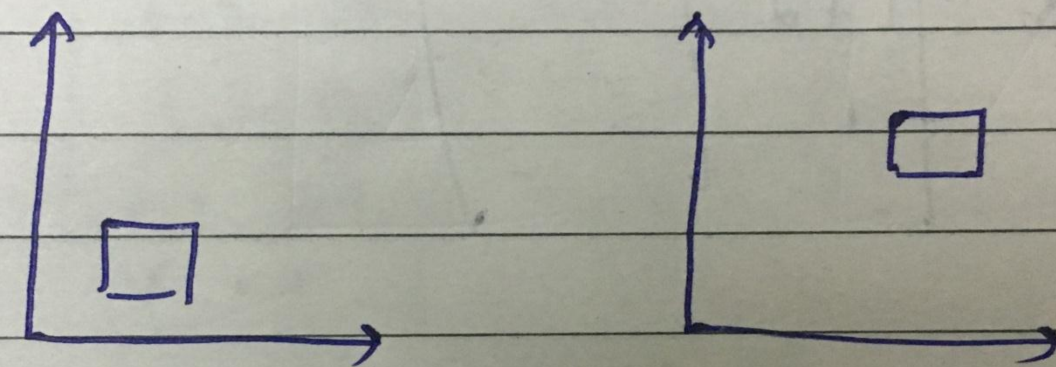
# Translate

$(t_x)\ \&\ (t_y)$



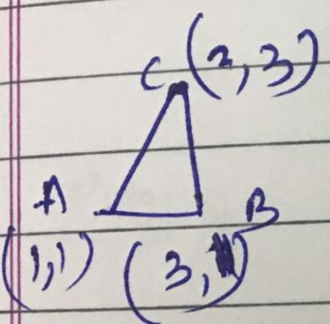
$$x' = x + t_x$$

$$y' = y + t_y.$$

for Point

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

for full object $\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} A(x) & B(x) & C(x) \\ A(y) & B(y) & C(y) \\ 1 & 1 & 1 \end{bmatrix}$




Translate $3i + 2j$.

$$\begin{bmatrix} 1. & 0. & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+0+3 & 3+0+3 & 2+0+3 \\ 0+1+2 & 0+1+2 & 0+3+2 \\ 0+0+1 & 0+0+1 & 0+0+1 \end{bmatrix}$$

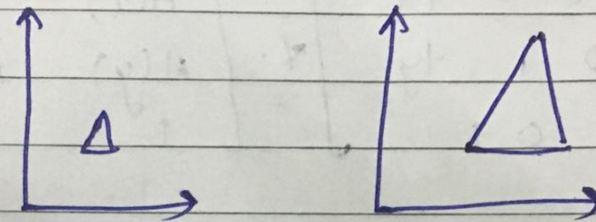
$$\Rightarrow \begin{bmatrix} 4 & 6 & 5 \\ 3 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$A' \Rightarrow 4, 3$

$B' = 6, 3$

$C' = 5, 5$

## Scaling: $(S_x)$



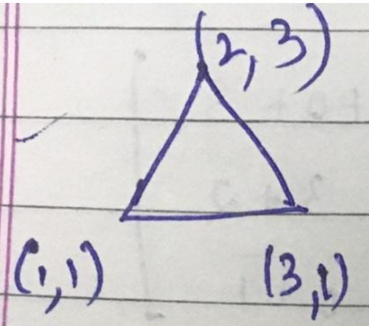
$$x' = S_x * x$$
$$y' = S_y * y.$$

for Point

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

for fig.

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A(x) & B(x) & C(x) \\ A(y) & B(y) & C(y) \\ 1 & 1 & 1 \end{bmatrix}$$




scale by (2,2)

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+0+0 & 6+0+0 & 4+0+0 \\ 0+2+0 & 0+2+0 & 0+6+0 \\ 0+0+1 & 0+0+1 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

## Rotation ($R_\theta$)

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta.$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for Point

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A(x) & B(x) & C(x) \\ A(y) & B(y) & C(y) \\ 1 & 1 & 1 \end{bmatrix}$$ for full fig.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A(x) & B(x) & C(x) \\ A(y) & B(y) & C(y) \\ 1 & 1 & 1 \end{bmatrix}$$ for fig.

