# CS6015: Linear Algebra and Random Processes Programming Assignment 1

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**Note:** Colab is used for performing experiments and .pynb file is submitted

## Dimensionality Reduction

## 1 For the 2-dimensional dataset :

### 1.1 Plot the dataset and the singular vector

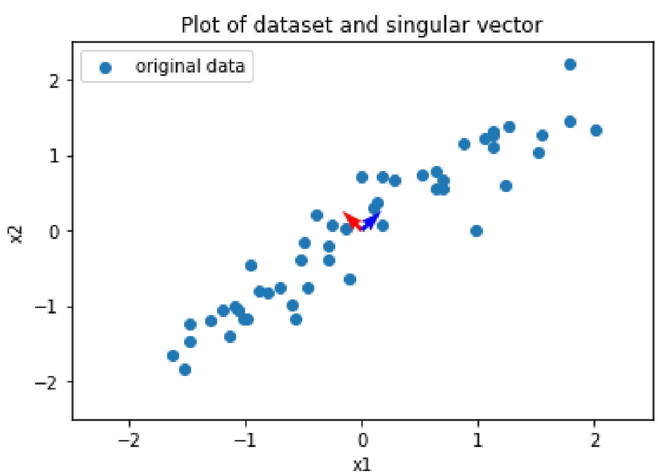



**Fig. 1.** Plot of dataset and singular vector

**Observation:**

– It is observed that the two eigen vectors are orthogonal to each other.
– The two eigen vector denotes the directions where data varies maximum.
– Blue arrow denotes the topmost eigen vector along which the data is maximum varies followed by the red arrow

### 1.2 Project the data points onto the top singular vector and plot the original as well as projected data.

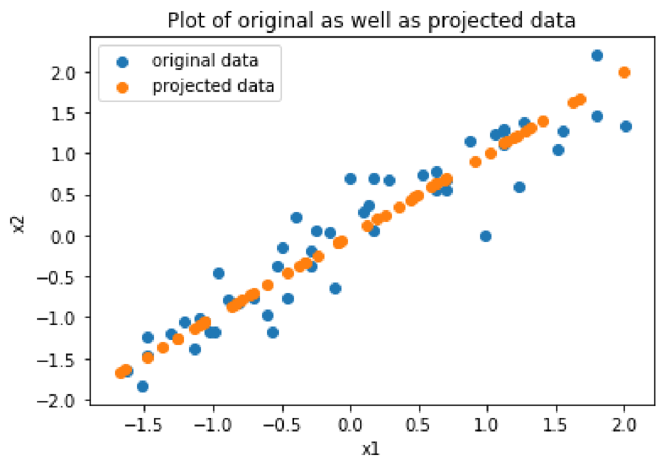



**Fig. 2.** Plot of data point projected on top singular vector and original data

**Observation:**
The projected data is along the top 1 eigen vector.

### 1.3 What is the percentage of information lost by reducing dimension?

- By reducing the dimension, the percentage of information lost is 5.834218550505044%
- This implies that the maximum information was along the top eigen vector.

### 1.4 Plot the dataset, the least squares approximation and the top singular vector in one figure.

- It is observed that the least square approximation line and top singular vector are not the same.
- Least square approximation error is 11.328056040059462%
- Dimensionality reduced error is 5.834218550505044%
- It is observed that when we reduce dimensionality the error is less compared to least square approximation because former method reduces the orthogonal distance from the data point whereas in latter method minimises the distance from the y-axis.

