamma\tau_t + \bar{P}_t \quad (4)$$

$$\tau_t = \tau_{t-1} + \eta_t \quad (5)$$

$$P_t = \Phi P_{t-1} + \tilde{u}_t \quad (6)$$

where γ is an N-vector with the loadings of the common trend, η_t is iid normal with variance σ_η^2, $\bar{P}_t$ are the N cycle components that have iid normal innovations with matrix $\widetilde{\Omega}_t$ and Φ is a mean-reversion matrix that has eigenvalues with modulus less than one so that $\tilde{P}_t$ is stationary.

Accordingly, the state variables $Z_t = (\tau_t P_t')'$ are cointegrated. It is assumed that innovations to trend and cycle are orthogonal, $E(\eta_t \tilde{u}_t) = 0$. These assumptions imply that the long-run trend components of P_t are:

$$P^* \equiv lim_{j\to\infty} E_t P_{t+j} = \bar{P} + \gamma\tau_t \quad (7)$$

The short-term interest rate is taken to be an affine function of P_t:

$$i_t = \delta_0 + \delta_1' P_t \quad (8)$$

It is assumed absence of arbitrage opportunities so that there exists a risk-neutral probability measure, given by $\mathbb{Q}$, which prices all financial assets. The risk-neutral dynamics are specified as:

$$P_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} P_{t-1} + u_t^{\mathbb{Q}} \quad (9)$$

where the innovations $u_t^{\mathbb{Q}}$ are *iid* normal (under $\mathbb{Q}$) with covariance matrix Ω. It is assumed that P_t is stationary under $\mathbb{Q}$ by imposing that $\Phi^{\mathbb{Q}}$ has eigenvalues less than one in absolute value. Equations (8) and (9) imply that yields are affine functions of the yield factors P_t, namely:

$$Y_t = A + BP_t \quad (10)$$

The loadings in *A* and *B* are determined by the risk-neutral parameters.

According to Bauer and Rudebusch (2020), two types of normalisations are required for the state variables and parameters to be identified.

The first set of normalisations uniquely pins down the yield factors, P_t. In their absence, an affine transformation of P_t would leave implied interest rates Y_t unchanged. Bauer and Rudebusch (2020) use the normalisation of Joslin et al. (2011), which imposes restrictions only on the $\mathbb{Q}$-dynamics and is ideally suited for the use of linear combinations of yields as yield factors. In this normalisation, four parameters determine the double-struck cap Q-dynamics: a scalar k to the double-struck cap Q and an N-vector lambda to the double-struck cap Q containing the eigenvalues of cap phi to the double-struck cap Q, which are assumed to be real, distinct and less than one. These four parameters and Ω fully determine the yield loadings A and B.

The second normalisation is needed to identify the unobserved trend τ_t. Without it, an affine transformation of τ_t would leave the yield factors P_t unchanged. For identification, it is required one linear constraint on each of the vectors $\bar{P}$ (to fix the level) and γ (to fix the scale). Bauer and Rudebusch (2020) identifies τ_t as the long-run mean of the short-rate:

$$\tau_t = i_t^* = \delta_0 + \delta_1' P_t^* \tag{11}$$

They use the normalisation $\delta_0 + \delta_1' P = 0$ and $\delta_1' \gamma = 1$. The parameters of the model are: $k^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \gamma, \bar{P}, \Phi, \Omega, \sigma_u^2$ and σ_e^2, where γ and $\bar{P}$ each have two degrees of freedom.

In contrast to the SE model, a conventional DTSM with stationary dynamics is labelled as a fixed endpoint (FE) model, which corresponds to a restricted special case of the model where $i_t^* = i^*$, i.e., $\sigma_\eta^2 = 0$ which is equivalent to the three-factor affine DTSM of Joslin et al. (2011). The underlying assumption for the FE model is that the underlying policy target for inflation is believed to be constant.