$A = (0,0)$, $B = (2,0)$ $C = (2,1)$.

$\theta = 90^\circ$.

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+0+0 & 0+0+0 & 0-1+0 \\ 0+0+0 & 2+0+0 & 2+0+0 \\ 0+0+1 & 0+0+1 & 1+0+1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

# Reflection $(M_x)$

About $x$-axis.

$$x' = x$$
$$y' = -y$$

About $y$-axis.

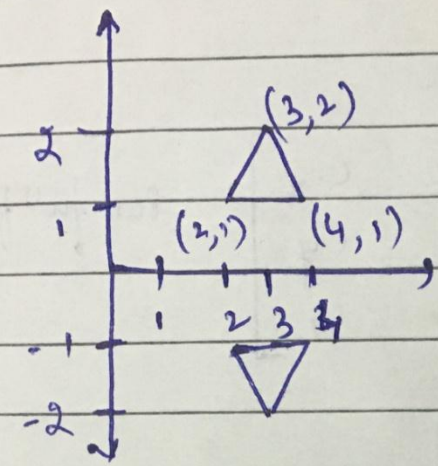
$$x' = -x$$
$$y' = y.$$

About $x$ axis.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

for Point

$$\begin{bmatrix} c(x-x_m) \\ c(y-y_m) \\ 1 \end{bmatrix}$$




$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+0+0 & 4+0+0 & 3+0+0 \\ 0-1+0 & 0-1+0 & 0-2+0 \\ 0+0+1 & 0+0+1 & 0+0+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 3 \\ -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

## Reflection about y-axis

$$\cancel{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}} \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

## Reflection of point about origin.

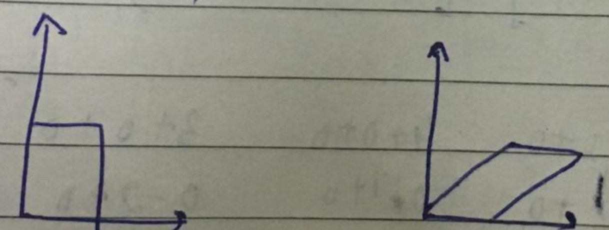
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

for Point

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ for full fig.

**Shearing** $[Sh_x, Sh_y]$

About $x$ axis.



$$x' = x + \underbrace{Sh_x}_{\text{shearing factor}} * y$$

$$y' = y$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for Point

$$\Rightarrow \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ ~~for fig~~

## Shearing about y axis

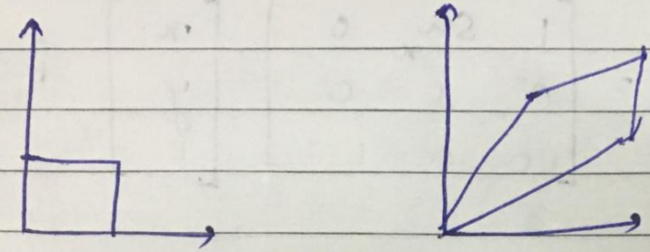
$x' = x$

$y' = y + \underbrace{sh_y}_{\text{shearing factor}} * x$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for Point

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ ~~for~~ fig.

## Shearing both x & y axis



$$x' = x + sh_x * y$$

$$y' = y + sh_y * x$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for Point.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

## Inverse Transformation

When geometric transformations are performed in opposite order.