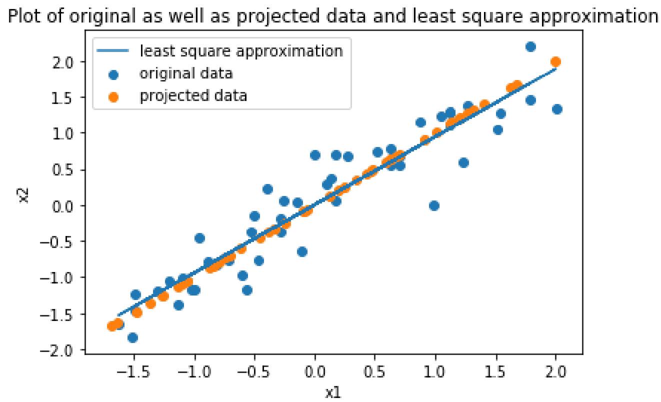



**Fig. 3.** Plot of data, projected data and least square approximation

## 2 For the 10-dimensional dataset

### 2.1 Project the data points onto the top two singular vectors

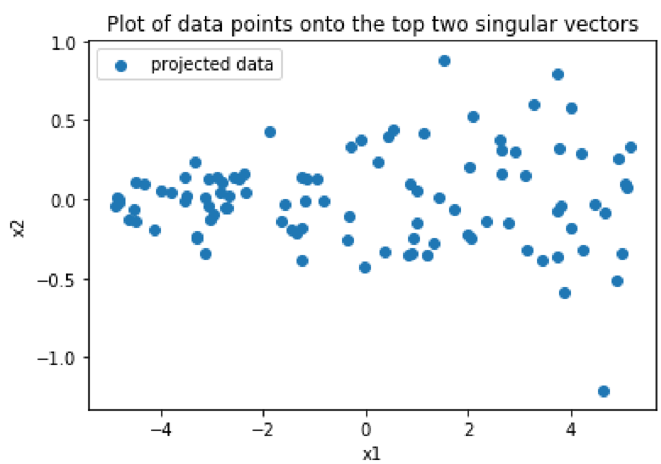



**Fig. 4.** Projection of data on top two singular vector

### Observation

– Fig.4 we observe that the data is projected along top 2 eigen vector.

### 2.2 Is the information lost by reducing to 2-dimension less than 10%

- The information lost by reducing to 2-dimension is not less than 10%. The information lost by reducing to 2-dimension is 41.62%
- To capture 90% of the information we need atleast 7 singular vectors.

## Image Compression

## 3 Square Image

### 3.1 Reconstruct the image using top N singular vector. Plot the reconstructed images along with their corresponding error image.

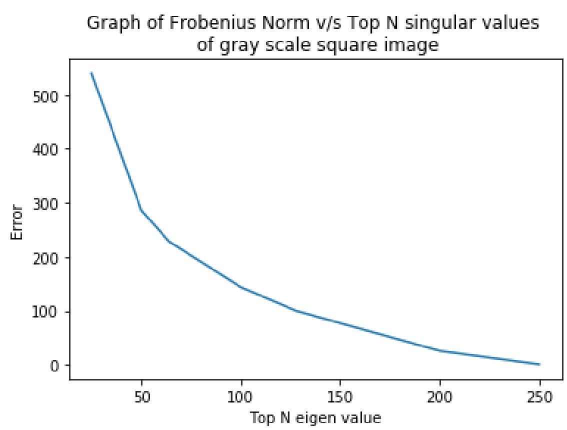



**Fig. 5.** Comparative graph of the reconstruction error vs N.

| Sr.No | N | Error |
|---|---|---|
| 1 | 25 | 539.58 |
| 2 | 64 | 228.12 |
| 3 | 128 | 99.64 |

**Table 1.** Quantitative Analysis of square image

### 3.2 Try random N (N = 25) singular vectors instead of top N.

## Observation :

- It is observed that as we increase the value of N we are able to reconstruct the image better.