For estimation of the SE model, similar to Beuer and Rudebusch (2020), data from long-run survey expectations of inflation (π_t^*) and the real rate (r_t^*) is used. Using a proxy for the trend interest rate (i_t^*) simplifies the estimation of the model, otherwise the common trend τ_t is unobserved, and the model is effectively an unobserved components model. As Bauer and Rudebusch (2020) explain, empirically uncovering long-run trends using limited time series is intrinsically difficult and estimating an unobserved components model with maximum likelihood and no external additional information is problematic and unreliable. So, one approach used to ease computations is to use empirical proxies for the trend interest rate. For the OSE model they use $\tau_t = i_t^*$ and treat this state variable as observable.

3. Data

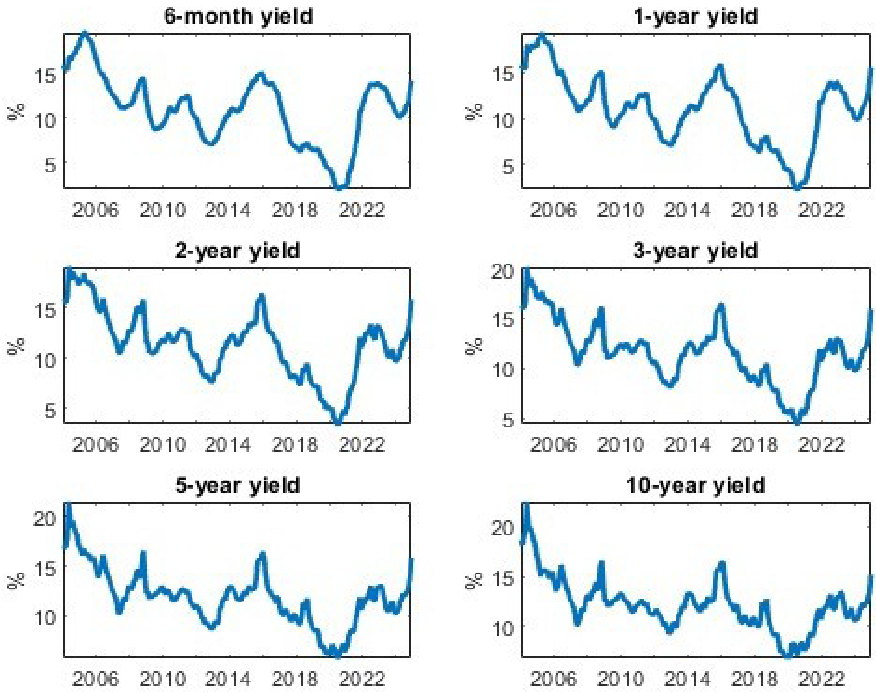
In order to estimate the OSE model, Bauer and Rudebusch (2020) use an empirical proxy for i_t^* equal to the sum of the perceived inflation target π_t^* and average of real-time estimates of r_t^*, some from established models in the literature and some from econometric estimates. Considering that it is usual in the literature to use data from long-run survey expectations, the estimation for Brazil uses data from the SGS database of the Central Bank of Brazil.

I transform these series in constant maturity according to the following equation: $forecast_{j+1}(month_i year_j) = \left(\frac{12-month(date)}{12}\right) * forecast(year_j) + \left(\frac{month(date)}{12}\right) * forecast(year_{j+1})$ This formula builds the constant maturity forecast as a weighted average of the forecasts of two subsequent years. In a given date, there are forecasts for up to 4 years ahead. The equation is used for each pair of subsequent years to create the constant maturity forecasts for 1, 2 and 3 years ahead (j = 0,1,2), respectively. The measure of i^* (i-star) used in the estimation is the resulting 3-year ahead Selic rate and is presented in the middle panel of Figure 2. Its components (π_t^* and r_t^*, namely pi-star and r-star) are depicted in the left panel of Figure 2.

Daily pre-DI swap data from 2004 to 2023 was retrieved from the R package RB3. Nominal yields were then interpolated for 3-month, 6-month, 1-year, 2-year, 3-year, 5-year and 10-year horizon. Since this package was discontinued in 2024, for this year the data come from Investing.com. The yields used in the estimation are displayed in Figure 1[3]. The sample period runs from January 2004 to December 2024, encompassing 252 observations.