## Inverse Translation

$$T_v^{-1} = T_v = \begin{bmatrix} 1 & 0 & -tx \\ 0 & 1 & -ty \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ for fig

## Inverse scaling

$$S_x^{-1} = \begin{bmatrix} \frac{1}{S_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{S_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ for full ~~pa~~fig

Inverse Rotation i.e $(-\theta)$

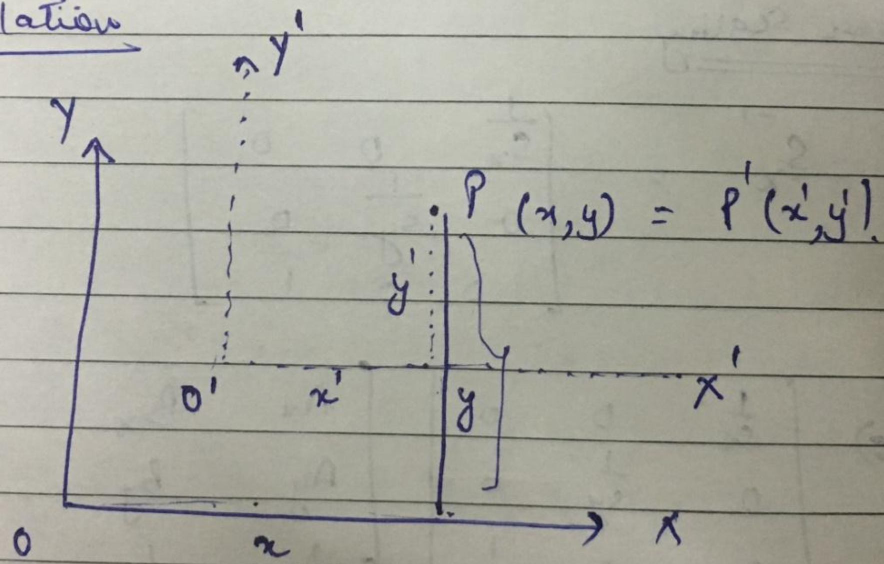
$$R_\theta^{-1} = R_{-\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

## Inverse Reflection: same as Reflection

## Coordinate Transformation

### Translation




$x' = x - t_x$

$y' = y - t_y$

$$x' = x - t_x$$

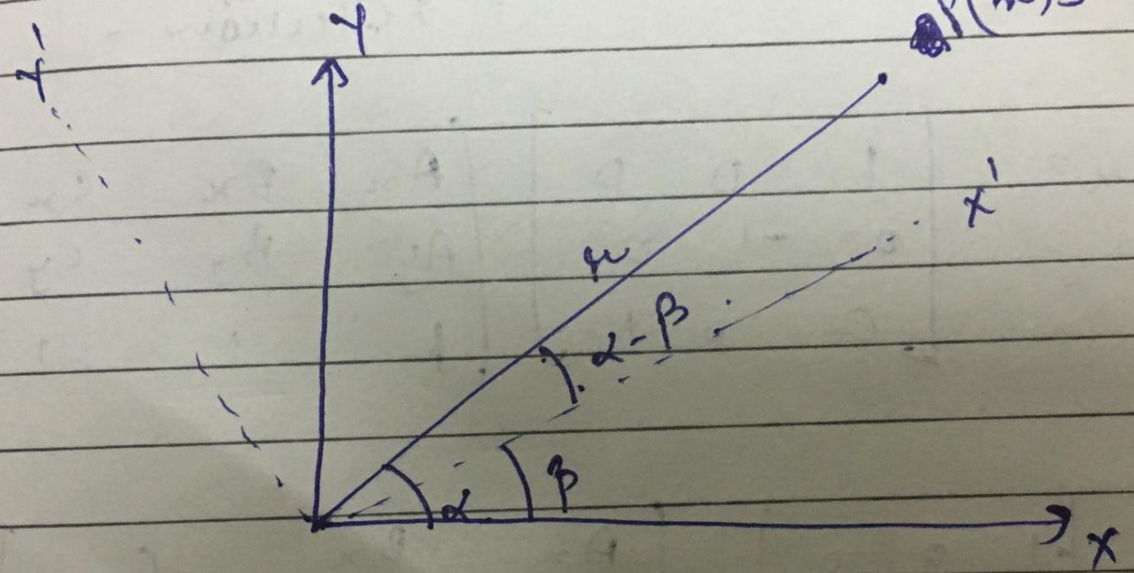
$$y' = y - t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for pt.