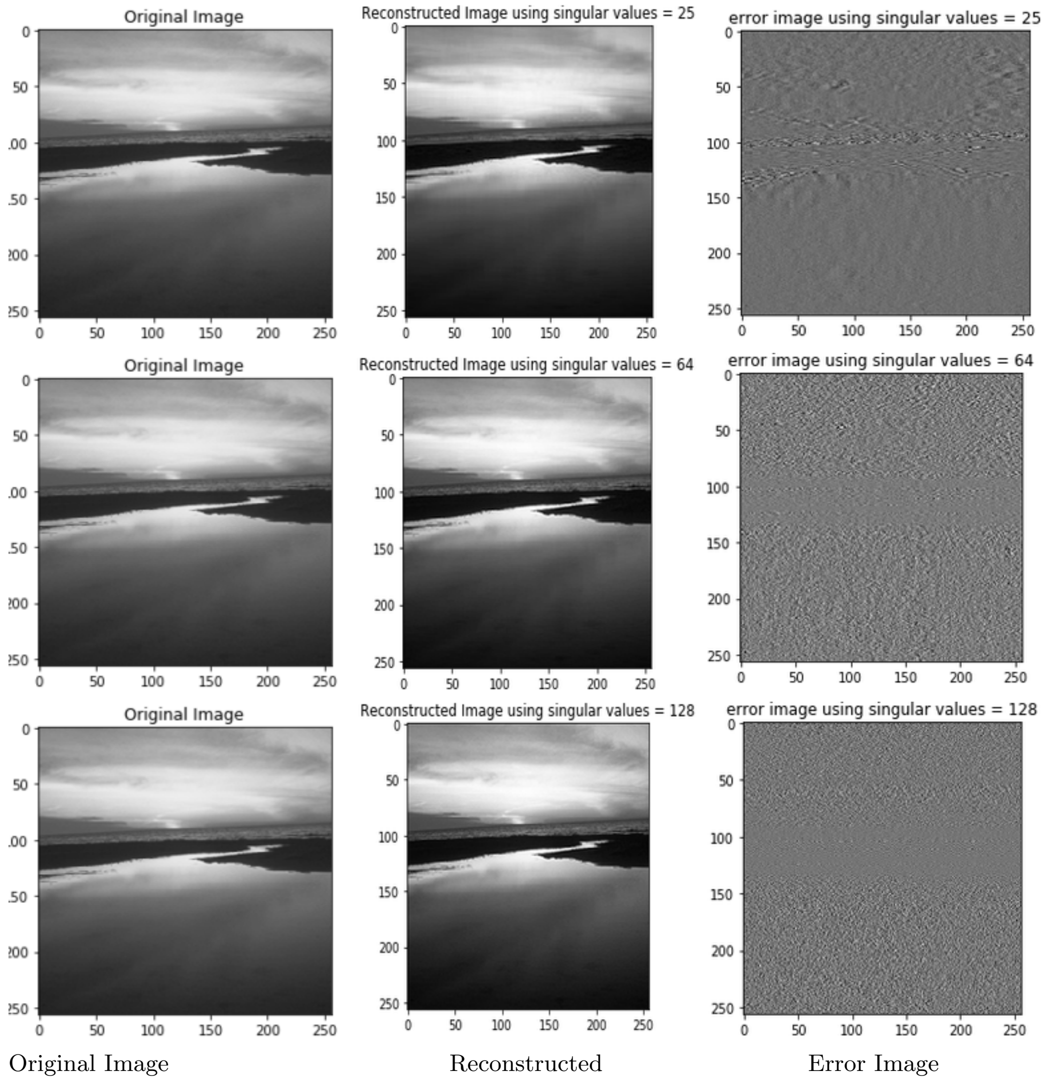
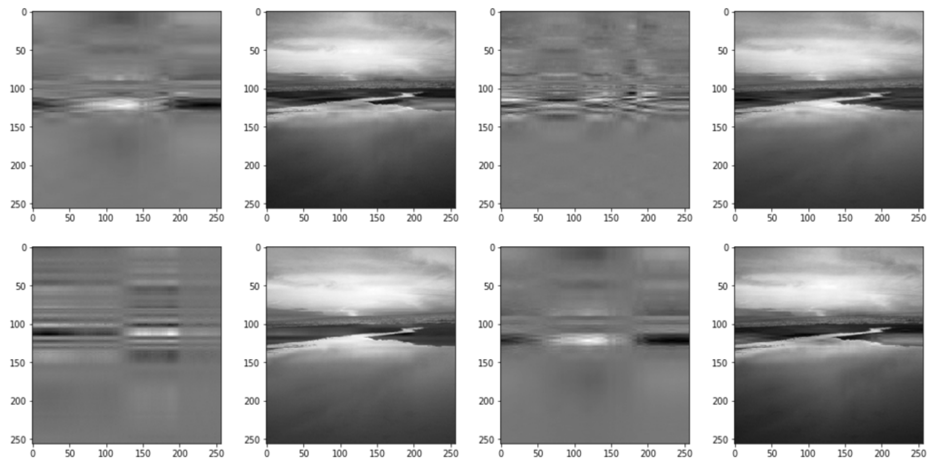



**Fig. 6.** Qualitative analysis of image reconstructed by using N =25, 64, 128 singular values and its corresponding error image.