[3] Spreadsheets with the dataset used in the estimations are available upon request.

Figure 1 - Data



4. Results

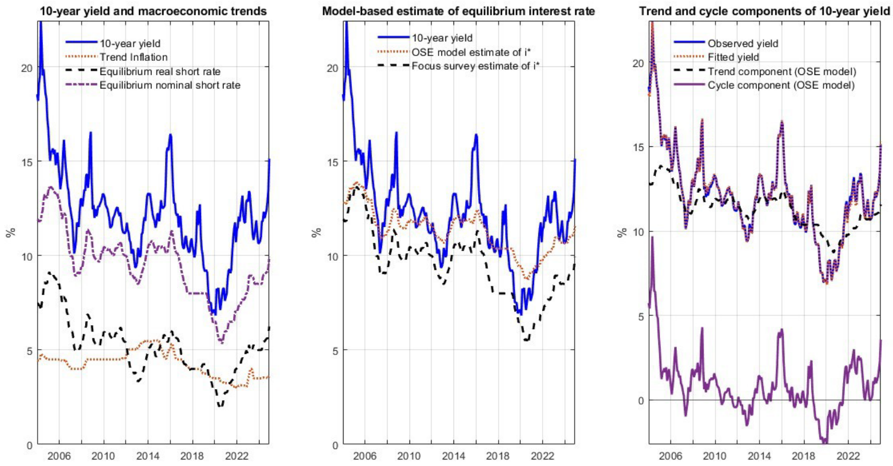
The left panel in Figure 2 plots the 10-year yield, along with the trend inflation measure (π^*), the equilibrium real short rate (r^*) and the equilibrium nominal short rate (i_t^*). The last three variables come from long-run Focus survey forecasts. Overall, the equilibrium nominal short rate tracks the 10-year yield with a correlation of 0.82. Over the period 2004-2024, the variability of i_t^* comes both from real rate component and from the trend inflation component. In the beginning of the sample, the real rate ranged between 7 and 9% p.a. From a peak in 2005, and except from a spike during the Global Financial Crisis (GFC) in 2008, it began a downward trend in the real rate, which lasted until 2013, when it reached a little above 3% p.a. before the Taper Tantrum. From then onwards it climbed to close to 6% p.a. in 2016, in the middle of the Brazilian recession of 2014-2016. The downward trend resumed after the recession, reaching a trough of 2% p.a. in 2020, during the Covid-19 pandemic. Afterwards the real rate began increasing again, reaching more than 6% p.a. in the end of the sample in 2024.

Likewise, the behaviour of trend inflation is more stable over time. It oscillated between 4 and 4.5 until 2011, increasing to 5.5% between 2012 and 2014, due to unanchored inflation expectations. After the 2014-2016 recession there is a downward trend, due to lower inflation targets and lower inflation readings, gravitating between 3 and 4% p.a.

The middle panel in Figure 2 shows the 10-year yield, along with the OSE model estimate of i_t^* and the Focus survey estimate of i_t^*. The model estimate of the equilibrium rate tracks very closely the survey estimate, but in a scaled-up manner. The equilibrium rate ranged between 12.5% p.a. between 2004 and 2008. From 2009 to 2016, it averaged 11.6% p.a. After the recession it began a downward trend, reaching a trough of a little below 9% p.a. in 2020, during the pandemic. After the pandemic it began an upward trend, averaging 10% p.a. between 2021 and 2023. At the end of the sample period, in December 2024, it was rising again, reaching 11.60% p.a.

The right panel of Figure 2 depicts the 10-year yield, the fitted yield of the model, the equilibrium (trend) component of the nominal rate, and the cycle component. The latter component was high in the beginning of the sample in 2004, during the GFC in 2008, during the 2014-2016 Brazilian recession, during the 2018 elections and in 2022 and 2024. On the contrary, the cycle component was low in 2007, in 2011-2013 due to unconventional monetary policies in advanced economies and in 2020, during the Covid-19 pandemic outbreak.