$$\Rightarrow \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

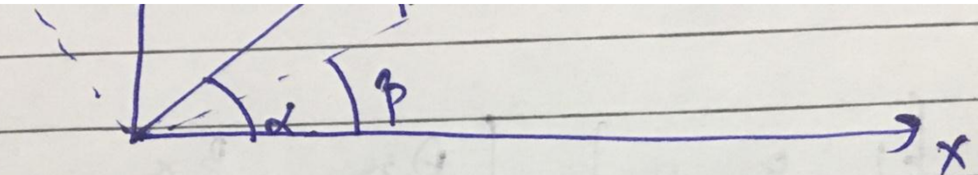
## Rotation about origin)

$P(x, y) = P'(x', y')$




$x' = r\cos\alpha\cos\beta + r\sin\alpha\sin\beta$

$y' = r\sin\alpha\cos\beta - r\cos\alpha\sin\beta$




$$x' = r\cos\alpha\cos\beta + r\sin\alpha\sin\beta$$

$$y' = r\sin\alpha\cos\beta - r\cos\alpha\sin\beta$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for pt.

$$\Rightarrow \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ for full fig.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for pt.

$$\Rightarrow \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ for full fig.

<u>scaling co-ordinates</u> : means reducing the scale of coordinate system

$$S_{\frac{1}{s_x}, \frac{1}{s_y}} = \begin{bmatrix} \frac{1}{S_x} & 0 & 0 \\ 0 & \frac{1}{S_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$ for full fig.

Coordinate Reflection = Same as geometric reflection.

$$\bar{M}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

$$\bar{M}_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

## Composite Transformation

A number of transformations can be combined together into one matrix to make things easy.

$$M = ABC$$

$$v' = Mv$$

$$v' = (ABC)v$$

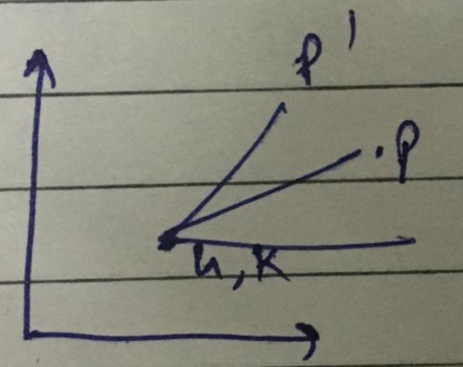
$$v' = AB(Cv)$$

$$v' = A(B(Cv))$$

$$v' = [A(B(Cv))]$$

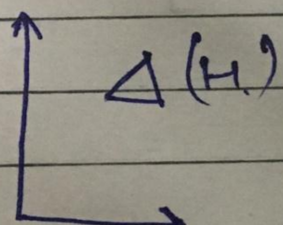
Right to left.

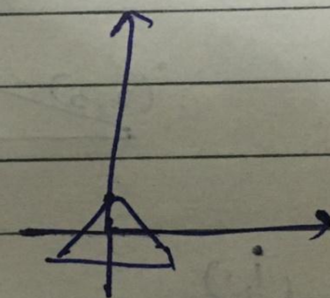
Rotate about Arbitary pt. (h,k)



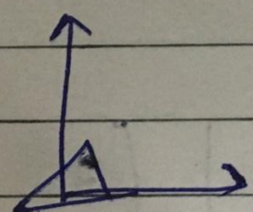

1) Translate pt (h,k) to origin.
2) Rotate around origin.
3) Translate back to point.

1) Translate pt $(h, k)$ to origin.
2) Rotate around origin.
3) Translate back to point.