- In error image we are able to observed the details which are not reconstructed in the image.
- From table.1 it is observed that the error decreases as N increases.
- We observed that when we take top 25 singular vector we are able to reconstruct the image better whereas when we take random singular vector we are not able to reconstruct the image.



Reconstructed Error Image Reconstructed Error Image

**Fig. 7.** Qualitative analysis of image reconstructed by using random singular values and its corresponding error image.

This denotes that topmost singular vectors captures maximum information

# 4 Rectangular Image

## 4.1 Reconstruct the image using top N eigen vector. Plot the reconstructed images along with their corresponding error image.

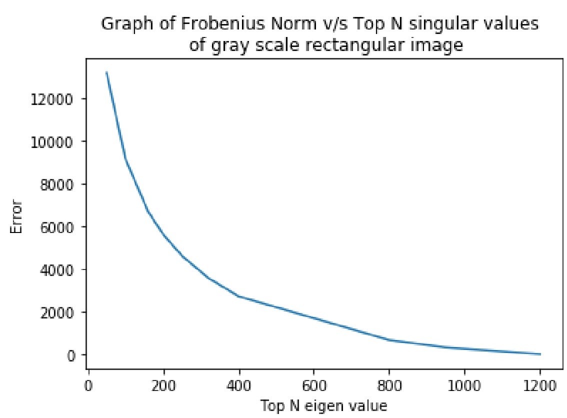
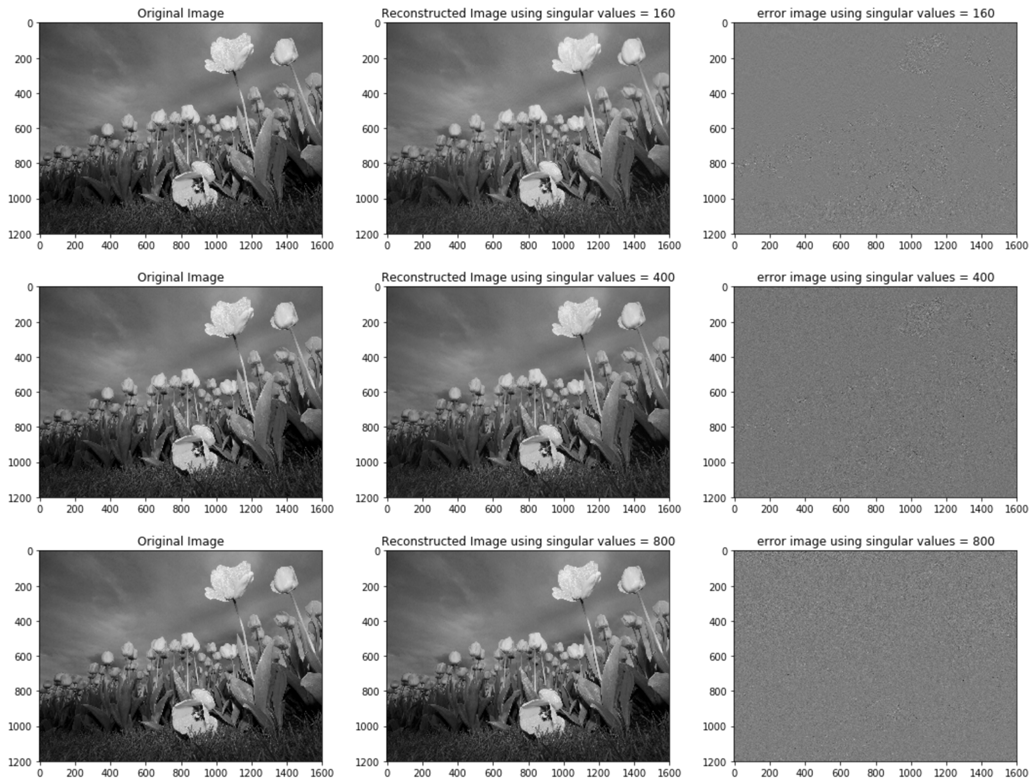