Figure 2

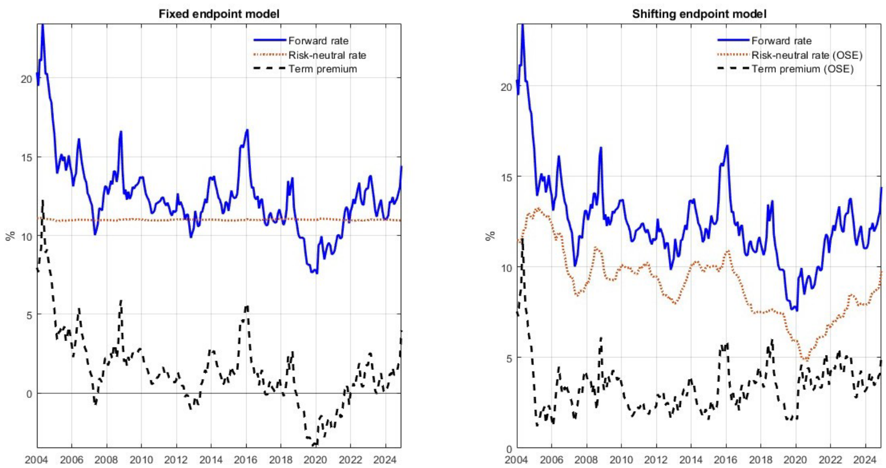


Left panel: 10-year yield and estimates of trend inflation π_t^*, the equilibrium real rate r_t^*, and the equilibrium nominal rate, i_t^*, $i_t^* = r_t^* + \pi_t^*$. Middle panel: 10-year yield, equilibrium nominal rate i_t^* obtained from the observed shifting endpoint (OSE) dynamic term structure model and the proxy estimate of i_t^* obtained from survey expectations, the sum of long-run inflation expectations π_t^* and the real rate r_t^*. Right panel: Model-based decomposition of the 10-year yield into trend and cycle components for the model with observed shifting endpoint (OSE).

Figure 3 presents the results from the fixed endpoint model in the left panel and the shifting endpoint model in the right panel. Each panel shows the five-to-ten year forward (5x10) rate, the risk-neutral rate and the estimated term premium. In the left panel the risk-neutral is roughly constant throughout the sample period, with an average of 11 % p.a. In the right panel, the risk-neutral rate has more volatility over time. There is a downward trend from 2004 to 2007. From 2008 to 2016 the average is 9.6 % p.a. The risk-neutral rate descends from 2017 onwards, reaching a trough of 5% p.a. in 2020, during the pandemic. Afterwards it climbs again, reaching an average of 9.7 % p.a. at the end of the sample period, in December 2024.

Since in the shifting endpoint model the risk-neutral rate is more volatile, less variability is due to the term premium. Noticeably, in the shifting endpoint model the term premium does not reach negative values in 2020, since the model understands that part of the decline in the forward rate was due to a lower risk-neutral rate.

Figure 3



Note: Five-to-ten year forward rate with estimated expectations component (risk-neutral rate) and term premium. Left panel: conventional dynamic term structure model (DTSM) with a fixed endpoint $\boldsymbol{i}^*$. Right panel: DTSM with a common stochastic trend, where the shifting endpoint is taken as observed using a proxy of $\boldsymbol{i}_t^*$ (OSE, observed shifting endpoint).

Table 1 shows the regression models that try to explain the 10-year yield using the nominal equilibrium rate from long-run Focus survey forecasts (i_t^*) along with the long-run forecasts for inflation (π_t^*) and consistent real rate (r_t^*). The following specifications were used in columns (1), (2), (3) and (4) respectively:

$$y10 = \alpha + \beta i_t^* + \varepsilon_t \tag{12}$$

$$y10 = \alpha + \beta \pi_t^* + \varepsilon_t \tag{13}$$

$$y10 = \alpha + \beta r_t^* + \varepsilon_t \tag{14}$$

$$y10 = \alpha + \beta \pi_t^* + \gamma r_t^* + \varepsilon_t \tag{15}$$

Column 1 shows that there is a unitary impact from the i-star to the 10-year bond yield. The results in column 2 show that the proxy for trend inflation explains only 13.6% of the variability of the 10-year yield.