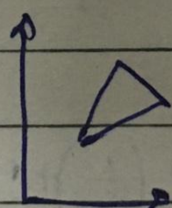


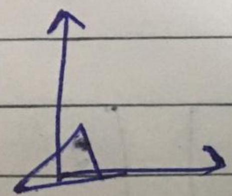
$T_{(-h,-k)}(H)$

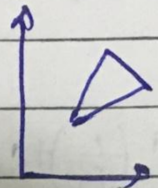


$R_\theta T_{(-h,-k)}(H)$



$T_{(h,k)} R_\theta T_{(-h,-k)}(H)$





$R_\theta \, T_{(-h,-k)} \; (H)$ $\qquad$ $T_{(h,k)} \, R_\theta \, T_{(-h,-k)} \; (H)$

$$f_\theta(h,k) = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

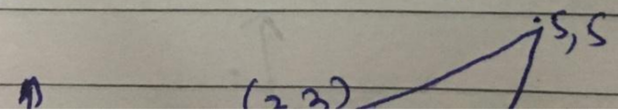
$$\Rightarrow \begin{bmatrix} \cos\theta & -\sin\theta & -h\cos\theta + k\sin\theta + h \\ \sin\theta & \cos\theta & -h\sin\theta - k\cos\theta + k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

Q Perform a counterclock. $45^\circ$ rotation of $\Delta$

$A = (2,3)$
$B = (5,5)$
$C = (4,3)$
about pt. $(1,1)$

Ans

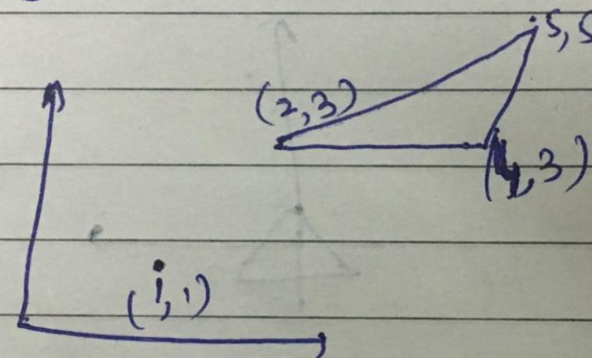
$$\begin{bmatrix} \cos\theta & -\sin\theta & -h\cos\theta + k\sin\theta + h \\ \sin\theta & \cos\theta & -h\sin\theta - k\cos\theta + k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$



about pt. (1, 1)

Ans

$$\begin{bmatrix} \cos\theta & -\sin\theta & -h\cos\theta + k\sin\theta + h \\ \sin\theta & \cos\theta & -h\sin\theta - k\cos\theta + k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$




$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -1\left(\frac{1}{\sqrt{2}}\right) + 1\left(\frac{1}{\sqrt{2}}\right) + 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -1\left(\frac{1}{\sqrt{2}}\right) - 1\left(\frac{1}{\sqrt{2}}\right) + 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 \\ 3 & 5 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$(1,1)$

$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -1\left(\frac{1}{\sqrt{2}}\right)+1\left(\frac{1}{\sqrt{2}}\right)+1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -1\left(\frac{1}{\sqrt{2}}\right)-1\left(\frac{1}{\sqrt{2}}\right)+1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 \\ 3 & 5 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\sqrt{2}+1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 \\ 3 & 5 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2} - \frac{3}{\sqrt{2}} + 1 & \frac{5}{\sqrt{2}} - \frac{5}{\sqrt{2}} + 1 & \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} + 1 \\ \sqrt{2} + \frac{3}{\sqrt{2}} - \sqrt{2} + 1 & \frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}} - \sqrt{2} + 1 & \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} - \sqrt{2} + 1 \\ 0 + 0 + 1 & 0 + 0 + 1 & 0 + 0 + 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{-1}{\sqrt{2}} + 1 & 1 & \frac{1}{\sqrt{2}} + 1 \\ \frac{3}{\sqrt{2}} + 1 & \frac{8}{\sqrt{2}} + 1 & \frac{5}{\sqrt{2}} + 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{-1}{\sqrt{2}}+1 & 1 & \frac{1}{\sqrt{2}}+1 \\ \frac{3}{\sqrt{2}}+1 & \frac{8}{\sqrt{2}}+1 & \frac{5}{\sqrt{2}}+1 \\ 1 & 1 & 1 \end{bmatrix}$$