**Fig. 8.** Comparative graph of the reconstruction error vs N.

| Sr.No | N | Error |
|---|---|---|
| 1 | 160 | 6685.48 |
| 2 | 400 | 2711.71 |
| 3 | 800 | 660.03 |

**Table 2.** Quantitative Analysis of rectangular image



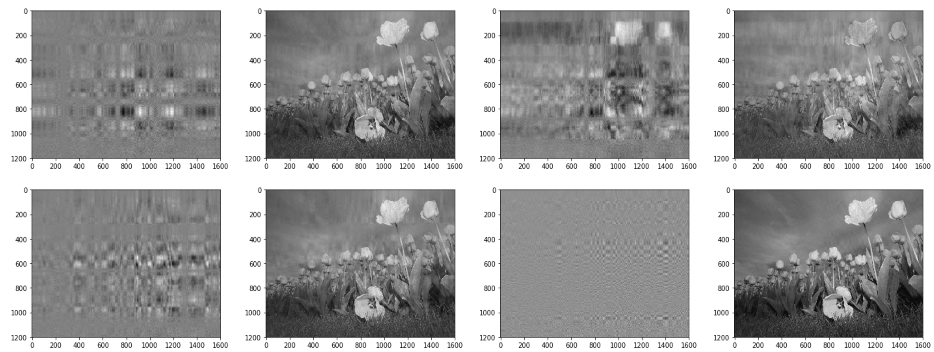

Original Image Reconstructed Error Image

**Fig. 9.** Qualitative analysis of image reconstructed by using N =160, 400, 800 singular values and its corresponding error image.

### 4.2 Try random N (N = 160) eigen vectors instead of top N.

## Observation:

- It is observed that as the value of N increases the reconstruction is better both qualitatively and quantitatively.
- Details which are not preserved while reconstructing are present in their corresponding error image.



Reconstructed Error Image Reconstructed Error Image

**Fig. 10.** Qualitative analysis of image reconstructed by using random eigen values and its corresponding error image.

- It's observed that when we take topmost 160 eigen values we are able to reconstruct the image better than when we take random 160 eigen values. This implies that topmost eigen values contains maximum information.

## 5 Save the water

### 5.1 Formulate the problem in Ax = b form.

**Procedure of framing the problem in AX = B**

As we have given NxN matrix each element of matrix represent a tap. Initially all taps are open.We will define an operation matrix for each of the tap according to the position of the tap. Ex.- for tap at position (1,2) operation matrix will be O(1,2) =

$$\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

G + O(0,0) + O(0,1) + ... + O(N-1,N-1) = 0 (modulo 2)
where G is the initial configuration of the matrix and O(i,j) is the operation matrix for the tap at position (i,j)
G + $sum_{i,j}$ a(i, j) O(i, j) = 0 (modulo 2), where a(i,j) is the coefficient saying whether the operation O(i,j) is performed or not to reach the goal.AS we are working Under GF(2) Airthmetic so we

can write $sum_{i,j}$ a(i, j) O(i, j) = G.
From here now flattening all the operation NxN matrx to vector of size NxN*1 and putting all of them in matrix of size (NxN,NxN) called A and flattening G to NxN*1 called B and putting all coefficients a(i,j) in a vector of size NxN*1 called X. So finally we get AX = B.