Column 3 shows that most of the explanatory power (70%) is due to r-star. Finally, column 4 presents the results of the regression of the 10-year yield on pi-star and r-star. Although the former is statistically significant, a comparison of R2 of columns 3 and 4 shows that most explanatory power comes from r-star. Noticeably, the estimated coefficient on inflation reduces from $\hat{\beta} = 1.46$ to $\hat{\beta} = 0.39$, implying that there is a high partial effect of r_t^* on the 10-year yield y10. Taking together the results of columns 2 to 4, the real rate component overshadows the proxy for trend inflation when it comes to explain the 10-year yield.

In some sense this result can be viewed positively, since in this period long-run inflation expectations were most of the time anchored in the inflation targeting regime. This can be related to Gürkaynak, Levin and Swanson (2010) who, using data from bond yields, find evidence that the inflation targeting regime anchored long-run inflation expectations in Sweden and the U.K. post central bank independence.

Finally, compared to Bauer and Rudebusch (2020), the explanatory power of the regression models is also high in Brazil, but a bit lower than for the U.S.

Table 1 -Regression models

Variables	(1) y10	(2) y10	(3) y10	(4) y10
i_t^*	1.14***			
	(0.059)			
π_t^*		1.46***		0.39***
		(0.22)		(0.11)
r_t^*			1.42***	1.36***
			(0.076)	(0.074)
Constant	1.36**	5.98***	4.79***	3.41***
	(0.54)	(0.90)	(0.36)	(0.54)
Observations	252	252	252	252
Adj. R2	0.678	0.136	0.706	0.714

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5. Conclusion

Recent papers have explored the role of macroeconomic trends in bond yields. This paper sought to contribute to the literature on the yield curve and the term premium for Brazil, by estimating the model of Bauer and Rudebusch (2020). This model incorporates shifting endpoints, which can be understood as an attractor for interest rates over time. It is consistent with the notion that agents reassess the equilibrium rate over time, and this is reflected in long-term bond yields.

One of the main results is that long-term nominal yields in Brazil reflect not only trend inflation, but more importantly, changes in the equilibrium real rate. Regression models that use both trend inflation and the proxy of the equilibrium rate to explain the 10-year yield show that the bulk of the explanatory power comes from the real rate (r-star), with little gain coming from the inclusion of trend inflation. This means that the inflation targeting regime has been successful in anchoring long-run inflation expectations in Brazil.

Previous research has found that real rate shocks are a sizeable determinant of business cycles in emerging markets in a general equilibrium context (Neumeyer and Perri, 2005; Souza Sobrinho, 2011). The result of this paper suggests that the impact of real rate shocks on long-term yields can be an important transmission channel in emerging markets.

Reductions in the equilibrium real rate coincided with periods of abundant international liquidity from Large Scale Asset Purchases (LSAP) in advanced economies, for instance in 2011-2012 and in 2020, following the Covid-19 outbreak. On the contrary, periods with heightened risk premiums such as during the GFC of 2008 and the 2014-2016 recession are accompanied by higher real rates, which is reflected in the equilibrium nominal rate obtained from the model.

These results are important for policymakers trying to assess the equilibrium rate of the economy and investors. With the terminal rate allowing to shift, term premiums tend to be less volatile. For emerging market like Brazil, risk assessment also has to take into account the possibility of changing equilibrium rate.

Unlike Bauer and Rudebusch (2020), which used quarterly data, this paper adapted and estimated their model at the monthly frequency. Since yield curve data is promptly available, this provides a way to track the equilibrium rate of the economy in almost real-time, which is an important input for monetary policy deliberations. Therefore, policymakers can monitor the changes in the equilibrium rate implied in the yield curve, assess the cyclical and trend components, and evaluate how the term premium have changed from one month to the next.

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