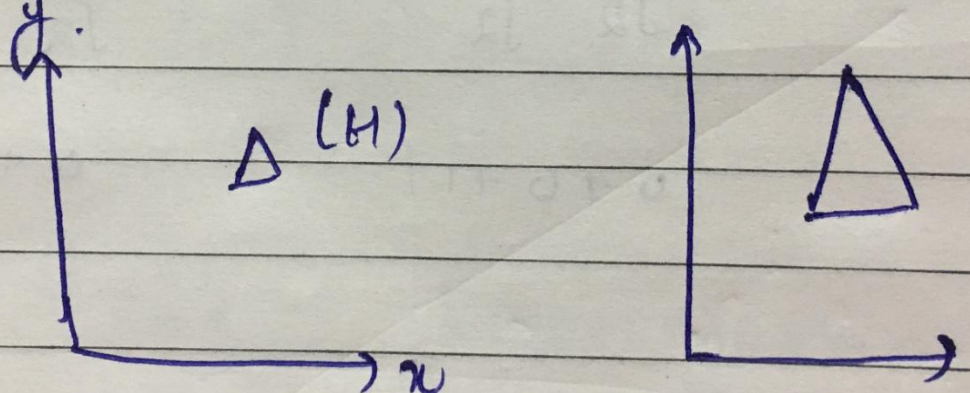
$A' = \frac{-1}{\sqrt{2}}+1, \ \frac{3}{\sqrt{2}}+1$

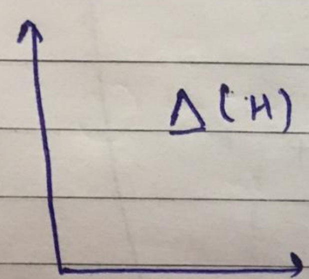
$B' =$ [illegible] $+1, \ \frac{8}{\sqrt{2}}+1$

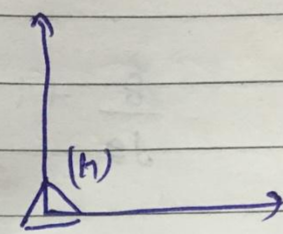
$C' = \frac{1}{\sqrt{2}}+1, \ \frac{5}{\sqrt{2}}+1$

## Scaling about Arbitary Point (h,k)

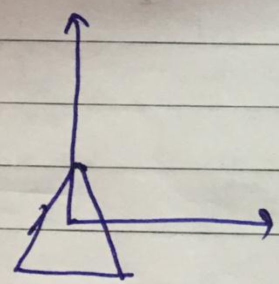
1) Translate pt (-h,-k) to origin.
2) Scale around origin.
3) Translate back to pt.



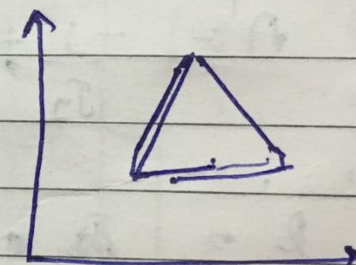



$T_{-v}(-h,-k)(H)$



$(S_x)(T_{-v}(-h,-k))(H)$



$T_v(h,k)(S_x)(T_{-v}(-h,-k))(H)$

$$\Rightarrow \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S_x & 0 & -h(S_x)+h \\ 0 & S_y & -K(S_y)+k \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & -h(S_x)+h \\ 0 & S_y & -k(S_y)+k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ for Point.

$$\Rightarrow \begin{bmatrix} S_x & 0 & -h(S_x)+h \\ 0 & S_y & -K(S_y)+k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}.$$

for full fig.

for full fig.

Q Magnify a $\Delta$ ABC, A (0,0); B (1,1) & C (5,2) twice keeping pt. (5,2) as fixed.