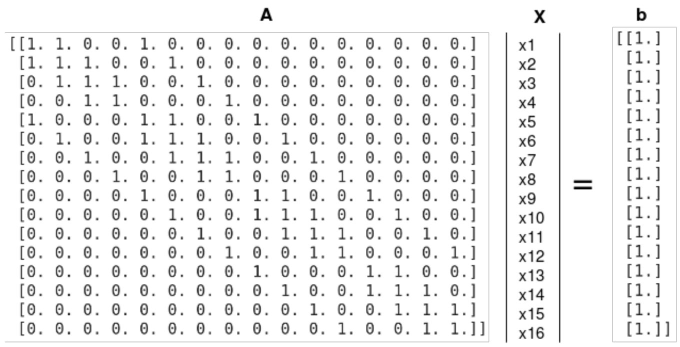



**Fig. 11.** Matrix A and B

### 5.2 Find the solution sequence.

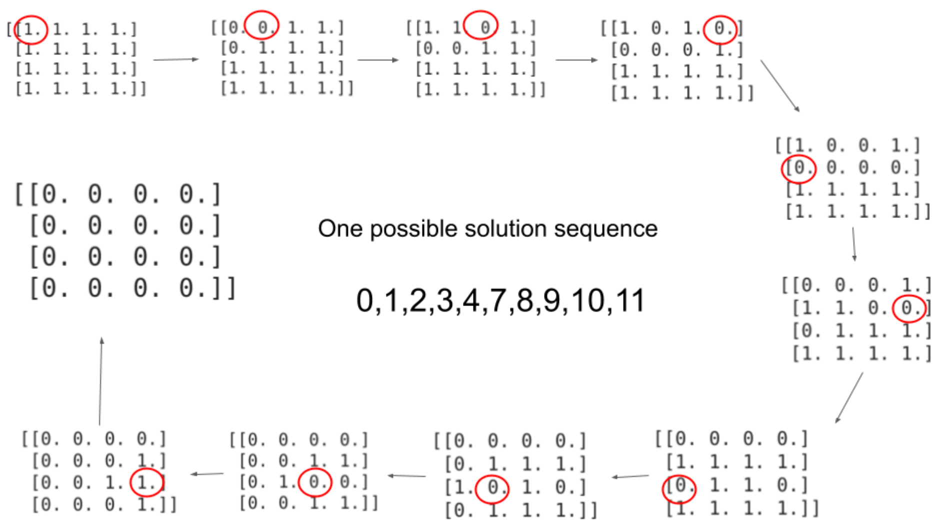



**Fig. 12.** Sequence number is assigned in row wise

### 5.3 Is the solution unique? if not, how many solutions are there?

NO, solution is not unique. There are 16 possible solutions.

**Reason :** After applying Gaussian elimination we find rank of the matrix is 12. so $\mathbf{AX} = \mathbf{0}$ have nullspace of dimension 4.So it means 4 independent vectors possible in null space.So solution will be $p1 + \alpha v1 + \beta v2 + \gamma v3 + \eta v4$.
where p1 is aparticular solution and v1,v2,v3,v4 are independent vectors in nullspace and $\alpha, \beta, \gamma, \eta$ are binary values

## 6 Ciphertext

### 6.1 What is the encoding of the message "LINEAR" using both X1 and X2?

The encoding of the message "LINEAR" using X1 : $[897, 1417, 3734, 3793, 229, 862]$
The encoding of the message "LINEAR" using X2 : $[897, , 1656, 2760, 6603, 229, 458]$

### 6.2 Decode 927 1345 4006 3913 and 927 1445 3811 3665 708 1081 1778 using both X1 and X2?

The decoding of message [927 ,1345, 4006, 3913] using X1 is COOL
The decoding of message [927, 1445, 3811 ,3665 ,708 ,1081 ,1778] using X1 is MATHFUN
Decoding of the message is not possible as in the case of the matrix X2 is not invertible. Overall matrix X2 is not a good choice for key for encoding and decoding because X2 is singular so there exist null space of X2. It means there are many messages possible which will give the same encoded message. So, if we try to decode the message using X2 we will not be able to get the original message.

### Reference


– Help from StackOverflow was taken to convert the image from RGB to Grayscale.
– For question 3 http://www.keithschwarz.com/interesting/code/?dir=lights-out