$h = 5; \quad K = 2$

(H)

$$\Rightarrow \begin{bmatrix} 2 & 0 & -5(2)+(5) \\ 0 & 2 & -(2)(2)+2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

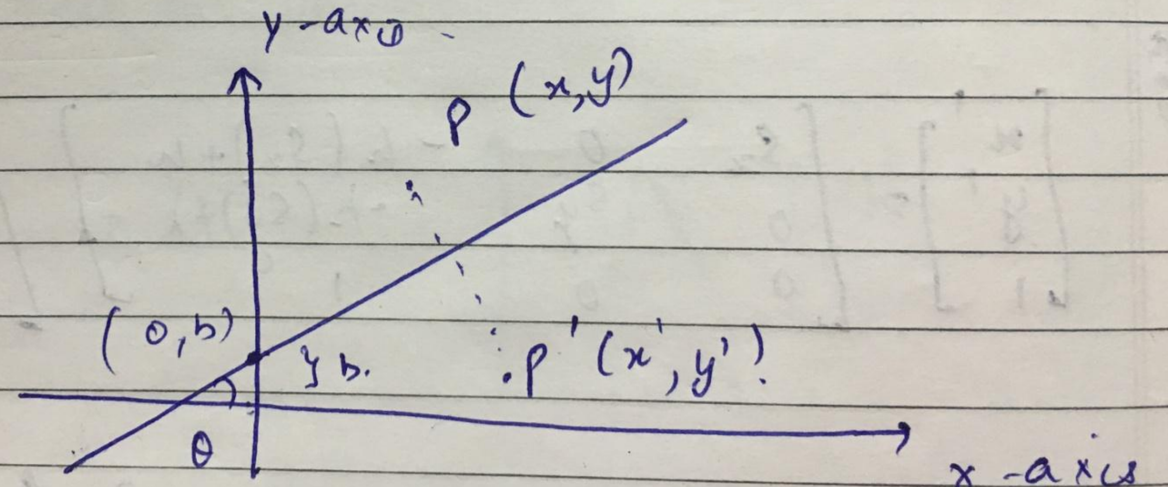
$$\Rightarrow \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0-5 & 2-5 & 10-5 \\ -2 & 2-2 & 4-2 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

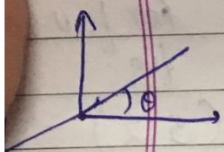
## Reflection about line

L:

$y = mx + b.$

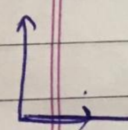


① Translate $(0,b)$ to origin so that the line passes through origin.



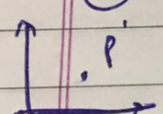

i.e $T_v = (-h, -k)(P)$

② Rotate by an angle of $(-\theta)$ so that the line aligns with +ve x-axis




$$R_{(-\theta)}\left(T_v(-h, -k)\right)(P)$$

③ Take mirror reflection about x-axis

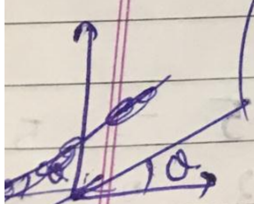



$$(M_x)(R_{(-\theta)})\left[(T_v)(-h, -k)\right](P)$$

P'

④ Re-rotate line back to angle θ

$$(R_\theta)(M_x)(R_{(-\theta)})\left[T_v(-h, -k)\right](P)$$

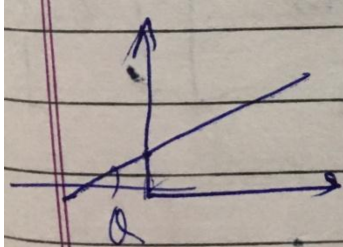



$\begin{bmatrix} 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

Put

(5) Re-translate to (0, b) Back.

$$(T_v)(R_\theta)(M_x)(R_{-\theta})[T_v(-h,-k)][P]$$




$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & -b\\0 & 0 & 1\end{bmatrix}$$

Multiplication with ~~by~~ be done from Right to Left.

$$\Rightarrow \begin{bmatrix} \cos 2\theta & \sin 2\theta & -b\sin 2\theta \\ \sin 2\theta & -\cos 2\theta & b\cos 2\theta + b \\ 0 & 0 & 1 \end{bmatrix}$$

Putting :- $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ ; $\sin 2\theta = \dfrac{\tan^2\theta}{1+\tan^2\theta}$

& $\tan\theta = m$.

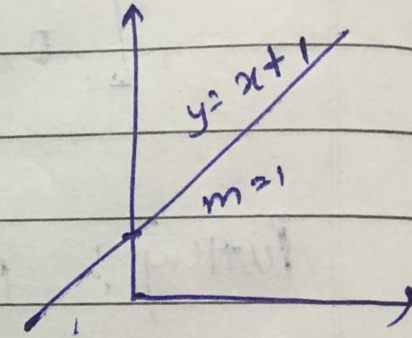
$$\Rightarrow \begin{bmatrix} \dfrac{1-m^2}{1+m^2} & \dfrac{2m}{1+m^2} & \dfrac{-2bm}{1+m^2} \\ \dfrac{2m}{1+m^2} & \dfrac{m^2-1}{1+m^2} & \dfrac{2b}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2bm}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2b}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

for point.

$$\Rightarrow \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2bm}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2b}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix}$$

Q. Reflect a sq. A(1,1), (3,1), (3,3) & (1,3) about line $y = x+1$




Ans

$y = x + 1$

$m = 1$

$b = 1$

$$\begin{bmatrix} \frac{1-1}{1+1} & \frac{2}{1+1} & \frac{-2}{1+1} \\ \frac{2}{1+1} & \frac{1-1}{1+1} & -\frac{2}{1+1} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 & 1 \\ 1 & 1 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

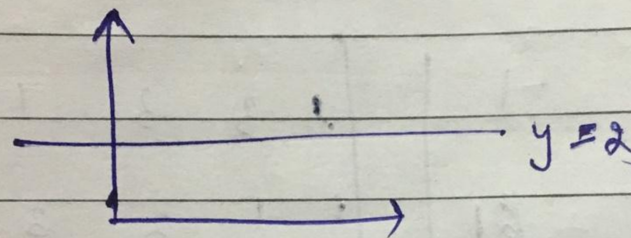
$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 & 1 \\ 1 & 1 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \cancel{\begin{bmatrix} 1-1 & 1-1 & 3-3 & 3-3 \\ 1+\frac{1}{2} & 3+\frac{1}{2} & 3+\frac{1}{2} & 1+\frac{1}{2} \end{bmatrix}}$$

$$\Rightarrow \cancel{\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{3}{2} & \frac{7}{2} & \frac{7}{2} & \frac{3}{2} \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 0 & 0 & 2 & 2 \\ 1+1 & 3+1 & 3+1 & 1+1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 2 & 2 \\ 2 & 4 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Horizontal line $(y=2)$




$(T_y)(m_x)(T_{-y})[\text{Object}]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

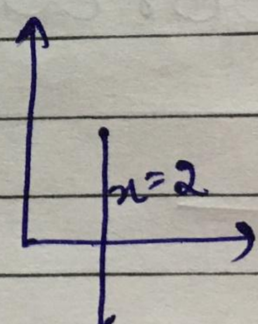
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2+2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{object} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2+2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 4 & 6 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

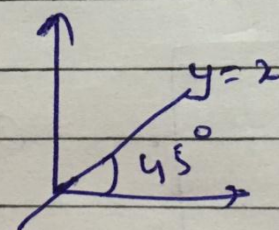
Vertical Line ( $x=2$ )




$$\begin{bmatrix} T_x \end{bmatrix} \begin{bmatrix} M_y \end{bmatrix} \begin{bmatrix} T_{-x} \end{bmatrix} \cdot \begin{bmatrix} \text{Object} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -tx \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{Object} \end{bmatrix}$$

**Q** Obtain reflection about line $y = x$




$$\left(R_{-45^\circ}\right)\left(M_y\right)\left(R_{45^\circ}\right)\begin{bmatrix} \text{Object matrix} \end{bmatrix}$$